UvA

Multi-Agent Systems

Auctions

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What are Auctions About?

- **Auctions** are used to allocate resources among selfish agents in a multi-agent system
  - And you have lots of different ones!
- (At least) two good reasons to study auctions:
  - Auctions are widely used in real life
    - To trade goods online (for example: eBay)
    - Used by governments around the world to sell public resources (such as oil and mineral rights)
  - General theoretical framework to understand resource allocation problems

Example

- Consider an outdoor market
- You are selling your old skateboard
- As the seller, you set a price
- First come, first serve: you sell to the first person who shows up and expresses interest
- That person gets the item
- They pay the price set by you (we ignore haggling here)
- And everyone lives happily ever after
    - Except…

Image: clipartkey.com

Example

- **Pros**: Good for getting rid of things fast
- **Cons**: Not great for seller, as there might be other buyers willing to pay more
    - Not great for buyer, as seller might have been willing to sell for less

Image: clipartkey.com

Auctions

- There are two goals when designing an auction
- On the one hand, **allocative efficiency**: the item should go to the bidder who derives the highest value
- On the other hand, **revenue maximization**: maximize the (expected) revenue to the seller
- In general, efficiency and revenue cannot both be optimized, resulting in trade-offs

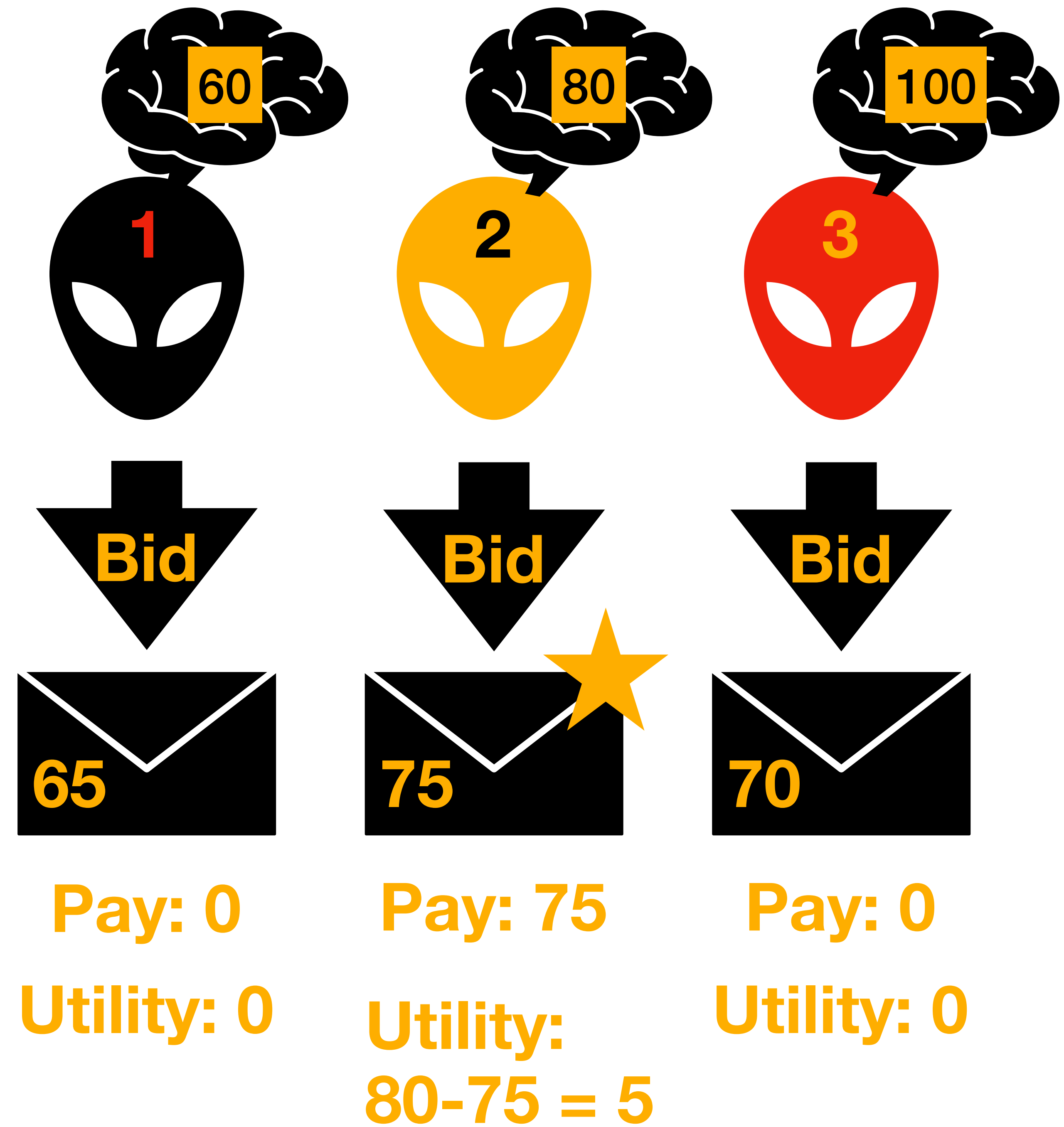
Single-Good Auctions

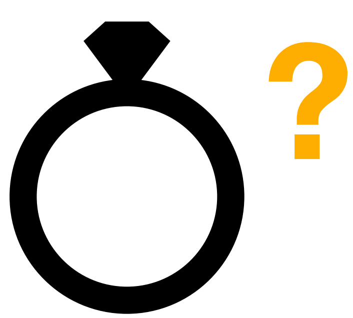
- One **seller** is auctioning off a single **good**, with many potential **buyers** interested
- Buyers have **private valuations**, i.e., how much they are willing to pay for the item
- Buyers submit **bids**, which can be **secret** (**sealed-bid**) or **public** (**open outcry**)
- The outcome, i.e., who gets the good, is determined according to an **allocation rule**
- Payments, i.e., how much everyone pays, is determined according to a **payment rule**
- Agents derive **utilities**, typically their valuation minus the price paid, if getting the item, and 0 if they don't get the item

Let’s first look at sealed-bid auctions.

First-Price (Sealed Bid) Auction

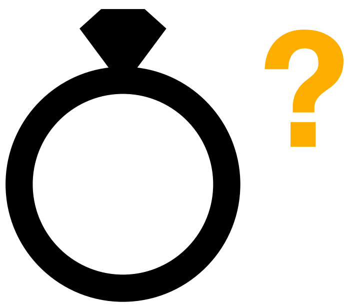
- Each agent submits a secret bid
- The winner is the highest bid
- The price paid is the highest bid

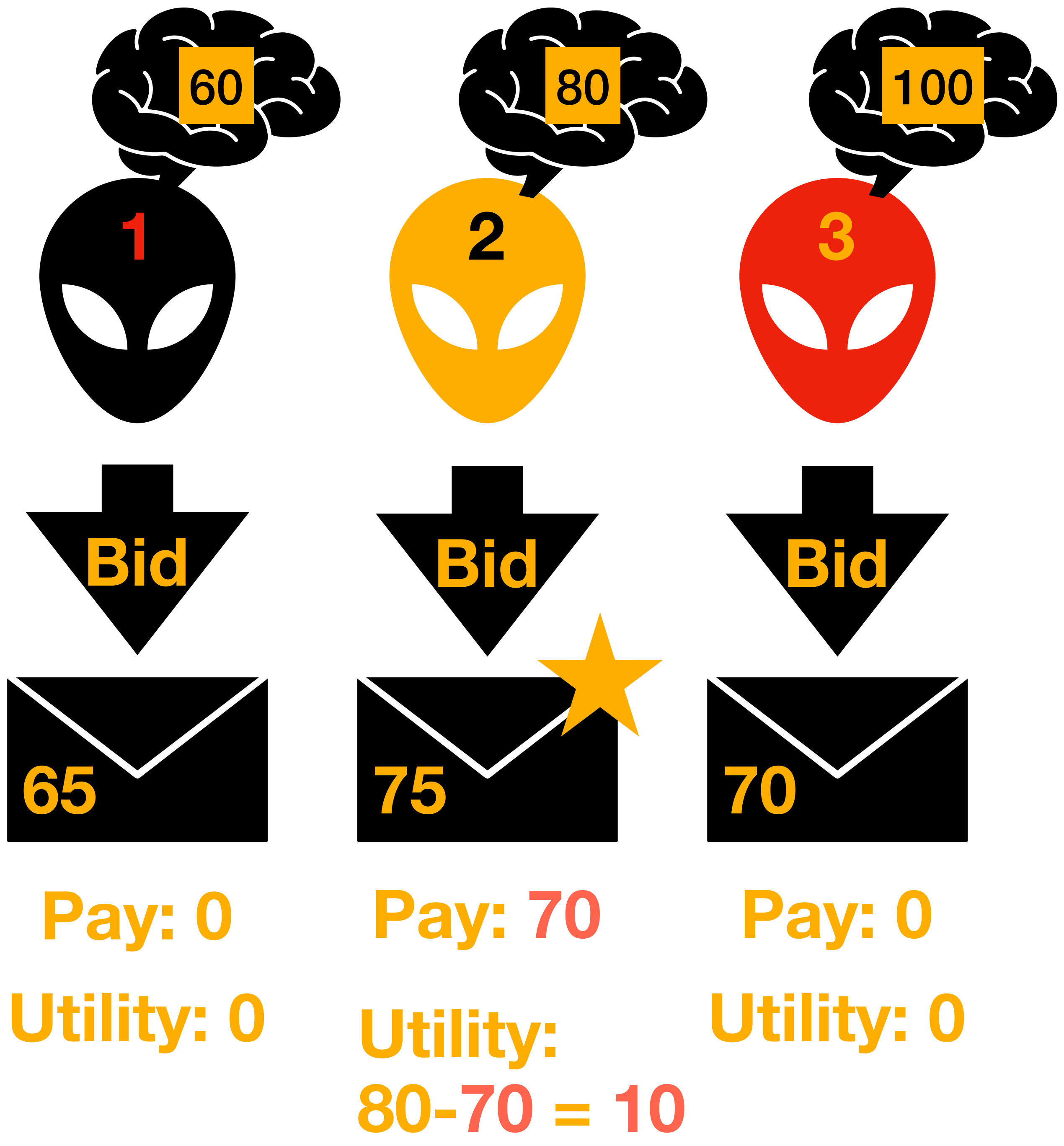




Second-Price (Vickrey) Auction

- Each agent submits a secret bid
- The winner is the highest bid
- The price paid is the second-highest bid





