Homework 6

Coalition Logic, Computation-Tree Logic

Due: December 12, 2025, by 23:59

Exercise 1. Let $N = \{1, 2\}$ be the set of agents, and let $A = \{p, q\}$ be the set of atomic propositions we are interested in modeling. Consider a coalition model $M = (W, E, V)$ where $W = \{w_1, w_2, w_3, w_4\}$ and V is such that $V(p) = \{w_1, w_2\}$ and $V(q) = \{w_1, w_3\}$. In each one of the cases described below, you should provide a definition of an effectivity function E, with the specified properties, under which the given formulas are satisfied (do not forget to satisfy the two requirements any effectivity function should satisfy). If you consider that it is not possible to find such an effectivity function, explain why.

(1) We want for agent 1 to have full control over p, without her having any control over q, and for agent 2 to have full control over q, without her having any control over p. Thus, the effectivity function should be such that the following formula is true:

$$\langle\langle\{1\}\rangle\rangle p \wedge \langle\langle\{1\}\rangle\rangle \neg p \wedge \neg\langle\langle\{1\}\rangle\rangle q \wedge \neg\langle\langle\{1\}\rangle\rangle \neg q \wedge \langle\langle\{2\}\rangle\rangle q \wedge \langle\langle\{2\}\rangle\rangle \neg q$$

$$\wedge \neg\langle\langle\{2\}\rangle\rangle p \wedge \neg\langle\langle\{2\}\rangle\rangle \neg p$$

1pt

(2) We want for agent 1 to have the power to guarantee p and also the power to guarantee $p \to q$, without her having the power to guarantee q. Thus, the effectivity function should be such that the following formula is true:

$$\langle\langle\{1\}\rangle\rangle p \wedge \langle\langle\{1\}\rangle\rangle (p \to q) \wedge \neg\langle\langle\{1\}\rangle\rangle q$$

But note: we want an effectivity function that is *closed under intersections*[1]. 1pt

Exercise 2. Let C be any coalition $C \subseteq N$, and consider each one of the following formulas of the coalition logic language:

(1) $(\langle\langle C\rangle\rangle \phi \wedge \langle\langle C\rangle\rangle \psi) \to \langle\langle C\rangle\rangle (\phi \wedge \psi)$ 1pt

[1] A collection of sets is closed under intersections if for any two sets in the collection, their intersection is also in the collection.

(2) $\neg\langle\langle\emptyset\rangle\rangle\neg\phi \to \langle\langle N\rangle\rangle\phi$ 1pt

Provide, in each case, a requirement the effectivity function should satisfy (for the given coalition C, and *for all formulas* ϕ, ψ) in order for the formula to be true. In other words, indicate in each case the requirement the effectivity function should satisfy (for the given coalition C) for the formula to be *always* true.

Exercise 3. The meaning of the temporal operators AU, EU, AG, EG, AF and EF in CTL was defined to be such that "the present includes the future". Often one would like corresponding operators such that the future excludes the present. Use suitable connectives of the language of CTL to define such (six) modified connectives as derived operators in CTL. 3pt

Exercise 4. Which of the following CTL formulas are equivalent? For those which are not, exhibit a model of one of the pair which is not a model of the other:

(1) $EF\phi$ and $EG\phi$ 0.5pt

(2) $EF\phi \vee EF\psi$ and $EF(\phi \vee \psi)$ 0.5pt

(3) $AF\phi \vee AF\psi$ and $AF(\phi \vee \psi)$ 0.5pt

(4) $AF\neg\phi$ and $\neg EF\phi$ 0.5pt

(5) $EF\neg\phi$ and $\neg AF\phi$ 0.5pt

(6) $A[\phi_1 U A[\phi_2 U \phi_3]]$ and $A[A[\phi_1 U \phi_2] U \phi_3]$, hint: it might make it simpler if you think first about models that have just one path 0.5pt

(7) $\top$ and $AG\phi \to EG\phi$ 0.5pt

(8) $\top$ and $EG\phi \to AG\phi$ 0.5pt