MIST Memo 68
Fitting Parameters that Characterize a Frequency-dependent Beam Model

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1 Description

This memo describes a frequency-dependent beam model parameter fitting using an preconditioned MC algorithm.

The Haslam map is used to create a simple but realistic sky model which we then used in the parameter fitting. The destriped Haslam 408-MHz file and the corresponding galactic coordinate file were downloaded from NASA Lambda site, and the sky temperature in the Haslam map is scaled from 408 MHz to 10 - 100 MHz using the following formula:

$$T_{sky}(\nu) = T_{408} \times \left(\frac{\nu}{408}\right)^{-2.5} \tag{1}$$

where $T_{sky}(\nu)$ is the sky temperature at frequency ν, T_{408} is the sky temperature of the Haslam map at 408 MHz, and frequency ν is in MHz. Figure 1 shows the sky temperature map at 60 MHz.

The scaled Haslam map has 3,145,728 pixels (Nside = 512), which slows down the analysis and requires a significant memory when fitting the parameters. To make the program run in a reasonable time, we used the `pixelfunc.ud_grade` function from `healpy` to reduce its resolution to 786,432 pixels (Nside = 256).

The beam model used in this memo is a function of Galactic longitude (l), Galactic latitude (b), and the width (σ_G) according to this Gaussian formula:

$$G(\nu, l, b, \sigma_G) = A_0 \times e^{-\frac{(l-l_0)^2}{2\sigma_G^2}} \times e^{-\frac{(b-b_0)^2}{2\sigma_G^2}} \tag{2}$$

in which $A_0 = 5$, $l_0 = 0$, $b_0 = 0$. σ_G depends on frequency according to:

$$\sigma_G(\nu) = m \times \nu + n \tag{3}$$

where m is the slope and n is the intercept. In this memo, we used $m = 0.3$ and $n = 7.5$.

We convolved the sky model and the beam model to get a noiseless simulated measurement of the sky spectrum, T_a. This is shown in Figure 2.

We added noise to the simulated measurement. This noise is Gaussian and has a standard deviation σ_n computed using the Radiometer Equation:

$$\sigma_n = \frac{T_a}{\sqrt{dt \times d\nu}} \tag{4}$$

where $dt = 1$ s and $d\nu = 1$ MHz. Figure 3 shows σ_n.

`pocoMC` described here implements a preconditioned MC algorithm and is used to perform parameter fitting on m and n. It uses two steps to fit parameters. The first step is to run the sampler. It requires the inputs n_particles and n_dimension. The number of iterations is not a input parameter in the algorithm. We