Independent Private Value

- Typically, though, bidders don’t know each other’s valuations
    - And so, the resulting game is harder to analyze
- What helps is to add some assumption about the distribution of values
    - For example, that they are drawn independently from the same, commonly known distribution
- We call this the Independent Private Value (IPV)

Second-Price Auction Theorem

- We can show that bidding truthfully in a second-price auction is optimal (even when valuations are private)

Theorem

A second-price auction is strategyproof, i.e., bidding one's true valuation is a weakly dominant strategy. Additionally, second-price auctions are efficient, in the sense that the item is allocated to the bidder with the highest value.

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Proof

We write $b = (b_1, \ldots, b_n)$ for the vector of bids and v_i for the valuation of bidder i, in which case the utility of i with a specific bidding profile is:

$$u_i = \begin{cases} v_i - \max_{j \neq i} b_j, \text{ if } b_i > \max_{j \neq i} b_j, \\ 0, \text{ otherwise.} \end{cases}$$

Second-Price Auction Theorem

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Step 1 (Sketch)

We first show that overbidding, i.e., $b_i > v_i$, is dominated by bidding truthfully.

Shown by case analysis:

- If the agent's valuation is larger than all the other bids, the agent wins with truthful bid as well as overbid.
- If the agent's bid is smaller than some other bid, the agent loses anyway.
- If the second-highest bid is between the agent's bid and valuation, the agent wins but gets negative utility. With a truthful bid, utility would be 0.

Second-Price Auction Theorem

Theorem

A second-price auction is strategyproof, i.e., bidding one's true valuation is a weakly dominant strategy. Additionally, second-price auctions are efficient, in the sense that the item is allocated to the bidder with the highest value.

Step 2 (Sketch)

Showing that underbidding is dominated by truthful bidding is analogous.

Lastly, we can easily see that second-price auctions are efficient.

Second-Price Auctions

- Second-price auctions are good for sellers, who don’t have to worry that bidders will rip them off by bidding less than their true values
- Another advantage is that if a buyer who submitted a high bid does not get the item, then the seller, knowing their true preference, might be able to offer a similar object

What about first-price auctions?

First-Price Auctions

- So then, what are the optimal bids in a first-price sealed-bid auction?

Theorem

In a first-price auction with two bidders and private valuations drawn independently and uniformly from [0,1], the pair $(\frac{1}{2}v_1, \frac{1}{2}v_2)$ of bids is optimal.

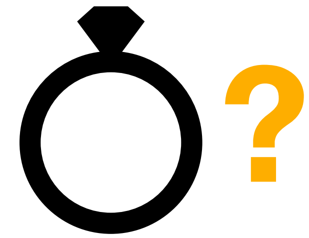
First-Price Auctions

- In first-price auctions for n bidders, the optimal bids in the IPV setting are $(\frac{n-1}{n}v_1, \ldots, \frac{n-1}{n}v_n)$
  - Outside the IPV setting, the optimal bids will be different

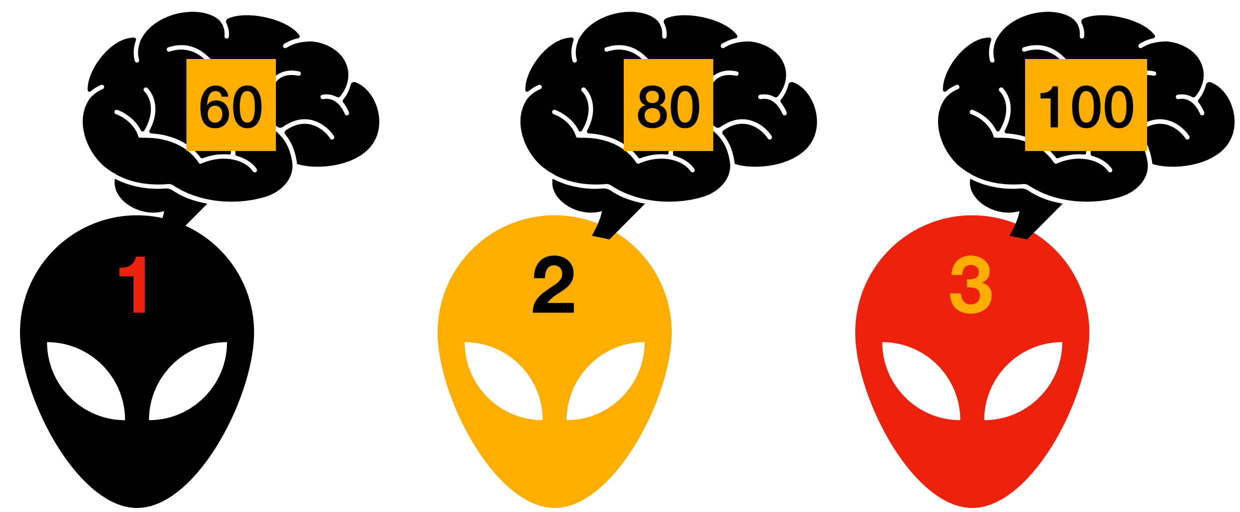
Next, we look at **open outcry** auctions (where bids are public).

Starting with **Japanese auctions**.

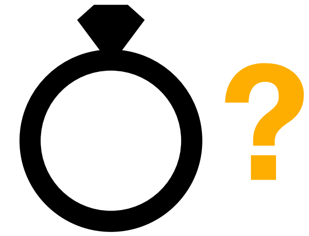
Japanese Auction



- Starting price is set by auctioneer
- Auctioneer successively announces increasing price
- Before each increase, agents announce whether they are in or out
    - Once an agent is out, they stay out
- Auction stops when only one agent is left
- Winner is the last agent left standing
- Price paid is the current price



Japanese Auction



Price List

1st price: 55

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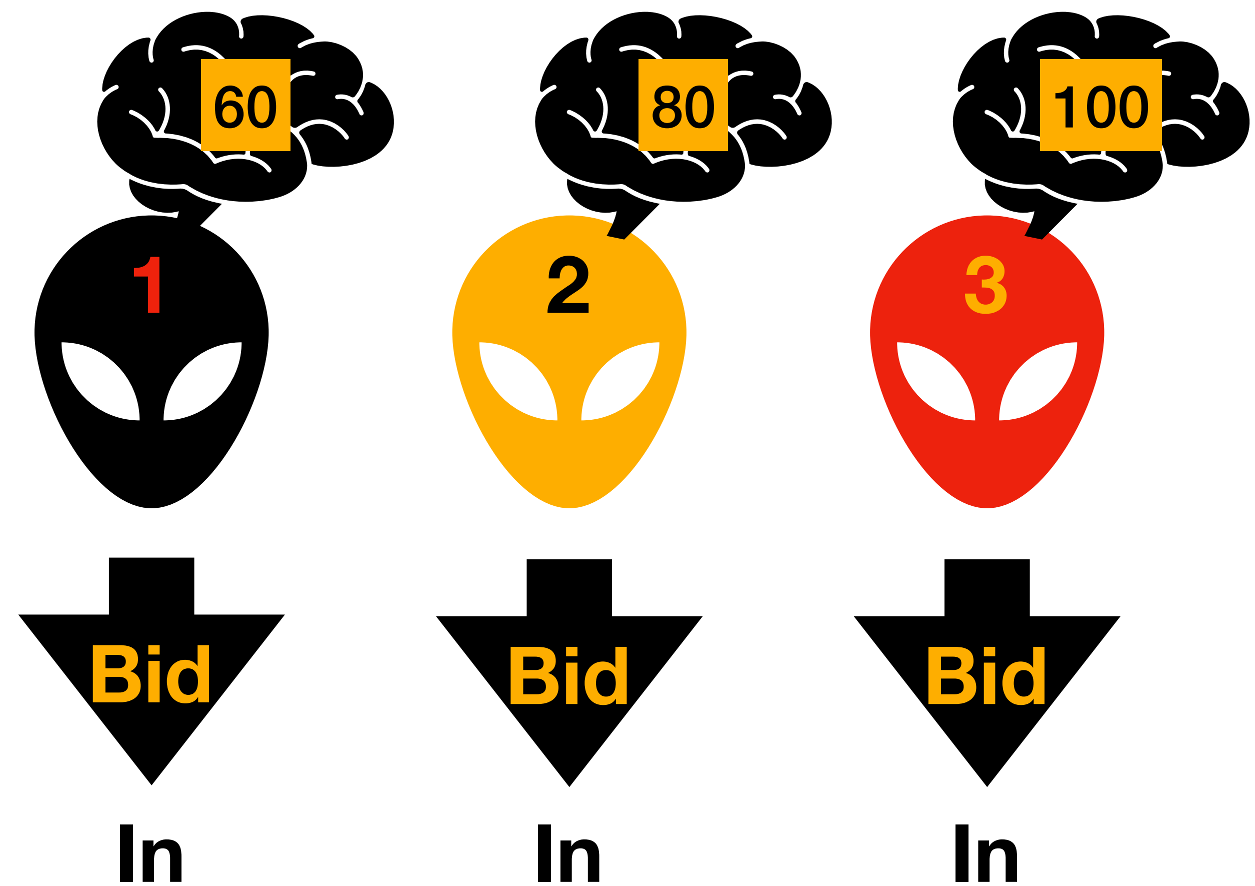
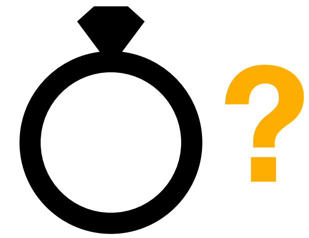


