Pattern Recognition HW1

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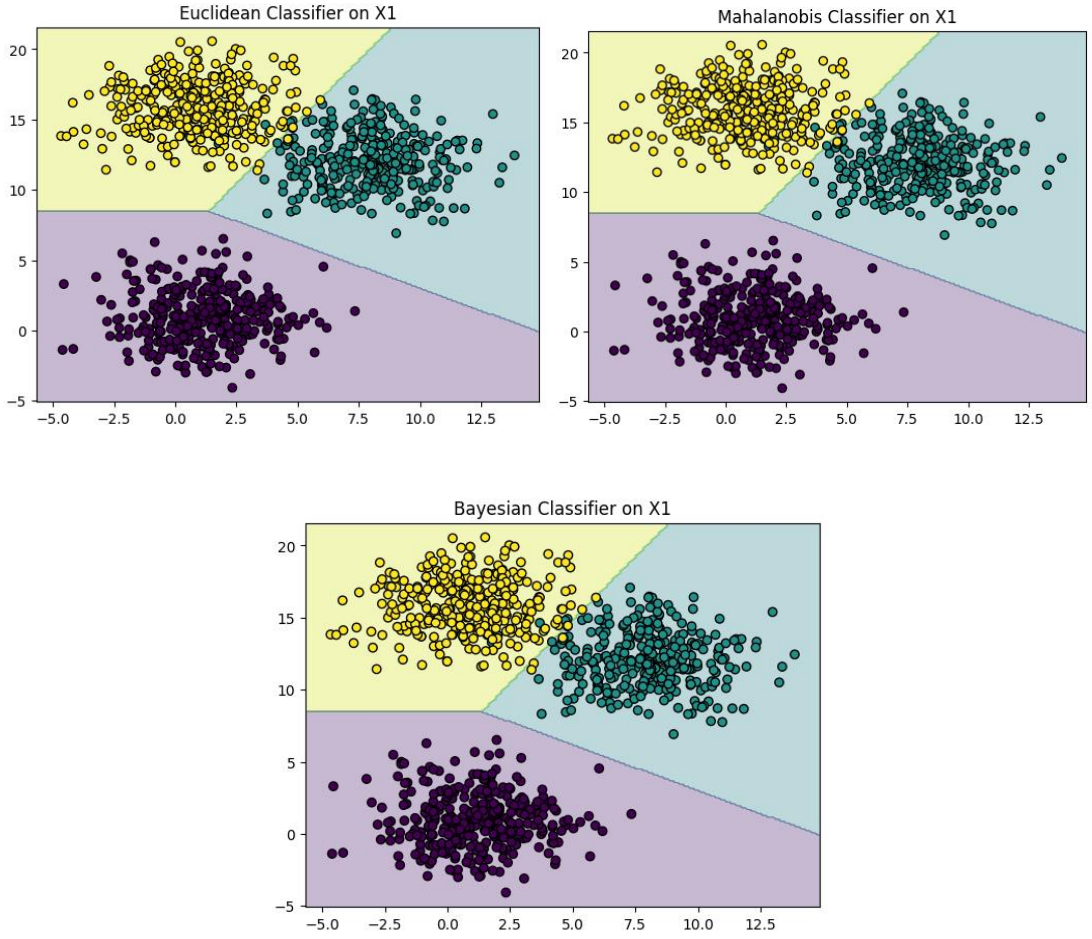
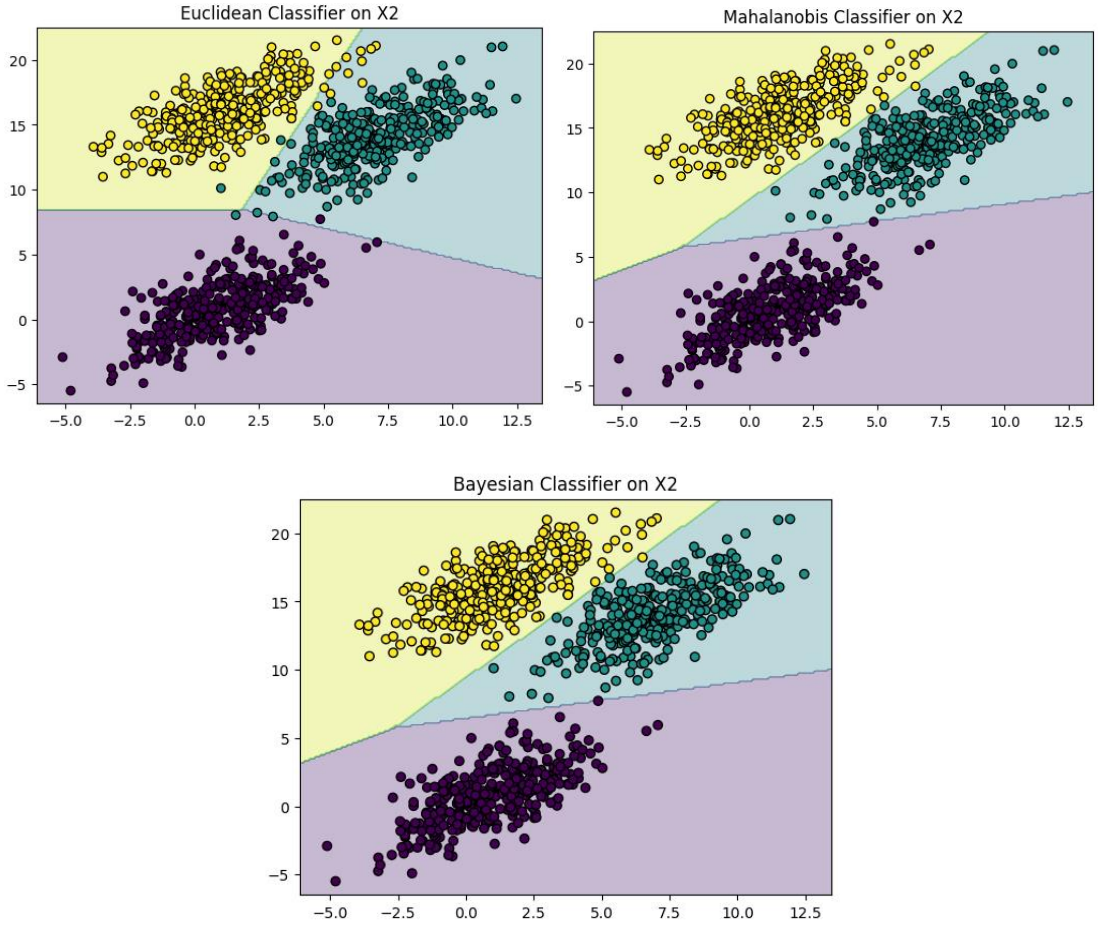
EXP1

