Homework 5

Modal Logic

Due: December 5, 2025, by 23:59

Exercise 1. Below there is a list of formulas of the basic modal language. For each one of them, build a pointed relational model in which the formula is true, and a pointed relational model in which the formula is false; in both cases, there is no restriction on the relation of the model. In case you consider one of the cases is impossible, explain why.

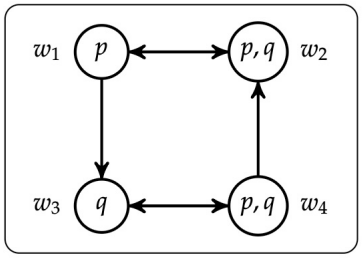
(1) $\Box p \to \Diamond p$ 1pt

(2) $\Box p \to p$ 1pt

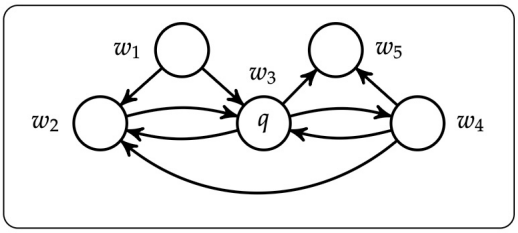
(3) $\Box(p \to q) \wedge \Box p \wedge \Diamond \neg q$ 1pt

Exercise 2. Below you find two relational models. For each one of them, find a formula in the basic modal language characterising each one of the model's worlds. In other words, for each listed model M, and for each one world w in M, find a formula χ_w that is true in (M, w) and false in *every other world in* M.

(1)



(2) 2pt



2pt

Exercise 3. Below you find a list of formulas of the basic modal language. For each one of them, prove that they are valid, or show that they are not valid.

(1) $\Box(p \to q) \to (\Diamond p \to \Diamond q)$ 1pt

(2) $\Diamond(p \to q) \to (\Box p \to \Box q)$ 1pt

(3) $\Diamond p \vee \neg\Diamond p$ 1pt

Exercise 4. Prove that a modal frame F is symmetric if and only if $F \Vdash p \to \Box\Diamond p$. Recall that a frame is symmetric if and only if for all $w, v \in W$: if Rwv, then Rvw. 2pt