- A probability distribution describes the probabilities of all possible outcomes of a random trial.
- Two events (A and B) are mutually exclusive if they cannot both occur (i.e., $\Pr[A \text{ and } B] = 0$). If A and B are mutually exclusive, then the probability of A or B occurring is the sum of the probability of A occurring and the probability of B occurring (i.e., $\Pr[A \text{ or } B] = \Pr[A] + \Pr[B]$). This is the addition rule.
- The general addition rule gives the probability of either of two events occurring when the events are not mutually exclusive:

  $$\Pr[A \text{ or } B] = \Pr[A] + \Pr[B] - \Pr[A \text{ and } B].$$

  The general addition rule reduces to the addition rule when A and B are mutually exclusive, because then $\Pr[A \text{ and } B] = 0$.
- Two events are independent if knowing one outcome gives no information about the other outcome. More formally, A and B are independent if $\Pr[A \text{ and } B] = \Pr[A]\Pr[B]$. This is the multiplication rule.
- Probability trees are useful devices for calculating the probabilities of complicated series of events.
- If events are not independent, then they are said to be dependent. The probability of two dependent events both occurring is given by the general multiplication rule: $\Pr[A \text{ and } B] = \Pr[A]\Pr[B \mid A]$.
- The conditional probability of an event is the probability of that event occurring given some condition.
- Probability trees and Bayes' theorem are important tools for calculations involving conditional probabilities.
- The law of total probability, $\Pr[A] = \sum_{\text{All values of } B} \Pr[B]\Pr[A \mid B]$, makes it possible to calculate the probability of an event (A) from all of the conditional probabilities of that event. The law multiplies, for all possible conditions (B), the probability of that condition ($\Pr[B]$) times the conditional probability of the event assuming that condition ($\Pr[A \mid B]$).

Online resources Learning resources associated with this chapter are online at https://whitlockschluter3e.zoology.ubc.ca/chapter05.html.

PRACTICE PROBLEMS

Answers to the Practice Problems are provided in the Answers Appendix at the back of the book.

1. **Calculation practice: Addition rule.** When women are asked to rate Brussels sprouts, 30% say sprouts are "very repulsive," 20% say that they are "somewhat repulsive," 43% are "indifferent," 6% say sprouts are "somewhat delicious," and 1% claim they are "especially delicious." Only one answer per woman was allowed. The data are from Trinkaus and Dennis (1991).
   a. Are these five possible answers mutually exclusive? Explain.

b. What is the probability that a woman would say that Brussels sprouts are either very repulsive or somewhat repulsive?
c. What is the probability that a woman would say that Brussels sprouts are anything other than especially delicious?

2. **Calculation practice: Law of total probability.** The survey in the previous problem was conducted on men as well: 34% say Brussels sprouts are "very repulsive," 19% say that they are "somewhat repulsive," 38% are "indifferent," 8% say they are "somewhat delicious," and 1% claim they are "especially delicious." Assume that in a given population, 52% of the adults are women. Use the following steps to build a probability tree and calculate the probability that a random adult says that Brussels sprouts are somewhat delicious or especially delicious using the law of total probability.
   a. What is the probability that a randomly chosen adult is a man? What is the probability that a randomly chosen adult is a woman? Draw the first part of the probability tree for these two events.
   b. What is the probability that a man says that Brussels sprouts are somewhat delicious or especially delicious?
   c. Write (b) as a shorthand probability statement. Hint: (b) can also be stated as: What is the probability that a randomly chosen adult says that Brussels sprouts are somewhat delicious or especially delicious, given that he is a man?
   d. What is the probability that a woman says that Brussels sprouts are somewhat delicious or especially delicious?
   e. Complete the probability tree for the events in parts (c) and (d).
   f. Apply the law of total probability to determine the probability that a random adult says that Brussels sprouts are somewhat delicious or especially delicious.

