User Authentication

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Contents

Introduction

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- Authentication implies a one-to-one test: the user asserts an identity and the server determines if that is true or false
- Human beings are very good at authenticating to each other by recognizing face, voice etc.
- The problem becomes more complex when a human must interact and authenticate with a machine

Introduction

A human can be authenticated to a machine using various methods:

- Use something he knows
  e.g. remember a password or PIN code to login to his email account
- Use something he has
  e.g. insert the smartcard to the bank ATM, open the car door with an RFID tag
- Use something he is
  e.g. scan his fingerprints or face to unlock a smartphone

Passwords

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- Passwords is the most common way humans use to authenticate to a system
- Passwords take various forms depending on the application context

  e.g. the email password consists of many alphanumeric characters, possibly together with special characters

  e.g. the PIN code consists of 4 digits, each from 0 until 9

  e.g. cryptocurrency wallets ask you to setup a lengthy passphrase for backup

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  e.g. cryptocurrency wallets ask you to setup a lengthy passphrase for backup
- In theory a user could use a random key as their password
- This can be hard to remember but still a valid option
- Passwords are easy to transfer, which is good and bad

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- It appears that brute-forcing a key and guessing a password has the same complexity

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- The probability of guessing a random 64-bit key with 2^{20} attempts is very low since $2^{20}/2^{64} = 2^{-44}$
- However, the probability of guessing the password with a 2^{20}-word dictionary can be fairly high due to the non-randomness

Passwords

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  - Select randomly a password for the user, instead of letting them choose
  - Require special characters, capital letters, impose minimal password length, avoid the username in the password etc.

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- User compliance to a password policy is hard to achieve
  - Select randomly a password for the user, instead of letting them choose
  - Require special characters, capital letters, impose minimal password length, avoid the username in the password etc.
- Users may still circumvent the password policy

  `Amsterdam1, Amsterdam2, Amsterdam3, Amsterdam1, ...`

Password Verification and Attacks

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- **Hash-based comparison.** Once the user registers and chooses a password x the system will apply a hash function to it. Instead of storing the password x, the system will store its hash value h.

$$\text{Compute: } h = hash(x), \quad \text{Store in database: } (userid, h)$$

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- To verify a user password x' the system will hash it and compare it to the hashed value in the database

$$\text{Compute: } h' = hash(x'), \quad \text{Compare: } h' == h$$

Password Verification and Attacks

- Notice that every time you reset your password you do not get the old one back, you must create a new one. By hashing the passwords we avoid storing directly and thus leaking them.
- **Preimage resistance.** Given h it is computationally difficult to find password x such that $hash(x) = h$

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- Still, assume an attacker that launches a dictionary attack and let a dictionary $\mathcal{D}$ that contains the b most common passwords. The attack cannot use $\mathcal{D}$ as is due to hashing.

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- Now the dictionary attack amounts to a single table lookup in $\mathcal{H}$ thus it is much faster than computing hashes on-the-fly

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- Thus when the hash tables are precomputed, the dictionary attack on the hash-based verification is trivial
- Can we prevent that?

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- Note that now we cannot precompute all the hashes in a password dictionary, since we have to first concatenate every password with a random salt
- The attacker must precompute the table for all salts in the database (number of different salts $\approx$ number of users) or perform online hash computations

Password Verification and Attacks

Exercise: Estimating the attack effort

- Let 8-character passwords using 128 choices per character i.e. there exist $128^8 = 2^{56}$ possible passwords
- The user database contains 2^{10} hashed passwords
- The attacker has a dictionary with 2^{20} common hashed values
- The attacker expects 25% of the passwords to be in the dictionary

  We want to estimate the average effort to to crack certain passwords in various cases (Case 1 until 3). We measure the effort in "number of hashes to compute" i.e. table lookups and comparisons are free.

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- This is equivalent to an exhaustive search across all possible 2^{56} passwords

$$\text{worst case effort} = \text{number of all possible hashes} =$$

$$2^{56} \text{ hashes}$$

$$\text{average effort} = \text{number of hashes computed s.t. probability} \geq 0.5 =$$

$$\frac{2^{56}}{2} = 2^{55} \text{ hashes}$$

- Since we do not use the dictionary, using salt (or not using salt) does not change the situation

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- With probability 25% the password will appear in the dictionary and can be recovered with a table lookup. With probability 75% it will not, requiring to compute the hashes.

$$\text{worst-case effort} = \frac{1}{4} * 0 + \frac{3}{4} * 2^{56} \approx 2^{55.6} \text{ hashes}$$

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- The effort is close to the effort for Case 1 (no dictionary), however the attacker can stop after trying all values in the dictionary and still have a 25% probability of success.

