MIST Memo 70
Fitting Parameters that Characterize a Frequency-dependent Beam Model using UltraNest

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1 Description

In this memo, we continue our study of fitting parameters of a frequency-dependent beam model using a nested sampling algorithm `UltraNest`. The sky temperature map, the beam model, and the Gaussian noise used in this memo are the same as the setup defined in memo 68. Here is the link to the memo for more information. As a reminder, the beam model has a beam width σ_G that is linear to the frequency:

$$\sigma_G(\nu) = m \times \nu + n \tag{1}$$

In this memo, we fit the slope m and the intercept n. The true value for the parameters m and n are 0.3 and 7.5 respectively.

`UltraNest` described here implements a nested sampling algorithm and is used to perform parameter fitting on m and n. It requires three inputs: parameters' names, a prior function, and a log-likelihood function. The parameters' names are in the list form ['m', 'n']. We define uniform priors between 0 and 1 for m and between 5 to 10 for n. Another input we can control is the min_num_live_points. Its default value is 400 and it sets the minimum existing live points in the sampler. Live points are the temporary sampled points that can be removed and sought by the sampler. The nested sampler manages the live point population, which affects the accuracy and duration of the parameter fitting process.

2 Results

After fitting using `UltraNest`, the results are $m = 0.30021 \pm 0.00014$ and $n = 7.4914 \pm 0.0059(1\sigma)$. Compared to the true values 0.3 and 7.5, the best-fit parameters are within 1 σ and are thus considered accurate. The triangle plot for m and n is shown in Figure 1.

The log-likelihood of the best fit model is -51.25, while that for the input model is -52.53. The log Bayesian Evidence estimated using `UltraNest` is -65.560 ± 0.300.

The overall run time for fitting the parameter using `UltraNest` is longer than that using `pocoMC`. One reason is that there are 7965 sampling points added compared to the 3000 sampling points that we used with `pocoMC`. The duration for one added sampling point is about the same. We can shorten the process by changing the min_num_live_points to a smaller number. Table 1 summarizes the comparison for two runs with 200 and 400 min num live points. When we change it from 400 to 200, it takes around 88 minutes to sample 3996 points, compared to the original 208 minutes. The new results are 0.30021 ± 0.00014 and $7.4914 \pm 0.0059(1\sigma)$. In both cases, the precision is the same, comparing the uncertainty. Both cases' distributions are about a Gaussian, which is a good sign that the simulations have largely converged. For the settings that we have used, the duration and sample points added are reduced, while the accuracy, precision, log_likelihood, BIC are the same. Thus,in our case, we can shorten the duration by reducing the min_num_live_points, which does not change the best-fit parameters.

min_num_live_points	400	200
duration	208 min	88 min
m (1σ)	0.30021 ± 0.00014	0.30021 ± 0.00014
n (1σ)	7.4914 ± 0.0059	7.4914 ± 0.0059
#samples	7965	3996
log_Z	-65.56 ± 0.30	-65.63 ± 0.22
log_likelihood	-51.25	-51.25
BIC	34.65	34.65

Table 1: Comparison between two runs with UltraNest with 200 and 400 min_num_live_points.

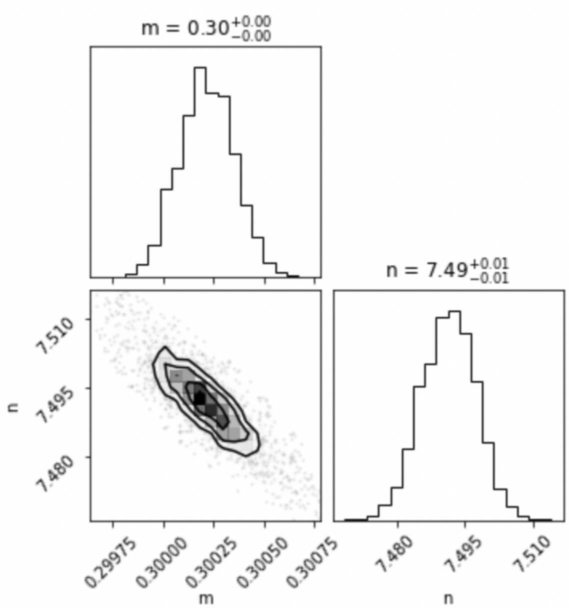

Figure 1: Posterior distributions for m and n of the beam model's σ_G from the parameter estimation done by UltraNest when the min_num_live_points is set to 400.

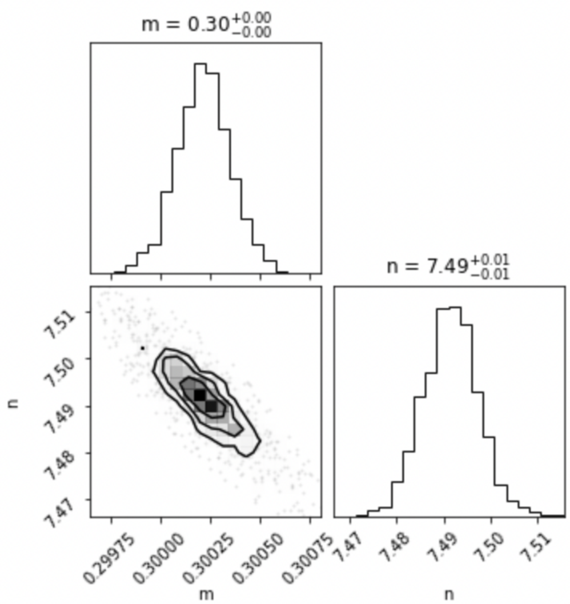

Figure 2: Posterior distributions for m and n of the beam model's σ_G from the parameter estimation done by UltraNest when the min_num_live_points is set to 200.