3. **Calculation practice: General addition rule.** Among women voluntarily tested for sexually transmitted diseases in one university, 24% tested positive for human papilloma virus (HPV) only, 2% tested positive for *Chlamydia* only, and 4% tested positive for both HPV and *Chlamydia* (Tábora et al. 2005). Use the following steps to calculate the probability that a woman from this population who gets tested would test positive for either HPV or *Chlamydia*.
   a. Write the goal of the question as a probability statement.
   b. Write the general addition rule with words specific to this example.
   c. Calculate the probability that a randomly sampled woman would test positive for HPV or *Chlamydia*.

4. **Calculation practice: General multiplication rule.** In the 1980s in Canada, 52% of adult men smoked. It was estimated that male smokers had a lifetime probability of 17.2% of developing lung cancer, whereas a nonsmoker had a 1.3% chance of getting lung cancer during his life (Villeneuve and Mao 1994).[8]
   a. What is the conditional probability of a Canadian man getting cancer, given that he smoked in the 1980s?
   b. Draw a probability tree to show the probability of getting lung cancer conditional on smoking.
   c. Using the tree, calculate the probability that a Canadian man in the 1980s both smoked and eventually contracted lung cancer.
   d. Using the general multiplication rule, calculate the probability that a Canadian man in the 1980s both smoked and eventually contracted lung cancer. Did you get the same answer as in (c)?
   e. Using the general multiplication rule, calculate the probability that a Canadian man in the 1980s both did not smoke and never contracted lung cancer.

5. **Calculation practice: Bayes' theorem.** Refer to Practice Problem 4. Use the following steps to calculate the probability that a Canadian man smoked, given that he had been diagnosed with lung cancer.
   a. Write Bayes' theorem for the specific case described in this question.
   b. Calculate the probability that a Canadian man in the late 1980s would eventually develop lung cancer. (Use the law of total probability.)

8. Actually, later studies showed that smokers were about 23 times more likely than nonsmokers to get lung cancer, but for the purpose of this problem, we'll use the numbers given in this study.

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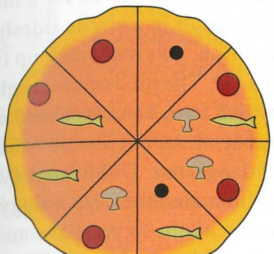
c. Use Bayes' theorem to calculate the probability that a man from this population smoked, given that he eventually developed lung cancer.

6. The pizza below, ordered from the Venn Pizzeria on Bayes Street, is divided into eight slices:



The slices might have pepperoni, mushrooms, olives, and/or anchovies. Imagine that, late at night, you grab a slice of pizza totally at random (i.e., there is a 1/8 chance that you grabbed any one of the eight slices). Base your answers to the following questions on the drawing of the pizza.

a. What is the chance that your slice had pepperoni on it?

b. What is the chance that your slice had both pepperoni and anchovies on it?

c. What is the probability that your slice had either pepperoni or anchovies on it?

d. Are pepperoni and anchovies mutually exclusive on the slices from this pizza?

e. Are olives and mushrooms mutually exclusive on the slices from this pizza?

f. Are getting mushrooms and getting anchovies independent when choosing slices from this pizza?

g. If I pick a slice from this pizza and tell you that it has olives on it, what is the chance that it also has anchovies?

h. If I pick a slice from this pizza and tell you that it has anchovies on it, what is the chance that it also has olives?

i. Seven of your friends each choose a slice at random and eat them without telling you what toppings they had. What is the chance that the last slice left has olives on it?

j. You choose two slices at random from this pizza. What's the chance that they both have olives on them? (Be careful—after removing the first slice, the probability of choosing one of the remaining slices changes.)

k. What's the probability that a randomly chosen slice does *not* have pepperoni on it?

l. Draw a pizza for which mushrooms, olives, anchovies, and pepperoni are all mutually exclusive.