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- Let 2^{10} distinct password hashes $v_1, v_2, \ldots, v_{2^{10}}$ stored in the database
- The attacker can perform an exhaustive search over the passwords x_i, $i = 1, 2, \ldots, 2^{56}$. Every $hash(x_i)$ will be compared to every hash v_j in the database.

$$\text{average effort} = \frac{2^{55}}{2^{10}} = 2^{45} \text{ hashes} + \text{comparison cost}$$

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- **Case 3B.** Let's assume 'salted' passwords
- Hash every password x_i together with every possible salt $salt_j$ in the database

$$\text{for all } i = 1, 2, \ldots, 2^{56} \text{ and all } j = 1, 2, \ldots, 2^{10}$$

$$\text{compute } hash(x_i || salt_j) \text{ and compare to all } v_j \text{ in database}$$

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- Attached to X is a **probability mass function** (p.m.f.)
  $p(x) = P(X = x)$ for all $x \in \mathcal{X}$

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  e.g. in the case of a cheating die: $p(1) = p(2) = p(3) = p(4) = \frac{1}{6}$

  but $p(5) = \frac{1}{12}$ and $p(6) = \frac{1}{4}$ making a 6-roll more likely

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- When computing the entropy $H(X)$ we do not consider cases of x with zero probability because of $0 * log_2 0$ values
- It holds that $H(X) \geq 0$ always

Entropy

Example 1: Fair die roll

.

- Compute the entropy $H(X)$ where X is the fair die roll example

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_2 p(x) = -\sum_{x \in \{1,2,3,4,5,6\}} p(x) \log_2 p(x) =$$

$$-\sum_{x \in \{1,2,3,4,5,6\}} \frac{1}{6} \log_2 \frac{1}{6} = -\left(\frac{1}{6} \log_2 \frac{1}{6} + \cdots + \frac{1}{6} \log_2 \frac{1}{6} \right) =$$

$$-\log_2 \frac{1}{6} = 2.585 \text{ bits}$$

Entropy

Example 2: Cheating die roll

- Compute the entropy $H(X)$ where X is the cheating die roll example

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_2 p(x) = -\sum_{x \in \{1,2,3,4,5,6\}} p(x) \log_2 p(x) =$$

$$-\left(p(1) \log_2 p(1) + \cdots + p(4) \log_2 p(4) + p(5) \log_2 p(5) + p(6) \log_2 p(6)\right) =$$

$$-\left(\frac{4}{6} \log_2 \frac{1}{6} + \frac{1}{12} \log_2 \frac{1}{12} + \frac{1}{4} \log_2 \frac{1}{4}\right) = 2.5221 \text{ bits}$$

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- Compute the entropy $H(X)$ where X is the cheating die roll example

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$$-\left(p(1) \log_2 p(1) + \cdots + p(4) \log_2 p(4) + p(5) \log_2 p(5) + p(6) \log_2 p(6)\right) =$$

$$-\left(\frac{4}{6} \log_2 \frac{1}{6} + \frac{1}{12} \log_2 \frac{1}{12} + \frac{1}{4} \log_2 \frac{1}{4}\right) = 2.5221 \text{ bits}$$

- The entropy of the 'cheating die' is less than the entropy of the 'fair die', i.e. the fair die has more uncertainty
- In general, entropy is maximized when all values of X are equiprobable

$$\text{Maximum entropy} \quad H(X) = \log_2 p, \quad \text{where } p \text{ the probability of all } x \in \mathcal{X}$$

Entropy

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- We will now use entropy to estimate the strength of passwords

Entropy

Example 3: Uniformly chosen passwords

- Let a 4-character password
- Assume a perfect user that chooses every password character according to a uniformly random distribution over the set $\mathcal{X} = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \ldots, \mathtt{z}\}$
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- Let random variable X that represents the choice of a password character
- The r.v. X is uniformly distributed thus:

$$P(X = x) = \frac{1}{|\mathcal{X}|}$$

$$\text{i.e.} \quad P(X = \mathtt{a}) = P(X = \mathtt{b}) = \cdots = P(X = \mathtt{z}) = \frac{1}{26}$$