Japanese Auction



Price List

1st price: 55
2nd price: 65

- Starting price is set by auctioneer
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- Auction stops when only one agent is left
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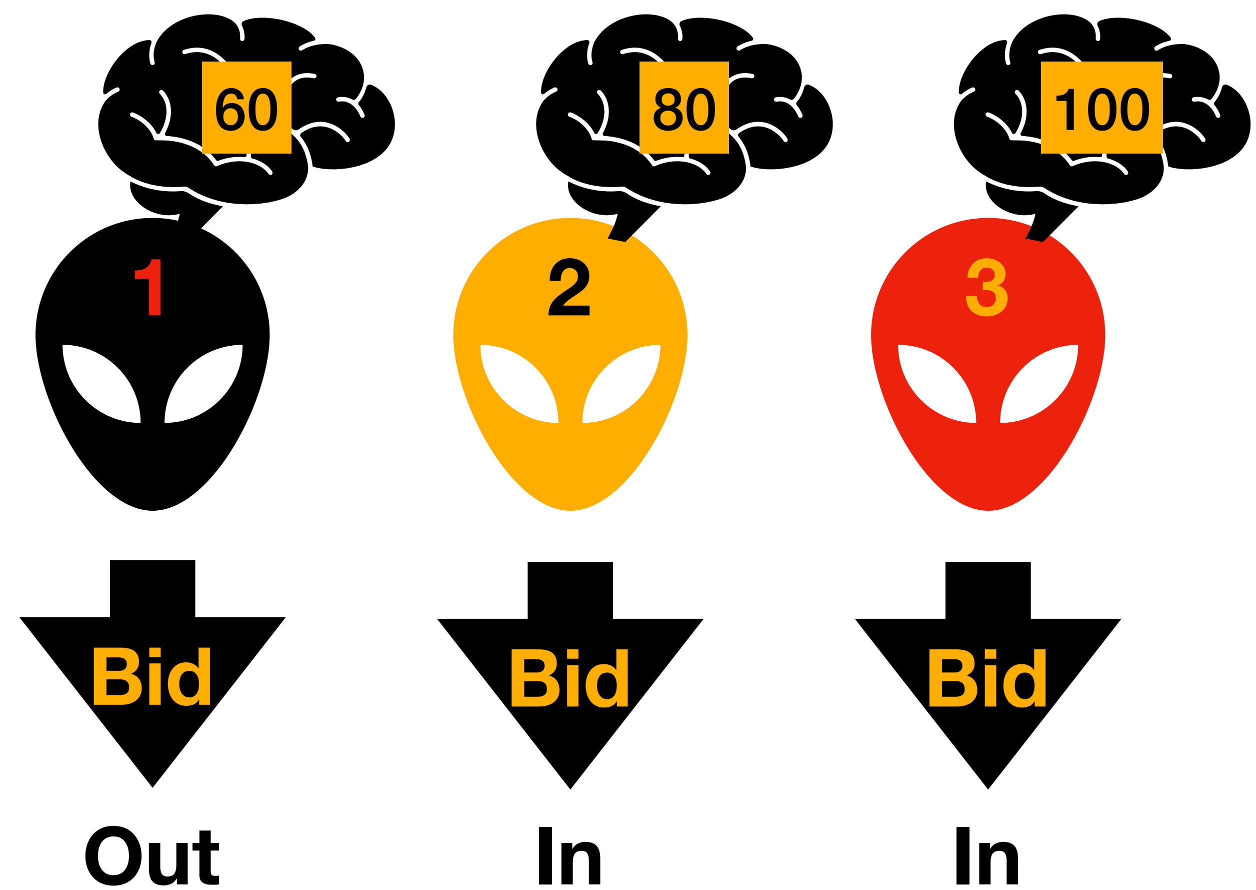

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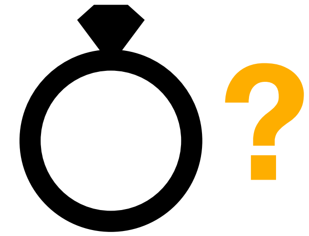

Price List

1st price: 55
2nd price: 65
3rd price: 75

- Starting price is set by auctioneer
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- Auction stops when only one agent is left
- Winner is the last agent left standing
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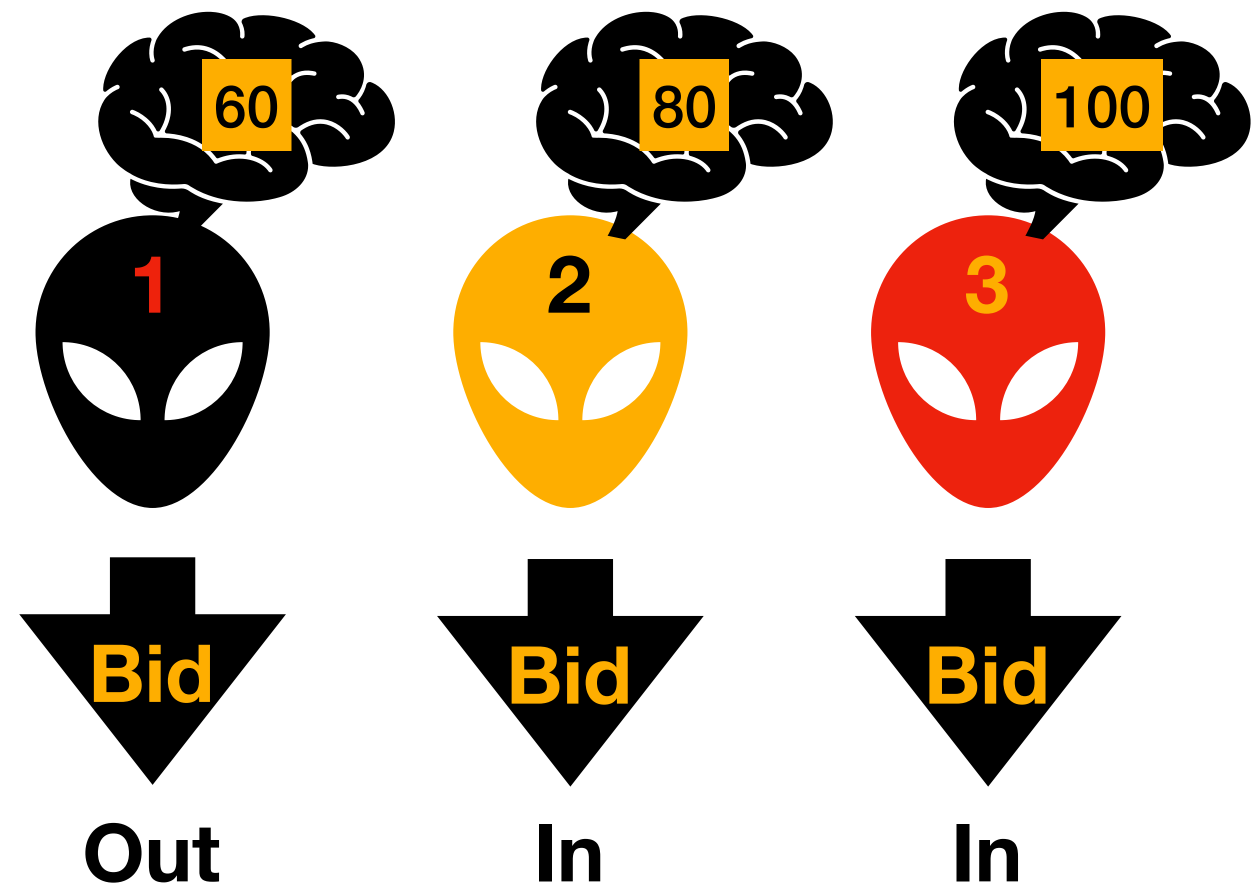
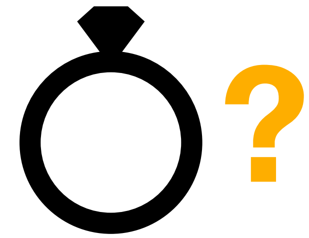