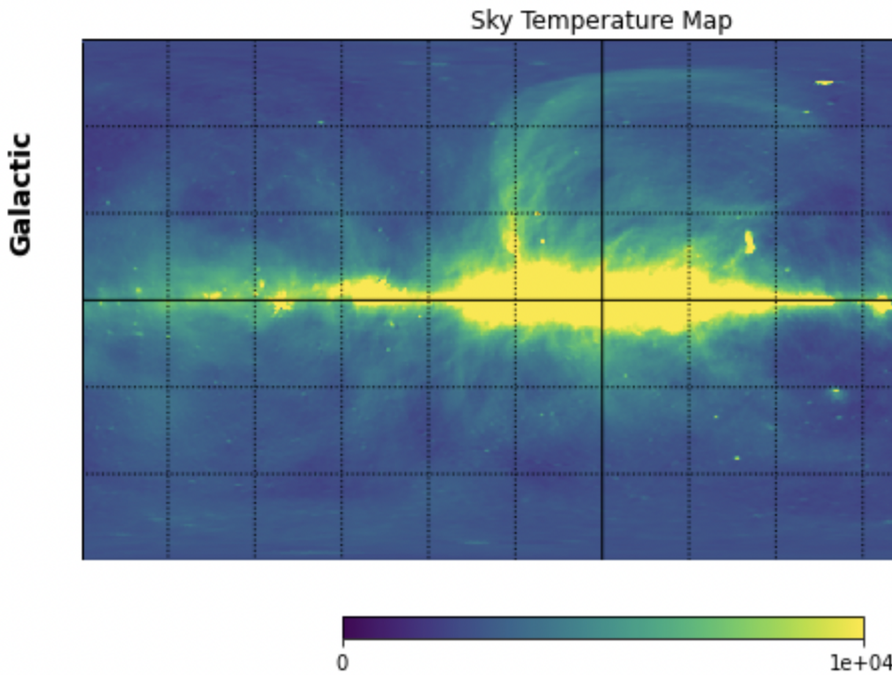

Figure 1: Example temperature map generated by scaling the destriped Haslam 408-MHz Map to 50 MHz using a power law and lowering the resolution by a factor of four.

chose n_particles = 4 because smaller values will underestimate the model's complexity and thus give a less accurate result, while larger values will take too long to run. n_dimension = 2 since there are two fitting parameters. We define uniform priors between 0 and 1 for m and between 5 to 10 for n. The second step is to add samples to the chains. This improves the parameter estimation. We added 3000 samples, which was enough to produce a Gaussian-like posterior. At the end of the process, the code outputs the log Bayesian Evidence.

2 Result

The preconditioned MC parameter fitting process gave us the result of $m = 0.2999 \pm 0.0001$ and $n = 7.4976 \pm 0.0053$ (1σ). Compared to the true values 0.3 and 7.5, the best fit parameters are considered accurate. The triangle plot for m and n is shown in Figure 4.

The log-likelihood of the best fit model is -44.65, while that for the input model is -47.52. The log Bayesian Evidence estimated using by `pocoMC` is -61.48.

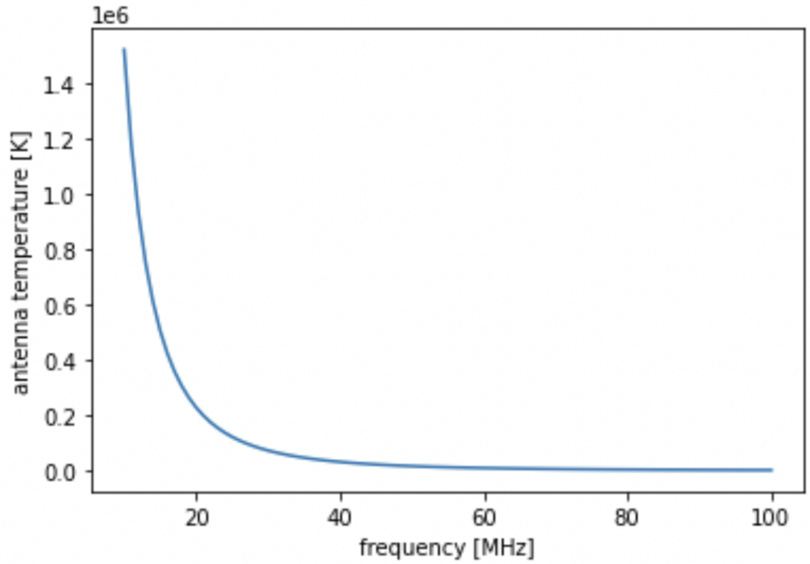

Figure 2: A noiseless simulated measurement of the sky spectrum generated by the convolution of the sky temperature model and the beam model.