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_2 p(x) = -\sum_{x \in \{\mathtt{a}, \ldots, \mathtt{z}\}} \frac{1}{26} \log_2 \frac{1}{26} = 4.70 \text{ bits}$$

Entropy

Example 3: Uniformly chosen passwords

- Compute the entropy of the 4-character password

Entropy

Example 3: Uniformly chosen passwords

- Compute the entropy of the 4-character password
- Let random vector **Y** that represents the 4 characters

$$Y = [X_1 \;\; X_2 \;\; X_3 \;\; X_4]$$

- The characters X_1, X_2, X_3, X_4 are chosen independently thus:

$$P(Y = y) = p(y) = P([X_1 \;\; X_2 \;\; X_3 \;\; X_4] = [x_1 \;\; x_2 \;\; x_3 \;\; x_4]) =$$

$$P(x_1)P(x_2)P(x_3)P(x_4) = \left(\frac{1}{|\mathcal{X}|}\right)^4 = \left(\frac{1}{26}\right)^4$$

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$$P(x_1)P(x_2)P(x_3)P(x_4) = \left(\frac{1}{|\mathcal{X}|}\right)^4 = \left(\frac{1}{26}\right)^4$$

- The entropy is computed as a sum over 26^4 equiprobable passwords **y**

$$H(Y) = -\sum_{y \in \mathcal{Y}} p(\mathbf{y}) \log_2 p(y) = -\log_2 \left(\frac{1}{26}\right)^4 = 18.80 \text{ bits}$$

Entropy

Example 4: Biased passwords

- Assume a non-ideal user that does not choose the password characters randomly
- We know that this user is much more likely to pick a 4-character password that is an actual word
- There are approximately 4000 common 4-character words in the English language and the user has a 70% probability of choosing one of them randomly
- Compute the entropy of the 4-character password

Entropy

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- Let the set $\mathcal{Y}$ of all possible passwords and we split it to a set $\mathcal{C}$ of 4000 common passwords and a set $\mathcal{U}$ of uncommon passwords (the rest)

$$\mathcal{Y} = \mathcal{C} \cup \mathcal{U}, \quad |\mathcal{C}| = 4000, \ \ |\mathcal{U}| = 26^4 - 4000$$

Entropy

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- Passwords y in common set $\mathcal{C}$ are more likely than set $\mathcal{U}$

$$p(y \in \mathcal{C}) = 0.7, \quad p(y \in \mathcal{U}) = 0.3$$

Entropy

Let $\mathcal{C} = \{y_1, y_2, \ldots, y_{4000}\}$ passwords and assume that they are all equally likely

$$p(y \in \mathcal{C}) = 0.7 \iff p(y_1 \cup y_2 \cup \cdots \cup y_{4000}) = 0.7 \iff$$

$$p(y_1) + p(y_2) + \cdots + p(y_{4000}) = 0.7 \iff 4000p = 0.7 \iff p = 0.7/4000$$

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Thus we have: $$p(y) = \begin{cases} 0.7/4000, & \text{if } y \in \mathcal{C} \\ 0.3/(26^4 - 4000), & \text{if } y \in \mathcal{U} \end{cases}$$

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$$H(Y) = -\sum_{y \in \mathcal{Y}} p(y) \log_2 p(y) = -\left(\sum_{y \in \mathcal{C}} p(y) \log_2 p(y) + \sum_{y \in \mathcal{U}} p(y) \log_2 p(y) \right) =$$

$$-\left(\sum_{i=1}^{4000} \frac{0.7}{4000} \log_2 \frac{0.7}{4000} + \sum_{i=1}^{26^4 - 4000} \frac{0.3}{26^4 - 4000} \log_2 \frac{0.3}{26^4 - 4000} \right) =$$

14.89 bits $<$ 18.80 bits for the uniform password example

Entropy

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Cumulative probability of success (CPS). Assume an attacker that has identified a password guessing strategy i.e. to guess among n passwords, he produced an ordering of passwords from most likely to least likely.

$$x_1, x_2, x_3, \ldots, x_n, \quad \text{such that} \quad p(x_1) \geq p(x_2) \geq p(x_3) \geq \cdots \geq p(x_n)$$

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CPS(b) gives the probability of success of our strategy when trying b out of n passwords. Note that $CPS(n) = 1$ i.e. trying all passwords guarantees success.