Japanese Auction



Price List

1st price: 55
2nd price: 65
3rd price: 75
4th price: 85

- Starting price is set by auctioneer
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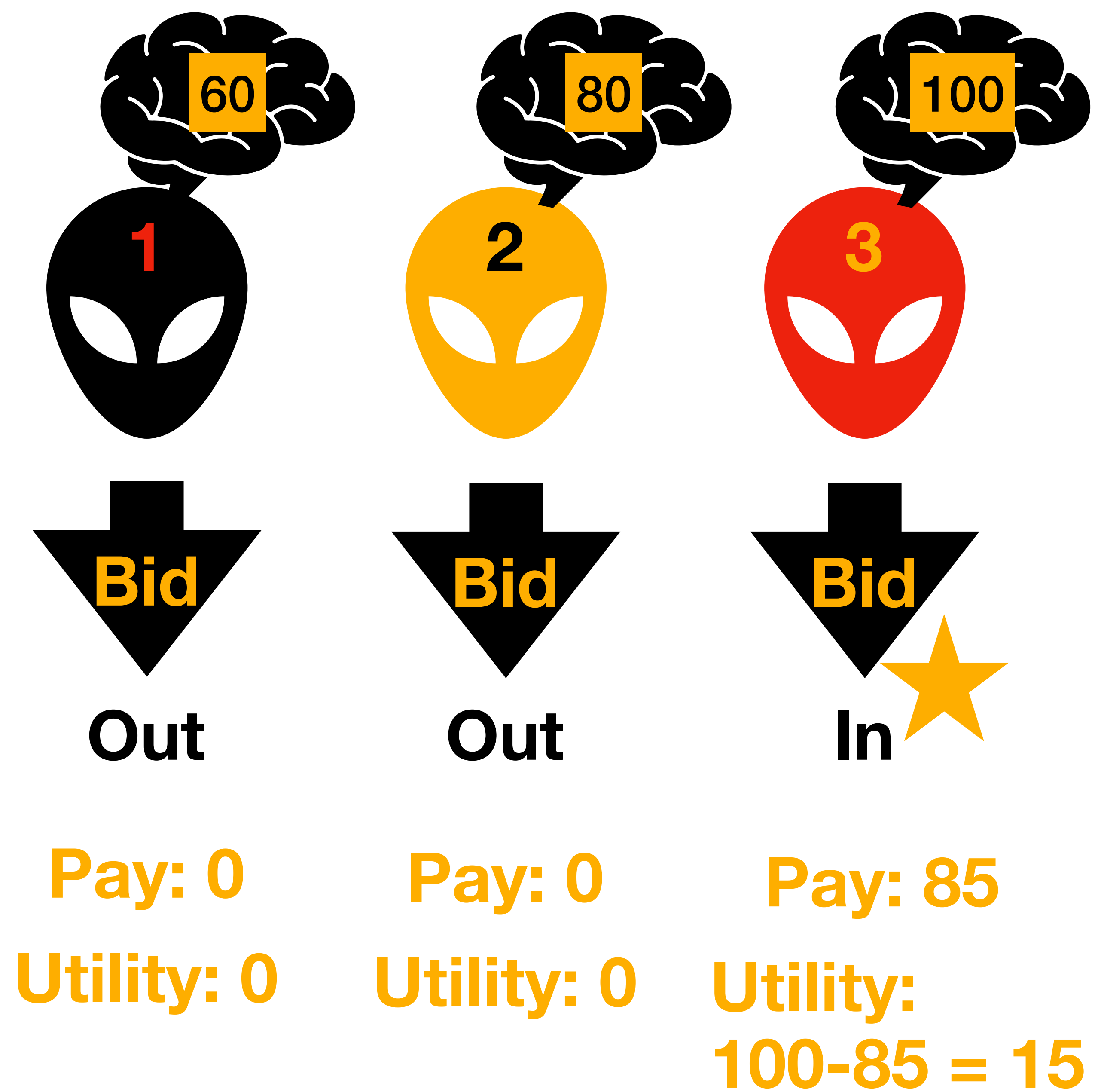
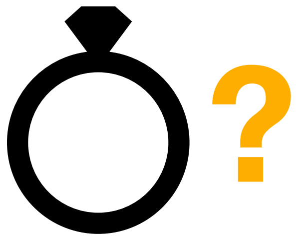
- Price paid is the current price

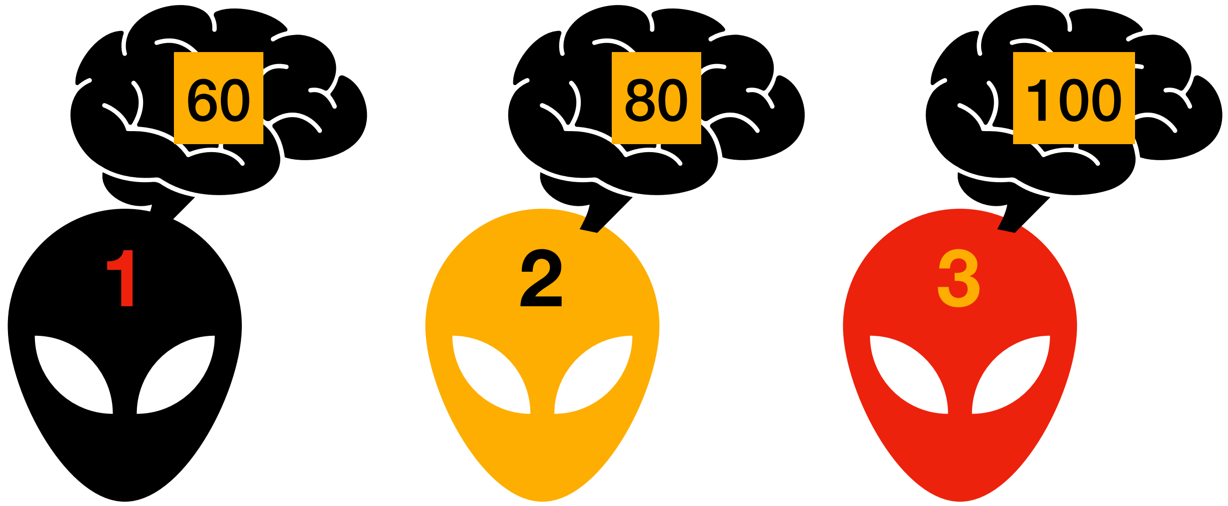


What Kind of Game is a Japanese Auction?

- First of all, note that with Japanese auctions the strategy space is more complicated
  - Because bidders can observe information about how the other bidders behave, and can condition their bids on these observations
- That being said, each bidder has to pick a drop-out number (like in a sealed-bid auction bidders pick a bid amount)
- Bidder with the highest number wins and pays approximately the amount selected by the second-highest bidder
  - Can you see why?
- With independent private values, truthful bidding is a dominant strategy

