Homework 1

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Solution 1. **(1)** Eliminate in order: B (dominated by T); R (by C); M (by T). With only T left, column chooses L.
Result: (T, L).

(2) Eliminate B (by T). No further strict dominance for column.
Result: (T, L) and (T, R).

(3) Start with weak dominance for column: eliminate L (weakly dominated by R). Then eliminate B (by T).
Result: (T, R). So order matters with weak dominance.

Solution 2. If $a < b$, owner 1 serves $[0, \frac{a+b}{2}]$ so $u_1(a,b) = \frac{a+b}{2}$ and $u_2(a,b) = 1 - \frac{a+b}{2}$.

1. $(a,b) = \left(\frac{1}{2}, \frac{3}{4}\right)$: $u_1 = \frac{1/2+3/4}{2} = \frac{5}{8}, \quad u_2 = 1 - \frac{5}{8} = \frac{3}{8}$.
2. $(a,b) = \left(\frac{1}{2}, \frac{5}{8}\right)$: $u_1 = \frac{1/2+5/8}{2} = \frac{9}{16}, \quad u_2 = 1 - \frac{9}{16} = \frac{7}{16}$.
3. Compare $(\frac{1}{4}, \frac{3}{4})$ vs. $(\frac{1}{2}, \frac{3}{4})$: $u_1(\frac{1}{4}, \frac{3}{4}) = \frac{1}{2}$, $u_1(\frac{1}{2}, \frac{3}{4}) = \frac{5}{8} \Rightarrow$ owner 1 prefers $(\frac{1}{2}, \frac{3}{4})$. $u_2(\frac{1}{4}, \frac{3}{4}) = \frac{1}{2}$, $u_2(\frac{1}{2}, \frac{3}{4}) = \frac{3}{8} \Rightarrow$ owner 2 prefers $(\frac{1}{4}, \frac{3}{4})$.
4. $a = \frac{1}{2}$ is nether weakly nor strictly dominant for owner 1. Best response on u_1 depends on b (e.g. to $b = \frac{3}{4}$, choosing $a = 0.74$ yields $\frac{0.74+0.75}{2} = 0.745 > \frac{5}{8}$), so $a = \frac{1}{2}$ is neither weakly nor strongly dominant.
5. Dominating strategies? None for either player; each player's best u_1/u_2 location depends on the opponent's b/a.

Solution 3. **1.** Choose

$$\begin{array}{cc} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} T \\ B \end{array} & \begin{bmatrix} (1,0) & (0,3) \\ (0,1) & (3,0) \end{bmatrix} \end{array}$$

Row's best reply:to L row plays T; to R row plays B. Column's best reply: to T column plays R; to B column plays L. There's never a best pure strategy for both players. Hence no pure Nash Equilibria.

2. Choose

$$\begin{array}{cc} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} T \\ B \end{array} & \begin{bmatrix} (1,0) & (2,3) \\ (0,2) & (0,1) \end{bmatrix} \end{array}$$

Row's best reply: to L pick T; to R pick T. Column's best reply: to T pick R; to B pick L. Unique pure Nash Equilibria: (T, R). It also *Pareto*-optimal. No player can better off without making other players worse off, so it is the unique Pareto optimal profile.

3. Take constant sum $s = 1$ and set every cell to $(\frac{1}{2}, \frac{1}{2})$.

$$\begin{array}{cc} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} T \\ B \end{array} & \begin{bmatrix} (\frac{1}{2}, \frac{1}{2}) & (\frac{1}{2}, \frac{1}{2}) \\ (\frac{1}{2}, \frac{1}{2}) & (\frac{1}{2}, \frac{1}{2}) \end{bmatrix} \end{array}$$

Then the game is constant–sum and every cell is a best reply, so all 4 cells are pure Nash Equilibria; every cell is Pareto optimal, as no player can better off, so there are 4 Pareto optimal profiles as well.

4. Let row have probability p on T, $1-p$ on B, , column have q on L, $1-q$ on R. To make choice indifferent:

$$\begin{aligned}
&u_1(T) = u_1(B) \Rightarrow \\
&u_1(T) = q, \quad u_1(B) = 3(1-q) \;\Rightarrow\; q = \tfrac{3}{4}, \\
&u_2(L) = u_2(R) \Rightarrow \\
&u_2(L) = 1-p, \quad u_2(R) = 3p \;\Rightarrow\; p = \tfrac{1}{4}.
\end{aligned}$$

Thus the mixed Nash Equilibria is $(p, q) = \left(\frac{1}{4}, \frac{3}{4}\right)$.

5. Yes. In zero/constant–sum games, increasing one player's payoff necessarily decreases the other's, so no player can better off without making other worse off; hence every profile is Pareto optimal.