Entropy

Example 5: CPS computation on a password guessing strategy

- Assume that a non-ideal user chooses a 4-character password i.e. $n = 26^4$
- Prior experience shows that users choose passwords from the following 3 disjoint sets $\mathcal{A}, \mathcal{B}, \mathcal{C}$
- The user may choose (with probability 50%) password from set $\mathcal{A}$ that contains only 0.1% of the passwords
- The user may choose (with probability 30%) password from set $\mathcal{B}$ that contains only 2% of the passwords
- The user may choose (with probability 20%) choose a password from the set $\mathcal{C}$ that contains the remaining 97.9% of the passwords

Entropy

Example 5: CPS computation on a password guessing strategy

- Sets $\mathcal{A}, \mathcal{B}, \mathcal{C}$ contain 456, 9139 and 447381 passwords respectively
- The guessing strategy will start guessing passwords from $\mathcal{A}$. If it fails it continues with set $\mathcal{B}$ and if it fails again it tries set $\mathcal{C}$.
- Individual passwords in sets $\mathcal{A}, \mathcal{B}, \mathcal{C}$ have probability of 0.5/456, 0.3/9139 and 0.2/447381 respectively

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- Compute $CPS(100)$, $CPS(10^3)$ and $CPS(10^4)$

$$CPS(100) = \sum_{i=1}^{100} p_i = \sum_{i=1}^{100} \frac{0.5}{456} \approx 0.10$$

$$CPS(10^3) = \sum_{i=1}^{10^3} p_i = \sum_{i=1}^{456} \frac{0.5}{456} + \sum_{i=457}^{10^3} \frac{0.3}{9139} \approx 0.51$$

$$CPS(10^4) = \sum_{i=1}^{10^4} p_i = \sum_{i=1}^{456} \frac{0.5}{456} + \sum_{i=457}^{9595} \frac{0.3}{9139} + \sum_{i=9596}^{10^4} \frac{0.2}{447381} \approx 0.80$$

Rainbow Tables

Rainbow Tables

- Let an 8-character password, whose digits are chosen over 256 values

$$\text{number of passwords } = 256^8 = 2^{64}$$

- We managed to gain access to the `/etc/shadow` file and obtained the hashed value h of this password
- We will start a brute-force attack to recover the password from its hash h

$$\text{We have the hashed value:} \quad h = hash(x)$$

$$\text{We want the preimage:} \quad x = hash^{-1}(h)$$

Rainbow Tables

Attack extremas

1. **Online computation.** Iterate over 2^{64} passwords and hash each one until we find the password that hashes to the recovered value h
   - Modern Intel/AMD processor have instruction set extensions for the $SHA256$ hash function
     `https://bench.cr.yp.to/`
   - Such implementations can reach 1.6 clock cycles per byte
   - Hashing 8 bytes at 4GHz takes:
     $8 * 1.6/(4 * 10^9) = 3.2 * 10^{-9}$ seconds
   - Total time is approximately $1.6 * 10^6$ hours i.e. 182 years

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- It requires no memory except for the current hash value
- It needs a huge amount of time, so we should consider parallel processing

Rainbow Tables

2. **Lookup table (Offline computation).** To save time we can precompute the hash of 2^{64} passwords and store the hashed values in memory
    - This step can be done before any attack
    - Every value hashed with $SHA256$ is 32 bytes, thus storing them takes $32 * 2^{64} = 2^{69}$ bytes i.e. 590 exabytes

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- It needs a huge amount of memory

- **Rainbow tables** are a **time-memory tradeoff** between online and offline attacks
- The online computation can be viewed as an attack that needs $T = 2^{64}$ hash operations and $M = 1$ memory unit of 32 bytes
- The lookup table can be viewed as an attack than needs $T = 1$ lookup operation and $M = 2^{64}$ memory units of 32 bytes

Rainbow Tables

Rainbow table functions. To generate the rainbow tables we use two functions.

