Extractors: QLDPC Architectures for Efficient Pauli-Based Computation

Zhiyang He (Sunny), Alexander Cowtan, Dominic Williamson, Theodore Yoder

I. Motivation: A QLDPC-Based Quantum Computer

II. Code Surgery and Extractors

III. Extractor Architecture and Compilation

IV. Building an Extractor with Graph Theory

V. Discussions and Outlooks

The Promise of QLDPC Codes

Surface code is the leading candidate for building a large-scale, fault tolerant quantum computer.

Amazing properties: high threshold, 2D connectivity, fast decoding, transversal gates, lattice surgery…
Challenge: Significant asymptotic space overhead, ~1000x for factoring.

Quantum LDPC codes promise to implement fault-tolerant computation with O(1) space overhead.

➢ At what scale can we fulfill this promise to gain a practical advantage?

* Surface code figure cred. Niel de Beaudrap.

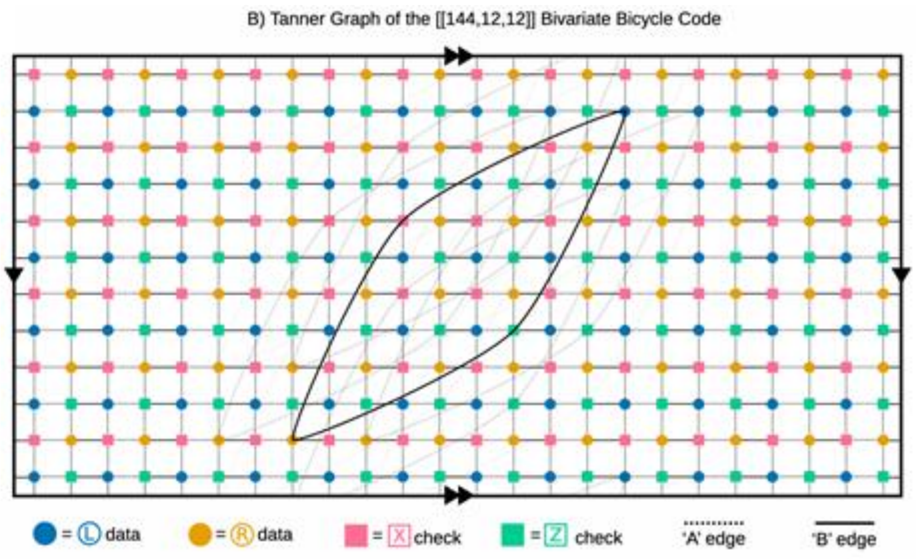
Fast Progress in QLDPC Memory

Quantum Low-Density Parity-Check (LDPC) Codes:
stabilizers of O(1) weight, qubits in O(1) stabilizers.
Better encoding rate than surface code!

Recent constructions, [n, k, d]:
- Bivariate Bicycle code [144, 12, 12] *
- Hypergraph product code [2500, 100, 12] **
- Lifted/balanced product code [544, 80, ≤ 12] **

Surface code: [265, 1, 12].

Memory: Decoding algorithm, threshold and logical error rate, hardware.
From memory to computer: logical computation.
➢ Long-standing challenge and many works.



* [Bravyi et al. 2308.07915]. ** [Xu et al. 2308.08648].

Extractor Architecture for QLDPC Computation

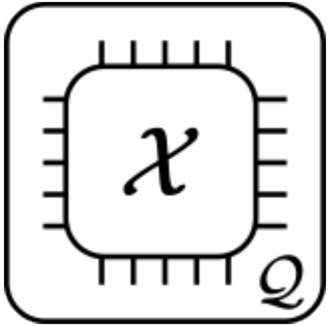
In this work, we present a solution to the QLDPC computation challenge: Extractors. Our solution has a few distinctive features:

1. **Any** quantum code can be augmented by an extractor system to become a computational block. I.e., extractors augment memories into processors.

2. Given **any** magic state factory, can implement universal quantum circuits via parallelized logical operations.

3. Can be implemented with fixed, constant degree connectivity (having movable qubits is certainly helpful but not necessary).

4. Highly optimizable, practical space and time overheads.



An Extractor-augmented computational (EAC) block.

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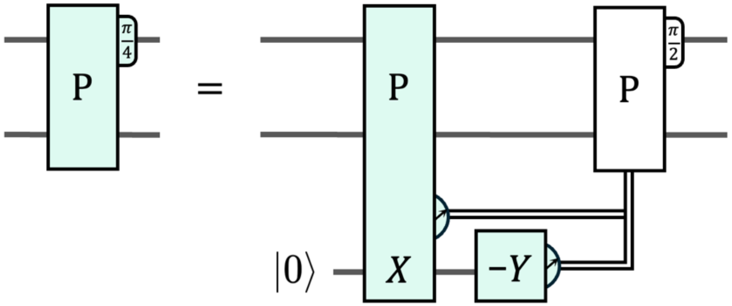
Universal Computation via Logical Measurements

A Clifford + T circuit can be written in terms of Pauli rotations, where:

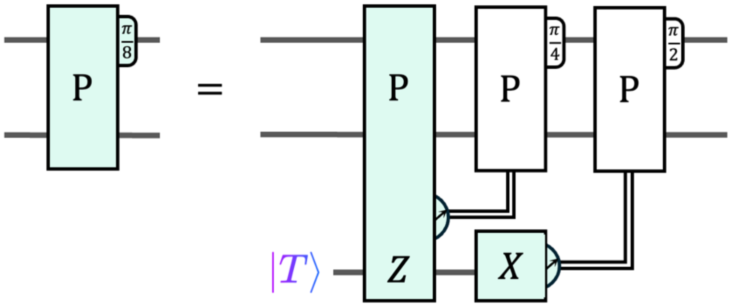
- Pauli gates → Pauli $\pi/2$ rotations,
- Clifford gates → Pauli $\pi/4$ rotations,
- T gates → Pauli $\pi/8$ rotations.

