Tutorial 7

Computation-Tree Logic and Alternating-Time Temporal Logic

11 December, 2025

Exercise 1. *Let $\mathcal{M} = (S, \to, L)$ be any model for CTL and let $[\![\phi]\!]$ denote the set of all $s \in S$ such that $\mathcal{M}, s \Vdash \phi$. Prove the following set identities by referring to the semantics of CTL:*

(1) $[\![\top]\!] = S$

(2) $[\![\bot]\!] = \emptyset$

(3) $[\![\neg\phi]\!] = S - [\![\phi]\!]$

(4) $[\![\phi_1 \vee \phi_2]\!] = [\![\phi_1]\!] \cup [\![\phi_2]\!]$

(5) $[\![\phi_1 \wedge \phi_2]\!] = [\![\phi_1]\!] \cap [\![\phi_2]\!]$

(6) $[\![\phi_1 \to \phi_2]\!] = (S - [\![\phi_1]\!]) \cup [\![\phi_2]\!]$

(7) $[\![AX\phi]\!] = S - [\![EX\neg\phi]\!]$.

Exercise 2. Prove the following equivalences:

(1) $\neg AF\phi \equiv EG\neg\phi$.

(2) $\neg AX\phi \equiv EX\neg\phi$.

Exercise 3. Consider the concurrent game model $\mathcal{M}$ in Figure 1. There are two agents, a and b. Each agent has two actions in each state, 0 and 1. Transitions are by a pair of actions by a and b, as indicated in edge labels: for example, from q_0, there is a transition by $\langle 0, 0\rangle$ to q_1 and by $\langle 1, 1\rangle$ to q_2. There are propositional variables p which is true in q_0 and q_1, and r which is true only in q_2.

(1) Based on Figure 1, give a formal definition of each component in $\mathcal{M} = (\mathbb{A}gt, St, Act, d, o, V)$.

(2) Express in ATL: Agent a has a strategy to reach a state where it has a strategy to make p true forever. Is this formula true in q_0? Justify your answer by referencing to the semantics for ATL. If the formula is true, give a strategy.

(3) Express in ATL: Agent b has a strategy to maintain p until it reaches a state where b has a strategy to make r true in the next state. Is this formula true in q_0? Justify your answer by referencing to the semantics for ATL. If the formula is true, give a strategy.

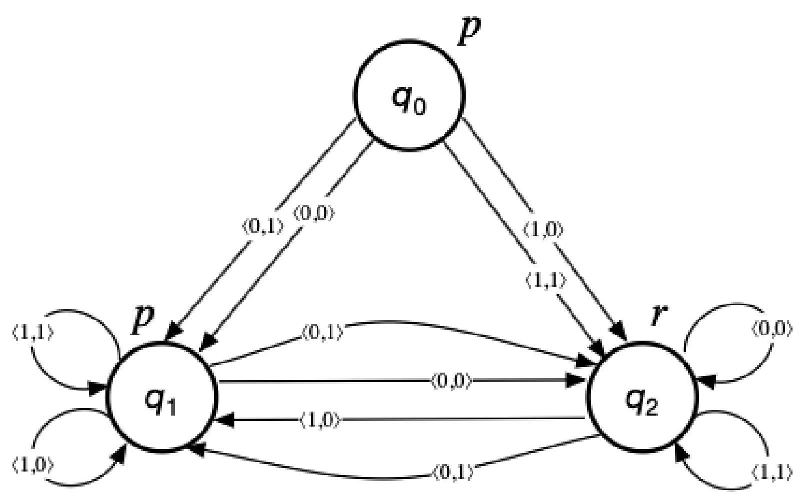

Figure 1: A concurrent game model $\mathcal{M}$.

Exercise 4. Is the following formula valid? If yes, prove it. If no, show a counter-example.

$$(p \wedge \langle\langle\{a\}\rangle\rangle G \langle\langle\{a\}\rangle\rangle X p) \rightarrow \langle\langle\{a\}\rangle\rangle G p$$