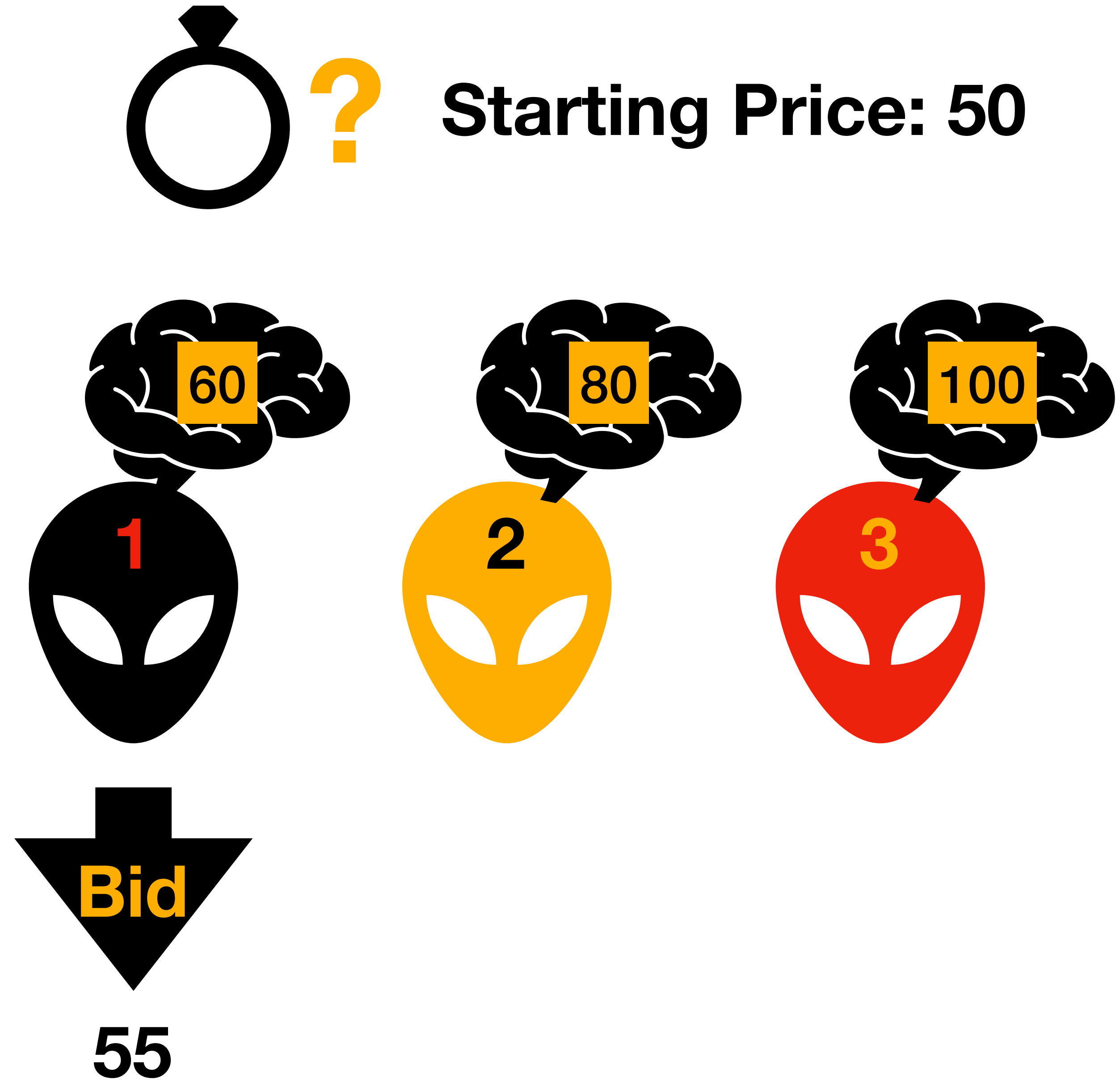
English Auction





- Starting price is set by auctioneer
- Agents announce successive bids, each higher than the last
- Stop when no new bids are made
    - Or after a fixed time, or after a fixed period when no new bids have been made
- Winner is highest bid
- Price paid is highest bid

English Auction



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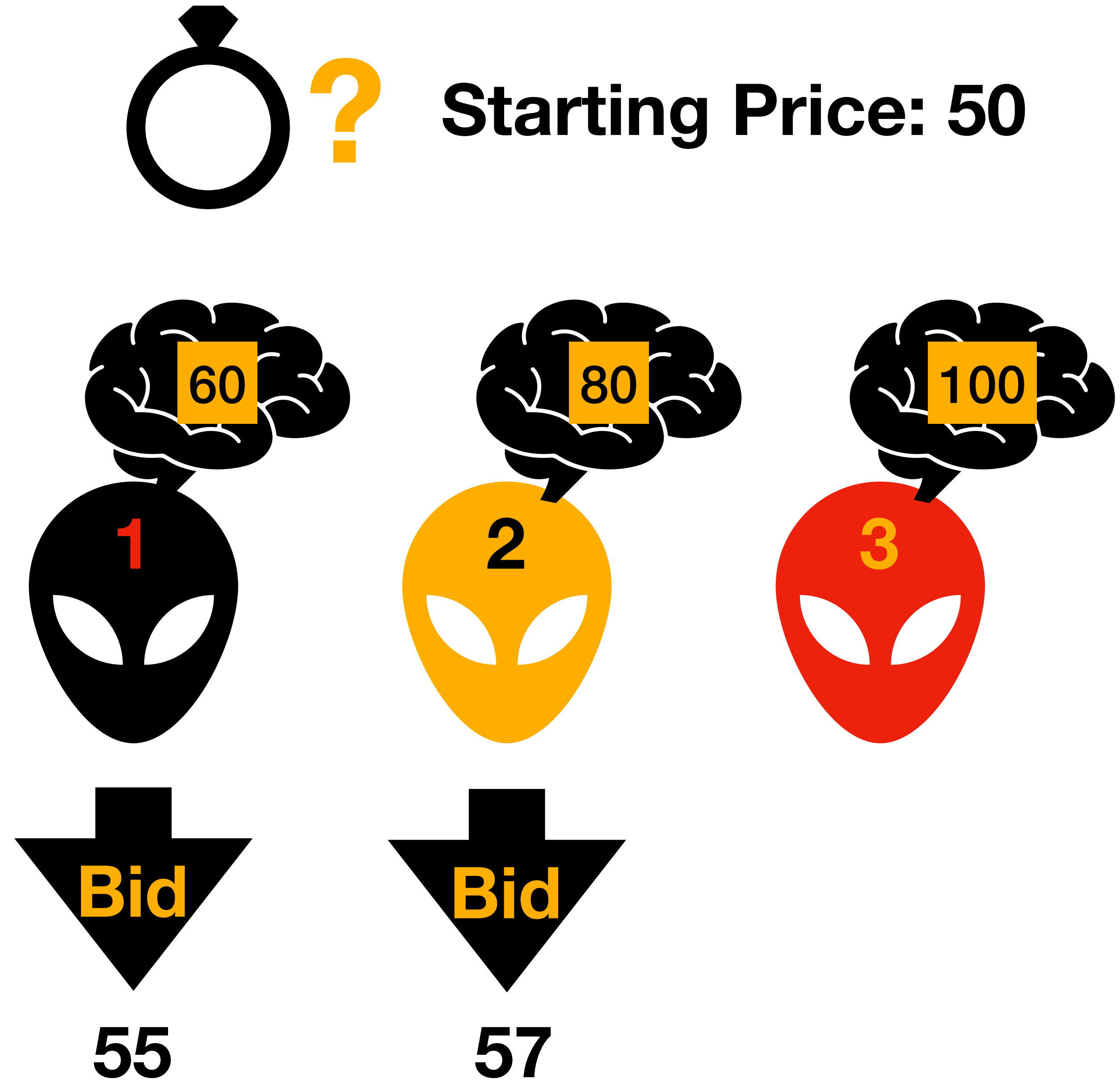

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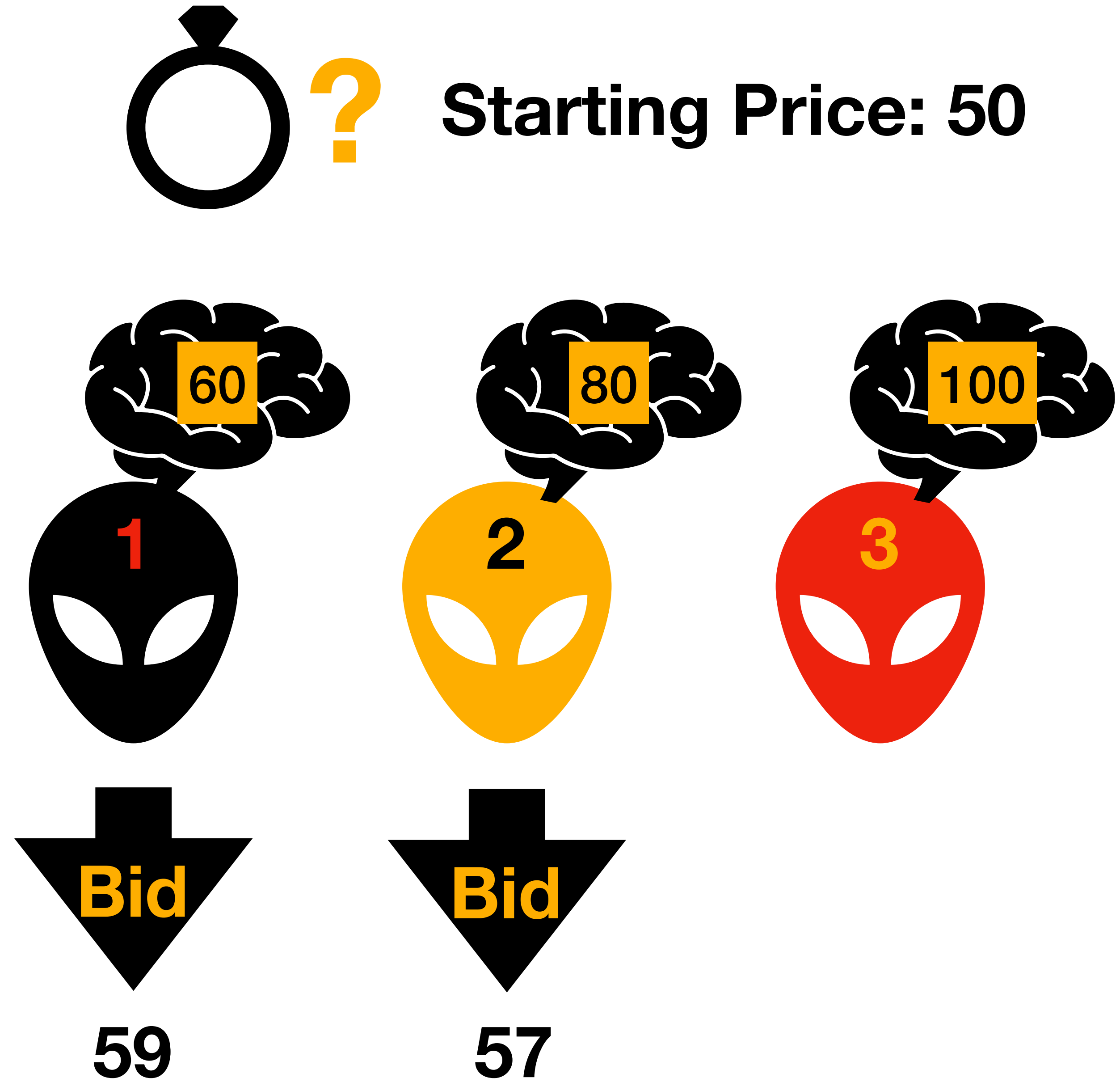