Pauli rotations can be implemented with Pauli measurements.



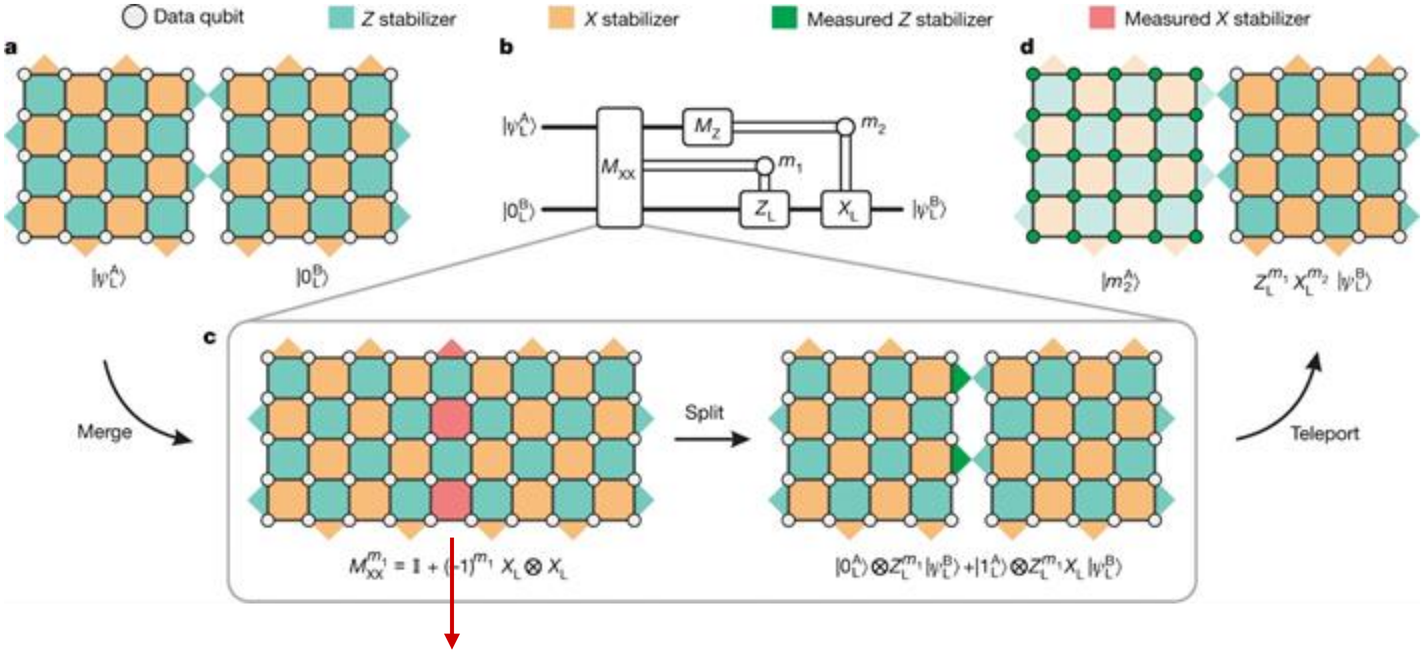
Pauli-based computation: Pauli measurements + magic states = universal computation.

- Fault-tolerant measurements + magic state factory = universal FT computation!



Surface Code Lattice Surgery

Logical measurements on surface codes: lattice surgery, [Horsman et al, 1111.4022].



Product of red X-checks = $X_L \otimes X_L$ – obtain logical measurement result by measuring new stabilizers.

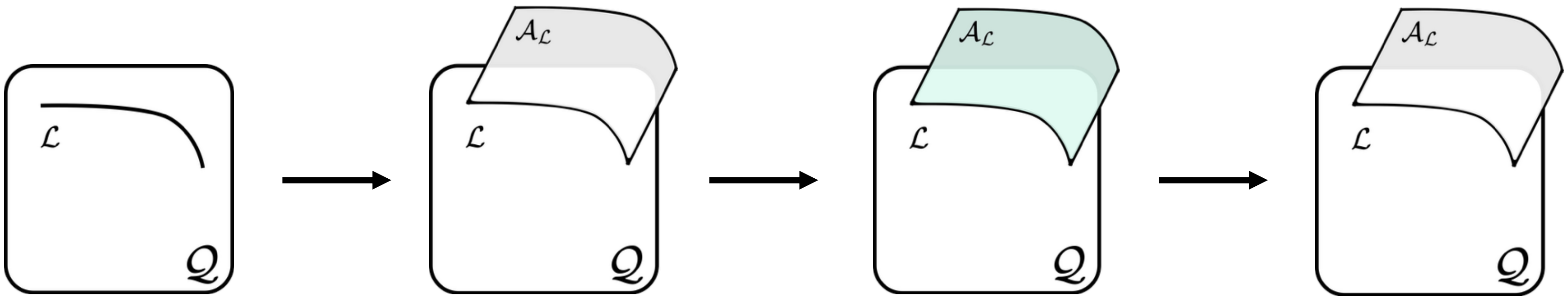
* Figure from [Erhard et al 2020].

QLDPC Code Surgery

First proposed by [Cohen et al., 2110.10794], > 10 papers on surgery in the past year.

- ➢ Section 3.2 of the present work [2503.10390] is a 2-page review.

High level description: for a quantum LDPC code $\mathcal{Q}$, for every logical operator $\mathcal{L}$, can construct ancilla system $\mathcal{A}_{\mathcal{L}}$ such that $\mathcal{Q}$ augmented by $\mathcal{A}_{\mathcal{L}}$ can be used to measure $\mathcal{L}$.



(a) Start: Code $\mathcal{Q}$ and operator $\mathcal{L}$.

