NRES 776 Lecture 19

GLM - multiple variables and Simpson’s paradox

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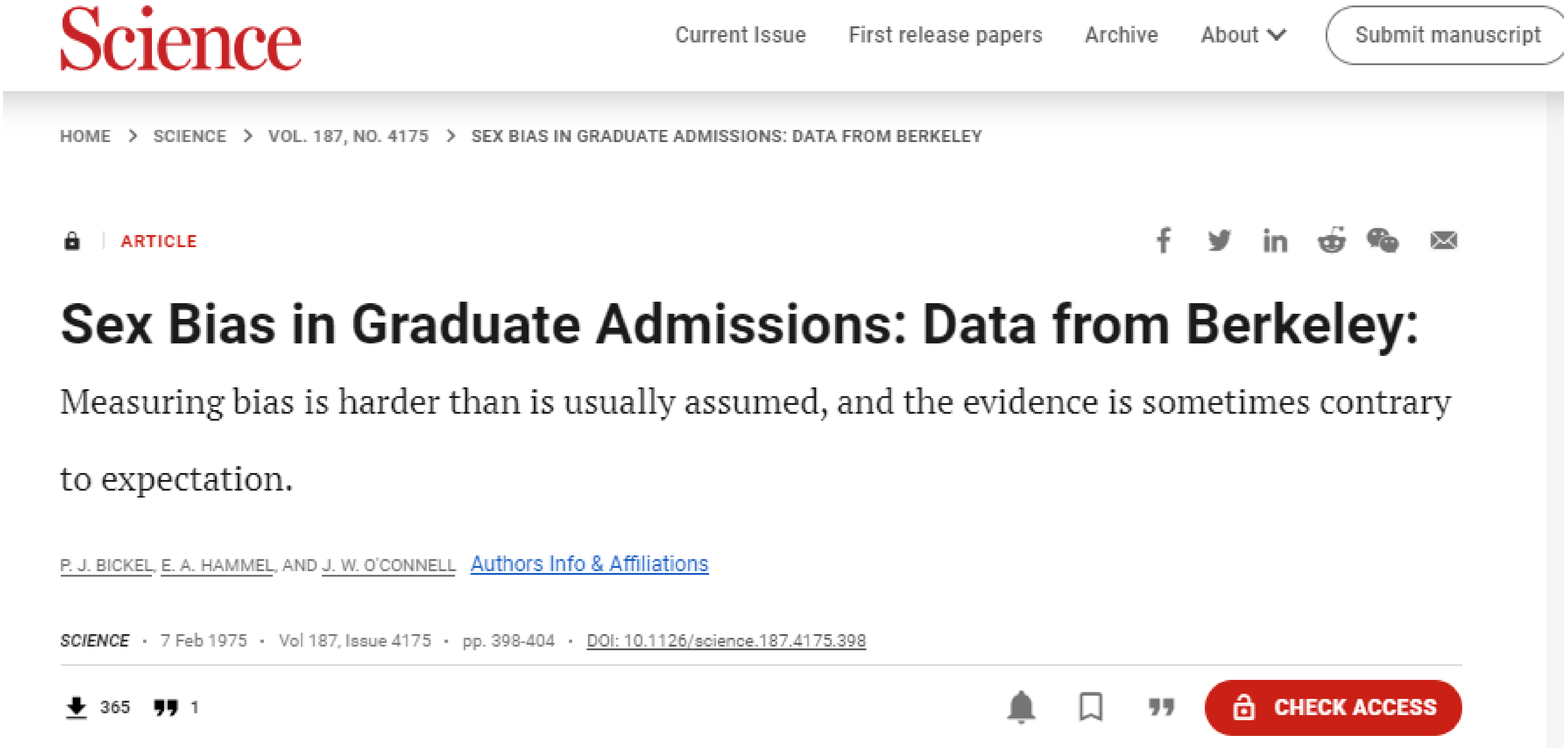
Our schedule today

- Announcement (3 min)
  - zoom recording
  - topic for next week's lab
- Binomial regression with multiple variables (30 min)
- Wrap up (5 min)

UCB Admission data set

Aggregate data on applicants to graduate school at Berkeley for the six largest departments in 1973 classified by admission and sex.



Science

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ARTICLE

Sex Bias in Graduate Admissions: Data from Berkeley:

Measuring bias is harder than is usually assumed, and the evidence is sometimes contrary to expectation.

