Signal Processing - Global Coordinates and Filtering

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Global Frame of Reference

Yaw is defined as the rotation of a body along the y-axis, which is the axis along which the gravity vector runs. As such, it is logical to assume that yaw is zero due to the fact that if the subject were to rotate around the gravity vector, no accelerometer readings would be identified by the measurement device. Thus, gravitational pull does not affect the readings along this axis, and resultantly, it is reasonable to assume that no yaw was present when data collection occurred.

System Equations

The system equations, assuming no noise in the system, are illustrated below.

The desired linear acceleration signal is measured directly by the accelerometer, and converted from IMU coordinates to global coordinates for analysis by determining pitch and roll values.

$$a(t) = a_{global}(t)$$

The system equation for velocity is established by integrating the equation for global linear acceleration function with respect to time.

$$v(t) = \int_0^t a(t)\, dt$$

Similarly, the system equation for position is determined by integrating the velocity signal with respect to time.

$$x(t) = \int_0^t v(t)\, dt$$

The angular velocity signal is measured directly by the gyroscope.

$$\omega(t) = \omega(t)$$

The orientation, or angle, is determined by integrating the measured angular acceleration with respect to time.

$$\theta(t) = \int_0^t \omega(t)\, dt$$

The system equations rewritten with noise factors included are represented below for linear acceleration, velocity, position, angular velocity, and angle.

The system equation for global linear acceleration, $a(n)$, can be represented by the signal recorded by the accelerometer, $a_n(t)$, with the noise terms removed. Several noise terms influence the linear acceleration, including constant bias, white noise, bias instability, temperature effects, and calibration errors. A constant bias noise term exists in the data recorded by the accelerometer, represented by the constant ε in the equation. Furthermore, the signal has white noise that has zero-mean, standard deviation σ, and can be numerically evaluated by determining the Allan variance of the signal. Due to its zero-mean, the white noise term does not have a net disturbance over the signal, and merely affects the signal at a given points, and thus can be simply represented in the system equation by its standard deviation. The variable BRW_a represents the bias random walk in the signal, or bias instability. k is a variable that represents all other noise factors that may influence the signal including temperature effects and calibration that are not removed in Part 2 of the project.

$$a(t) = a_n(t) - \varepsilon - \sigma - BRW_a - k$$

The velocity of the subject is represented by $v(t)$, which is evaluated by integrating the linear acceleration data recorded by the accelerometer after conversion to global coordinates, $a_n(t)$, and removing the noise factors. By integrating the constant bias present in the accelerometer data, a linear error is a noise factor in the velocity signal, represented by εt in the system equation. When integrated, white noise produces a velocity random walk with standard deviation of $\sigma\sqrt{\delta t \cdot t}$. As the original noise term has zero-mean, the velocity random walk noise term will as well, and can be simply represented by its standard deviation as it has no net influence on the signal. The variable BRW_v represents a second-order random walk in the system, due to bias instability in the accelerometer data. h is a variable that represents all other noise factors including temperature effects and calibration that are not removed in Part 2 of the project.

$$v(t) = \int_0^t a_n(t)\,dt - \varepsilon t - \sigma\sqrt{\delta t \cdot t} - BRW_v - h$$

Various noise factors can influence the signal representing position, and can be integrated into the system equation. The system equation for position, $x(t)$, can be represented by integrating the signal for velocity, $v_n(t)$ with respect with time, and removing the noise terms involved. When integrated twice with respect to time, the constant bias originally in the data recorded by accelerometer becomes a noise term varying quadratically by time. The white noise factor in the data recorded by the accelerometer becomes a position random walk term, with standard deviation of $\sigma \cdot t^{3/2}\sqrt{1/3\ \delta t}$ when integrated twice. The variable BRW_x represents a third order random walk in the system, due to bias instability in the accelerometer data. i is a variable that represents all other noise factors including temperature effects and calibration errors that are not removed in Part 2 of the project.

$$x(t) = \int_0^t v_n(t)\, dt \; - 0.5\varepsilon t^2 - \sigma \cdot t^{3/2} \sqrt{1/3\ \delta t} \; - \; BRW_x - i$$

Several noise factors influence the angular velocity signal recorded by the gyroscope, other than the angular velocity itself, including constant bias, white noise, bias instability, temperature effects, and calibration. As such, the system equation for the angular velocity $\omega(t)$ desired for signal analysis can be represented by the signal recorded by the gyroscope, $\omega_n(t)$, without the noise terms. There exists a constant bias factor, represented by the constant ε in the equation. Furthermore, the signal has white noise that has zero-mean, standard deviation σ, and can be numerically evaluated by determining the Allan variance of the signal. Due to its zero-mean, the white noise does not have a net disturbance over the signal, merely affecting the signal at a given points, and thus can be simply represented in the system equation by its standard deviation. The variable BRW_{av} represents the bias random walk in the system, or bias instability. k is a variable that represents all other noise factors that may influence the signal including temperature effects and calibration errors that are not removed in Part 2 of the project.

$$\omega(t) \; = \; \omega_n(t) \; - \varepsilon \; - \sigma \; - BRW_{av} - k$$

The orientation of the foot is represented by $\theta(t)$, which is evaluated by integrating the angular velocity recorded by the gyroscope, $\omega_n(t)$, and removing the noise factors. By integrating the constant bias present in the gyroscope data, a linear angular error is a noise factor in the orientation of the foot, represented by εt in the system equation. When integrated, white noise produces an angle random walk with standard deviation of $\sigma \sqrt{\delta t \cdot t}$. As the original noise term has zero-mean, the angle random walk noise term will also, and can be simply represented by its standard deviation. The variable BRW_o represents the bias random walk in the system, or the bias instability factor from the gyroscope data integrated with respect to time. h is a variable that represents all other noise factors including temperature effects and calibration that are not removed in Part 2 of the project.

$$\theta(t) = \int_0^t \omega_n(t)\, dt \; - \; \varepsilon t - \sigma \sqrt{\delta t \cdot t} - BRW_o \; - h$$

Analysis of Filtered Signal in Global Coordinates

A complete collection of plots for data in the x, y, and z directions for gyroscope and accelerometer data can be found in the Jupyter Notebook file.