(b) Init: Initialize ancilla in product state.

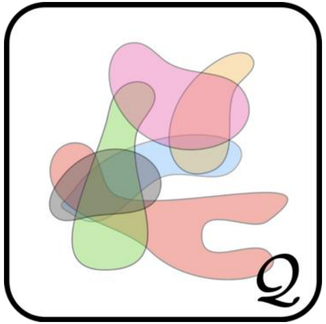
(c) Merge: Code deformation by measuring new stabilizers

(d) Split: Measure out ancilla and return to $\mathcal{Q}$.

Challenge: Compact Memory Has Many Operators

Challenge: High-rate codes have many operators, and they overlap.
Prior works: for every logical operator $\mathcal{L}$, construct an ancilla system for measurement.

- Building many ancilla systems will quickly blow up space and connectivity overhead.

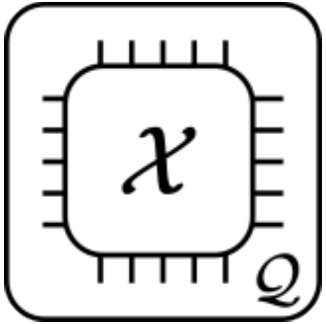


Extractors: one ancilla system $\mathcal{X}$, can measure any logical operator.

- For *any* code of n qubits, can built LDPC extractor of size $\tilde{O}(n)$.
- In practice, expect space overhead to be a small constant. E.g., 103-qubit (partial) extractor for [144, 12, 12] code. [2407.18393]
- Any operator can be measured with $O(d)$ syndrome rounds.

<u>Def</u> [Extractors]: extract logical Pauli observables from the memory.

- Built using tools developed in [2407.18393], [WY 2410.02213]*, and [SJOY 2410.03628].



An Extractor-augmented computational (EAC) block.

* See also [Ide et al. 2410.02753].

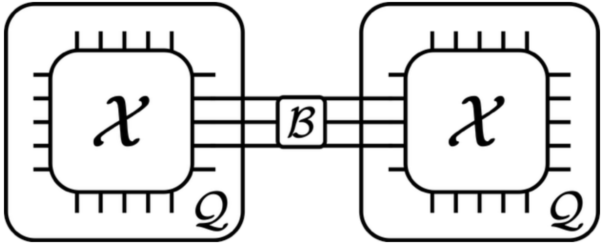
Modularity: Bridges and Adapters

Bridge/Adapter: primitive developed in [2407.18393] and [2410.03628].

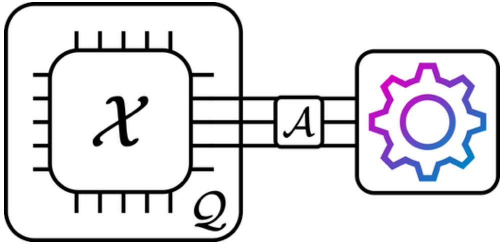
- LDPC ancilla system that can connect two extractors into a bigger extractor. Enables Pauli measurements across connected blocks.

Two names for the same system:

- If it connects blocks of the same code, we call it a bridge.
- If it connects blocks of different codes, we call it an adapter.



Two EAC blocks joined by a bridge $\mathcal{B}$.



An EAC blocks connected to a source of magic states by an adapter $\mathcal{A}$.

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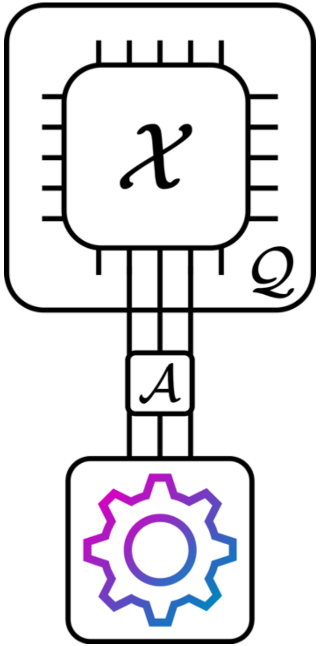
Extractor Architecture: MWE

Let's start with a minimal working example (MWE) of extractor architectures.

A [n, k, d] code $\mathcal{Q}$, augmented by an extractor $\mathcal{X}$. This is an EAC block.

The extractor $\mathcal{X}$ is connected to the factory by an adapter.

An arbitrary $|T\rangle$ state factory.



Features & Comments

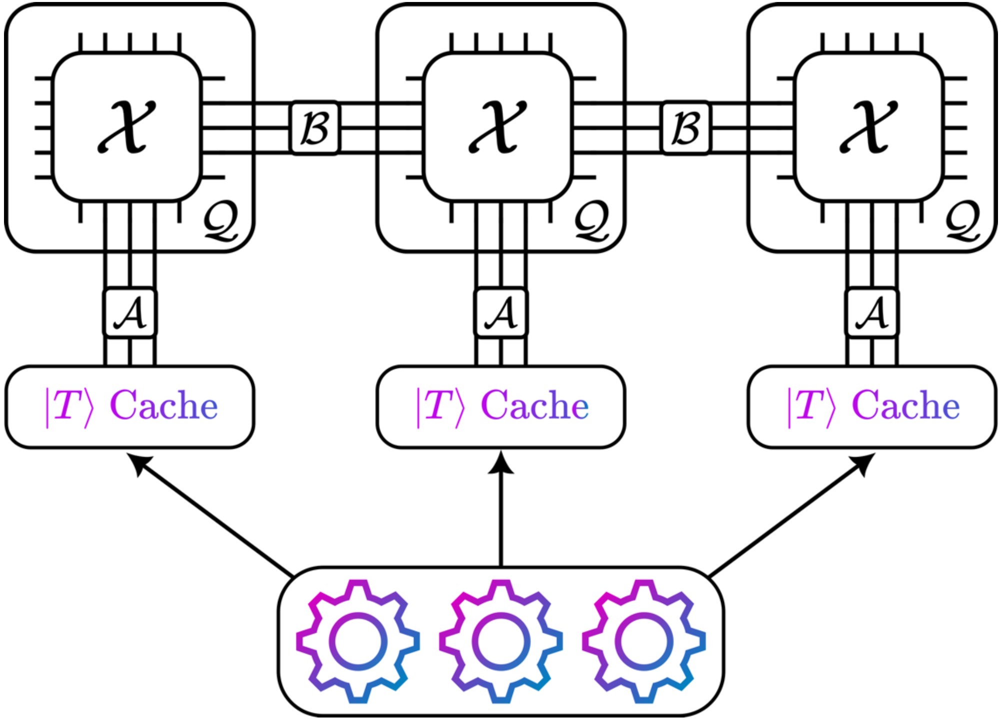
- Every logical measurement takes O(d) syndrome cycles.
- Can be built with *any* code $\mathcal{Q}$ and *any* $|T\rangle$ state factory.
- For near-term, can use small QLDPC code + magic state cultivation.
- Entire system has fixed, constant-degree connectivity.