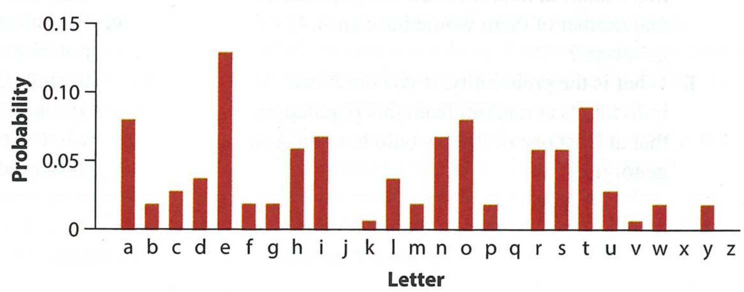
7. In the first hour of a hunting trip, the probability that a pride of Serengeti lions will encounter a Cape buffalo is 0.035. If it encounters a buffalo, the probability that the pride successfully captures it is 0.40 (numbers are from Scheel 1993). What is the probability that the next one-hour hunt for Cape buffalo by a pride of lions will end in a successful capture?

8. Cavities in trees are important nesting sites for a wide variety of wildlife, including the white-breasted nuthatch shown on the first page of this chapter. Cavities in trees are much more common in old-growth forests than in recently logged forests. A recent survey in Missouri found that 45 out of 273 trees in an old-growth area had cavities, while the rest did not (Fan et al. 2005). What is the probability that a randomly chosen tree in this area has a cavity?

9. The accompanying bar graph gives the relative frequency of letters in texts from the English language. Such charts are useful for deciphering simple codes.

a. If a letter were chosen at random from a book written in normal English, estimate by eye (and a bit of calculation) the probability that it is a vowel (i.e., A, E, I, O, or U).

FIGURE FOR PROBLEM 9



b. Estimate by eye the probability that five letters chosen independently and at random from an English text would spell out (in order) "S-T-A-T-S."

c. Estimate by eye the probability that two letters chosen at random from an English text are both E's.

10. The gene *Prdm9* is thought to regulate hotspots of recombination (crossing over) in mammals, including humans. In the people of Han Chinese descent living in the Los Angeles area there are five alleles at the *Prdm9* gene, labeled A_1, A_2, A_3, A_4, and A_5. The relative frequencies with which these alleles occur in that population are 0.06, 0.03, 0.84, 0.03, and 0.04, respectively (Parvanov et al. 2010). Assume that in this population, the two alleles present in any individual are independently sampled from the population as a whole (this can happen if people in the community marry and produce children randomly with respect to *Prdm9* genotype).

a. What is the probability that a single allele chosen at random from this population is either A_1 or A_4?

b. What is the probability that an individual has two A_1 alleles (i.e., what is the probability that its first allele is A_1 and its second allele is also A_1)?

c. What is the probability that an individual has one A_1 allele and one A_3 allele? (Note that this can happen if the first allele drawn is A_1 and the second is A_3, or if the first allele is A_3 and the second is A_1. A probability tree will help to keep track of all the possibilities.)

d. What is the probability that an individual is not A_1A_1 (i.e., does not have two A_1 alleles)?

e. What is the probability, if you drew two individuals at random from this population, that neither of them would have an A_1A_1 genotype?

f. What is the probability, if you drew two individuals at random from this population, that at least one of them would have an A_1A_1 genotype?

g. What is the probability that three randomly chosen individuals would have no A_2 or A_3 alleles? (Remember that each individual has two alleles.)

11. After graduating from your university with a biology degree, you are interviewed for a lucrative job as a snake handler in a circus sideshow. As part of your audition, you must pick up two rattlesnakes from a pit. The pit contains eight snakes, three of which have been defanged and are assumed to be harmless, but the other five are definitely still dangerous. Unfortunately, budget cuts have eliminated the herpetology course from the curriculum, so you have no way of telling in advance which snakes are dangerous and which are not. You pick up one snake with your left hand and another snake with your right.

