Part 4-1
Response Surface Methodology

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University of Edinburgh

January, 2013

Response Surface Methodology - Why?

Necessity:

Response Surface Methodology - Why?

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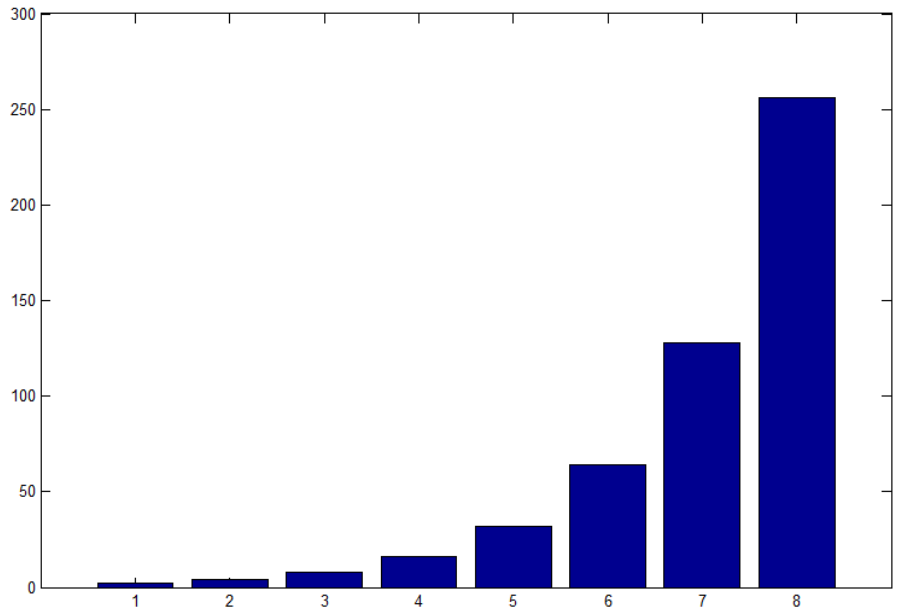

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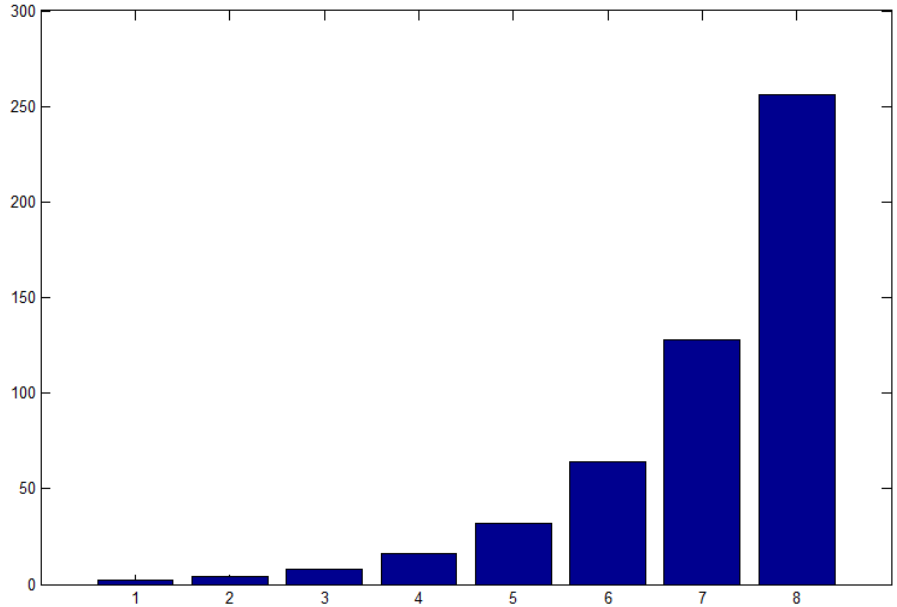

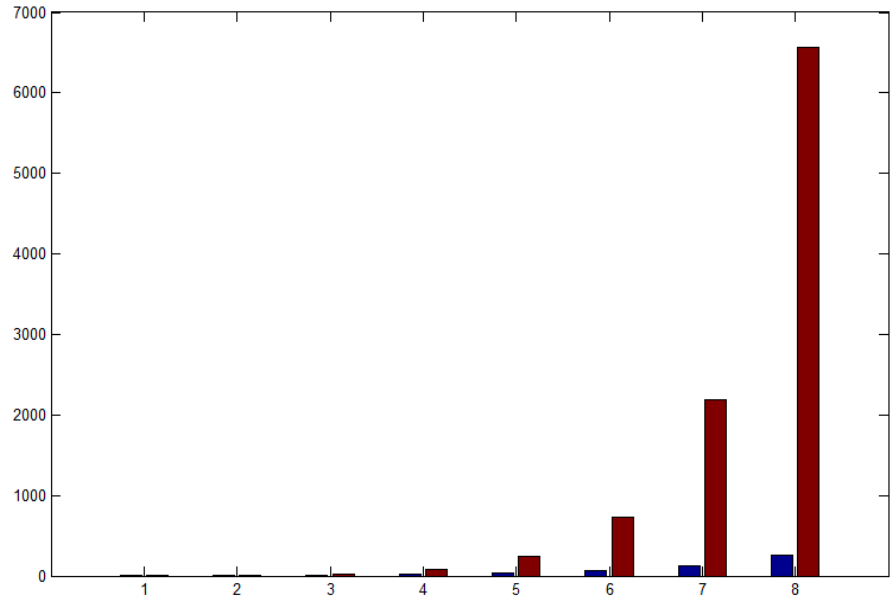

Response Surface Methodology

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- Create specialised experimental models

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- Perform a regression on results

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- Create a simplified mathematical model:

$$Y = a_0 + a_1 X1 + a_2 X2 + a_3 X1 X2 + a_4 X1^2 + a_5 X2^2 + Error$$

Response Surface Methodology

Theoretical Model:

$$Y = \frac{15.0X1^2}{1.0 + 1.5e^{2.5X2}}$$

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Quadratic Approximation:

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Response Surface Methodology

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Quadratic Approximation:

$$Y = a_0 + a_1X1 + a_2X2 + a_3X1X2 + a_4X1^2 + a_5X2^2$$

$a_0 = 14.92$ $a_1 = 7.65$ $a_2 = -16.76$
$a_3 = 0.77$ $a_4 = -6.53$ $a_5 = 6.31$

$$2 \leq X1 \leq 3 \qquad 0 \leq X2 \leq 1$$

Response Surface Methodology

Quadratic Approximation:

$$Y = 14.92 + 7.65X1 - 16.76X2 + 0.77X1X2 - 6.53X1^2 + 6.31X2^2$$

$$2 \leq X1 \leq 3 \qquad 0 \leq X2 \leq 1$$

Response Surface Methodology

Quadratic Approximation:

$Y =$

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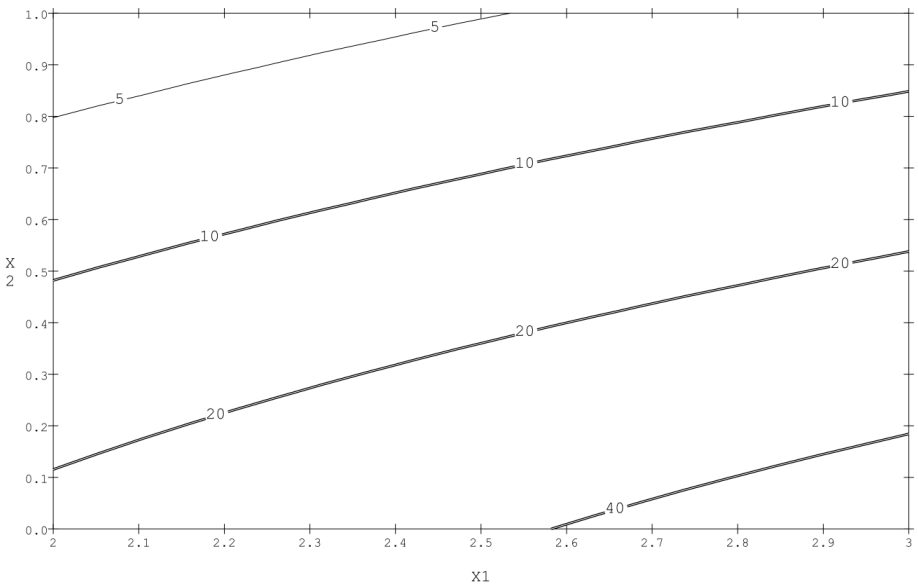