1. The hash function $hash(\cdot)$
   - We use the common hash function $SHA256(\cdot)$ and we choose password:

     $$x = 12345678$$

   - Converting the ASCII characters to hex we get:

     $$x = 31\ \ 32\ \ 33\ \ 34\ \ 35\ \ 36\ \ 37\ \ 38$$

   - Applying $SHA256$ on the password we get:

     $$h = \texttt{ef797c8118f02dfb649607dd5d3f8c76...}$$

     $$\texttt{...23048c9c063d532cc95c5ed7a898a64f}$$

Rainbow Tables

2. The reduction function $reduce(\cdot)$

   - Note that the output of $SHA256$ is 32 bytes

   $$h = \texttt{ef797c8118f02dfb649607dd5d3f8c76...}$$

   $$\texttt{...23048c9c063d532cc95c5ed7a898a64f}$$

   - We want to reduce this output to a new password x' that is 8 bytes long
   - To do this we can simply drop the last 24 bytes and generate a new password

   $$x' = reduce(h) = \texttt{ef 79 7c 81 18 f0 2d fb}$$

Rainbow Tables

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$$f(x) = reduce(hash(x))$$

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- Starting with password x_1, we can apply repeatedly the function $f(\cdot)$ to create a hash chain of length t

$$x_1 \xrightarrow{f} x_2 \xrightarrow{f} x_3 \xrightarrow{f} \ldots \xrightarrow{f} x_t$$

$$x_1 \xrightarrow{hash} h_1 \xrightarrow{reduce} x_2 \xrightarrow{hash} h_2 \xrightarrow{reduce} x_3 \ldots \xrightarrow{hash} h_{t-1} \xrightarrow{reduce} x_t$$

- We refer to x_1 as the **start point** and to x_t as the **end point**

Rainbow Tables

Precomputation Step (offline)

- We choose m different passwords as start points

$$\text{start points} \;= \{x_{1,1}, x_{2,1}, x_{3,1}, \ldots, x_{m,1}\}$$

- For every start point we create a hash chain of length t

$$x_{1,1} \xrightarrow{f} x_{1,2} \xrightarrow{f} x_{1,3} \xrightarrow{f} \ldots \xrightarrow{f} x_{1,t}$$

$$x_{2,1} \xrightarrow{f} x_{2,2} \xrightarrow{f} x_{2,3} \xrightarrow{f} \ldots \xrightarrow{f} x_{2,t}$$

$$x_{3,1} \xrightarrow{f} x_{3,2} \xrightarrow{f} x_{3,3} \xrightarrow{f} \ldots \xrightarrow{f} x_{3,t}$$

$$\ldots$$

$$x_{m,1} \xrightarrow{f} x_{m,2} \xrightarrow{f} x_{m,3} \xrightarrow{f} \ldots \xrightarrow{f} x_{m,t}$$

Rainbow Tables

- From this $m \times t$ table we only store the start and end points
- Thus every hash chain (row) is computed on the fly, keeping only the first and the last password

$$\mathbf{x_{1,1}} \xrightarrow{f} x_{1,2} \xrightarrow{f} x_{1,3} \xrightarrow{f} \ldots \xrightarrow{f} \mathbf{x_{1,t}}$$

$$\mathbf{x_{2,1}} \xrightarrow{f} x_{2,2} \xrightarrow{f} x_{2,3} \xrightarrow{f} \ldots \xrightarrow{f} \mathbf{x_{2,t}}$$

$$\mathbf{x_{3,1}} \xrightarrow{f} x_{3,2} \xrightarrow{f} x_{3,3} \xrightarrow{f} \ldots \xrightarrow{f} \mathbf{x_{3,t}}$$

$$\ldots$$

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$$
\begin{array}{c}
\mathbf{x_{1,1}} \xrightarrow{f} x_{1,2} \xrightarrow{f} x_{1,3} \xrightarrow{f} \dots \xrightarrow{f} \mathbf{x_{1,t}} \\
\mathbf{x_{2,1}} \xrightarrow{f} x_{2,2} \xrightarrow{f} x_{2,3} \xrightarrow{f} \dots \xrightarrow{f} \mathbf{x_{2,t}} \\
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\dots \\
\mathbf{x_{m,1}} \xrightarrow{f} x_{m,2} \xrightarrow{f} x_{m,3} \xrightarrow{f} \dots \xrightarrow{f} \mathbf{x_{m,t}}
\end{array}
$$

