Tutorial 4

Voting, Auctions

November 19

Exercise 1. Consider the set of alternatives (or candidates) $A = \{a, b, c, d, e\}$ and 100 voters in N who express their preferences over the alternatives as per the following profile:

16 voters	$e \succ c \succ b \succ a \succ d$
20 voters	$b \succ d \succ c \succ e \succ a$
19 voters	$d \succ c \succ e \succ b \succ a$
31 voters	$a \succ d \succ c \succ e \succ b$
14 voters	$c \succ e \succ d \succ b \succ a$

(1) Recall that STV is the voting rule that at each step eliminates the candidate ranked first by the fewest voters, and redistributes the votes of these voters to their next preferred candidate, repeating the process until only one candidate remains. Compute the winner of STV for the above profile.

(2) Consider a social choice function, similar to STV, which does the following: if there is a *majority* winner in the profile, it elects it. Otherwise, it eliminates all candidates except for the *two plurality frontrunners*, transferring the agents' votes to them, and outputs the plurality winner. Compute the winner of this voting rule for the above profile.

(3) Consider the *anti-plurality* rule, which for every ranking submitted by a voter, gives 1 point to every candidate, except for the candidate ranked last (which receives 0 points). Compute the winner of the anti-plurality rule for the above profile.

Exercise 2. If the following statements are true, you should provide a formal argument in support of the claim; if the fact is false, you should provide a counter-example (and explain why it is a counter-example). Assume that the preferences of the voters are strict rankings and do not consider the case of ties.

(1) Does Borda satisfy *Pareto efficiency*? Y

(2) Does Borda satisfy *independence of irrelevant alternatives*? N

(3) Is it possible that *plurality* selects a *Condorcet loser* as the outcome?

Exercise 3. Consider the resolute social function F depicted below, for two voters, v_1 and v_2, and three alternatives:

		v_2					
F		abc	acb	bac	bca	cab	cba
v_1	abc	a	a	a	b	c	a
	acb	a	a	b	a	a	c
	bac	b	a	b	b	b	c
	bca	a	b	b	b	c	b
	cab	a	c	c	b	c	c
	cba	c	a	b	c	c	c

The entries in the table are the results of F for the profile consisting of the votes of the two voters. For instance, $F(bca, abc) = a$.

Show that for both voters 1 nor 2 there is at least one situation where they can gain by misrepresenting their preferences, with respect to F. Fix one order as the truthful preference, and see if agents can do any better by lying. Give details.

Exercise 4. You want to participate in a sealed-bid auction for a painting of your favourite painter. Offers are made as multiples of 1k euros (e.g., 150k euros, 151k euros, etc). You know that you and four other buyers (Anne, Beatrix, Carl and Daan) will be participating in the auction. You manage to obtain some information about which offers the others will make:

- Anne will offer 150k euros,
- Beatrix will offer 100k euros,
- Carl will offer 200k euros,
- Daan will offer 100k euros.

Your offer is of 170k euros. Moreover, you have a budget of 250k euros and you think that the painting has a value of 220k euros.

(1) If a *first-price auction* is used, who will get the painting and how much will they pay?

(2) If a *second-price (Vickrey) auction* is used, who will get the painting and how much will they pay?

Exercise 5 (Optimal bids in sealed-bids first-price auctions)**.** Assume you are one of two bidders in a sealed-bid first-price auction where the item up for

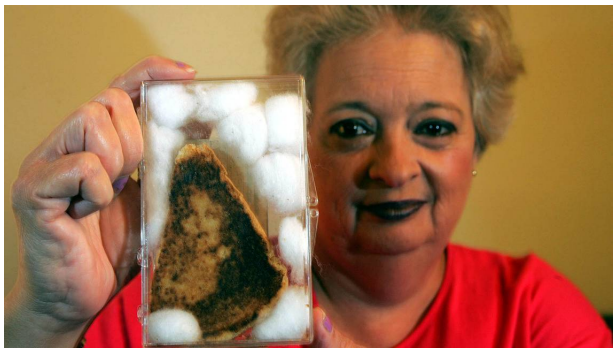

Figure 1: Lady with toast.

auction is a grilled cheese sandwich which, you are told, bears the face of the Virgin Mary.[1]

Your valuation for the item is $v_1 = \$10000$. You do not know the valuation of your rival, just that it is somewhere in between \$0 and \$40000, uniformly distributed (every value has equal probability). But you can assume that your rival is bidding optimally.

(1) What is your optimal bid?

(2) What is your expected payoff?

(3) Suppose you learn that the sandwich is worth \$14000 to the second buyer. The second buyer still has no idea how much the sandwich is worth to you, so they stick to their optimal strategy. What is your optimal bid in this case? What is your expected payoff?

(4) Suppose you learn that the sandwich is worth \$22000 to the second buyer. The second buyer still has no idea how much the sandwich is worth to you, so they stick to their optimal strategy. What is your optimal bid in this case? What is your expected payoff?

[1] This is based on a true story. A piece of toast believed to hold an image of the Virgin Mary was sold for \$28000 dollars in 2004 to an online casino. The lady who had made the toast, ten years earlier, said it never went moldy. Read more here: http://news.bbc.co.uk/2/hi/americas/4034787.stm