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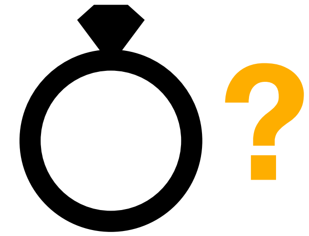

Starting Price: 50

- Starting price is set by auctioneer
- Agents announce successive bids, each higher than the last
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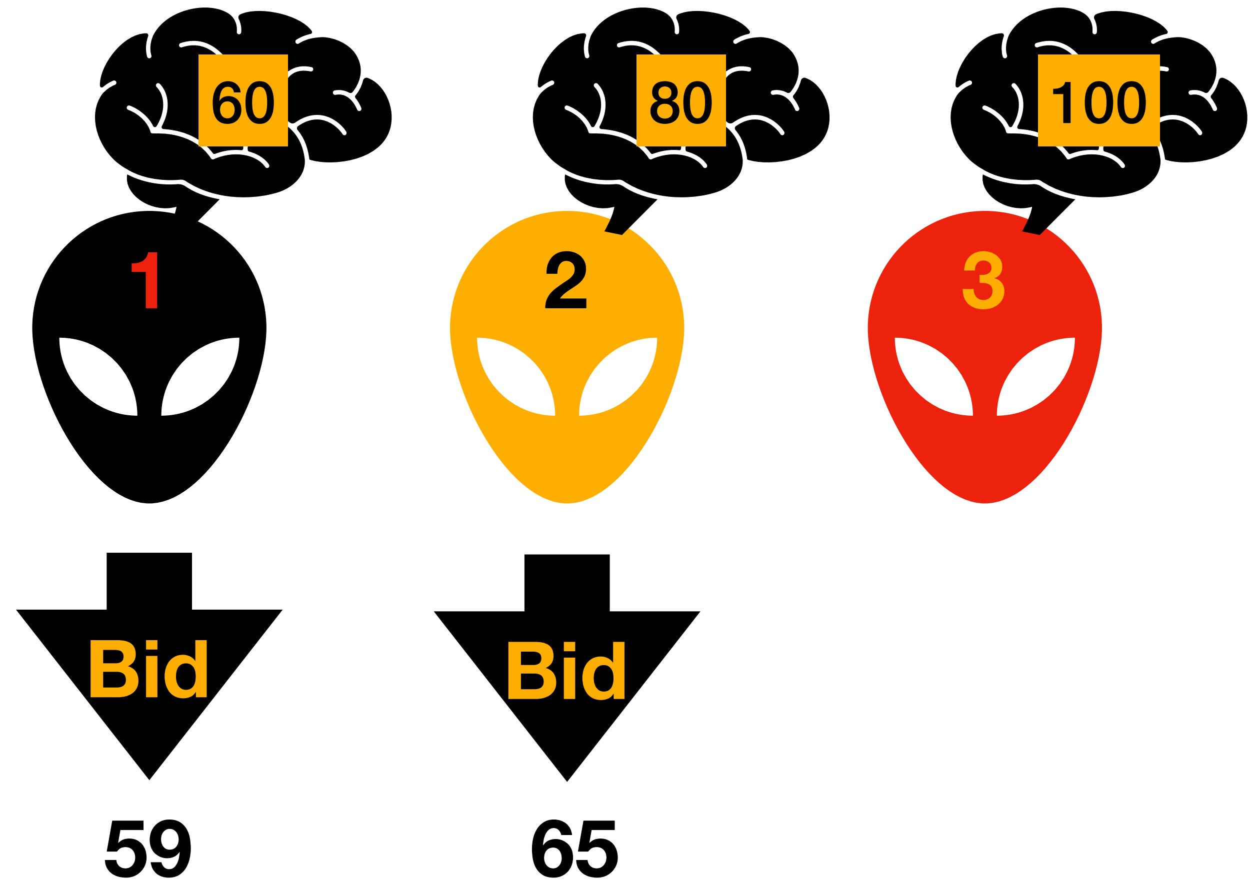
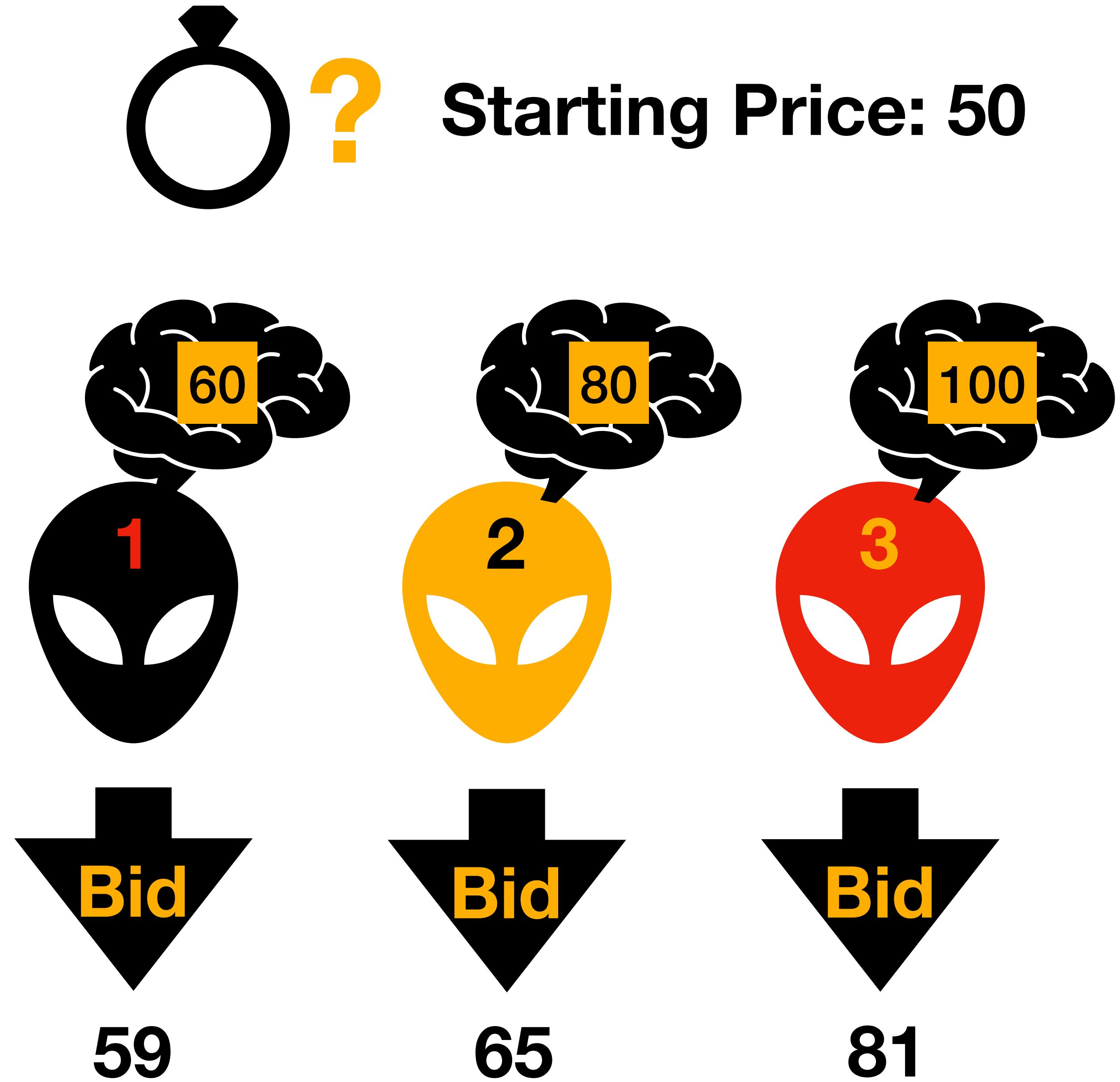
- Winner is highest bid
- Price paid is highest bid