Extractor Architecture

EAC blocks connected by bridges

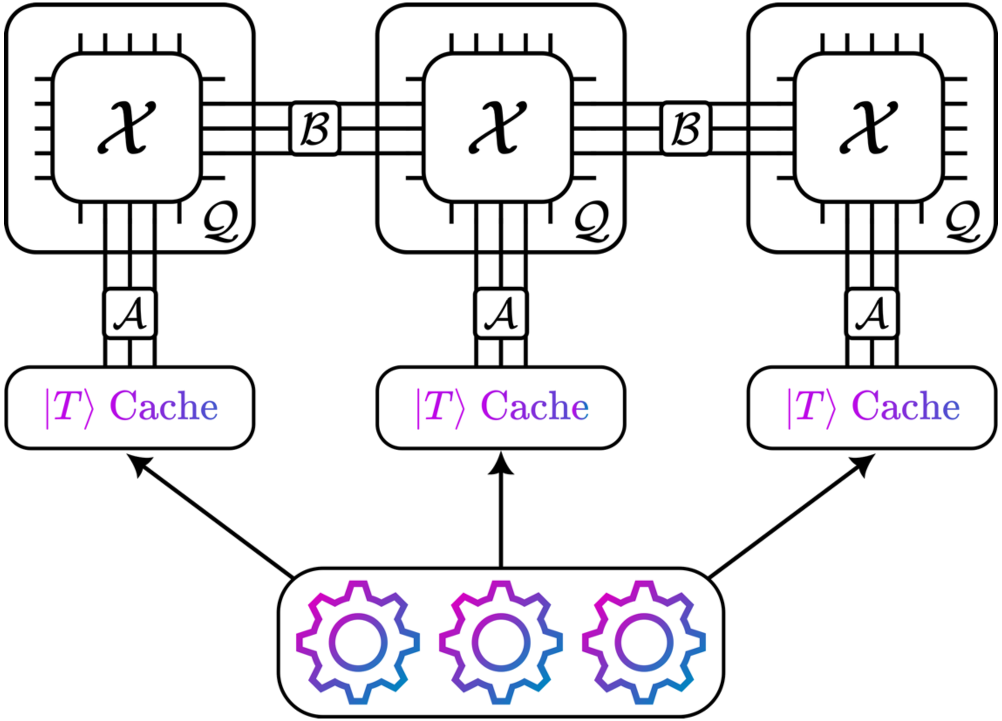
Storage for $|T\rangle$ states, connected to EAC blocks by adapters.

Centralized magic state factory



Extractor Architecture

- Any logical Pauli supported on blocks and caches connected by bridges and adapters can be measured in O(d) syndrome rounds, with fault distance d.
- Operators supported on disjoint blocks can be measured in parallel by deactivating bridges/adapters.
- Flexibility: global architecture can be tailored to hardware or application.

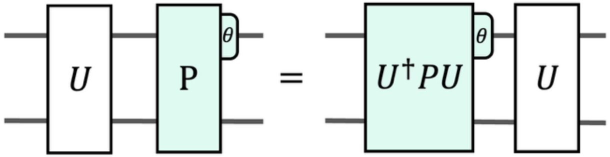


Compilation for an Extractor Architecture

Compilation similar to Game of Surface Code [Litinski 1808.02892]. Given a logical circuit of Pauli rotations, we consider three types of gates:

1. Pauli π/8 rotations,
2. Pauli π/4 rotations supported within one EAC block (in-block Cliffords),
3. Pauli π/4 rotations supported on two EAC blocks connected by bridges (cross-block Cliffords).

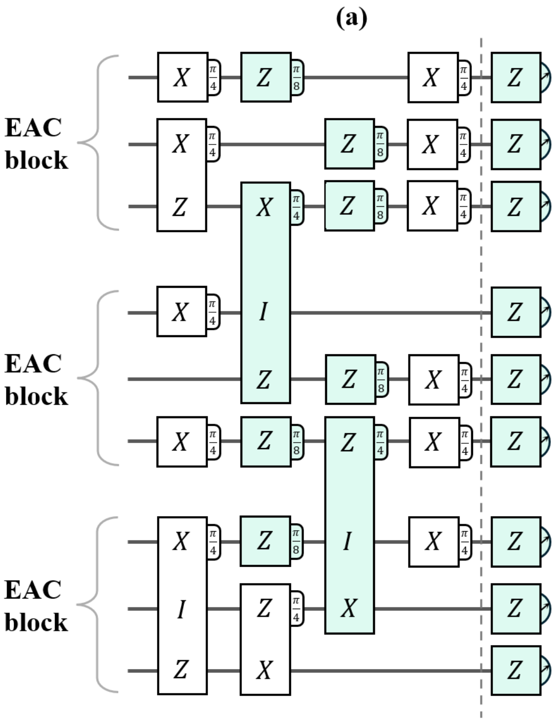
We conjugate all in-block Cliffords (type 2) to the end of the circuit.