- The precomputation step results in the tuple:

$$(\text{first password, last password}) = (x_{i,1}, x_{i,t}), \quad \text{for } i = 1, 2, 3, \dots, m$$

Rainbow Tables

Attack Step (online)

- We want to find which password results in hashed value h under $SHA256$
- We apply the $reduce(\cdot)$ function on h, resulting in password y

$$y = reduce(h)$$

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$$y \in \{x_{1,t}, x_{2,t}, x_{3,t}, \ldots, x_{m,t}\}$$

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- Assume that password y is found at the endpoint indexed j, i.e. $y = x_{j,t}$
- Looking at the chain we see that the hash value $h_{j,t}$ is reduced to password y

$$x_{j,1} \xrightarrow{hash} h_{j,1} \xrightarrow{reduce} x_{j,2} \xrightarrow{hash} \ldots \xrightarrow{reduce} x_{j,t-1} \xrightarrow{hash} \mathbf{h_{j,t-1}} \xrightarrow{reduce} \mathbf{x_{j,t}} = \mathbf{y}$$

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- The password $x_{j,t-1}$ hashes to value $h_{j,t-1}$, thus $x_{j,t-1}$ is the correct password!

$$x_{j,1} \xrightarrow{hash} h_{j,1} \xrightarrow{reduce} x_{j,2} \xrightarrow{hash} \ldots \xrightarrow{reduce} \mathbf{x_{j,t-1}} \xrightarrow{hash} \mathbf{h_{j,t}} \xrightarrow{reduce} x_{j,t} = y$$

Rainbow Tables

Attack Step (online)

- **Case 1 (continued).** Note that the rainbow table has stored only the start and end points $(x_{j,1}, x_{j,t})$
- To compute the right password $x_{j,t-1}$ we start at with the start point $x_{j,1}$ which is stored in the rainbow table
- We apply the function $f(\cdot)$ $t-1$ times, reaching the password

$$x_{j,1} \xrightarrow{f} x_{j,2} \xrightarrow{f} x_{j,3} \xrightarrow{f} \dots \xrightarrow{f} x_{j,t-1}$$

- It is possible that the end point $x_{j,t}$ has more than a single preimage. We refer to this as a **false alarm**.

Rainbow Tables

Attack Step (online)

- **Case 2.** The password y is not one of the end points of the rainbow table

$$y \notin \{x_{1,t}, x_{2,t}, x_{3,t}, \ldots, x_{m,t}\}$$

- Then we apply the function $f(\cdot)$ on y and repeat until it matches any of the end points

$$y = f(y) = reduce(hash(y))$$

Rainbow Tables

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- The process is probabilistic

$$\text{probability of success } \geq \frac{1}{n} \sum_{i=1}^{m} \sum_{j=0}^{t-1} \left(1 - \frac{it}{n}\right)^{j+1},$$

where n the total amount of passwords and m, t the rainbow table parameters

- Typically we construct different tables with different reduction functions thus naming the process rainbow tables

Rainbow tables

How to limit password attacks

- **Policy.** Good password policies or system-assigned passwords: enforce long passwords and special characters, notify the user about the password strength, generate random passwords
- **Salting.** Hashing passwords with salt makes precomputed hash tables obsolete and forces the attacker to compute hashes online
- **Iterated hashing.** To slow down an attacker that is hashing passwords, the system can apply the hash multiple times when registering a user i.e. compute $hash^{1000}(x)$ for user password x
- **Special hash functions.** General cryptographic hash functions are fast, use slower ones on purpose such as $Argon2$ and $bcrypt$

One-Time Passwords

OTP

- A major password problem is their static nature
- If somehow recovered, passwords can be easily re-used to establish authentication

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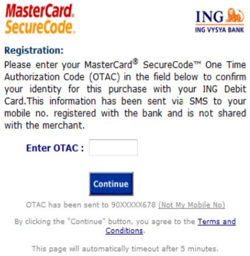
- We can use **one-time passwords (OTP)** that are valid for one use only
- The OTPs are pre-shared between the party that must be authenticated and the verifier that confirms it

OTP

- A major password problem is their static nature
- If somehow recovered, passwords can be easily re-used to establish authentication

- We can use **one-time passwords (OTP)** that are valid for one use only
- The OTPs are pre-shared between the party that must be authenticated and the verifier that confirms it