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Quadratic Approximation:

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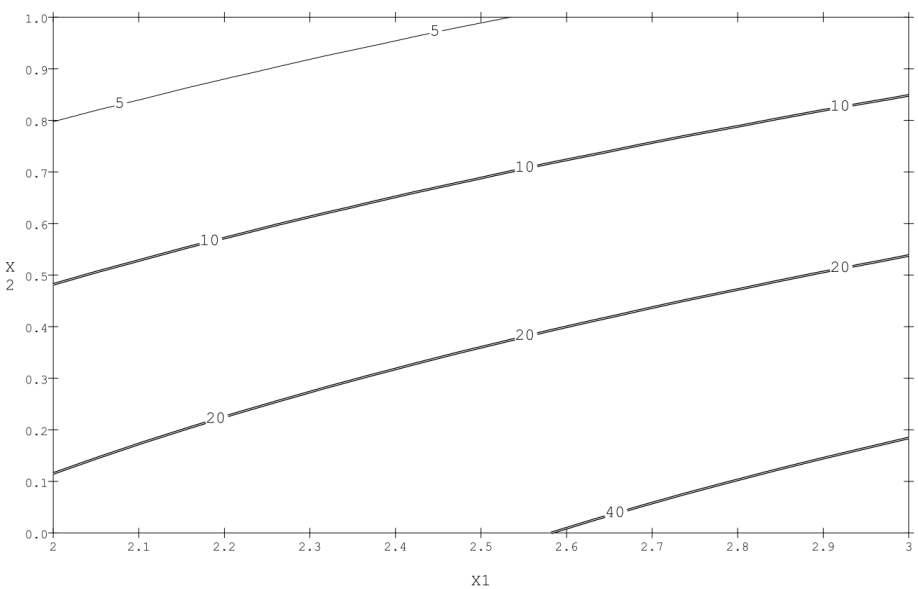
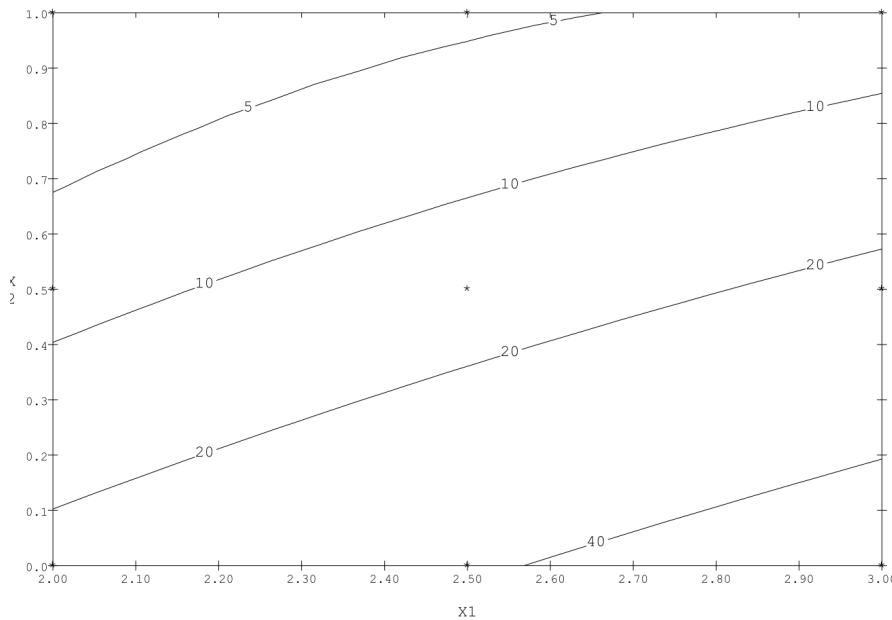


Approximation:



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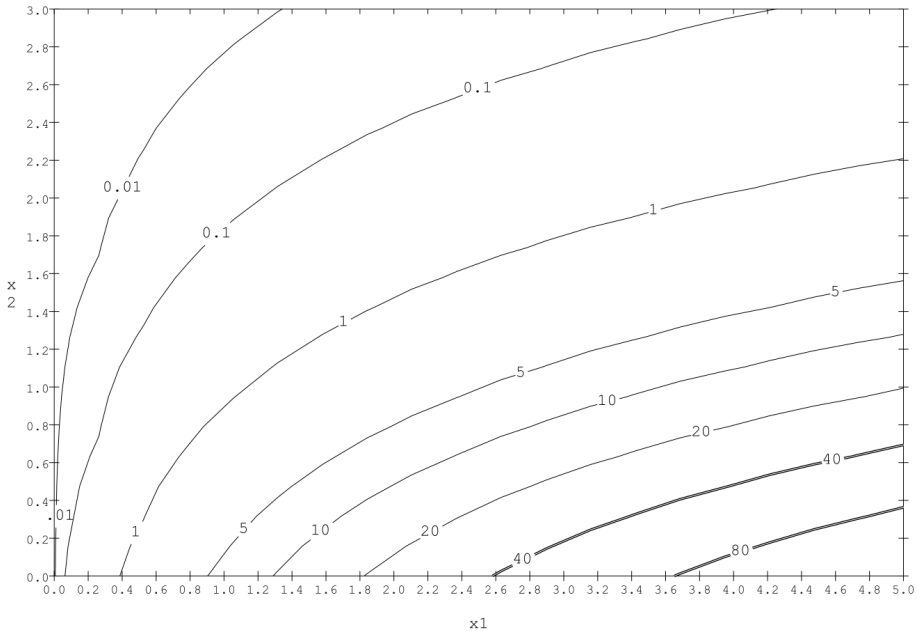

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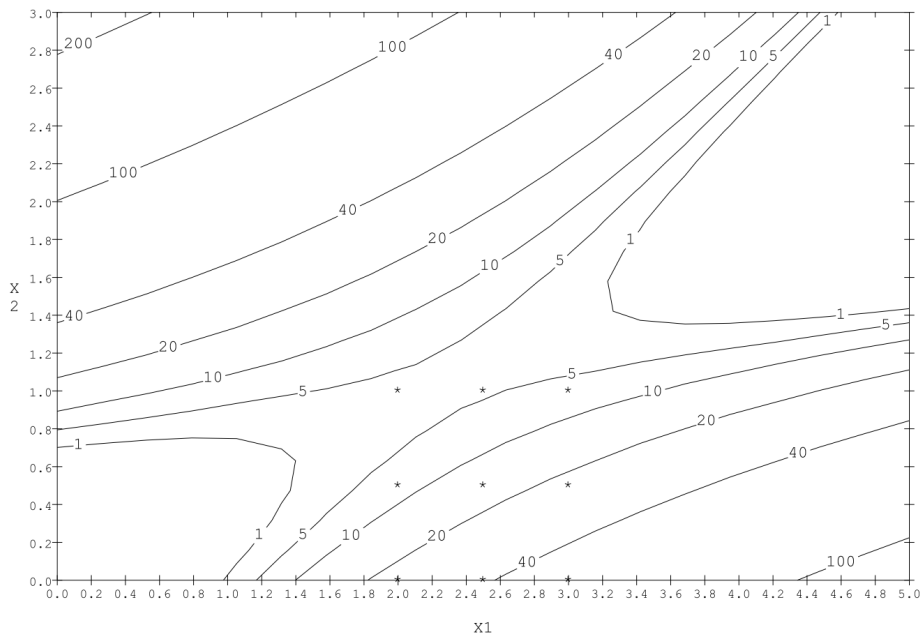

Central Composite Designs

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- Based on Fractional Factorial Designs

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- Based on Fractional Factorial Designs
- Creates a new design suitable for regression

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Good basis for regressions: Rotatable Designs