They will be absorbed by a round of final read-out. Type 1 and 3 rotations will then be implemented with logical measurements.

Circuit Example

1. Conjugate all in-block Cliffords to the end of the circuit.

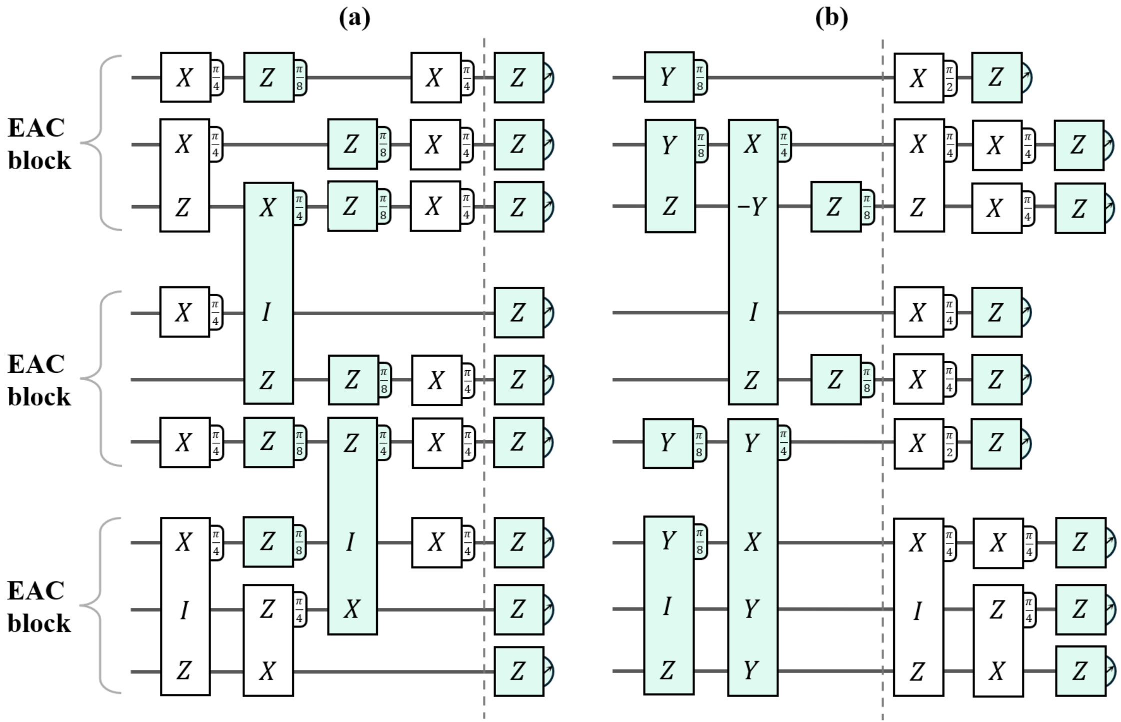


Green operations are what we compile and implement. White operations are in-block Clifford that are compiled away.