P. J. BICKEL, E. A. HAMMEL, AND J. W. O'CONNELL Authors Info & Affiliations

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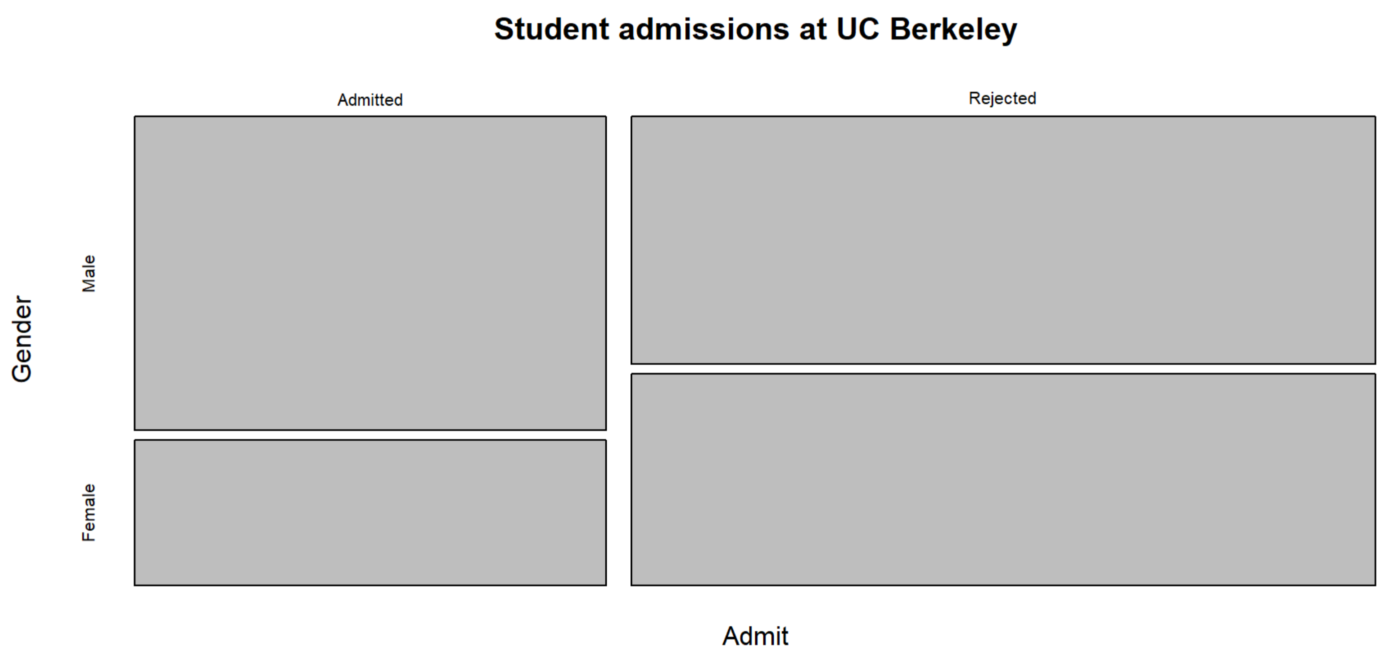
Exploratory data visualization

```r
apply(UCBAdmissions,
      c(1, 2),
      sum)
```

```
           Gender
Admit       Male Female
  Admitted 1198    557
  Rejected 1493   1278
```

```r
mosaicplot(apply(UCBAdmissions, c(1, 2), sum),
           main = "Student admissions at UC Berkeley")
```



Model formulation (glm_1)

$$logit(admission_i) = \beta_0 + \beta_1 genderM_i$$

```r
glm_1 <- glm(formula = cbind(Admitted, Rejected) ~ Gender,
             data = UCBAdmissions_clean,
             family = "binomial")

glm_1 %>% summary
```

```

Call:
glm(formula = cbind(Admitted, Rejected) ~ Gender, family = "binomial",
    data = UCBAdmissions_clean)

Deviance Residuals:
     Min        1Q    Median        3Q       Max
-16.7915   -4.7613   -0.4365    5.1025   11.2022

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.83049    0.05077 -16.357   <2e-16 ***
GenderMale   0.61035    0.06389   9.553   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 877.06  on 11  degrees of freedom
Residual deviance: 783.61  on 10  degrees of freedom
AIC: 856.55
```

```
Number of Fisher Scoring iterations: 4
```