Rotatable Designs

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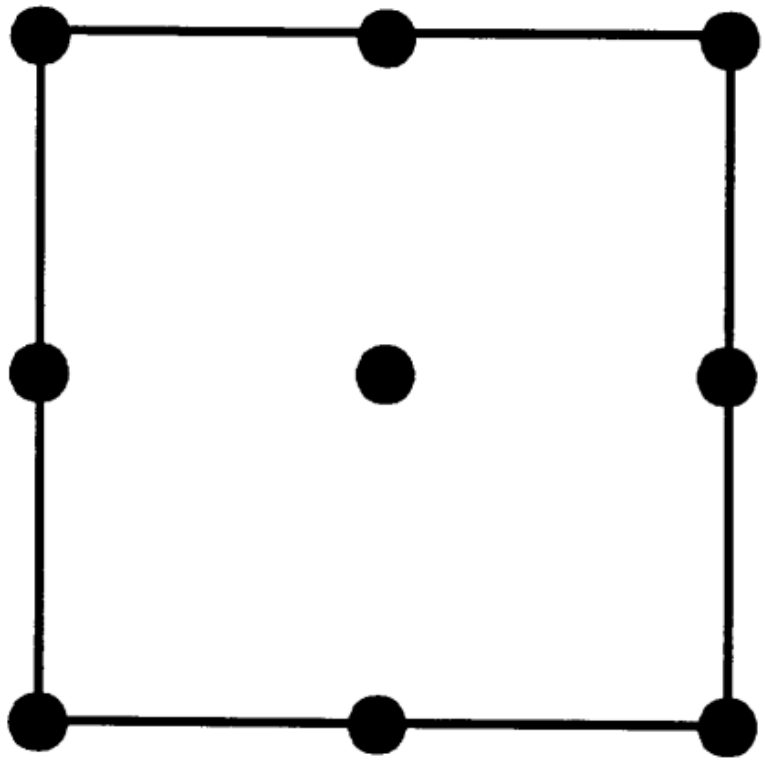
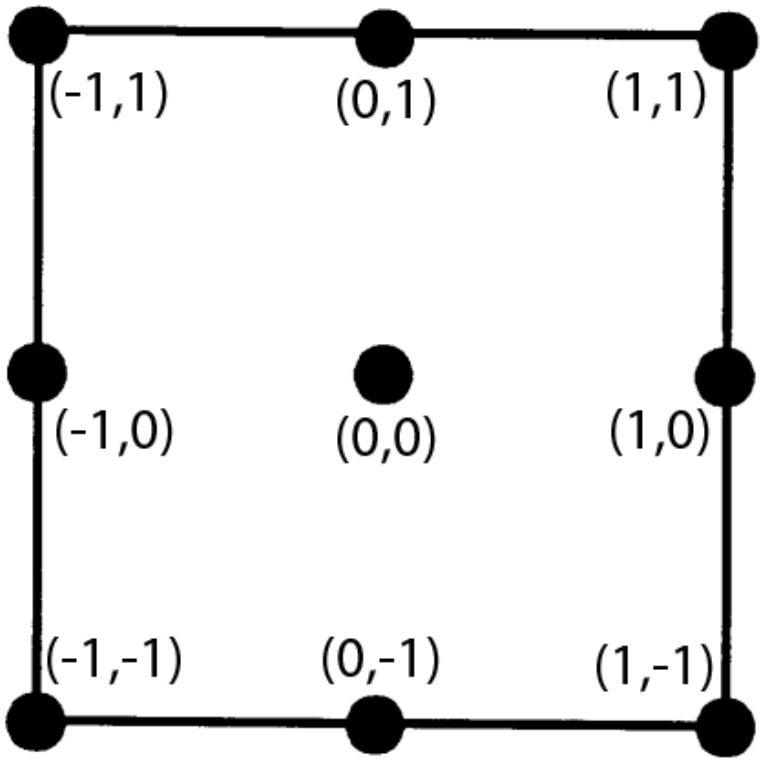


Rotatable Designs



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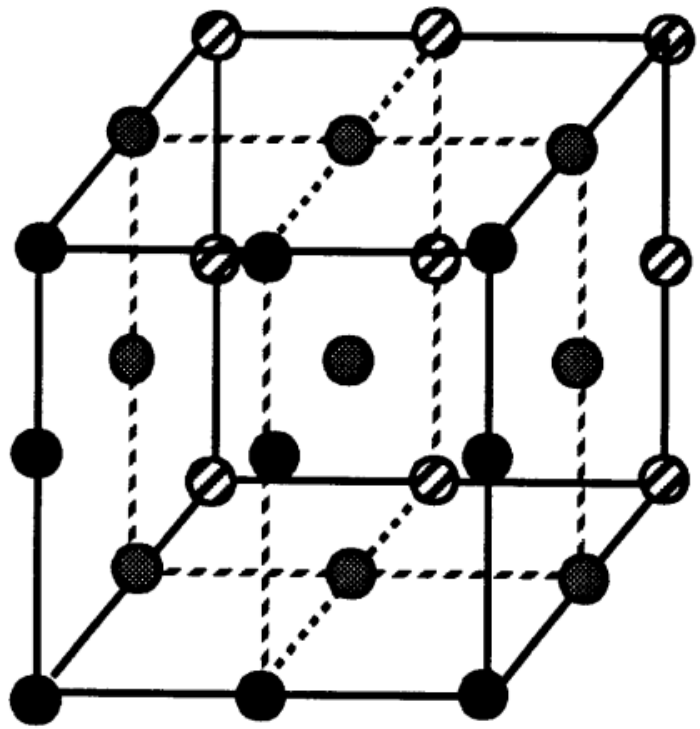

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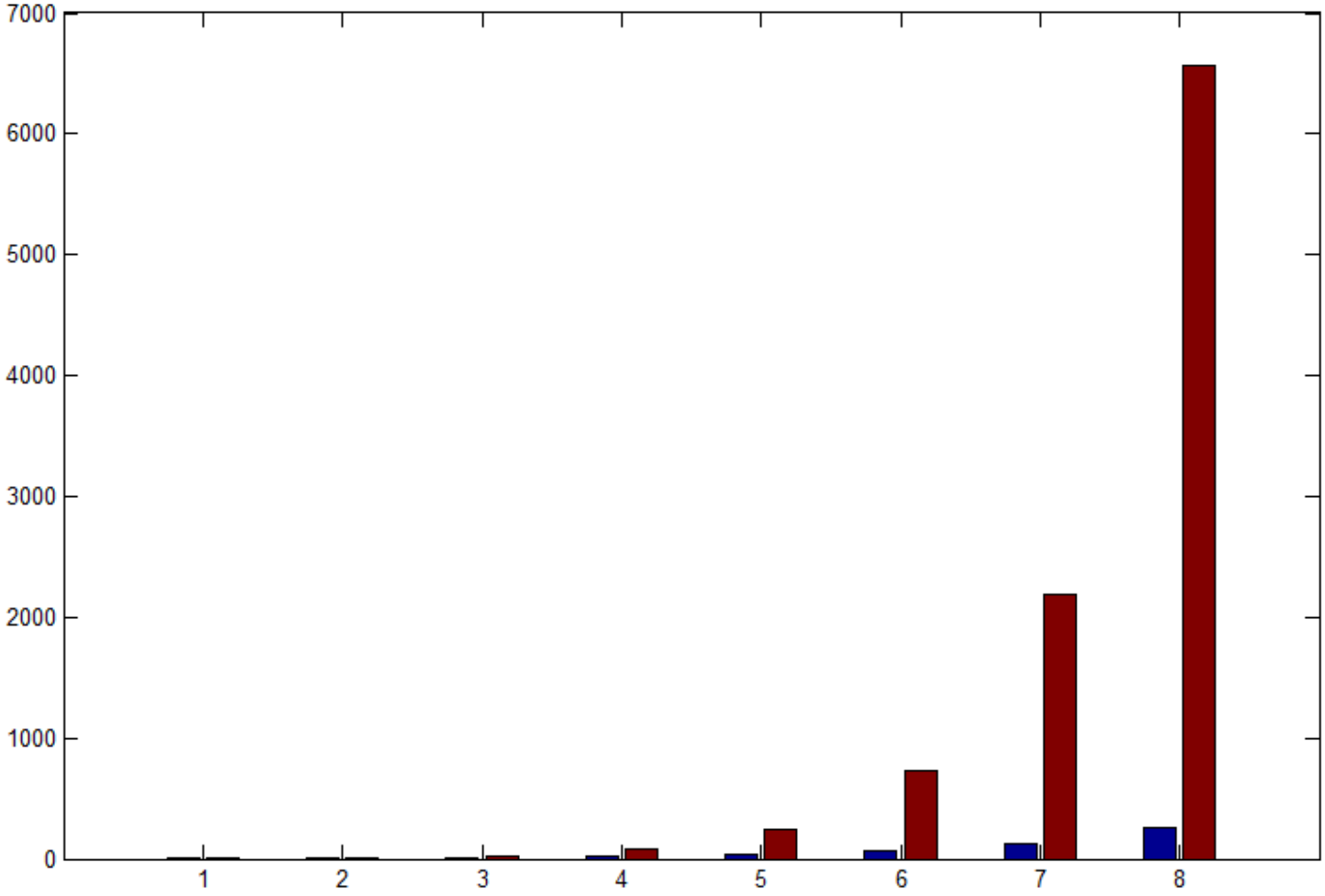

