Homework 1

Games in Normal Form

Due: November 7, 2025, by 18:00

Exercise 1 (Eliminating dominated strategies)**.** In this exercise you will practice eliminating dominated strategies.

1. Consider the payoff matrix below:

$$\begin{array}{cc} & \begin{array}{ccc} L & \quad \cancel{C} & \quad \cancel{R} \end{array} \\ \begin{array}{c} T \\ \cancel{M} \\ \cancel{B} \end{array} & \begin{bmatrix} (10,4) & \cancel{(5,3)} & \cancel{(3,2)} \\ \cancel{(0,1)} & \cancel{(4,6)} & \cancel{(6,0)} \\ \cancel{(2,1)} & \cancel{(3,5)} & \cancel{(2,8)} \end{bmatrix} \end{array}$$

   What do we get by applying Iterated Elimination of Strictly Dominated Strategies (IESDS) to this game? 1p

2. Consider the following payoff matrix:

$$\begin{array}{cc} & \begin{array}{cc} L & \quad R \end{array} \\ \begin{array}{c} T \\ \cancel{B} \end{array} & \begin{bmatrix} (10,4) & (5,4) \\ \cancel{(0,1)} & \cancel{(4,6)} \end{bmatrix} \end{array}$$

   What do we get by IESDS? 1p

3. For the payoff matrix at point (2), note that player 2, i.e., the column player, has a weakly dominated strategy. What do we get if we start by eliminating that instead of the strictly dominated strategies? 1p

Exercise 2 (Where to set up shop)**.** Consider two food-truck owners, 1 and 2, facing a decision about where to place their food-truck on a populated street. We represent the street as the $[0,1]$ interval, and the positions chosen by owners 1 and 2 as points a and b, respectively, in this interval.

People on the street prefer to go to the food-truck closest to them, such that the owners' utilities are calculated as the fraction of people they are able to attract given their respective positions. We think of these fractions as the lengths of the intervals given by the points closer to one truck than to the other. For instance, if the food-truck owners set up shop at $a = \frac{1}{4}$ and $b = \frac{3}{4}$, then the utility of owner 1 is:

$$u_1(\frac{1}{4}, \frac{3}{4}) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

This is because owner 1 attracts the points to the left of a, which make up $\frac{1}{4}$ of the people on the street, plus half the people between a and b, which is $\frac{1}{4}$ of the population. See Figure 1 below for a depiction of the points that are closer to a than to b:[1]

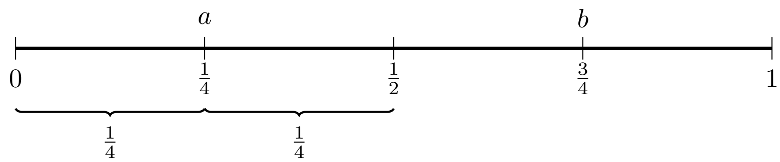

Figure 1: Food-trucks on a street: position a attracts the fraction of people closer to a than to b.

Finally, let us agree that if the two owners place their food-trucks in the same spot then customers flip a coin about where to go and hence are divided equally among the two food-trucks. Thus, for instance:

$$u_1(0,0) = u_2(0,0) = \frac{1}{2}.$$

$u_1 = \frac{5}{8}$ $u_2 = \frac{3}{8}$

1. What is the utility of each owner if they place themselves at $a = \frac{1}{2}$ and $b = \frac{3}{4}$? 1p
2. What is the utility of each owner if they place themselves at $a = \frac{1}{2}$ and $b = \frac{5}{8}$? 1p $u_1 = \frac{9}{16}$ $u_2 = \frac{7}{16}$
3. The vendors, of course, want to maximize their own utilities. Of the two pure strategy profiles $(\frac{1}{4}, \frac{3}{4})$ and $(\frac{1}{2}, \frac{3}{4})$, which one does owner 1 prefer? Which one does owner 2 prefer? 1p
4. Is it a (weakly, or strongly) dominant strategy for owner 1 to set up shop at $\frac{1}{2}$? 1p
5. Does anyone have a (weakly/strongly) dominating strategy? 1p

Exercise 3. In this exercise you are the game master, tweaking utilities to draw out certain outcomes. Consider the payoff matrices (a), (b) and (c) below:

$$\begin{array}{cc} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} T \\ B \end{array} & \begin{bmatrix} (1, \_) & (\_, 3) \\ (\_, 1) & (3, \_) \end{bmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} T \\ B \end{array} & \begin{bmatrix} (1, \_) & (\_, 3) \\ (\_, \_) & (\_, \_) \end{bmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} T \\ B \end{array} & \begin{bmatrix} (\_, \_) & (\_, \_) \\ (\_, \_) & (\_, \_) \end{bmatrix} \end{array}$$

(a) (b) (c)

[1]Formally, a point x is closer to a than to b if $|a - x| < |b - x|$. The point at $\frac{1}{2}$ is at equal distance to a and b, and we can assume that a person living at that point flips a coin about where to go, but whether it goes to a or to b does not matter for the length of interval. We will generally make this assumption.

1. Complete matrix (a) with the missing utilities such that there is *no pure Nash equilibria* in the resulting game. Justify your answer. 1p

2. Complete matrix (b) with the missing utilities such that there are *exactly one pure Nash equilibria and one Pareto optimal* profile in the resulting game. Justify your answer. 1p

3. Complete matrix (c) with the missing utilities such that the resulting game is *constant-sum* (i.e., the payoffs in every cell add up to the same number $s \neq 0$) and there are *as many pure Nash equilibria as Pareto optimal* profiles. Justify your answer. 1p

4. For the game that you described for payoff matrix (a), find a mixed-strategy Nash equilibrium. 1p

5. If $G = (N, A, \boldsymbol{u})$ is a zero-sum game (i.e., payoffs in every cell add up to 0), does it follow that all strategy profiles are Pareto optimal? 1p