Circuit Example

1. Conjugate all in-block Cliffords to the end of the circuit.

2. Absorb in-block Cliffords by the final measure-out.



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Circuit Example

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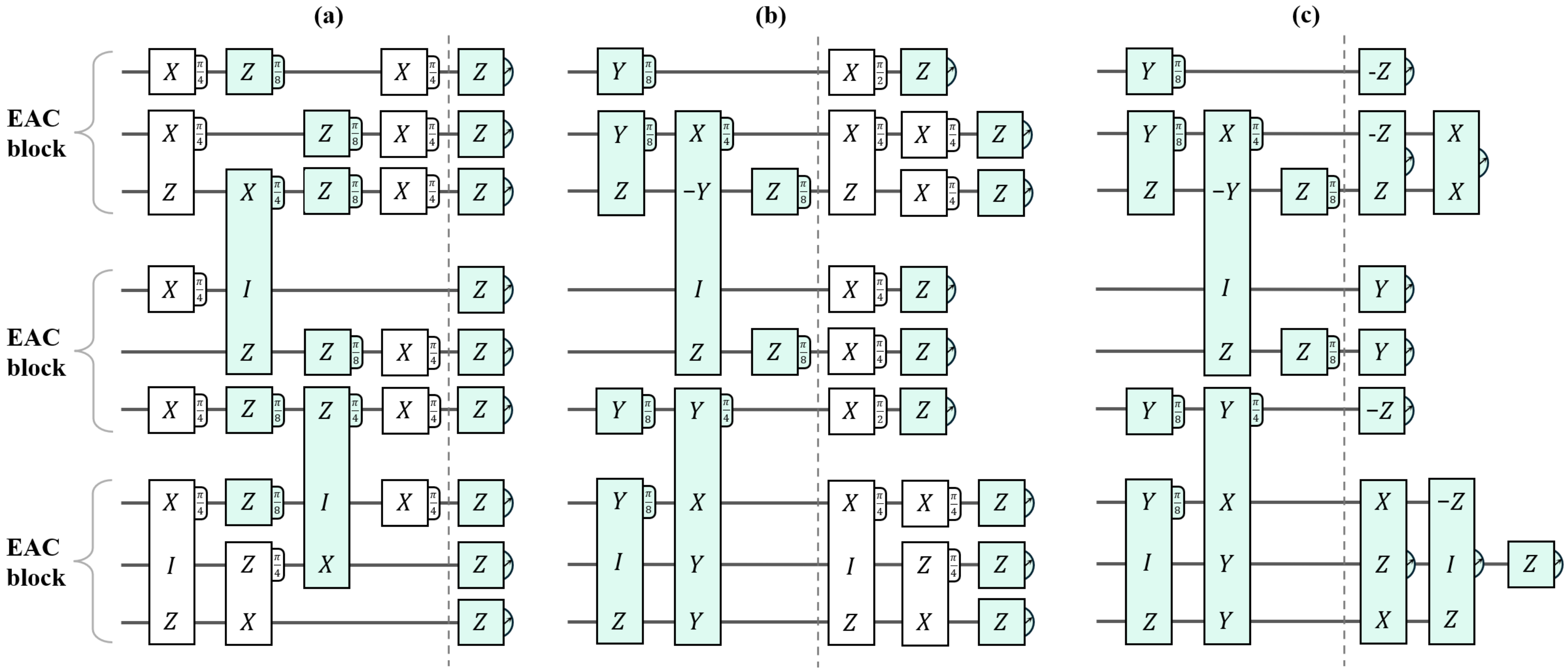
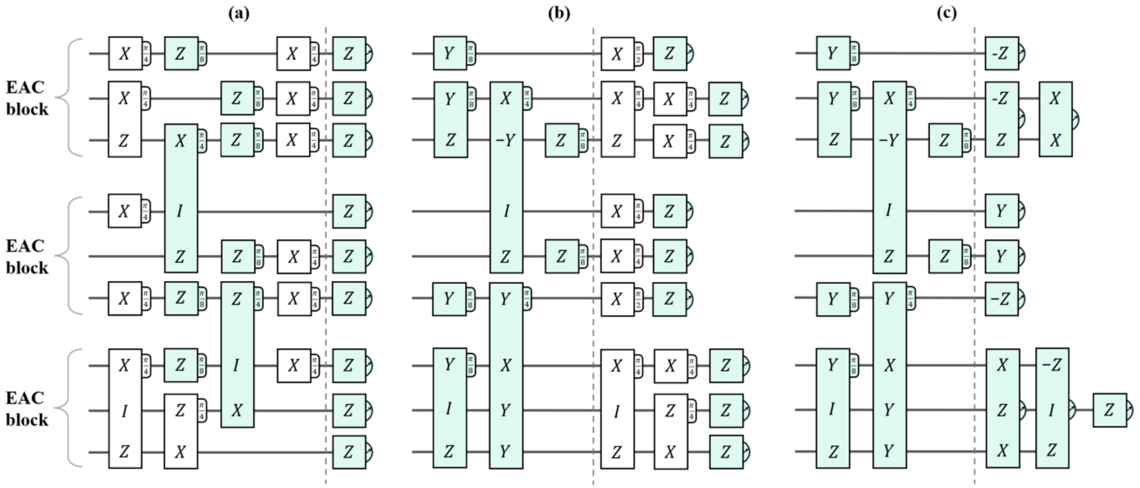


Green operations are what we compile and implement. White operations are in-block Clifford that are compiled away.

Remarks

- In-block Clifford gates are essentially free.
- This compilation heavily relies on the fact extractors can measure any logical Pauli.
- Bottleneck: magic state supply speed and number of cross-block gates.
- Highly optimizable for specific applications.



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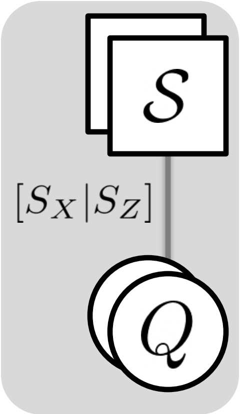
Scalable Tanner Graphs