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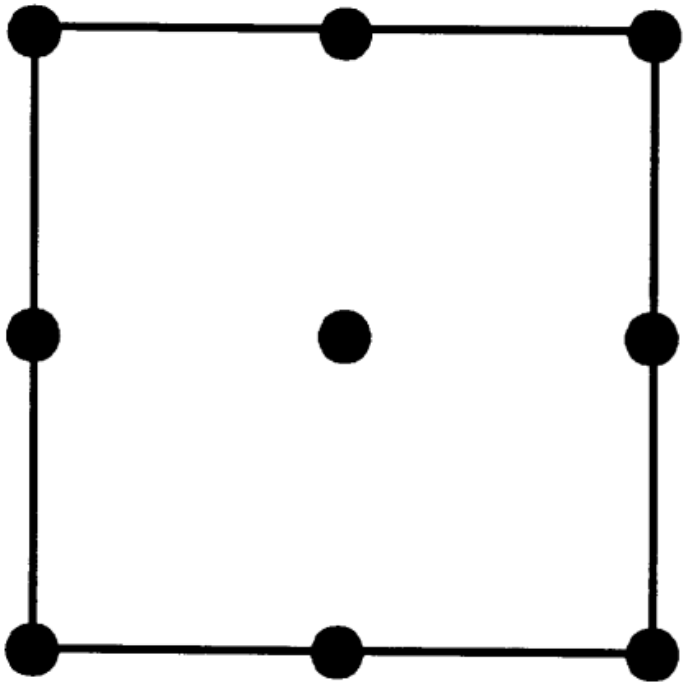

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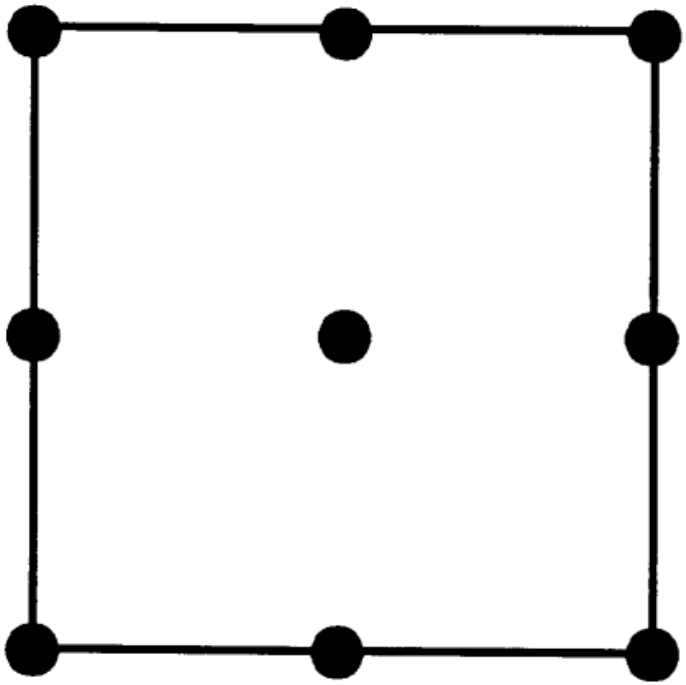

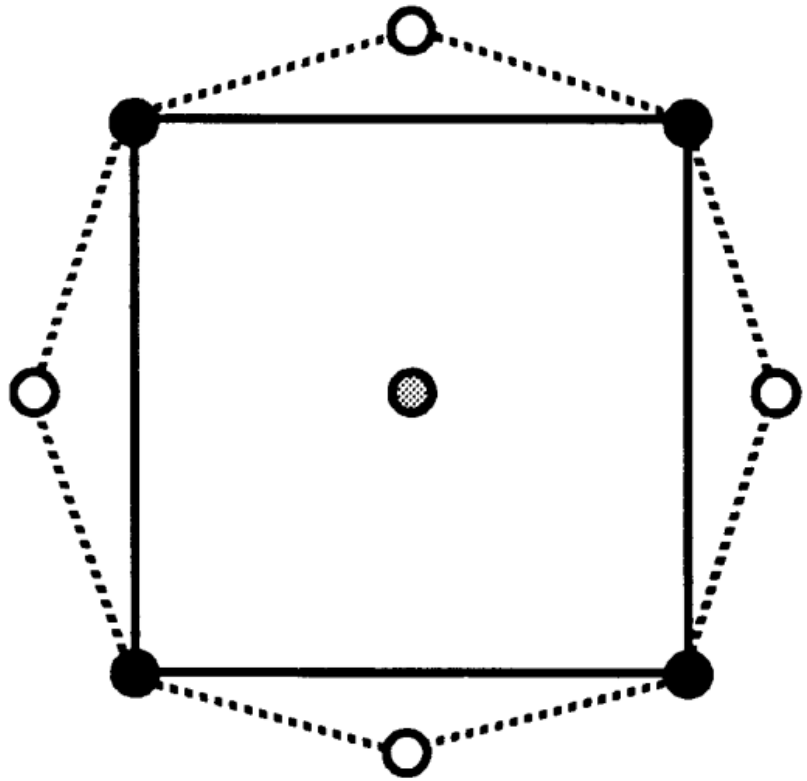

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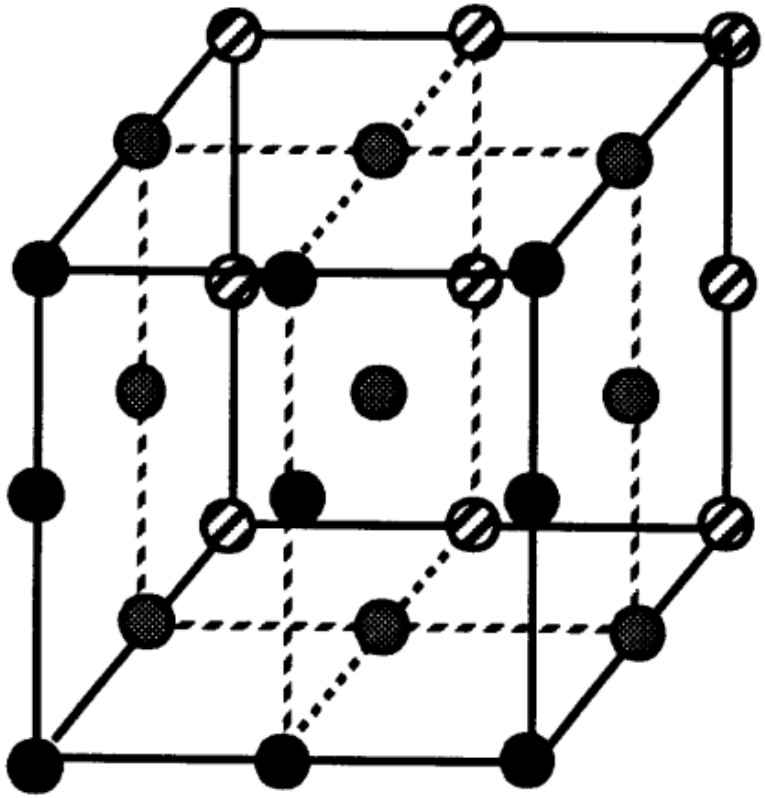