© Mark Laita, courtesy of Fahey Klein Gallery

a. What is the probability that you picked up *no* dangerous snakes?

b. Assume that any dangerous snake you pick up has a probability of biting you. This probability is the same for each snake: 0.8. The defanged snakes do not bite. What is the chance that, in picking up your two snakes, you are bitten at least once?

c. Still assume that the defanged snakes do not bite and the dangerous snakes have a probability of 0.8 of biting. If you picked up only one snake and it did not bite you, what is the probability that this snake is defanged?

12. Five different researchers independently take a random sample from the same population and calculate a 95% confidence interval for the same parameter.
- **a.** What is the probability that all five researchers have calculated an interval that includes the true value of the parameter?
- **b.** What is the probability that at least one does *not* include the true parameter value?

13. Schrödinger's cat lives under constant threat of death from the random release of a deadly poison. The probability of release of the poison is 0.01 per day, and the release is independent on successive days.
- **a.** What is the probability that the cat will survive 1 day?
- **b.** What is the probability that the cat will survive 7 days?
- **c.** What is the probability that the cat will survive a year (365 days)?
- **d.** What is the probability that the cat will die by the end of a year?

14. Rapid HIV tests allow for quick diagnosis without expensive laboratory equipment. However, their efficacy has been called into question. In a population of 1517 tested individuals in Uganda, 4 had HIV but tested negative (false negatives), 166 had HIV and tested positive, 129 did not have HIV but tested positive (false positives), and 1218 did not have HIV and tested negative (Gray et al. 2007). Assume that these proportions represent the probabilities of the corresponding outcomes.
- **a.** What was the probability of a false positive (also called the false-positive rate)?
- **b.** What was the false-negative rate?
- **c.** If a randomly sampled individual from this population tests positive on a rapid test, what is the probability that he or she has HIV?

15. Medical diagnostic tests have "sensitivity" and "specificity." The sensitivity of a test for a given medical condition is the proportion of all afflicted individuals (i.e., those who have the condition) for whom the test gives a positive result (i.e., the test indicates they indeed have the condition). The specificity is the probability that a person not afflicted with the disease gets a negative result from the test. In other words, both the sensitivity and the specificity give the probabilities that the test gives the correct result to patients who have and do not have the condition, respectively.
- **a.** Refer to the rapid HIV test in Practice Problem 14. What is the sensitivity of this HIV test?
- **b.** What is the specificity of this HIV test?

16. Kalani et al. (2008) discovered cells responsive to *Wnt* proteins in the subventricular zone of developing brains of mouse embryos. These cells included a high fraction of self-renewing stem cells, which suggested that *Wnt* signaling occurs during brain cell self-renewal. In a particular cell preparation in vitro, 9% of subventricular brain cells were *Wnt*-responsive. If six cells are sampled randomly from the cell preparations, what is the probability of sampling *Wnt*-responsive (W) and nonresponsive (L) cells in the following orders, from a large population of cells?
- **a.** WWLWWW
- **b.** WWWWWL
- **c.** LWWWWW
- **d.** WLWLWL
- **e.** WWWLLL
- **f.** WWWWWW
- **g.** What is the probability of at least one nonresponsive brain cell when six cells are randomly sampled?

17. Studies have shown that the probability that a man washes his hands after using the restroom at an airport is 0.74, and the probability that a woman washes hers is 0.83 (American Society for Microbiology 2005). A waiting room in an airport contains 40 men and 60 women. Assume that individual men and women are equally likely to use the restroom. What is the probability that the next individual who goes to the restroom will wash his or her hands?

18. If you have ever tried to take a family photo, you know that it is very difficult to get a picture in which no one is blinking. It turns out that the probability of an individual blinking during a photo is about 0.04 (Svenson 2006).
- **a.** If you take a picture of one person, what is the probability that she will *not* be blinking?
- **b.** If you take a picture of 10 people, what is the probability that at least one person is blinking during the photo?