With the current graphs of the data, it is possible to identifying landings, stairs, and floors. For example, analyzing the orientation or angle about the x-axis signal assists in locating the landings. As the IMU was placed at the subject's right ankle, and climbing the stairs of E5 results in counterclockwise motion, periodic troughs in the angle about the x-axis occur when the subject turns at the landings as shown belows shown below in Figure 1. This is due to the subject pushing off with their right foot to initiate the turn; two troughs can be occasionally detected in close proximity due to the fact that two turns occur at each landing. By understanding the difference between the time required to climb a flight of stairs and to run across a landing, it is possible to determine where the stairs and landings are. The time between two troughs close in proximity represent the landings, whereas a longer timespan represents the stairs.

Figure 1 - Signal for angle about x-axis

The occurrence of landings at these times is further supported by the signal illustrating the angular velocity about the y-axis, as demonstrated in Figure 2. As the subject rotates counterclockwise when turning at each landing, troughs exist in the data. Floors can be identified by understanding that a floor exists at every two landings.

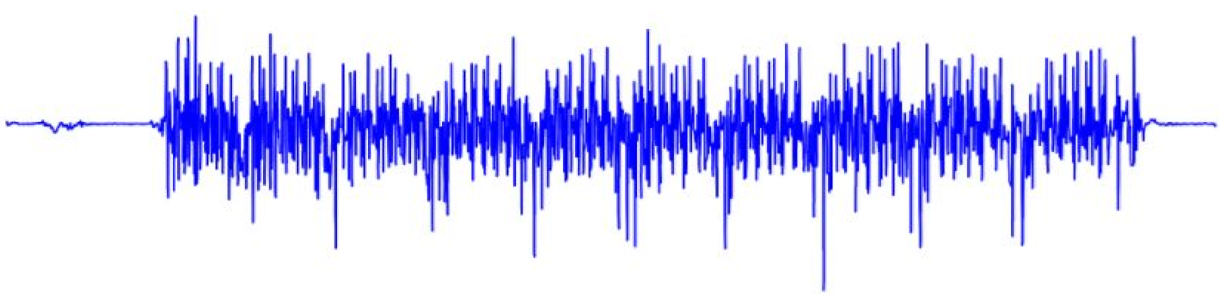

FIgure 2 - Signal for angular velocity about y-axis

It is also possible to discern where the floors are by examining the signal illustrating the linear acceleration in the z-axis, as demonstrated by Figure 3. Particularly at the beginning of the signal, the recorded data is centered periodically above and below the x-axis of the graph, depending on the location of the subject; floors can be identified as they follow a concentration of data below the x-axis.

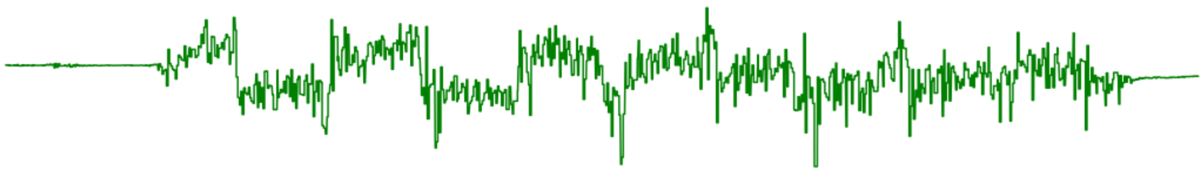

Figure 3 - Signal for linear acceleration in z-axis

The location of the IMU does affect the data readings, particularly by influencing the strength of signal recorded and the amount of noise present in the recording. For example, attaching the IMU to the foot will produce a more distinct signal with greater change in various readings, such as the change in angle. This benefit, however, would come at the cost of an increase in noise of the data, due to closer placement to the ground, less stability, and a larger effect of impact of the foot hitting the ground. On the other hand, placing the IMU on the thigh would reduce the strength of gyroscope data measured when compared to the ankle and foot, due to less movement in this area of the body. Placement of the IMU on the thigh would, however, also reduce the noise in the signal recorded as the impact of the foot hitting the ground would be dispersed by two joints rather than one, which occurs when recording data at the foot or ankle. By placing the IMU higher on the body would significantly decrease the strength of the signal, as in certain directions, little gyroscope data will be present. Varying the placement of the IMU does not ultimately affect the direction of data collection, as any such effects would be negated when the data is transferred into global coordinates. Based on this information, the data recorded for this project was recorded by placing the IMU on the ankle to provide an optimal balance between increasing signal strength and minimizing noise.

Interestingly, after accounting for errors in the data, it is difficult to discern any noticeable change in the x, y, and z data. The reason being that the overall noise that was identified was extremely small in comparison to that which was plotted. As such, only after a substantial decrease in the display time would the change be evident. Generally speaking, however, by accounting for the white noise and bias, the residual error of the IMU as well as the IMU's instability which would have an inherent improvement on the signal regardless of its visibility. Additionally, the application of the rotation matrix to the accelerometer data, thereby converting it to global from the phone's local coordinates, made the data significantly easier to visualize. Rather than simply having large accelerometer vectors with no distinguishable direction, the rotation matrix oriented the vectors to that of Earth coordinates which allowed for directional analysis of the vectors to a standard reference.