- Three classifier's classification Error
  i. Euclidean Classifier Error: 0.013
  ii. Mahalonobis Classifier Error: 0.013
  iii. Bayesian Classifier Error: 0.013
- Conclusion: Since the covariance of three distributions are the same and the covariance matrix of three distributions is an identity matrix * 4 (4I). All three classifier decision boundary is the same. Therefore, the decision boundary is the same for all three classifier. (See below plot)
- Plot of three distributions' decision boundary and dataset:



EXP2

- Three classifier's classification Error
  i. Euclidean Classifier Error: 0.0160
  ii. Mahalonobis Classifier Error: 0.001
  iii. Bayesian Classifier Error: 0.001
- Conclusion: The covariance matrix is the same for all three distributions. However, the covariance matrix is arbitrary, not a diagonal matrix. Therefore, euclidean classifier does not have an optimal decision boundary. Mahalanobis classifier and bayesian classifier has the same decision boundary. (see below plot)
- Plot of three distributions' decision boundary and dataset:



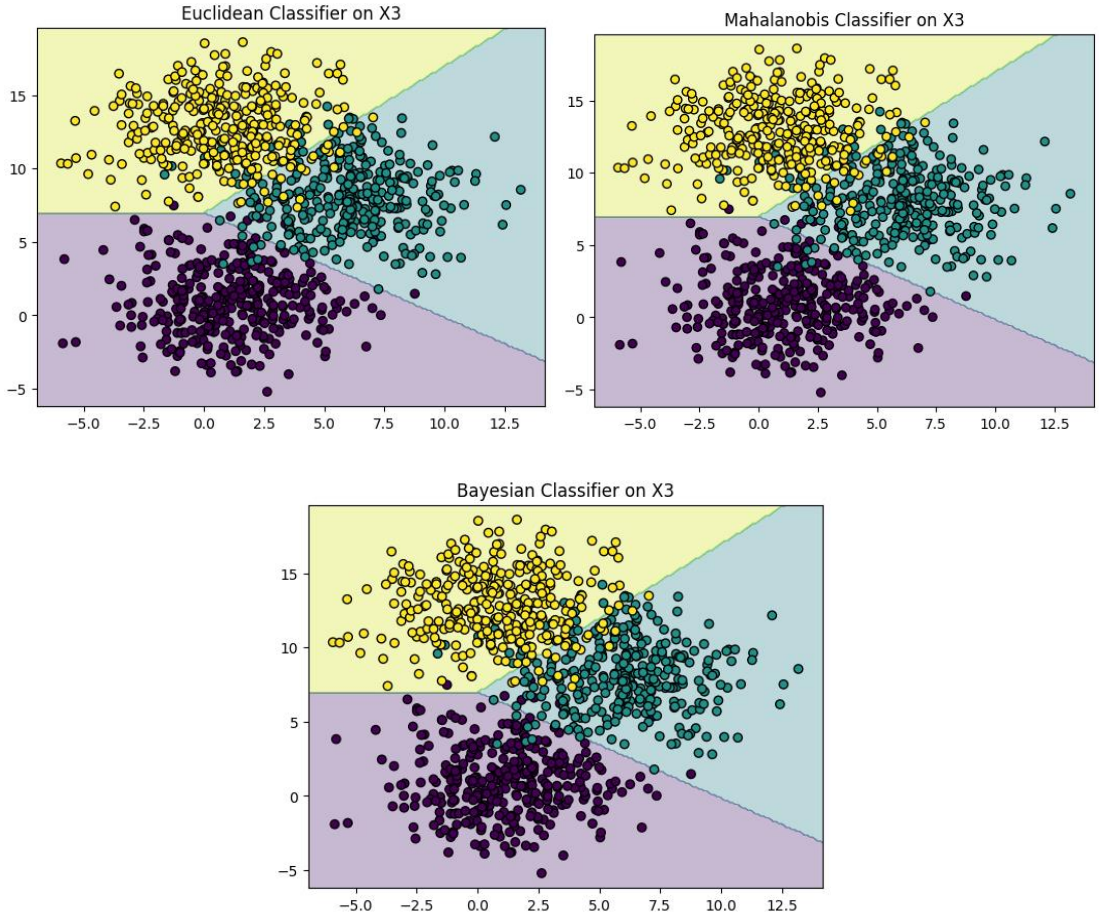
EXP3

- Three classifier's classification Error
  i. Euclidean Classifier Error: 0.0700

ii. Mahalonobis Classifier Error: 0.0700

iii. Bayesian Classifier Error: 0.0700

- Conclusion: Since the covariance of three distributions are the same and the covariance matrix of three distributions is an identity matrix * 6 (6I). All three classifier decision boundary is the same. Different from EXP1, the variance is bigger (4 from EXP1 vs. 6 from EXP3). This means that the **spread of data is wide** (from the plot below). Therefore, the classification error can be bigger than EXP1. Also, same as EXP1, the decision boundary is the same for all three classifier.
- Plot of three distributions' decision boundary and dataset:



EXP4

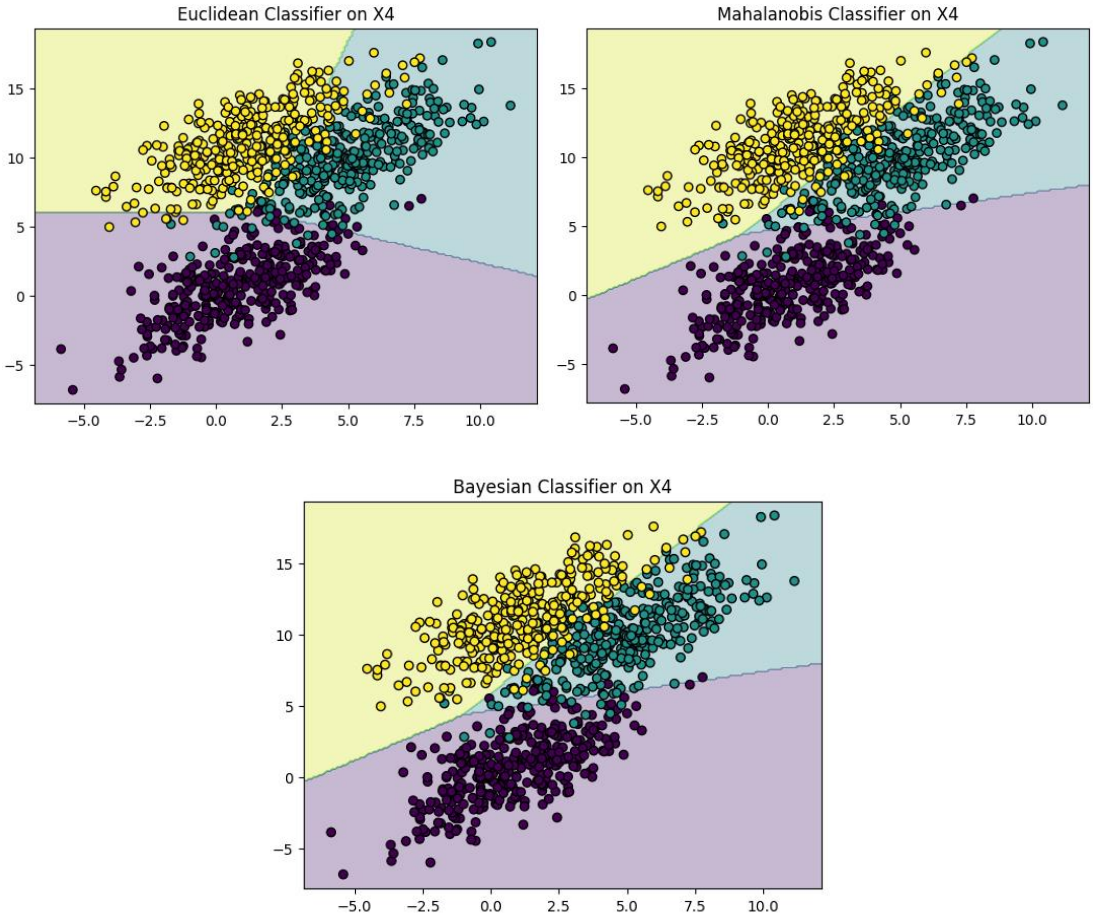
- Three classifier's classification Error

i. Euclidean Classifier Error: 0.1080

ii. Mahalonobis Classifier Error: 0.0790

iii. Bayesian Classifier Error: 0.0790

- Conclusion: Conclusion: The covariance matrix is the same for all three distributions. However, the covariance matrix is arbitrary, not a diagonal matrix. Therefore, same as EXP2, euclidean classifier does not have an optimal decision boundary. Also, we can observe that the variance is also bigger than EXP2. This means that the data spread wider than EXP2. Therefore, the classification error can be bigger than EXP2.
- Plot of three distributions' decision boundary and dataset:

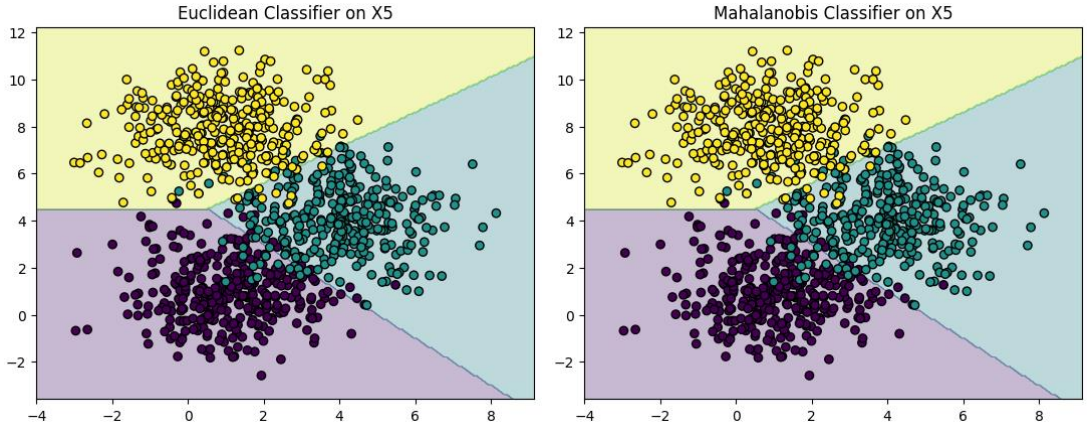


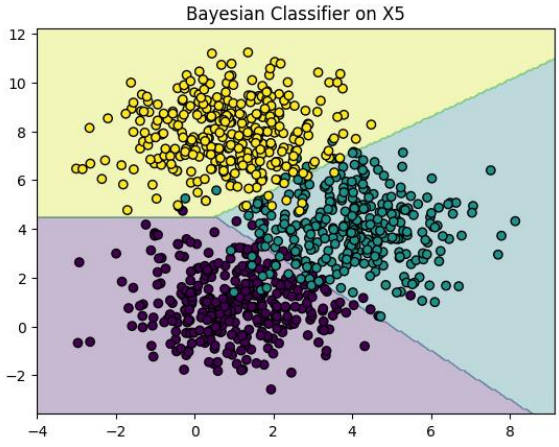
EXP5

- Three classifier's classification Error
  i. X5 Euclidean Classifier Error: 0.0560
  ii. X5 Mahalonobis Classifier Error: 0.0560
  iii. X5 Bayesian Classifier Error: 0.0560
  iv. X5' Euclidean Classifier Error: 0.0570
  v. X5' Mahalonobis Classifier Error: 0.0570
  vi. X5' Bayesian Classifier Error: 0.0320

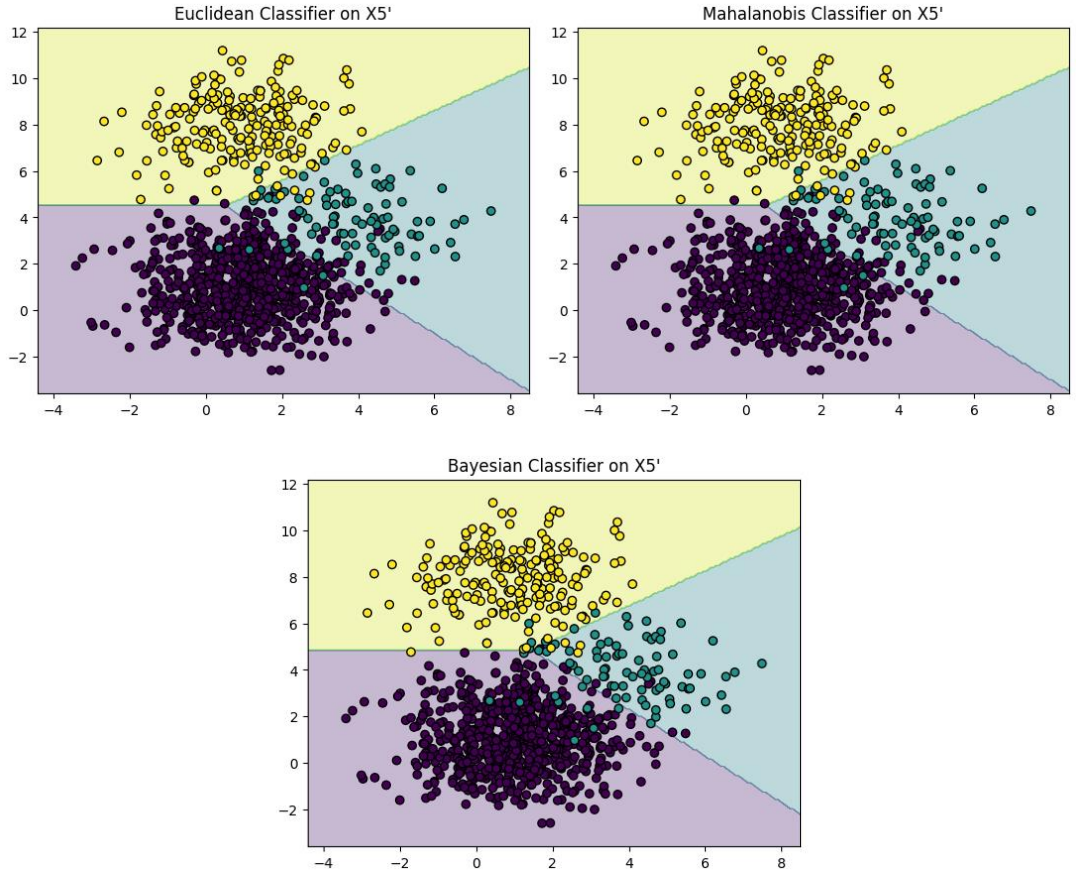
- Conclusion: The covariance matrix is the same for all three distributions and the covariance matrix of three distributions is an identity matrix * 2 (2I).
  For X5 data generation, prior probability is the same, three classifier has the same decision boundary.
  For X5' data generation, different from EXP1 to EXP4, the prior probability is not the same for all 3 classes. Euclidean classifier and mahalonobis cannot handle this kind of prior probability, only bayesian classifier has the optimal decision boundary.
- Plot of three distributions' decision boundary and dataset:

X5 decision boundary





X5' decision boundary



EXP6

- Three classifier's classification Error
  i. X6 Euclidean Classifier Error: 0.0340
  ii. X6 Mahalonobis Classifier Error: 0.0040
  iii. X6 Bayesian Classifier Error: 0.0040
  iv. X6' Euclidean Classifier Error: 0.0570
  v. X6' Mahalonobis Classifier Error: 0.0070
  vi. X6' Bayesian Classifier Error: 0.0060
- Conclusion: The covariance matrix is arbitrary, not a diagonal matrix. Different from EXP1 to EXP5, in EXP6, three distribution has their own covariance matrix.

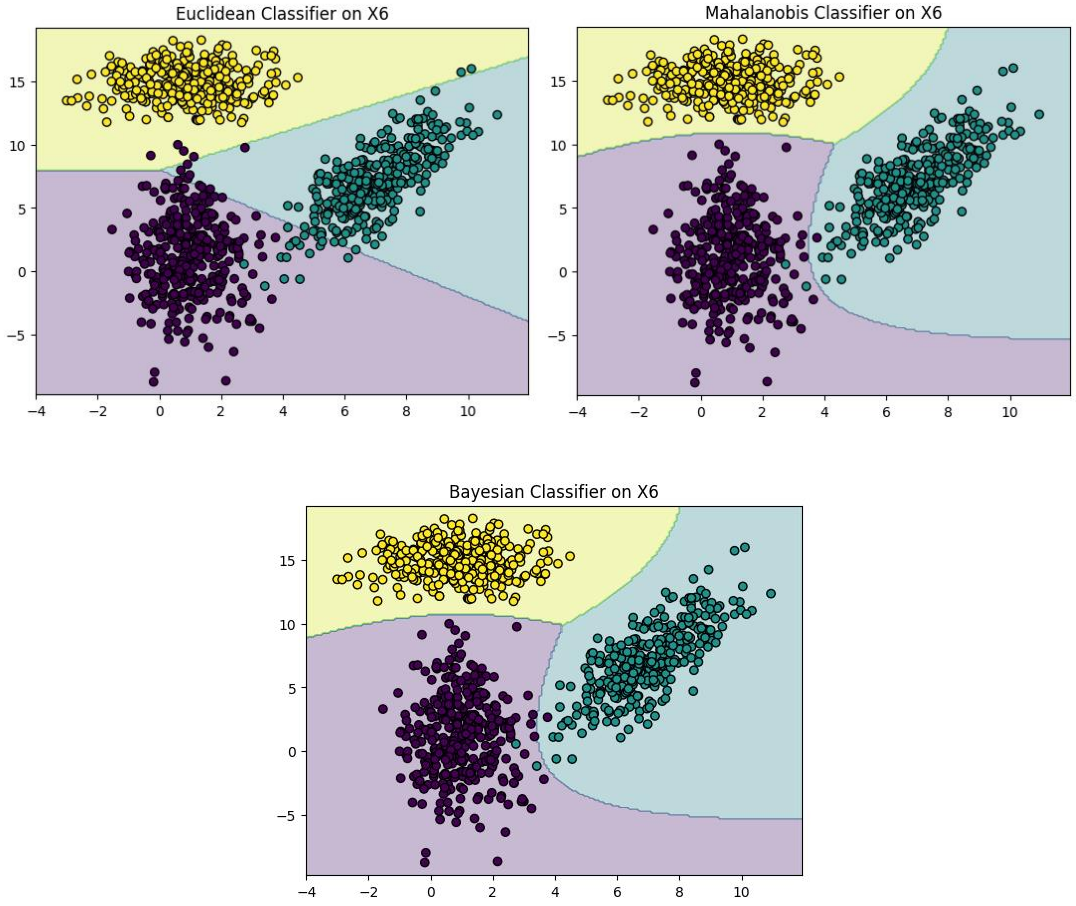
  For X6 data generation, prior probability is the same. Euclidean classifier's decision boundary is linear, whereas mahalonobis classifier and bayesian classifier's decision boundary is quadratic. Note that mahalonobis classifier and bayesian classifier has **different** decision boundaries. Only when the covariance matrices are the same for all three distributions and prior probabilities are the

