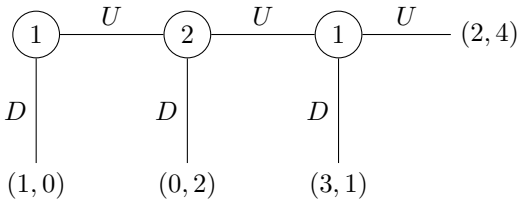
Tutorial 3

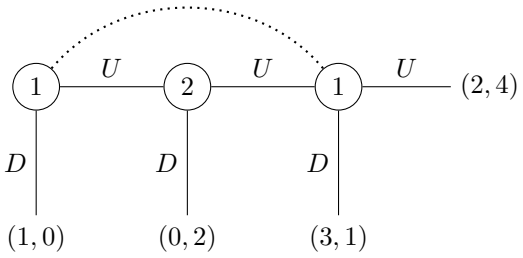
Imperfect Information, Voting

November 13

Exercise 1. Consider two mini-versions of the centipede game. The first, in Figure 1a, is like the one seen during the lecture, with complete information. Then, there is a version with incomplete information in Figure 1b, in which player 1 cannot distinguish between its two choice nodes.



(a) Centipede game with complete information



(b) Centipede game with incomplete information

Figure 1: Centipede game

pure NE: no one wants to deviate alone.

(1) For the complete information version in Figure 1a, write down the normal-form equivalent, with the payoffs and strategies.

(2) What are the pure Nash Equilibria for the normal-form game from (1)?

(3) Do the same for the incomplete information version in Figure 1b: write down the normal-form game, and identify the pure Nash Equilibria.

Exercise 2. Consider the set of alternatives (or candidates) $A = \{a, b, c, d, e\}$ and 100 voters in N who express their preferences over the alternatives as per the following profile:

16 voters	$e \succ c \succ b \succ a \succ d$
20 voters	$b \succ d \succ c \succ e \succ a$
19 voters	$d \succ c \succ e \succ b \succ a$
31 voters	$a \succ d \succ c \succ e \succ b$
14 voters	$c \succ e \succ d \succ b \succ a$

(1) Determine the (set of) winners according to the *plurality* rule. a

(2) Determine the (set of) winners according to the *Borda* rule.

(3) Determine if there is a *Condorcet winner* and/or a *Condorcet loser* in the profile. A Condorcet loser is a candidate that *loses* against every other candidate in a head-to-head contest based on the given profile.

Exercise 3. Consider two voters $N = \{1, 2\}$ and two alternatives $A = \{a, b\}$, i.e., $n = m = 2$. Assume that agents submit a profile of strict preferences $\boldsymbol{R}$. Recall that a social welfare function returns a linear order over A, a social choice function returns a subset of A, and a resolute social choice function returns a single element of A. Always justify your answers.

$F: L^n \to L$ profile (4)

(1) How many different *social welfare functions* exist for this domain? 8

(2) How many different *social choice functions* exist for this domain?

(3) How many different *resolute social choice functions* exist for this domain?