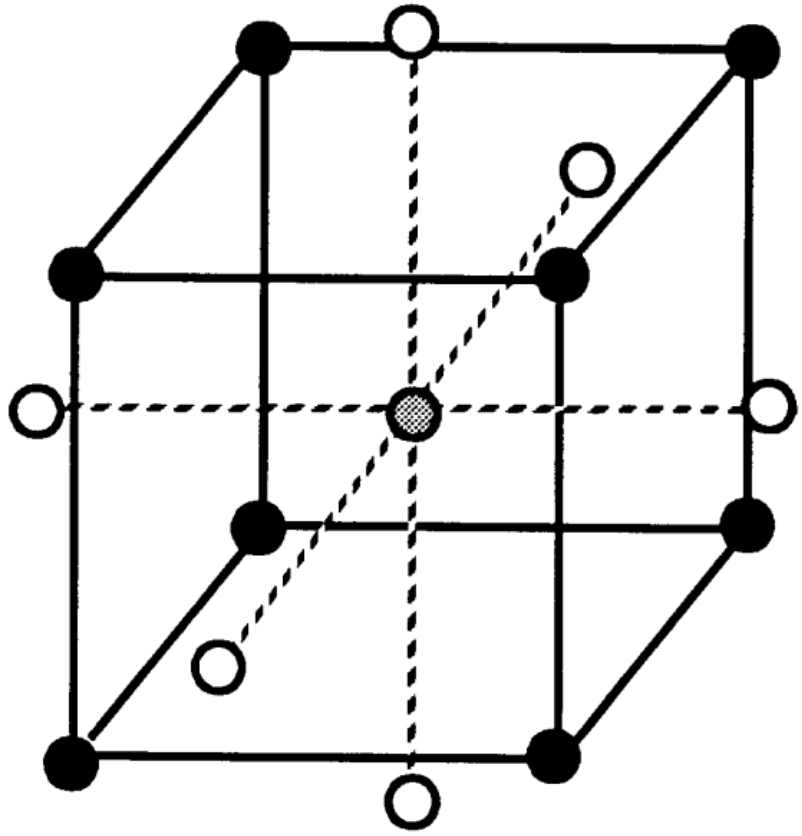

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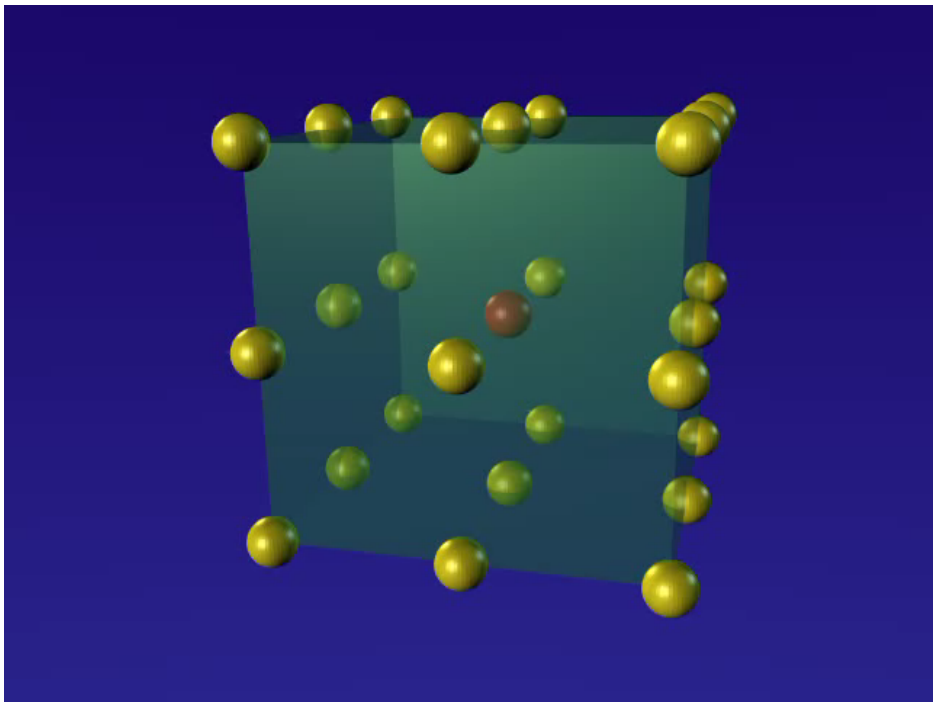
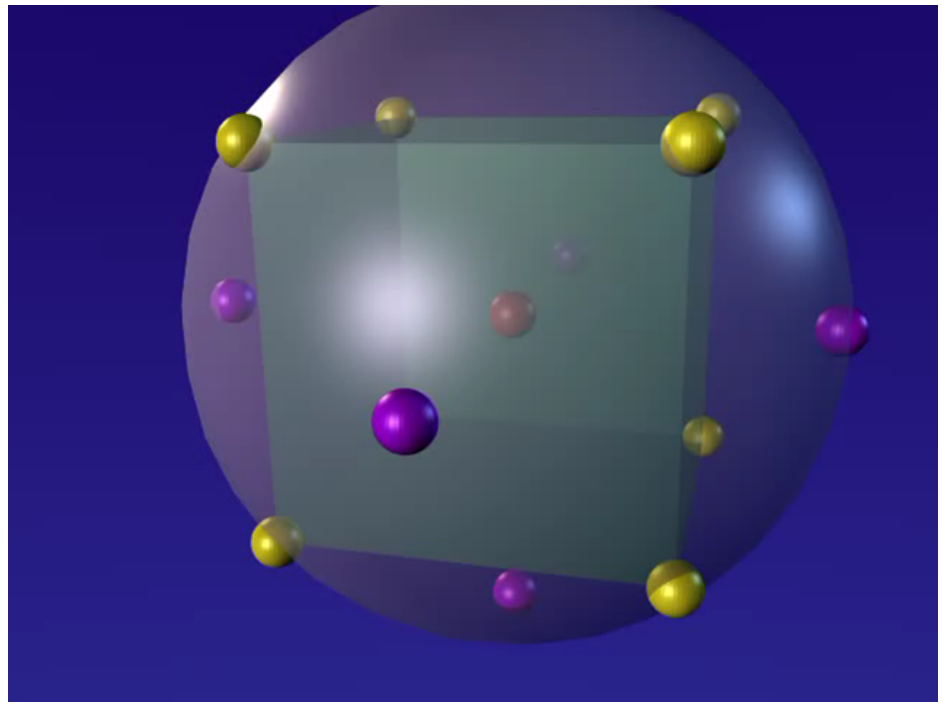
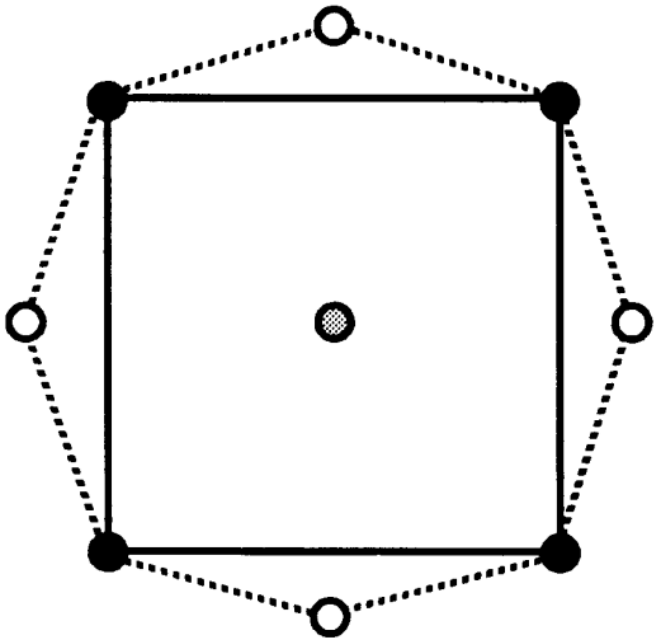
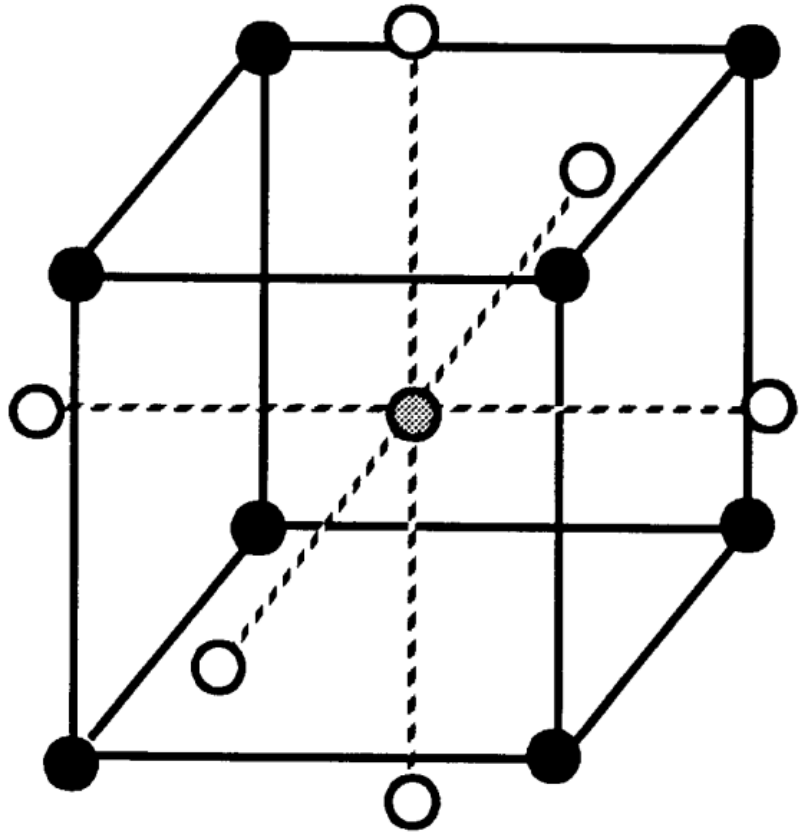