Model goodness of fit

```r
glm_null <- glm(formula = cbind(Admitted, Rejected) ~ 1,
                data = UCBAdmissions_clean,
                family = "binomial")

lrtest(glm_1, glm_null)
```

```
Likelihood ratio test

Model 1: cbind(Admitted, Rejected) ~ Gender
Model 2: cbind(Admitted, Rejected) ~ 1
  #Df  LogLik Df  Chisq Pr(>Chisq)
1   2 -426.27
2   1 -473.00 -1 93.449  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Coef. interpretation

$$logit(p_i) = \beta_0 + \beta_1 genderM_i$$

$$\beta_1 = -0.83$$

$$\beta_1 = 0.61$$

For female applicants:

$$logit(p_F) = log(\frac{p_F}{1-p_F}) = \beta_0$$

$$\frac{p_F}{1-p_F} = exp(\beta_0) = 0.43$$

Coef. interpretation (con'd)

For male applicants:

$$logit(p_M) = log(\frac{p_M}{1 - p_M}) = \beta_0 + \beta_1$$

$$\frac{p_M}{1 - p_M} = exp(\beta_0 + \beta_1)$$

$$\frac{Odd(p_M)}{Odd(p_F)} = exp(\beta_1) = 1.84$$

- Male students has 1.84 times higher odds in getting admitted in the university
- Evidence of sex bias in admission practices!

Model formulation (glm_2)

$$logit(admission_i) = \beta_0 + \beta_1 genderM_i + \beta_2 deptB_i + \beta_3 deptC_i + \beta_4 deptD_i + beta_5 deptE_i + beta_6 de$$

```r
glm_2 <- glm(formula = cbind(Admitted, Rejected) ~ Gender + Dept,
                data = UCBAdmissions_clean,
                family = "binomial")
glm_2 %>% summary()
```

```

Call:
glm(formula = cbind(Admitted, Rejected) ~ Gender + Dept, family = "binomial",
    data = UCBAdmissions_clean)

Deviance Residuals:
      1        2        3        4        5        6        7        8
-1.2487   3.7189  -0.0560   0.2706   1.2533  -0.9243   0.0826  -0.0858
      9       10       11       12
 1.2205  -0.8509  -0.2076   0.2052

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept)  0.68192    0.09911   6.880 5.97e-12 ***
GenderMale  -0.09987    0.08085  -1.235    0.217
DeptB       -0.04340    0.10984  -0.395    0.693
DeptC       -1.26260    0.10663 -11.841  < 2e-16 ***
DeptD       -1.29461    0.10582 -12.234  < 2e-16 ***
DeptE       -1.73931    0.12611 -13.792  < 2e-16 ***
DeptF       -3.30648    0.16998 -19.452  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 877.056  on 11  degrees of freedom
Residual deviance:  20.204  on  5  degrees of freedom
AIC: 103.14
```

Model goodness of fit

```r
glm_null <- glm(formula = cbind(Admitted, Rejected) ~ 1,
                data = UCBAdmissions_clean,
                family = "binomial")

lrtest(glm_1, glm_null)
```

```
Likelihood ratio test

Model 1: cbind(Admitted, Rejected) ~ Gender
Model 2: cbind(Admitted, Rejected) ~ 1
  #Df  LogLik Df  Chisq Pr(>Chisq)
1   2 -426.27
2   1 -473.00 -1 93.449  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Coef. interpretation

$$logit(admission_i) = \beta_0 + \beta_1 genderM_i + \beta_2 deptB_i + \beta_3 deptC_i + \beta_4 deptD_i + beta_5 deptE_i + beta_6 de$$

$$\beta_0 = 0.6$$

$$\beta_1 = -0.09$$

$$\beta_2 = -0.04$$

For female student in dept B:

$$logit(p_{FB}) = log(\frac{p_{FB}}{1 - p_{FB}}) = \beta_0 + \beta_2$$

$$\frac{p_{FB}}{1 - p_{FB}} = exp(\beta_0 + \beta_2) = 1.89$$

Coef. interpretation (con'd)

For male student in dept B:

$$logit(p_{MB}) = log(\frac{p_{MB}}{1 - p_{MB}}) = \beta_0 + \beta_1 + \beta_2$$

$$\frac{p_{MB}}{1 - p_{MB}} = exp(\beta_0 + \beta_1 + \beta_2)$$

$$\frac{Odd(p_{MB})}{Odd(p_{FB})} = exp(\beta_1) = 0.9$$

- Male students has 0.9 times less odds in getting admitted in the department B
- In general, male students has 0.9 times less odds in getting admitted in the university
- Evidence of sex bias in admission practices! <- no!

