Tutorial 1

Games in Normal Form, Domination Among Strategies

October 30

Exercise 1 (Dominating Strategies in a Numbers Game)**.** Consider a game played between two players, as follows. Each player secretly picks a number in $\mathbb{N} = \{0, 1, 2, \ldots\}$. The players then report the numbers they chose to each other. The player who chose the larger number gets 1 euro, while the other gets nothing. If they both choose the same number they both get nothing.

N= players 0, 1

Ai=N

$u_0 = \begin{cases} 1 & a_0 > a_1 \\ 0 & a_0 \leq a_1 \end{cases}$

$u_1 = \begin{cases} 0 & a_0 \geq a_1 \\ 1 & a_0 < a_1 \end{cases}$

1. Model this as a game in normal form. Who are the players? What are the actions? What are the utilities? What are the pure strategies?

2. In the game thus defined, does any player have a strictly dominant strategy? A weakly dominant strategy? No

3. Consider the same game as above, except that now players choose numbers from the set of negative integers plus 0, i.e., from the set $\{0, -1, -2, \ldots\}$. Any strictly/weakly dominating strategies here? have weakly dominant, choose 0,

Exercise 2. Consider the two games below:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} (-1,-1) & (-20,0) \\ (0,-20) & (-15,-15) \end{bmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} T \\ B \end{array} & \begin{bmatrix} (3,1) & (2,1) \\ (1,2) & (2,0) \end{bmatrix} \end{array}$$

(i) *(ii)*

2.1 D strictly dominate C
eliminate C
solution is (D, D)

2. NO

3. L weakly dominate R
eliminate R
T weakly dominant B
eliminate B
(T, L)

1. Apply the iterated elimination of strictly dominated strategies (IESDS) to the game shown in *(i)* and report the result.

2. Can you find a different order of eliminating strictly dominated strategies? Do the results differ?

3. Apply the iterated elimination of *weakly* dominated strategies to the game shown in *(ii)*.

4. Can you find a different order of eliminating weakly dominated strategies? Do the results differ?

4. OR. T weakly dominant B
eliminate B
R weakly dominant L / L weakly dominate R
(T, L) or (T, R)

Exercise 3. Consider the payoff matrix below:

$$\begin{array}{c c} & \begin{array}{cc} L & R \end{array} \\ \begin{array}{c} T \\ B \end{array} & \begin{bmatrix} (2,-4) & (6,3) \\ (1,2) & (8,1) \end{bmatrix} \end{array}$$

1. Any dominating strategies, either weak or strong? no

$\text{if } T = \frac{2+6}{2} = 4$

$\text{if } B = \frac{8+1}{2} = 4.5$

2. Suppose player 2, the column player, does not care very much about the game and chooses by flipping a coin, i.e., plays an action in $\{L, R\}$ with equal probability—and player 1 *knows this*. What should player 1 do? Hint: think about 1's expected payoff given 2's strategy. choose B.

3. What is 2's expected payoff given that 1 always takes the optimal action computed above? if $L = 2$ if $R = 1$ $\text{payoff}_2 = \frac{2+1}{2} = 1.5$

$\text{if } T = \frac{3}{4} * 2 + \frac{1}{4} * 6 = 3$

$\text{if } B = \frac{3}{4} * 1 + \frac{1}{4} * 8 = 2.75$

1 choose L, so for 2:

if $L = -4$, if $R = 3$

So no outsmart.

4. At this point it should be clear that if 2 chooses randomly as at (a), 1 will always play B. Suppose, now, that the players play this game for a while and 2 starts thinking like this:

    If player 1 keeps playing B, then I should play L more often because my payoff is higher there.

   So player 2 changes its strategy and starts playing L with probability $3/4$ and R with probability $1/4$. Has player 2 outsmarted player 1? Suppose 1 catches wind of player 2's new tactic and adapts its own strategy in response: what is 1's optimal action now?

$\text{payoff}_2 = -4 * \frac{3}{4} + 3 * \frac{1}{4} = -2.25$

5. What is player 2's expected payoff under player 1's new optimal action?
6. What should 2's probabilities be to make 1 indifferent between their two actions?

suppose $\text{Prob}(L) = p$, $\text{Prob}(R) = 1-p$

need $T = B$

$$2p + 6(1-p) = p + 8(1-p)$$

$$6 - 4p = 8 - 7p$$

$$3p = 2$$

$$p = \frac{2}{3}$$

so $\frac{2}{3} L$, $\frac{1}{3} R$.