Central Composite Circumscribed Design





Central Composite Circumscribed Design

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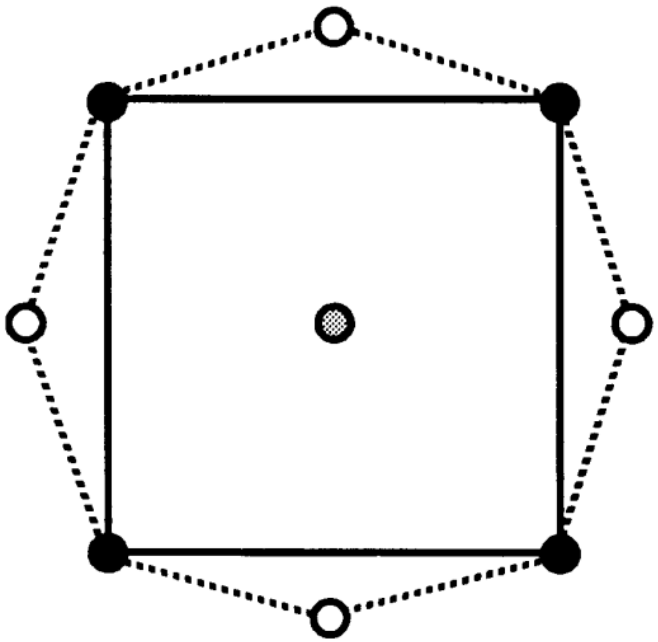
- Need access to 5 levels

Central Composite Circumscribed Design

- Need access to 5 levels
- Need access above and below previously defined Max/Min points

Central Composite Circumscribed Design



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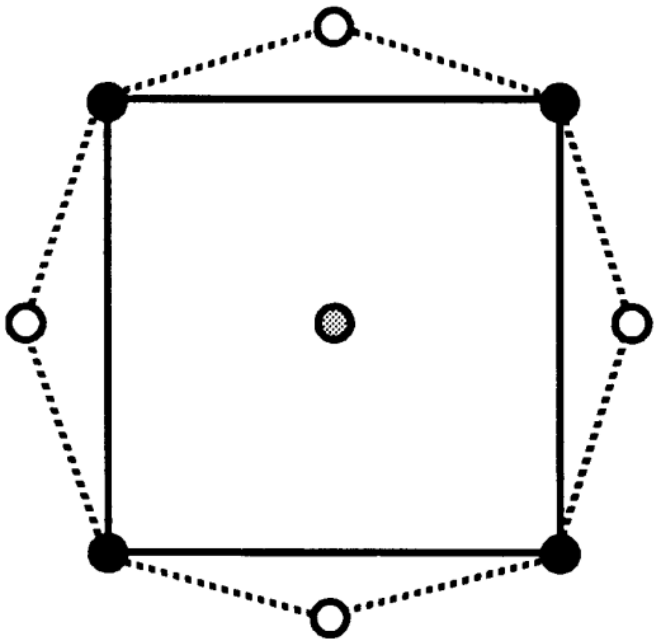
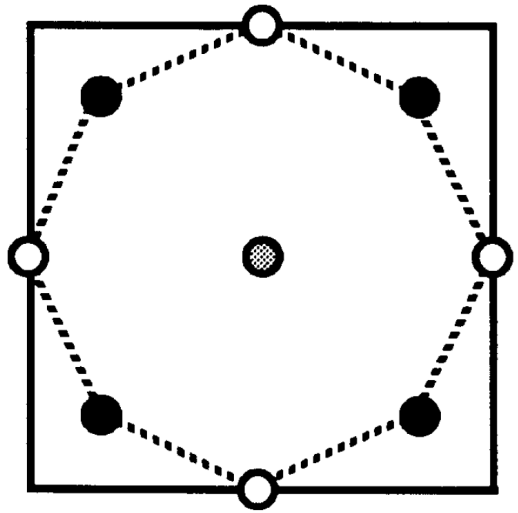
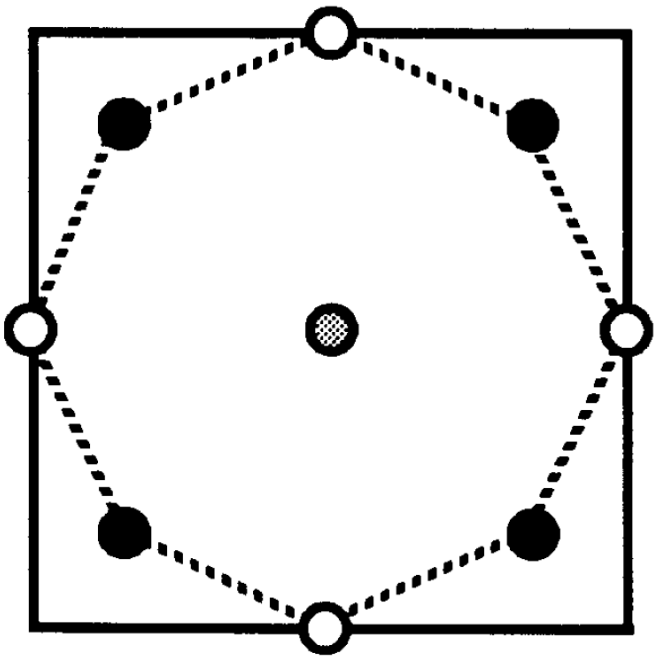




Central Composite Inscribed Design



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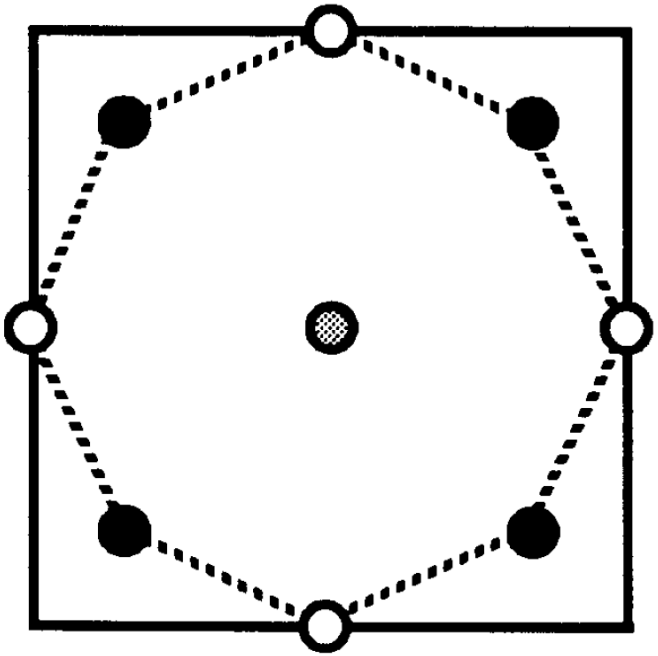

