Tutorial 6

Coalition Logic

4 December, 2025

Exercise 1. In Coalition Logic, and with any coalition of agents $C \subseteq N$ (with, recall N the set of all agents), formulas on the form $\langle\langle C \rangle\rangle\phi$ are read as "*coalition C can cooperate to ensure that ϕ holds*" (i.e., coalition C has the 'power' to guarantee that ϕ is the case). Consider now the two extreme coalitions, $\emptyset$ and N. What is the reading of the following formulas?

(1) $\langle\langle \emptyset \rangle\rangle\phi$

(2) $\langle\langle N \rangle\rangle\phi$

Exercise 2. Let C be any coalition $C \subseteq N$, and consider each one of the following formulas of the coalition logic language:

(1) $\neg\langle\langle C \rangle\rangle\bot$

(2) $\langle\langle C \rangle\rangle\top$

(3) $\langle\langle C \rangle\rangle(\phi \wedge \psi) \rightarrow \langle\langle C \rangle\rangle\phi$

Provide, in each case, a requirement the effectivity function should satisfy (for the given coalition C, and *for all formulas* ϕ, ψ) in order for the formula to be true. In other words, indicate in each case the requirement the effectivity function should satisfy (for the given coalition C) for the formula to be *always* true.

Exercise 3. Let $N = \{1, 2\}$ be the set of agents, and let $A = \{a, b\}$ be the set of atomic propositions we are interested in modeling. Consider a coalition model $M = (W, E, V)$ where $W = \{w_1, w_2, w_3, w_4\}$ and V is such that $V(a) = \{w_1, w_2\}$ and $V(b) = \{w_1, w_3\}$. For each one of the following formulas, provide a definition of an effectivity function E under which the formula is true (do not forget to satisfy the two requirements any effectivity function should satisfy). If you consider that it is not possible to satisfy a formula, explain why.

(1) $\neg\langle\langle \{1, 2\} \rangle\rangle a \wedge \neg\langle\langle \{1, 2\} \rangle\rangle b \wedge \neg\langle\langle \{1, 2\} \rangle\rangle(a \wedge b)$

(2) $\neg\langle\langle \{1\} \rangle\rangle a \wedge \neg\langle\langle \{1\} \rangle\rangle b \wedge \neg\langle\langle \{1\} \rangle\rangle(a \wedge b) \wedge \langle\langle \{1\} \rangle\rangle(a \vee b)$

need to define for all pose
collisions.

Exercise 4. Let $M = (W, E, V)$ be a coalition model. Suppose M satisfies *super-additivity*. Show that, for every world $w \in W$, any coalitions $C_1, C_2 \subseteq N$ with $C_1 \cap C_2 = \emptyset$, and any coalition logic formulas ϕ_1 and ϕ_2, we have:

$$M, w \Vdash (\langle\langle C_1 \rangle\rangle \phi_1 \wedge \langle\langle C_2 \rangle\rangle \phi_2) \rightarrow \langle\langle C_1 \cup C_2 \rangle\rangle (\phi_1 \wedge \phi_2)$$