- How do we share these passwords?
- We generate them on the system and then send them to the user
  e.g. get them in a letter from your bank, through SMS, using a custom device



OTP

Lamport OTP scheme

- We want to avoid transmitting OTPs in the clear thus we can use the **Lamport hash chain** between a user and a system
- The scheme can efficiently generate OTPs using an initial secret w and a hash function $hash(\cdot)$

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   Notice the reverse indexing of the hash chain
   - The user transfers in a secure manner $h_0 = hash^t(w)$ to the system and both parties initialize a counter $i = 1$

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2. **Authentication phase.** The user can access the system using the following hash value as the one-time password

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- The system checks that $v == h_{i-1}$ and if so the user is authenticated
- On a successful authentication, the counter i is incremented and the system saves h_i for the next verification

OTP

Lamport OTP scheme

- Let authentication session $i = 20$ out of $t = 100$ available
- The user will compute the hash value h_{20}:

$$h_{20} = hash^{100-20}(w) = hash^{80}(w)$$

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- Rephrasing, the attacker cannot use authentication session $i = 20$ in order to bypass authentication session $i = 21$ i.e. a past password cannot be used to derive a future password
- This is the reason of indexing the hashes in reverse

OTP

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- The user will send to the system the following triplet:

$$(userid, 20, h_{20})$$

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$$v = hash(h_{20})$$

- If the user provided the correct triplet, then the result should be h_{19}. The system has access to all preceding values in the chain.

$$hash(h_{20}) = hash(hash^{80}(w)) = hash^{81}(w) = h_{19}$$

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- On a successful authentication both parties increment the counter i to 21 and the system stores h_{20} for comparison during the next authentication session (session 21)

OTP

- A commercial application of OTPs is the **passcode generator**
- The passcode generator holds a user-specific secret and outputs passcodes
- Passcodes are a function of the user secret, timestamps and system challenges

OTP

- A passcode generator is often used in conjunction with a static password, offering in-depth security
- To login now I need *something that I know* and *something that I have* and we refer to this as a **two-factor authentication**

e.g. a password together with a passcode generator
e.g. a password together with a hardware security token
e.g. a password together with mobile authenticator app
e.g. a PIN together with a smartcard

Biometrics

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Desirable properties of biometrics

- Universal: everyone should possess this feature
  e.g. some people do not have fingerprints (adermatoglyphia)
- Distinguishing: we should be able to distinguish with certainty different users
  e.g. iris scans are very precise while walking (gait) recognition is less potent
- Permanent: the physical characteristic should not change over our lifespan
  e.g. facial features may change a lot (glasses, beard, aging)
- Collectable: easy to collect, without harm or huge effort
  e.g. camera-powered smartphones make faces easy to capture
- Reliable, robust, user-friendly
  e.g. face recognition became much less user-friendly after wearing a mask - iPhones degraded its security to make it usable again

Biometrics

Deploying biometrics

- Biometrics can be used for authentication but also for **identification**
- When identifying the goal is to find the subject from a list of many possible subjects
  e.g. DNA from a crime scene is matching a list of known suspects

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- Biometrics often requires an **enrollment** phase where the subject enters the biometric features to a database
- This is followed by a **recognition** phase that authenticates or identifies a subject
- Biometric recognition implies errors that can be analyzed using **true positive** and **false negative** rates
- Biometrics can be combined with other authentication methods for in-depth security

Biometrics

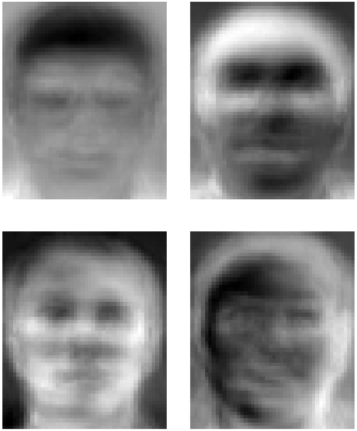
Fingerprints

- Fingerprints have been historically used by forensics investigations
- Distinguishing them requires to extract their special features known as ‘minutia’

Biometrics

Face recognition

- Widely deployed in smartphones and surveillance systems
- Various techniques including 'eigenfaces'



Biometrics

Gait analysis

- Analyzes the motion of the person or animal
- Temporal features and video analysis