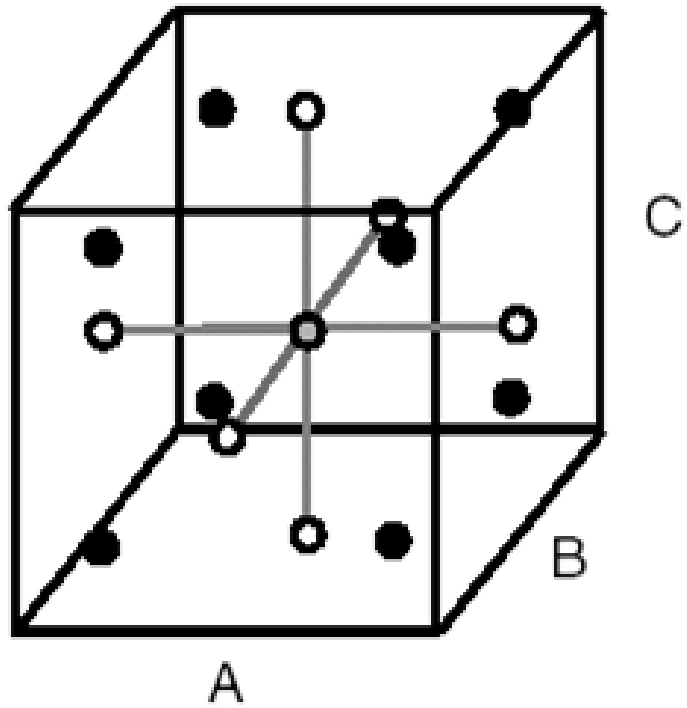

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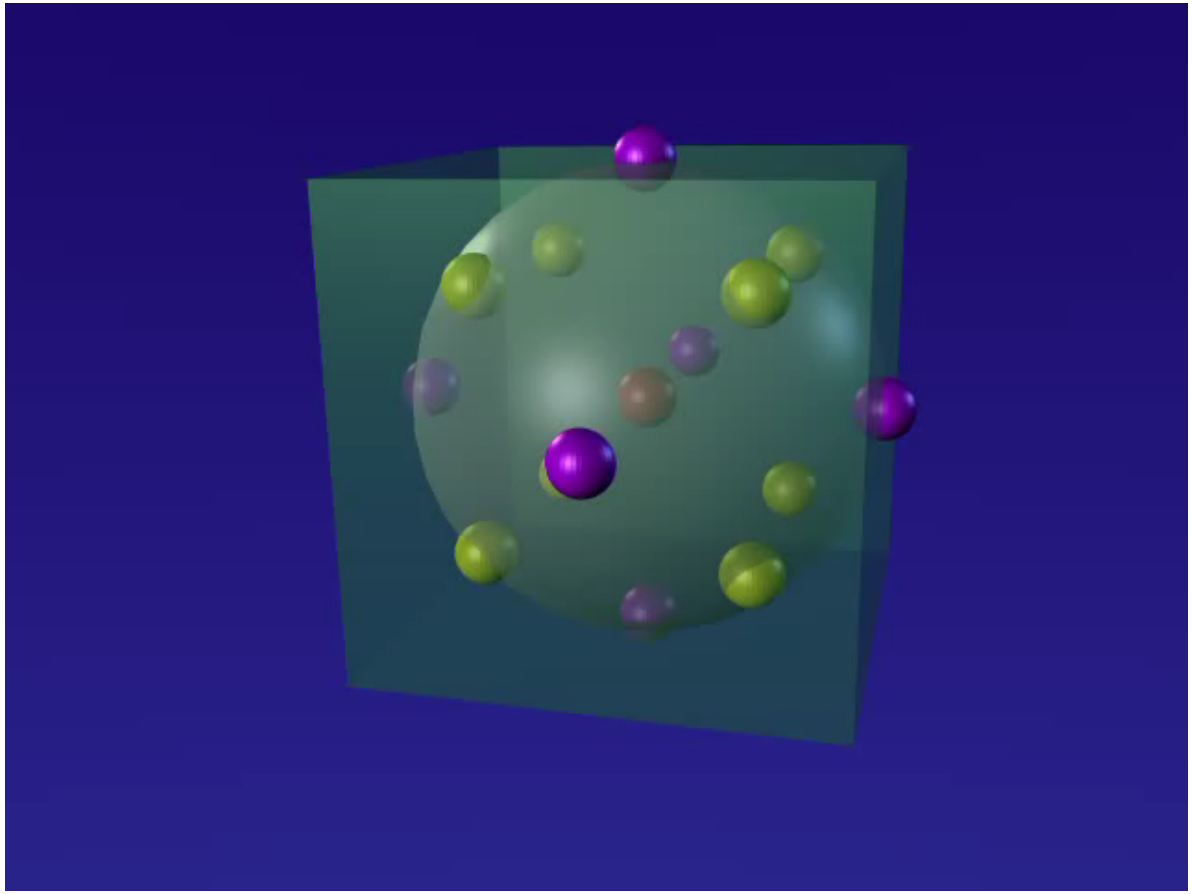

Central Composite Inscribed Design

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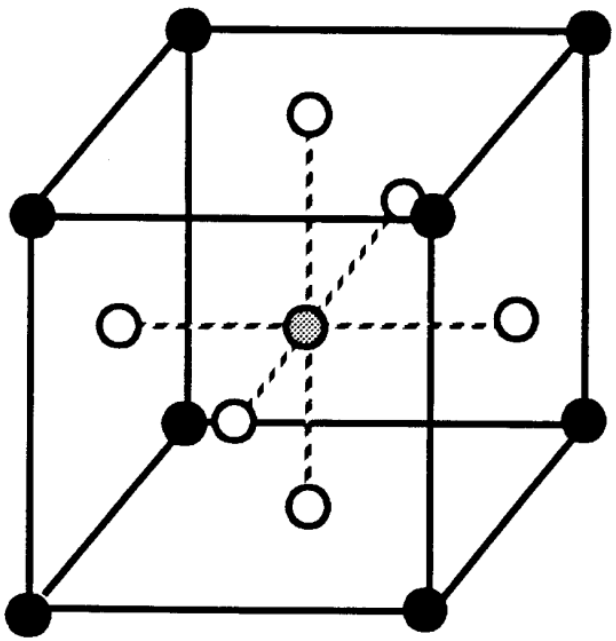
- Still need access to 5 levels

Central Composite Inscribed Design

- Still need access to 5 levels
- Fails to test extremities

Face Centred Cube

Face Centred Cube



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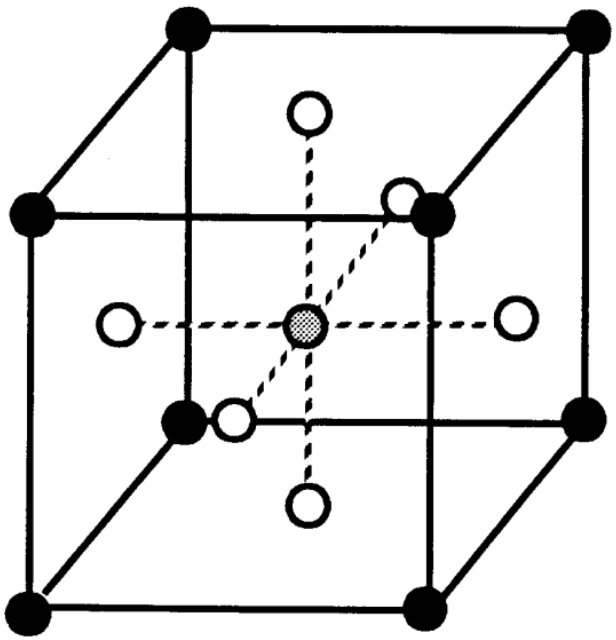