Comparison between models

```
glm_3 <- glm(formula = cbind(Admitted, Rejected) ~ Dept,
             data = UCBAdmissions_clean,
             family = "binomial")
```

- `glm_null`: null model
- `glm_1`: include Gender
- `glm_2`: include Gender and Dept
- `glm_3`: include Dept

Compare AIC values

GLM model	AIC
`glm_null`	948
`glm_1`	856
`glm_2`	103
`glm_3`	102

Comparison between models

Use likelihood ratio test for nested models

```r
lrtest(glm_1, glm_2) # select glm_2
```

```
Likelihood ratio test

Model 1: cbind(Admitted, Rejected) ~ Gender
Model 2: cbind(Admitted, Rejected) ~ Gender + Dept
  #Df  LogLik Df Chisq Pr(>Chisq)
1   2 -426.27
2   7  -44.57  5 763.4  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

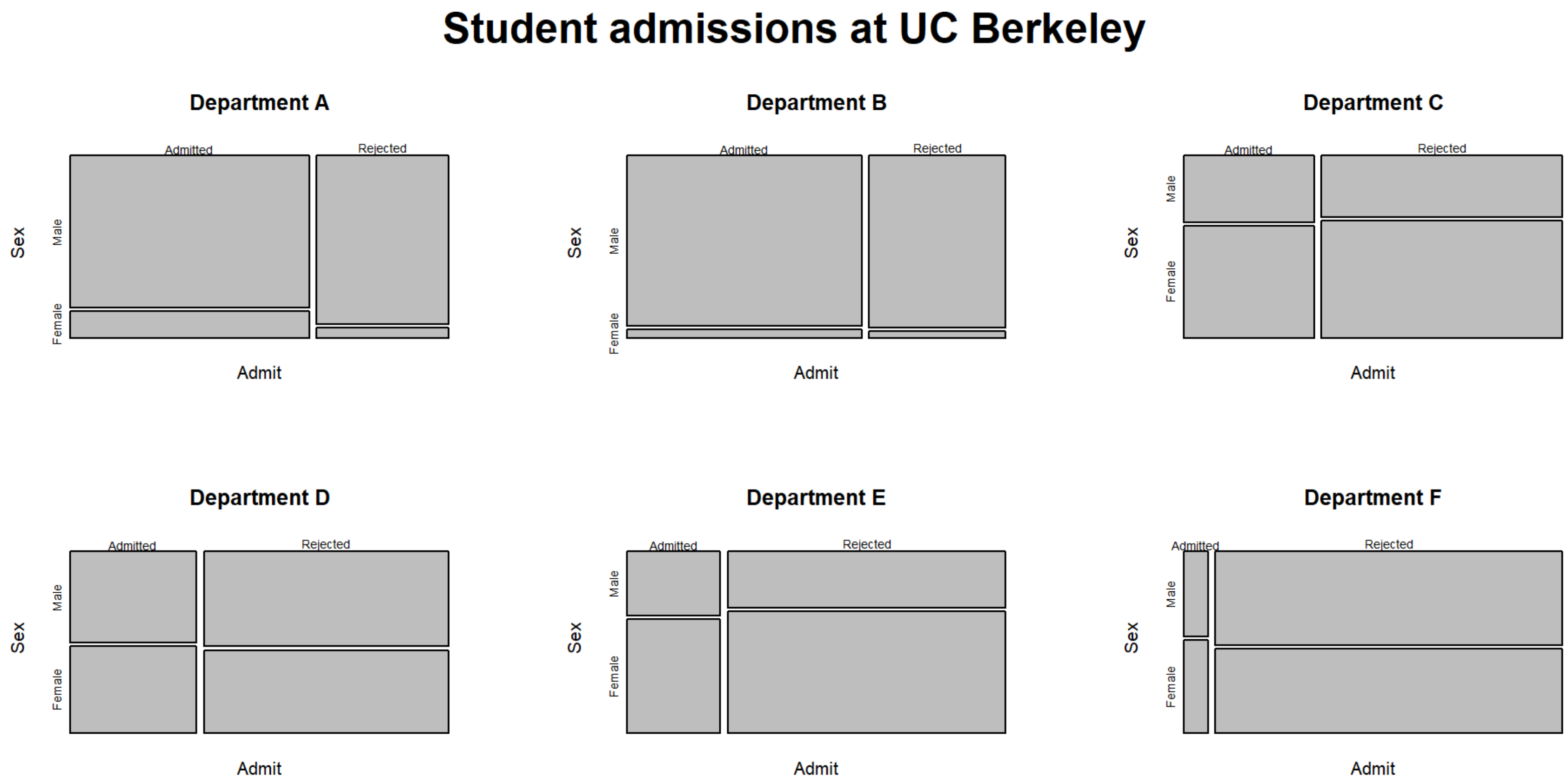
```r
lrtest(glm_2, glm_3) # select glm_3
```