English Auction



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English Auction

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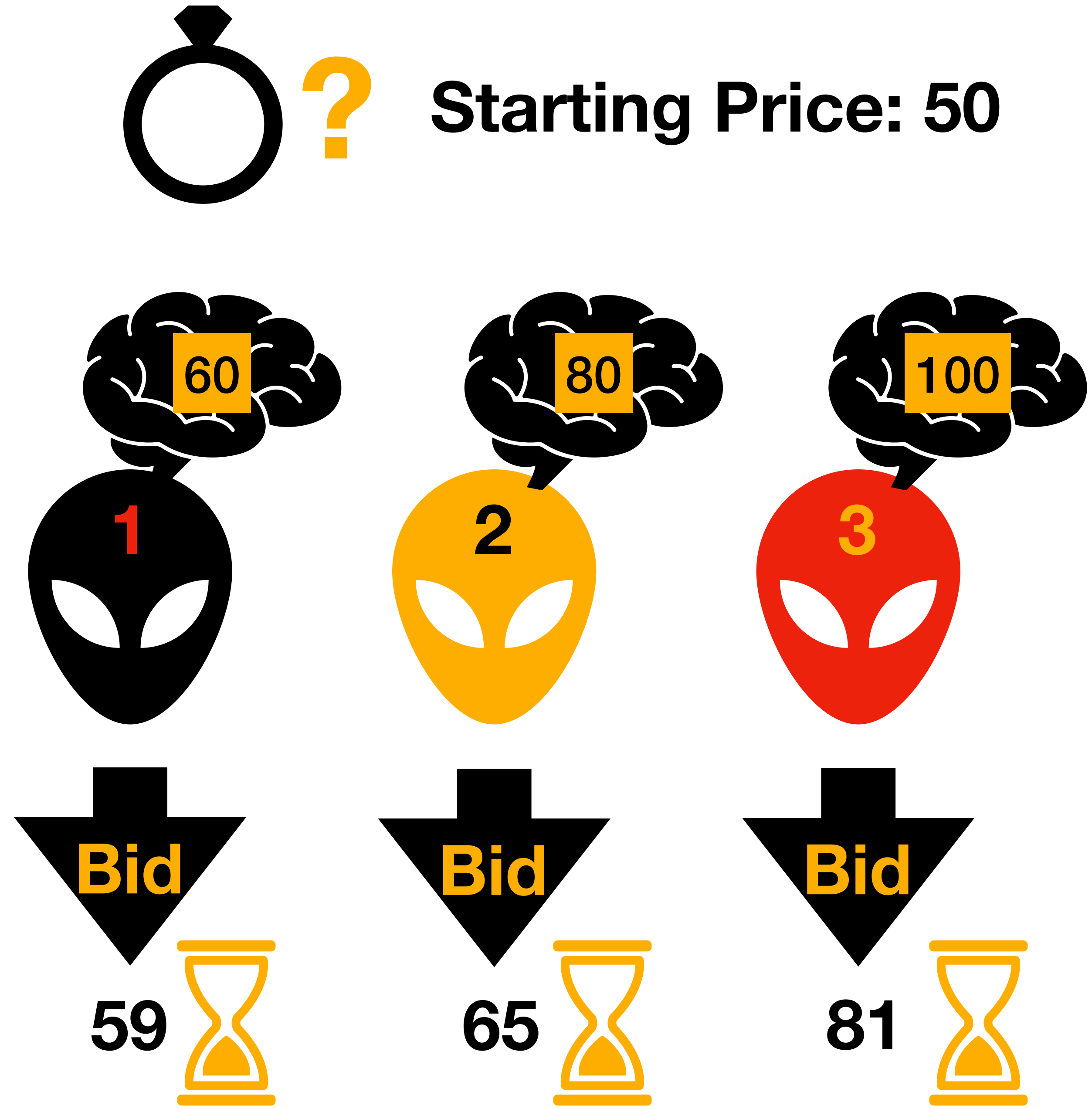

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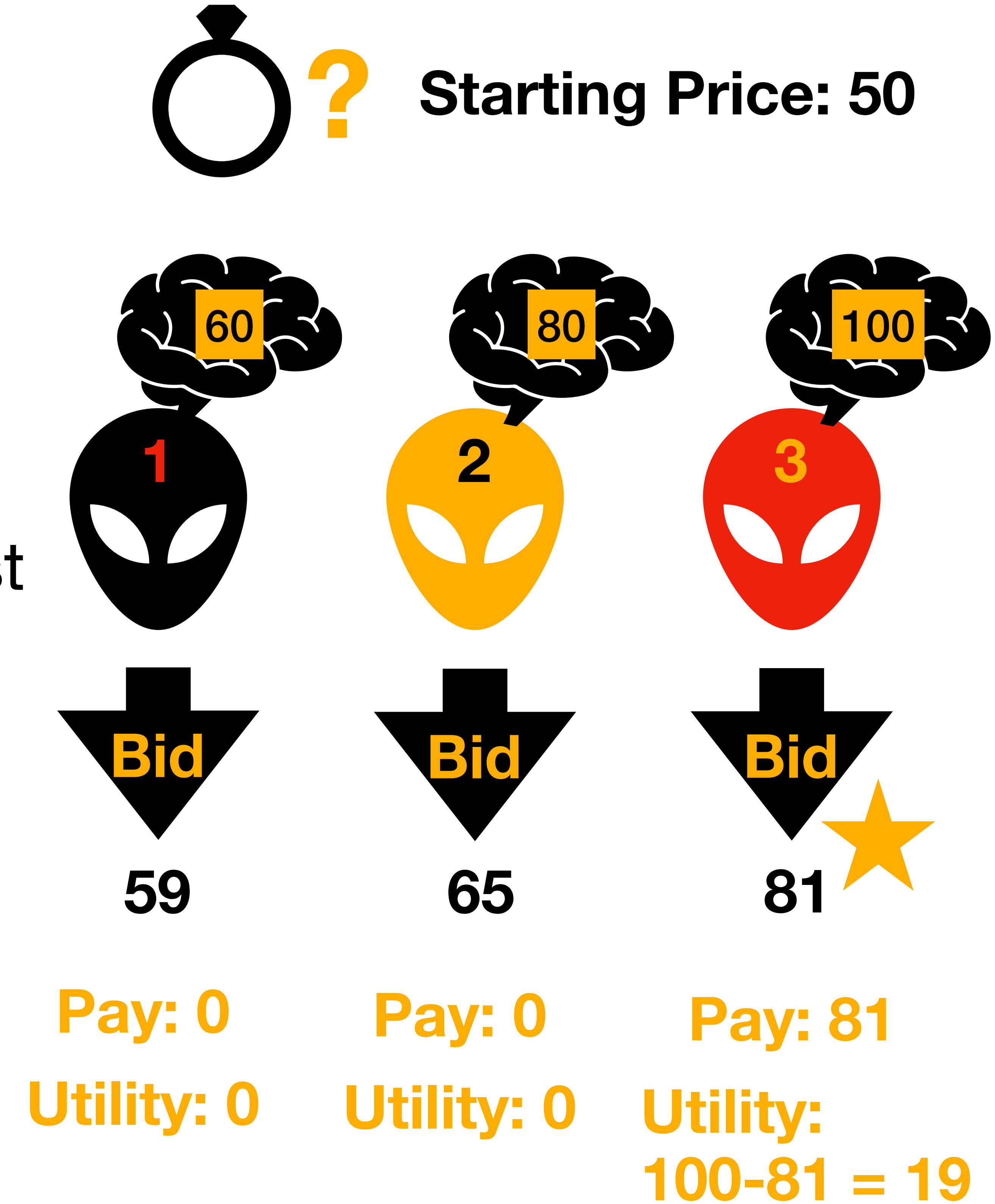
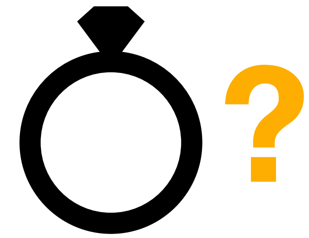


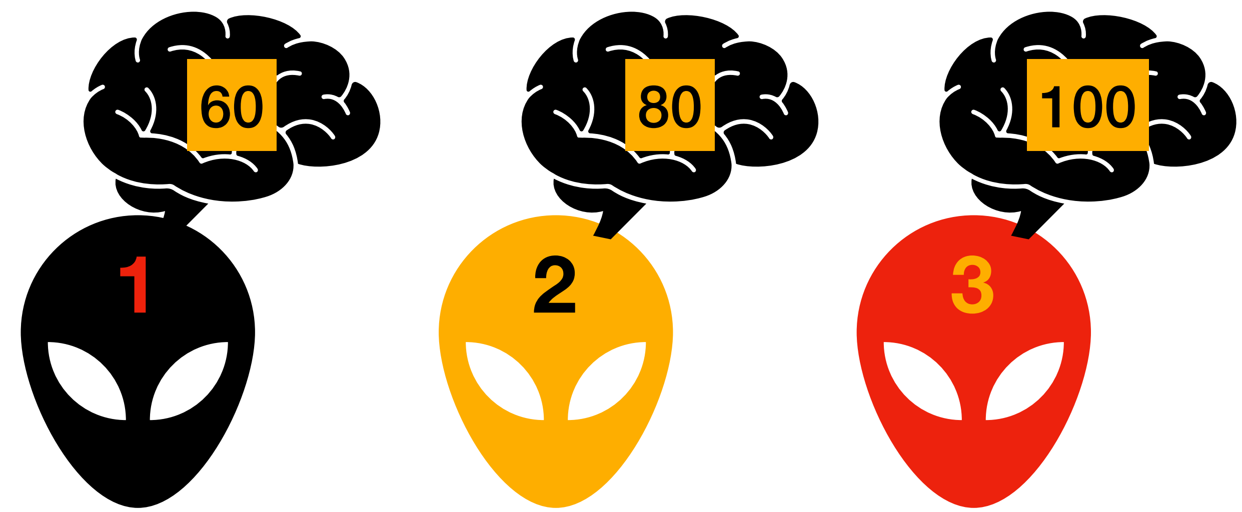
English Auctions

- English auctions turn out to also be similar to second-price auctions
- Think of **proxy bidding**, e.g., on eBay
    - Works by starting with a small bid, setting a maximum amount b_{max}, and leaving the rest to the computer
    - Every time current bid is outbid by some b', your proxy responds with $b' + \varepsilon$, up to b_{max}
    - If everyone does this what results is a second-price auction where the winner pays the second-highest bid, plus ε