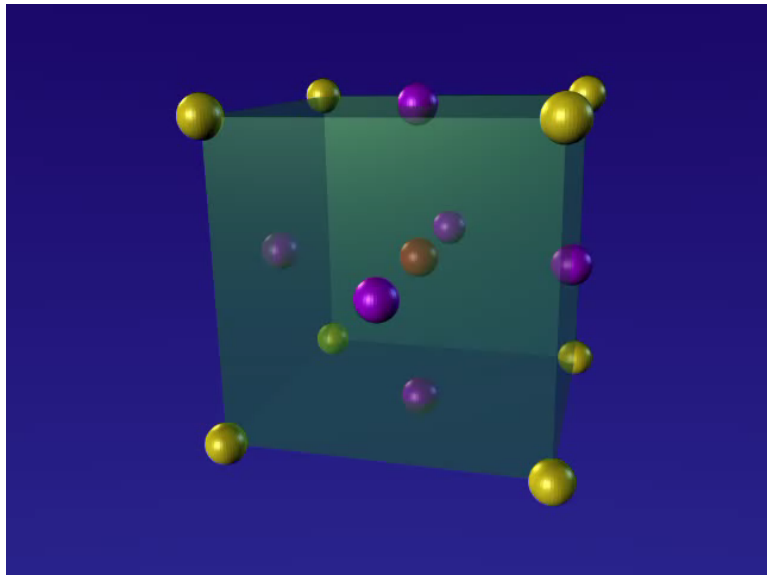

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Kind of like CCC/CCI Designs...

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Only requires three levels.

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One Major disadvantage:

Face Centred Cube

Kind of like CCC/CCI Designs...

Only requires three levels.

One Major disadvantage:
Not orthogonal!

Face Centred Cube

Kind of like CCC/CCI Designs...

Only requires three levels.

One Major disadvantage:
Not orthogonal!

Wait, what? Orthogonal?

Orthogonal Designs

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- Blockable

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- Blockable
- Can be run in batches

Orthogonal Designs

- Blockable
- Can be run in batches
- Divide experiments into groups

Orthogonal Designs

- Blockable
- Can be run in batches
- Divide experiments into groups

...even more options?

Box Behnken Design

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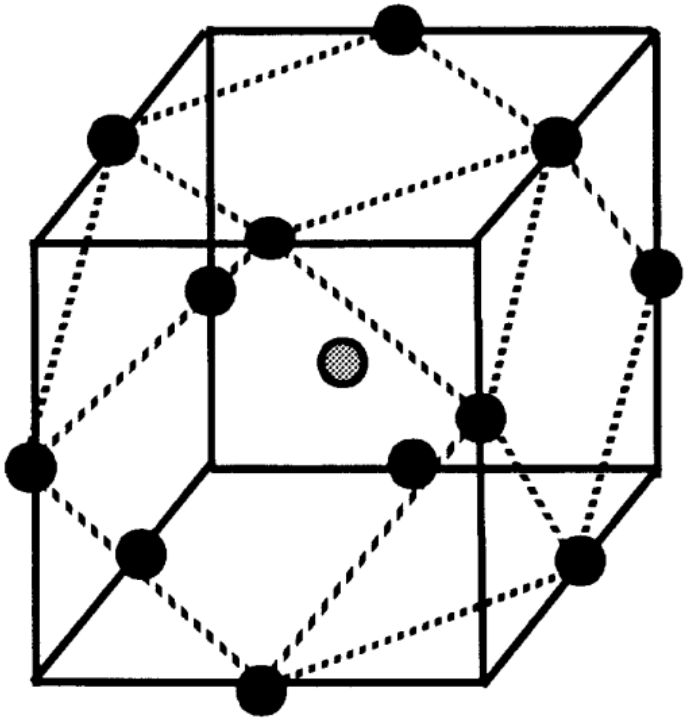

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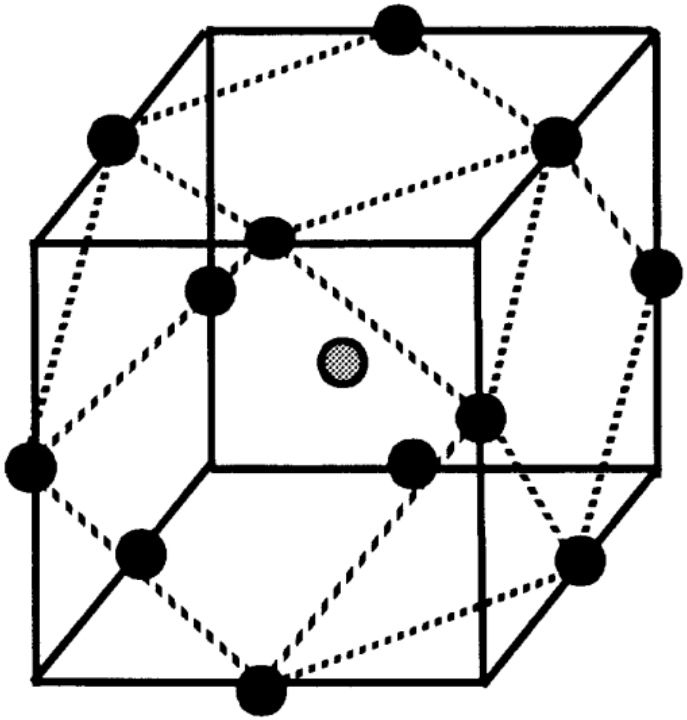

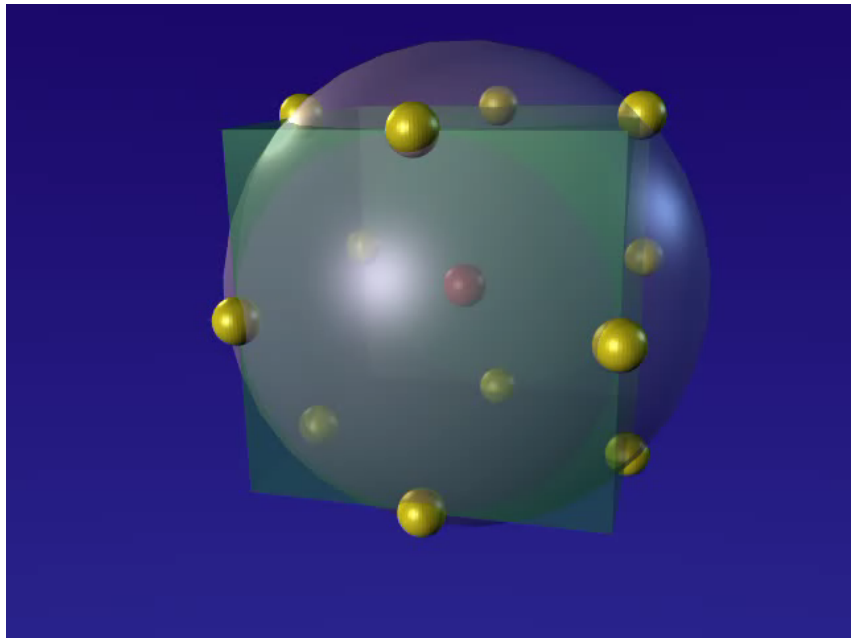

D-Optimal Designs

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The computer sorts out everything!

D-Optimal Designs

The computer sorts out everything!

Tell it how many runs you can afford, what regions you are interested in, and what model you prefer, and it will automatically design and optimal set of experimental runs for you.

Choices... Choices...

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How to choose?

Choices... Choices...

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- How close is the corner of the optimum operating region to the corner of the design region?

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- Do I care about orthogonality or rotatability?

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- How close is the corner of the optimum operating region to the corner of the design region?
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- Is it possible to experiment outside of the design-factor range?

Choices... Choices...

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Choices... Choices...

How to choose?

- How close is the corner of the optimum operating region to the corner of the design region?
- Do I care about orthogonality or rotatability?
- Is it possible to experiment outside of the design-factor range?
- How many runs can I afford?
- What is my personal preference?

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- The ultimate goal is to create an RSM model.

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- In the process of doing this we choose an experimental design to create a good RSM model.

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- Different designs are all different, but valid, ways of gathering data for this model.

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- The ultimate goal is to create an RSM model.
- In the process of doing this we choose an experimental design to create a good RSM model.
- Different designs are all different, but valid, ways of gathering data for this model.
- Once a good model exists, no more experiments are necessary. The magic of statistics lets you predict the likely outcome of any combination of factors without running an experiment.