Tutorial 2

Games in Normal Form, Games in Extensive Form

November 6

Exercise 1. In the *Battle of the movies* game we are considering a situation where a couple is deciding what to watch on a Friday evening. After 45 minutes of scrolling through a popular streaming platform, they narrow the choice down to two movies: a comedy and a documentary.[1] A : C B : D

However, one partner prefers the comedy to the documentary, and the other one the documentary to the comedy. Watching one's favourite movie gives a utility of 3, while watching what the other person prefers gives a utility of 2. If they ended up each on their own laptop, watching their favourite movie separately, they would only get a utility of 1. The absolute worst (and a bit silly too) would be for each person to watch their *least* favourite movie alone, giving a utility of 0.

		B	
		C	D
A	C	(3,2)	(1,1)
	D	(0,0)	(2,3)

1. Write the above game in normal-form, with agents, actions and payoff matrix.
2. For each pair of strategy profiles, find whether one Pareto dominates the other.
3. What are the Pareto optimal strategy profiles? Justify your answer.
4. Can you find pure(-strategy) Nash equilibria for the game? Justify your answer.

Exercise 2. Consider the following payoff matrix:

making P_1 indifferent,

$2q+(-1)(1-q) = 2q-(1-q)$

$q = \frac{1}{3}$ ✓

P_2 ✓

$$\begin{array}{ccc} & q & 1-q \\ & L & R \\ p \quad T & (2,-2) & (-1,1) \\ 1-p \quad B & (-2,2) & (1,-1) \end{array}$$

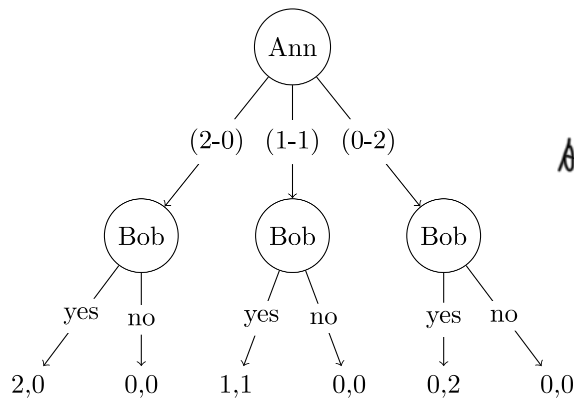
1. Show that the game given by the payoff matrix above does not have any pure Nash equilibria.
2. Find a mixed-strategy Nash equilibrium (guaranteed to exist by Nash's theorem).

[1] This game is known in the game theory literature as the *Battle of the sexes*, but we want to be more progressive here.

Exercise 3. During the lecture we looked at the *Ultimatum Game*. Let's call payers 1 and 2 as Ann and Bob, respectively. Recall that in the story, Ann has to split two euros and makes an offer to Bob: *(i)* she keeps both euros; *(ii)* they get one euro each; *(iii)* Bob gets both euros.

If Bob accepts the offer, they divide the money accordingly. Otherwise, nobody gets anything. The utility of Ann and Bob can be taken to be exactly the amount of money received.

This sequential game can be represented by the following tree:



		Bob	
		yes	no
Ann	(2-0)	(2,0)	(0,0)
	(1-1)	(1,1)	(0,0)
	(0-2)	(0,2)	(0,0)

YNY, YYN ..

1. Transform the game into normal-form, by presenting the corresponding payoff matrix.

2. Given the payoff matrix you found above, what are all pure-strategy Nash equilibria of the game?

3. Are there any *noncredible threats* among them? Why?

4. Apply the BACKWARDINDUCTION algorithm to the game, making each step explicit.

5. By choosing a different order when considering available actions, do you obtain a different result?

6. Given the outcome of BACKWARDINDUCTION, which strategies are subgame-perfect equilibria?

Exercise 4. If $(N, A, \boldsymbol{u})$ is a normal-form game where all profiles are Pareto optimal, does it follow that the game is constant-sum (i.e., in all cells the payoffs add up to the same number)?

Counter example:

	L	R
T	(4,0)	(2,1)
B	(1,2)	(0,3)