

Name : Rashika Khatri[®]

Father's Name: Mr. B. S. Khatri[®]

University Roll no.: 1918594

Class Roll no.: 23

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Sign: Rashika

$$\Downarrow u = \frac{4x^2y^3}{z^4}$$

$$x = y = z = 1$$

$$\delta u = \frac{\delta u}{\delta x} \delta x + \frac{\delta u}{\delta y} \delta y + \frac{\delta u}{\delta z} \delta z$$

$$\frac{4x^2(y^3)}{z^4} \delta x + \frac{4x^2 \cdot 3(y^2)}{z^4} \delta y + (-4x^2y^3z^{-5}) \delta z$$

$$\frac{8xy^3}{z^4} \delta x + \frac{12x^2y^2}{z^4} \delta y - \frac{16x^2y^3}{z^5} \delta z$$

Since, $\delta x, \delta y, \delta z$ may be (+ve) or (-ve). So.

$$(\delta u)_{\max} = \left| \frac{8xy^3 \cdot \delta x}{z^4} \right| + \left| \frac{12x^2y^2 \delta y}{z^4} \right| + \left| \frac{16x^2y^3 \delta z}{z^5} \right|$$

Putting values of $\delta x, \delta y, \delta z$ i.e., 0.01 and x, y, z i.e., 1

= .

$$8(0.01) + 12(0.01) + 16(0.01)$$

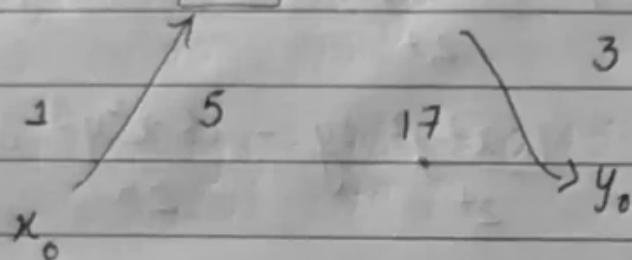
$$0.08 + 0.12 + 0.16$$

$$= 0.36$$

\leq Any -

2)

u	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-3	1	5				
-2	2	8	3	0		
-1	3	11	3	0		
0	4	14	3	0		



$$f(8.5) = ?$$

$$x_1 = 8.5 \quad x_0 = 4 \quad h = 1 \quad y_0 = 14$$

$$\frac{u = x - x_0}{h} = -0.5 \quad -1 < -0.5 < 0$$

$$[u = -0.5]$$

using Backward's interpolation formula

$$\begin{aligned}
 y_x = y_0 + u \Delta y_0 + \frac{(u+1)u}{2!} \Delta^2 y_{-1} + \frac{(u+1)(u+2)}{3!} u \Delta^3 y_{-2} \\
 + \frac{(u+2)(u+1)u(u-1)}{4!} \Delta^4 y_{-2}
 \end{aligned}$$

$$= 14 + 3(-0.5) + (-0.5+1)(-0.5) \cdot 0 + 0 + 0$$

^{2!}

$$14 - 1.5 = 12.5$$

= Ans

5>b> since regression lines always intersect at pt. (\bar{x}, \bar{y}) representing mean values of x and y

$$8x - 10y = -66 \quad - \textcircled{1}$$

$$40x - 18y = 214 \quad - \textcircled{2}$$

using eq. ① and ② we have,

$$82y = 544$$

$$\bar{y} = 17$$

$$\text{Then, } \bar{x} = (10\bar{y} - 66) / 8 = \frac{(10 \cdot 17 - 66)}{8} = 13$$

To find coefficient of correlation b/w x and y

$$8x - 10y = -66$$

$$-10y = 66 - 8x$$

$$y = \frac{66 + 8x}{10}$$

i.e.,

$$b_{yx} = \frac{8}{10} = 0.8$$

$$40x - 18y = 214$$

$$40x = 214 + 18y$$

$$x = \frac{214}{40} + \frac{18}{40} y$$

i.e., $b_{xy} = \frac{18}{40} = 0.45$

So, coefficient of correlation b/w x and y
is given by:

$$r = \sqrt{b_{xy} \times b_{yx}} = \sqrt{0.45 \times 0.80} = 0.60$$

For standard deviation of y

$$\sigma_y = \frac{b_{yx} \sigma_x}{r} = \frac{0.8 \times \sqrt{9}}{0.6} = 4$$

3) a) Difference table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.2	1.5095				
1.3	1.6984	0.1889	0.017	0.0021	
1.4	1.9043	0.2059	0.0191	0.0041	
1.5	2.1293	0.225	0.0253	0.0062	
1.6	2.3796	0.2503			

$$x_0 = 1.4 \quad h = 0.1 \quad x_0 = 1.4$$

$$y'_{1.4} = \frac{1}{h} \left(\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 \right)$$

$$y'_{1.4} = \frac{1}{0.1} \left(0.025 - \frac{1}{2} \times 0.0253 + \frac{1}{3} \times 0 - \frac{1}{4} \times 0 \right)$$

$$y'_{1.4} = 0.1235$$

$$y''_{1.4} = \frac{1}{0.61} \left(0.0253 - 0 + \frac{11}{12} \times 0 \right)$$

$$y''_{1.4} = 2.53$$

$$i) f'(a) = f(a, a+h) = \frac{f(a+h) - f(a)}{h}$$

$$f'(1.4) = f(1.4, 1.5) = \frac{f(1.4+0.1) - f(1.4)}{0.1}$$

$$= \frac{2.1293 - 1.9043}{0.1} = 2.250$$

$$E = |2.250 - 2.1235| = 0.1265$$

$$E = \frac{-1}{2} h f''(1.4) = \frac{-1}{2} \times 0.1 \times 2.53$$

$$= -0.1265$$

= equal
error

$$ii) f'(a) = f(a-h, a+h)$$

$$= \frac{2.1293 - 1.6984}{0.2}$$

$$f'(1.4) = 2.154500$$

$$E = |2.1545 - 2.1235| = 0.0310$$

$$E = -\frac{1}{2} h f''(1.4) = 0.0310$$

4) $\frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x} - y^2$

$$f(x, y) = \frac{1}{x^2} - \frac{y}{x} - y^2$$

$$x_0 = 1 \quad y_0 = -1$$

$$h = 0.5$$

$$x_1 = 1.5, y_1 = ?$$

We know that,

$$k_1 = h f(x_n, y_n)$$

Put $n=0$

$$k_1 = h f(x_0, y_0) = 0.5 \left(\frac{1}{x_0^2} - \frac{y_0}{x_0} - y_0^2 \right)^2$$

$$= 0.5 \left(\frac{1}{1^2} - \frac{(-1)}{1} - (-1)^2 \right)$$

$$= 0.5 (1+1-1) = 0.5$$

$$\begin{aligned}
 k_2 &= h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) \\
 &= 0.5 \left[\frac{1}{(x_0 + h/2)^2} - \frac{y_0 + 0.5/2}{x_0 + h/2} - (y_0 + k_1/2)^2 \right] \\
 &= 0.5 \left[\frac{1}{(1+0.25)^2} - \frac{(-0.75)}{1.25} - 0.5625 \right] \\
 &= 0.5 [0.64 + 0.6 - 0.5625]
 \end{aligned}$$

Since $y_{n+1} = y_n + k$, by runge kutta method,

$$\begin{aligned}
 y &= y_0 + k \\
 &= -1 + 0.334
 \end{aligned}$$

$$y_1 = 0.666$$

$$y(1.5) = -0.666$$

$$k_2 = 0.33875$$

$$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= 0.5 \left[\frac{1}{(x_0 + h/2)^2} - \frac{y_0 + k_2/2}{x_0 + h/2} - (y_0 + k_2/2)^2 \right]$$

Date. _____

Page No. _____

$$K_3 = 0.5 \left[\frac{1}{(1+0.25)^2} - (-0.831) - 0.690 \right] / 0.25$$

$$K_5 = 0.5 [0.64 + 0.665 - 0.690]$$

$$K_3 = 0.3075$$

$$K_4 = h f(x_0 + h, y_0 + K_3)$$

$$= 0.5 \left[\frac{1}{(x_0 + h)^2} - \frac{y_0 + K_3}{x_0 + h} - (y_0 + K_3)^2 \right]$$

$$K_4 = 0.5 [0.444 - (-0.462) - 0.480]$$

$$K_4 = 0.213$$

$$K = \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$

$$K = \frac{1}{6} (0.5 + 2 \times 0.339 + 2 \times 0.303 + 0.213)$$

$$K = 0.334$$