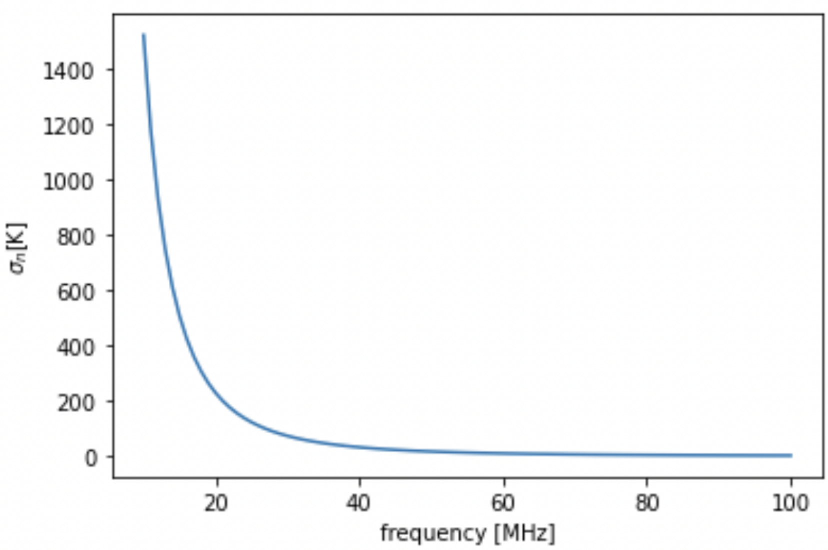

Figure 3: The σ_n of the Gaussian noise for Figure 2's noiseless sky spectrum. It is calculated using the Radiometer Equation.

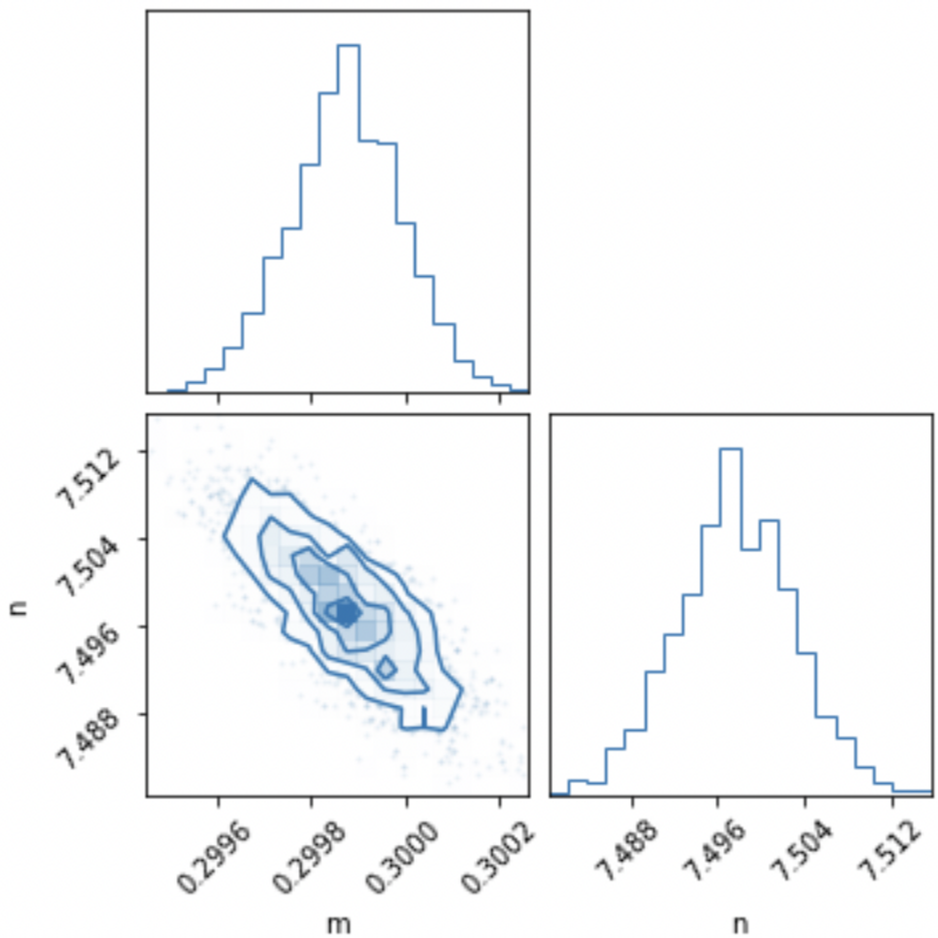

Figure 4: Posterior distributions for m and n of the beam model's σ_G from the parameter estimation done by pocoMC.