```
Likelihood ratio test

Model 1: cbind(Admitted, Rejected) ~ Gender + Dept
Model 2: cbind(Admitted, Rejected) ~ Dept
  #Df  LogLik Df  Chisq Pr(>Chisq)
1   7 -44.572
2   6 -45.338 -1 1.5312     0.2159
```

What is actually going on

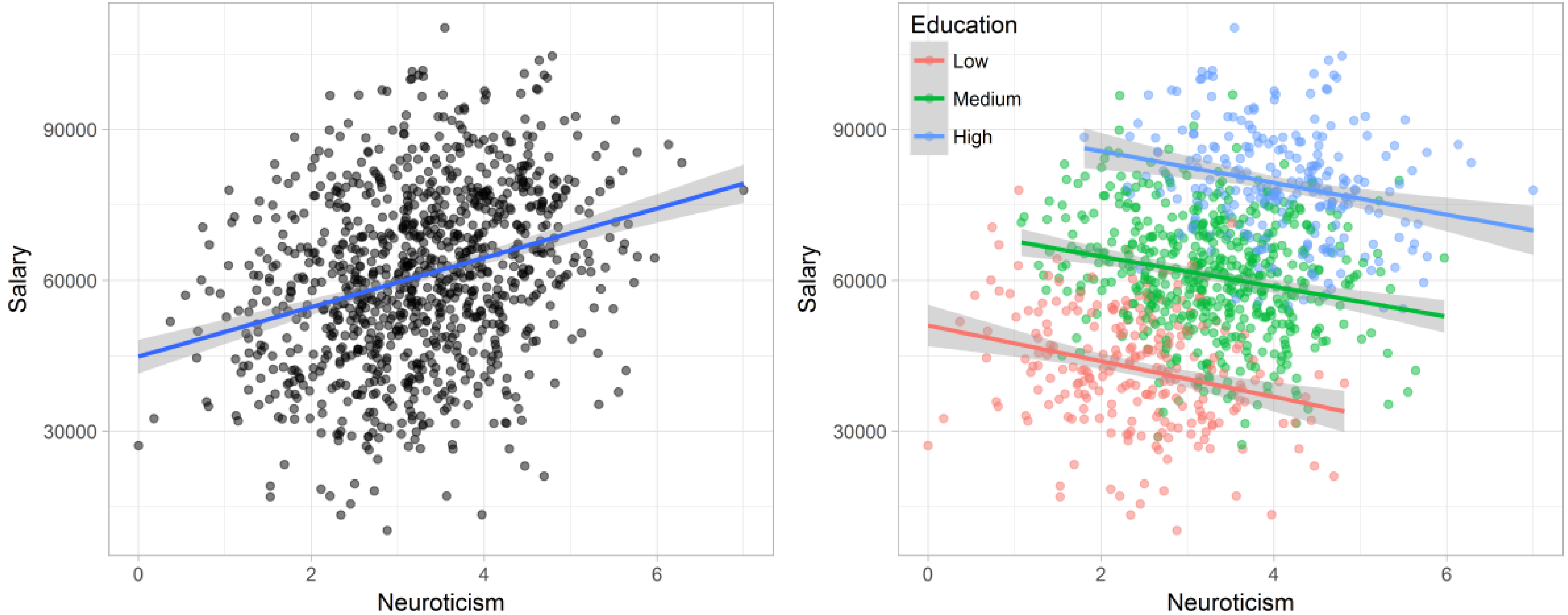
- Admission rate is actually influenced by department, not gender
- Just happen to be that more male students applied to the department with higher admission rate



Simpson’s Paradox

A phenomenon in probability and statistics in which a trend appears in several groups of data but disappears or reverses when the groups are combined.

Which means, in a easier language, missing of important variable in a model.



Wrap up

What we learned today

- Multiple GLM with Binomial regression
- Compare models using AIC and likelihood ratio test
- The importance of including critical variable in the model

Next time

- Next Tuesday in person lecture with Lisa
- Next Thursday lab 10, virtual on zoom