same mahalonobis classifier and bayesian classifier has the same decision boundary,

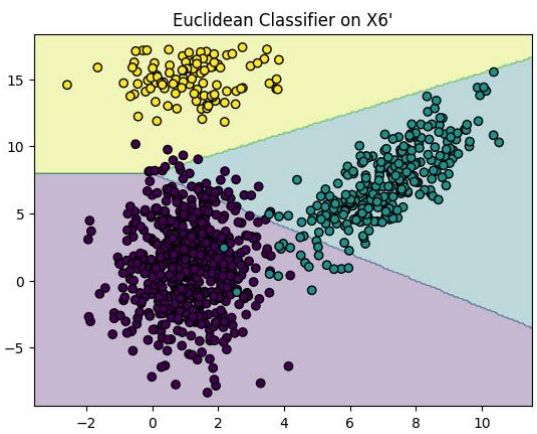
For X6' data generation, different from EXP1 to EXP4, the prior probability is not the same for all 3 classes. Euclidean classifier and mahalonobis cannot handle this kind of prior probability, only bayesian classifier has the optimal decision boundary. From the plot below, euclidean classifier's decision boundary is linear, whereas mahalonobis classifier and bayesian classifier's decision boundary is quadratic.

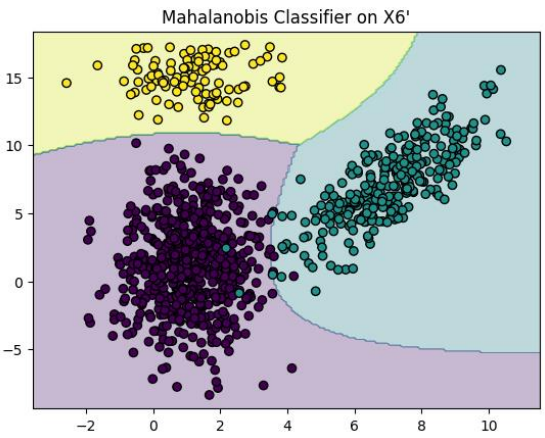
- Plot of three distributions' decision boundary and dataset:

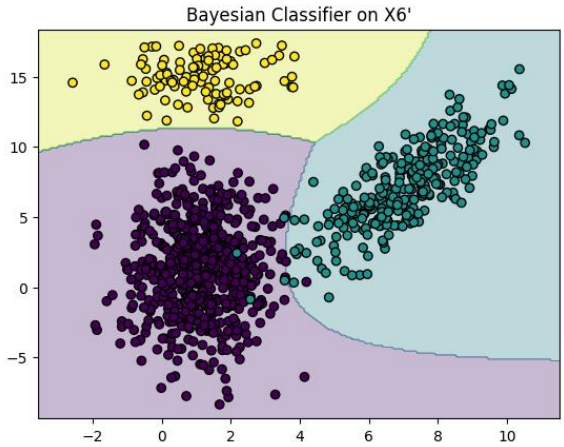
X6 decision boundary



X6' decision boundary


Euclidean Classifier on X6'
15
10
5
0
−5
−2
0
2
4
6
8
10


Mahalanobis Classifier on X6'
15
10
5
0
−5
−2
0
2
4
6
8
10


Bayesian Classifier on X6'
15
10
5
0
−5
−2
0
2
4
6
8
10