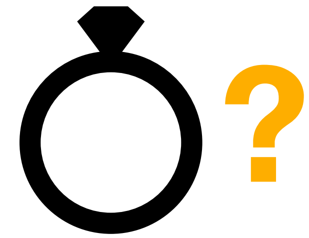
Dutch Auction



- Auctioneer sets high price at beginning
- Auctioneer successively announces decreasing prices
- Stop when first bidder signals “buy”
- Winner is this agent
- Price paid is the current price

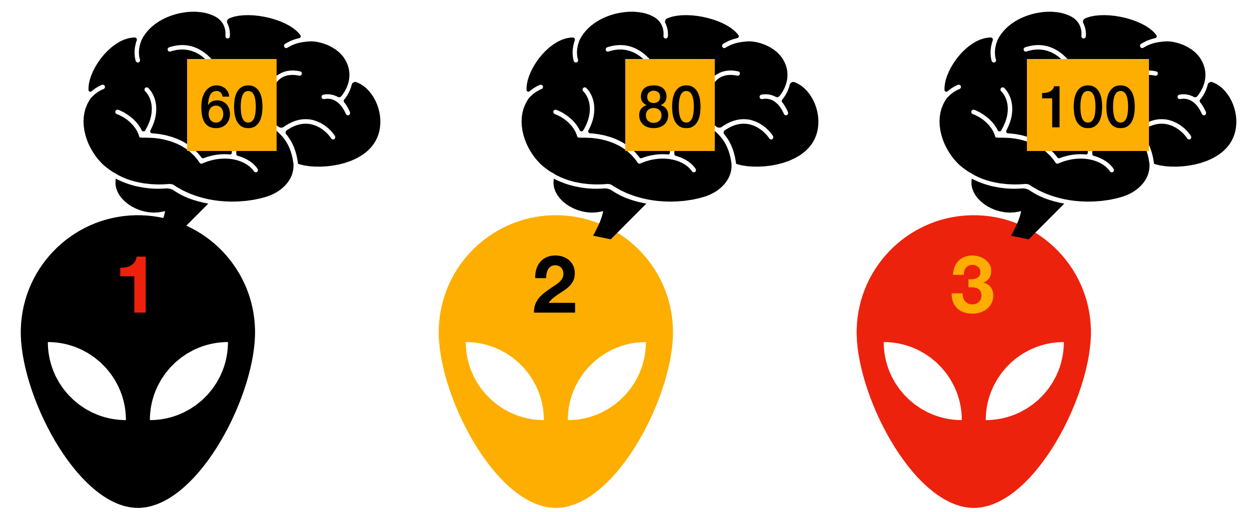


Dutch Auction



Price List

1st price: 120



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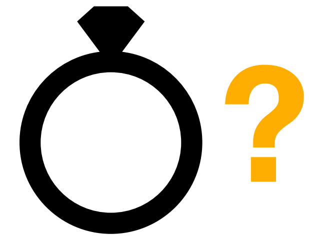
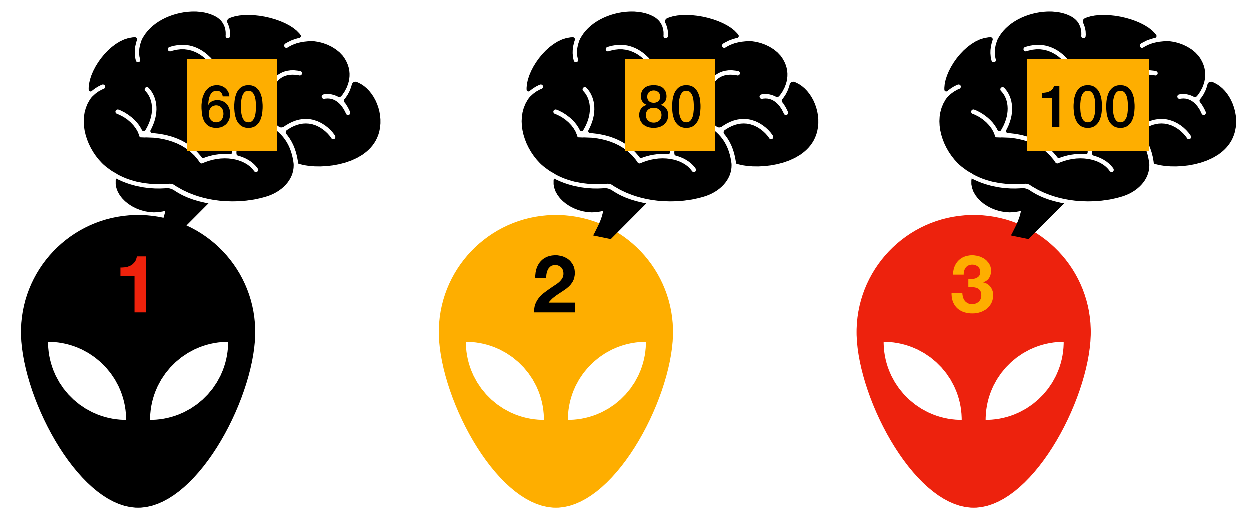
Dutch Auction



Price List

1st price: 120

2nd price: 105



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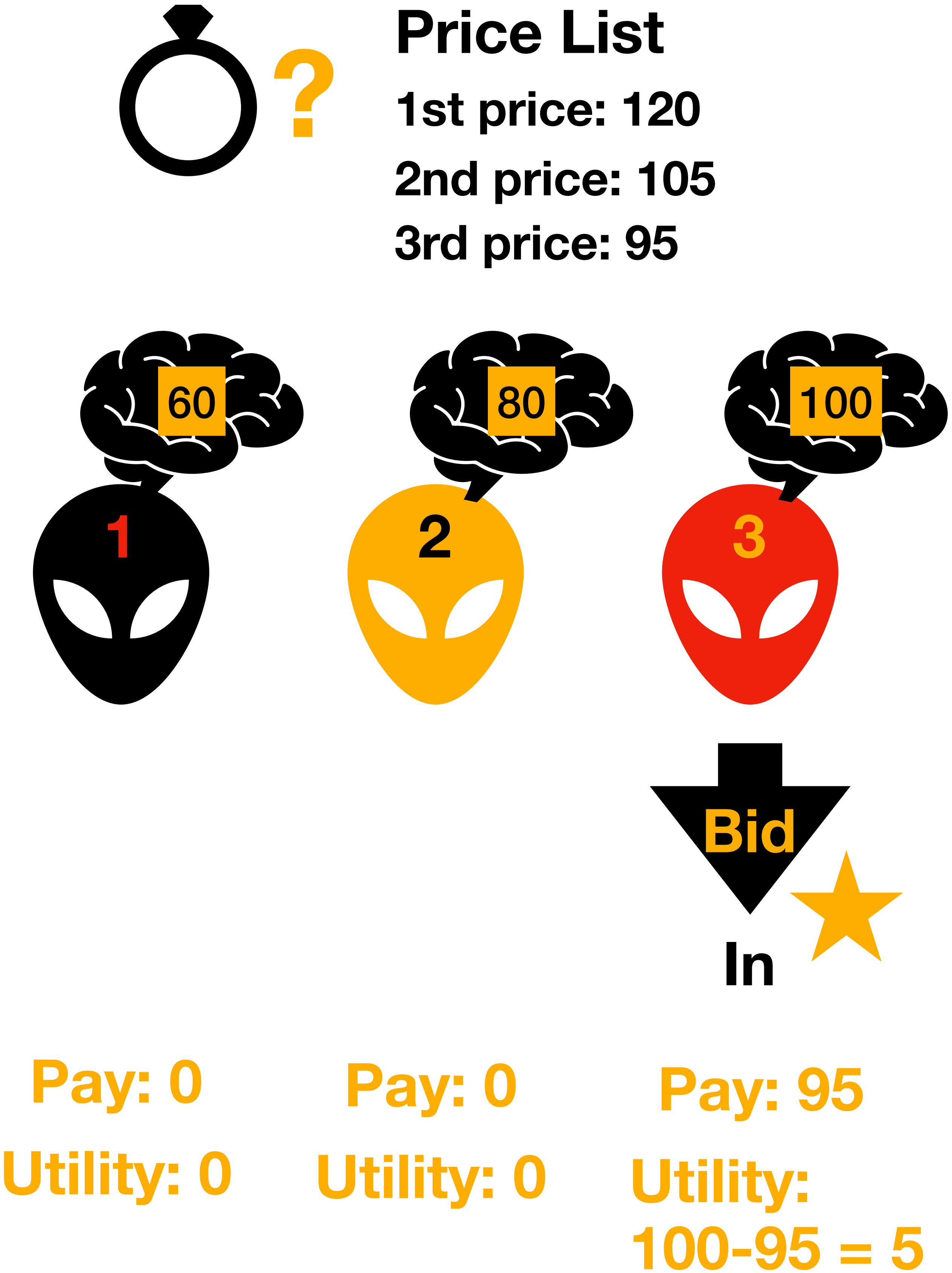

Dutch Auctions

- Dutch auctions are used to sell flowers to wholesalers in the Netherlands
- The descending prices are shown on a clock that everyone sees
- They are useful for selling goods quickly
- Each bidder selects an amount at which to yell "buy", without knowing the others' valuations, and the winner is the bidder with the highest amount
- Payment is the highest bid
- Dutch auctions turn out to be equivalent to sealed-bid first-price auctions