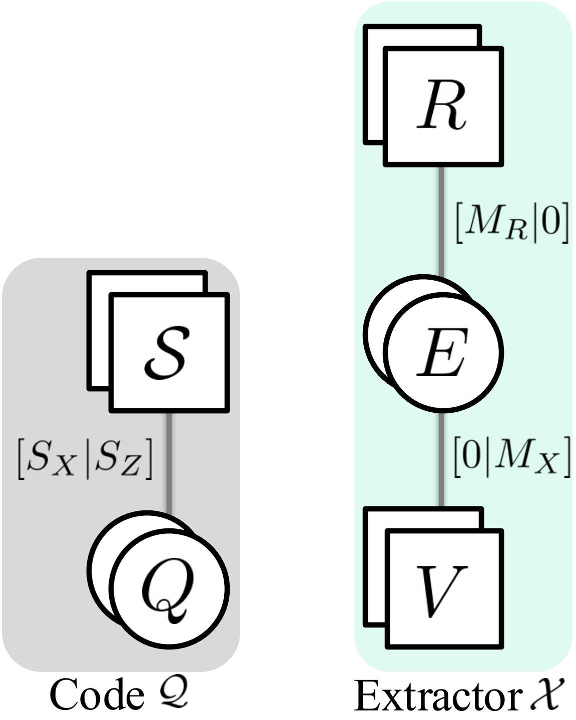
Code $\mathcal{Q}$

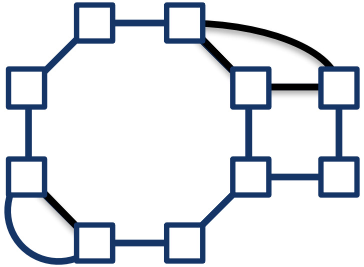
Stabilizers of the code $\mathcal{Q}$

Symplectic check matrix

Physical qubits of the code $\mathcal{Q}$

Building an Extractor from a Graph



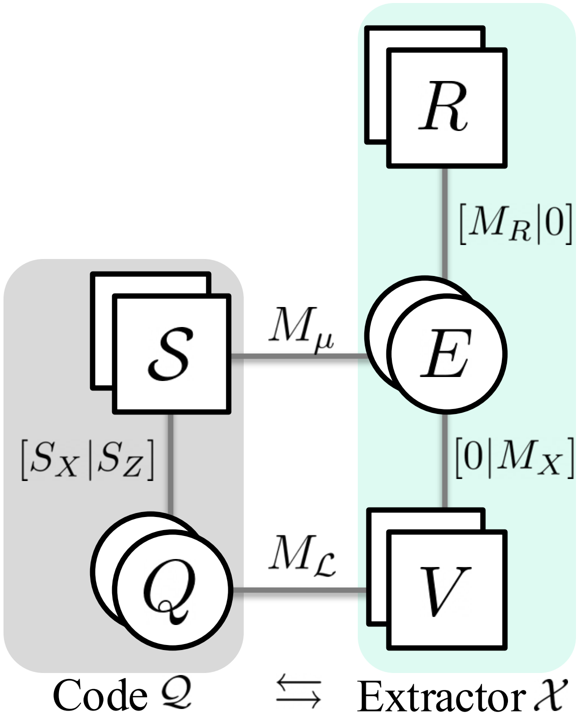


Let X = (V, E) be a graph.

1. For every edge in E, create an ancilla qubit.
2. For every vertex in V, create an ancilla check, which act on adjacent edge qubits by Pauli Z.
3. Pick a cycle basis R of X. For every cycle C in R, create an ancilla check, which act on edges in C by Pauli X.

This ancilla system, the extractor system, commutes.

Building an Extractor from a Graph



We will build fixed connections between:

1. Vertex checks V and qubits of $\mathcal{Q}$;
2. Stabilizers S and ancilla edge qubits E.

What's their Pauli action?

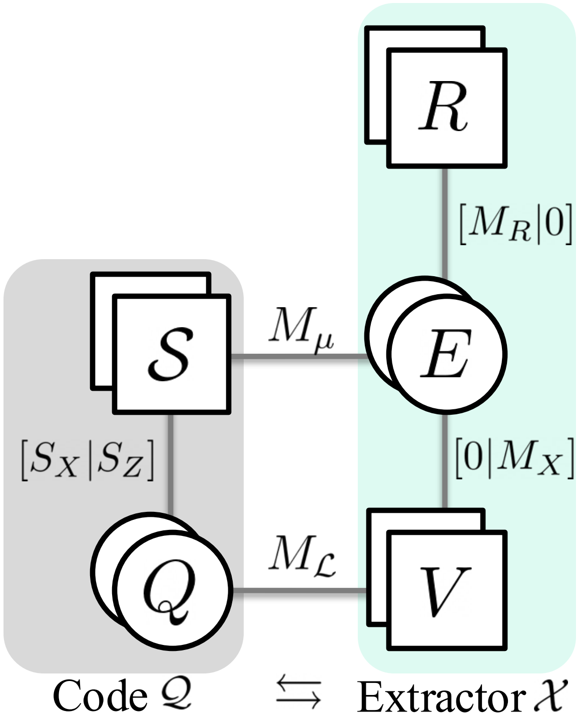
Depends on the operator we want to measure!

Given operator $\mathcal{L}$, we will pick symplectic matrices $M_\mathcal{L}$ and M_μ so that

1. Entire system in EAC block commutes. I.e, we have a well-defined measurement code $\mathcal{Q}_\mathcal{L}$.
2. Product of vertex checks V equals to $\mathcal{L}$.

Measuring stabilizers of $\mathcal{Q}_\mathcal{L}$ for O(d) rounds gives logical measurement of $\mathcal{L}$ fault-tolerantly.

Many Important Details…



Many details not discussed in this talk:

1. Why is this system LDPC?
2. How to connect S with E and Q with V?
3. How to choose matrices $M_\mathcal{L}$ and M_μ?
4. How to prove fault-tolerance of this code-switching process?
5. How to upper bound size of extractors by $\tilde{O}(n)$?
6. Most importantly, how to build this in practice?

All proved & discussed in the paper with graph theory.

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The Landscape of QLDPC Computation

Symmetry
- Transversal gates;
- Automorphisms gates;
- ZX Duality.

Teleportation-based
- Gate teleportation: magic states and Clifford states
- Homomorphic measurements

Code deformation
- Code surgery and extractors
- Punctures?
- Code-switching?

Universal Computation = Symmetry + magic state factory + transversal CNOT + (multiple) Clifford state factories.

➢ Standard solution: multiple factories for different gates incurs heavy overhead.

Universal Computation = QLDPC memory + surgery + surface code computation (magic state factory).

➢ Hybrid architecture: surface code computation will quickly erase space advantage.

Universal Computation = Magic state factory + extractors.

➢ Extractor architecture: in-block Cliffords are free, fixed & LDPC connectivity. Larger decoding instance.

Universal Computation = Symmetry + magic state factory + transversal CNOT/partial extractors.

➢ [Malcolm et al. 2502.07150]: inverse-exponential rate.

Where does automorphism gates fit?

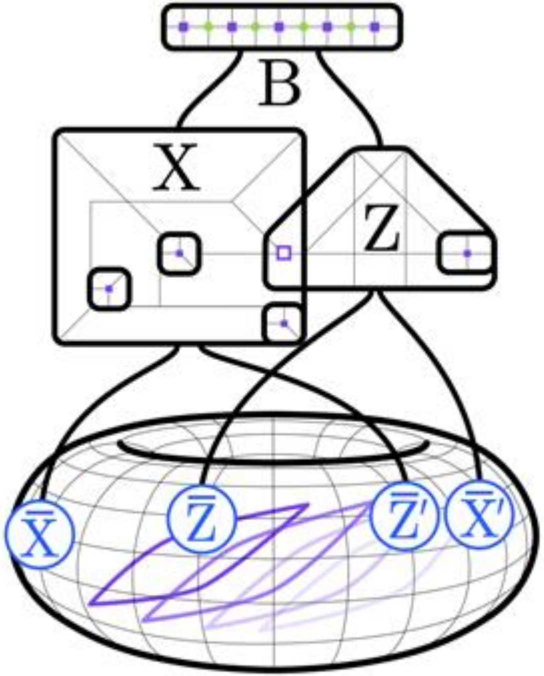
For a code with automorphism gates $\mathcal{U}$, we don't need to build a full extractor.

Instead, we can build a partial extractor which can measure Pauli operators in the set $\mathcal{O}$, such that $\mathcal{U}^{\dagger}\mathcal{O}\mathcal{U}$ generate the full k qubit Pauli group.

- All $\pi/4$ rotations on k-1 qubits!

This is similar to the 103-qubit system on the [144, 12, 12] gross code. [2407.18393]

Same applies if the code has other low-overhead logical operations.



Bridge used in partial extractor

Partial extractor

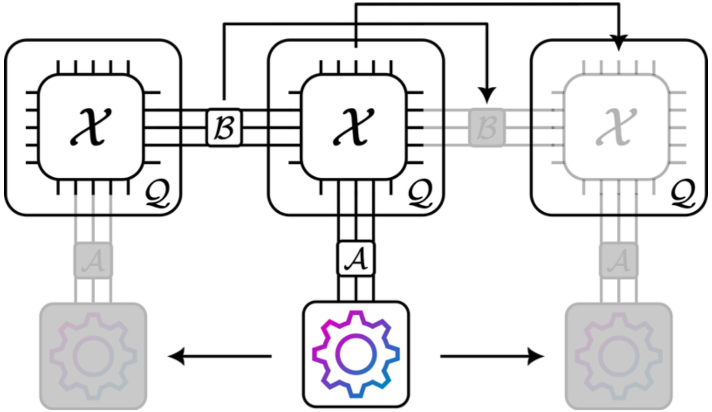
Gross code

What if the hardware supports qubits movement?

Everything can move: factories, bridges, adapters, extractors.

➢ Space overhead can be bounded by 'active' components.

Within one EAC block, a moving partial extractor can act as a full extractor.

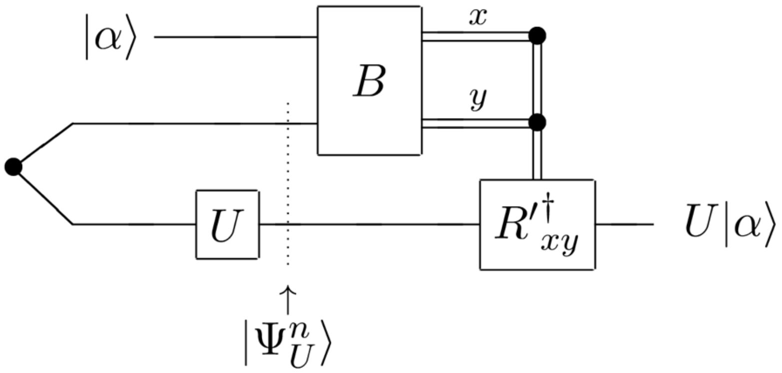


Movement makes transversal CNOT easy.

➢ Extractor can prepare arbitrary logical stabilizer state 'offline', effectively as a 'Clifford factory'.

➢ Gate teleportation lets us perform arbitrary Clifford operation in O(1) 'online' step.

➢ Universal = addressable non-Clifford* + extractor Clifford factory + transversal CNOT.



* See also discussions & constructions in [2502.01864]

Future Directions

Design of (partial) extractors on promising codes

- Reducing cost of extractors with automorphisms or code structure;
- Constant asymptotic/practical space overhead?
- Hardware layout and/or optimizations;

Global architectures design for specific hardware & applications

- Choice of code & block size, bridge connections, and magic state supply given specific circuit;
- Combination of extractor architecture with specialized algorithmic gadgets.

Resource Estimation

- Compilation of algorithms, such as factoring, to an extractor architecture.
- Hardware constraints, architecture design, EAC blocks, decoders...

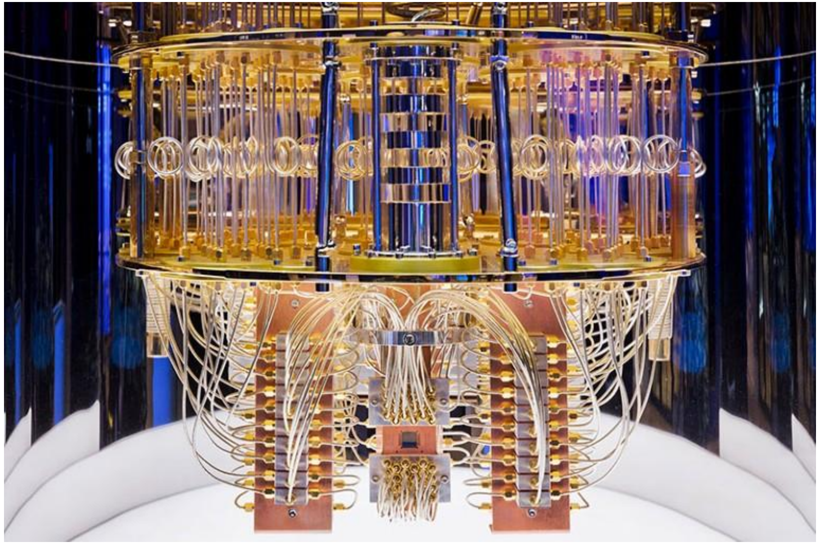
Ending Remarks

🌞 Assuming a source of magic states, Extractor is a new, one-stop solution to perform universal FT computation on any code with low overhead.

➢ A bridge from memories to processors.

🌞 An open and exciting frontier for theoretical and practical explorations.

🚜 Challenges ahead: LDPC hardware, fast and accurate decoding, many more…



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