Arithmetic

SMS Math Club

September 21, 2023

Problem 0.1. What is $2+8$? What is $36+64$? What about $18325+81625$?

Problem 0.2. Still too easy? Do these without using scratch paper:

- $26+87+74$
- $(21 \div 5) \cdot (25 \div 7)$
- $99+199+299+399+4$
- $2 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 2$

Problem 0.3. Find $1+2+3+4+5$. Simple? Now add more! $1+2+\cdots+10$. Remember the formula? If not, try to rediscover it by pairing up terms.

Problem 0.4. We had a formula for $1+2+3+\cdots+n$. How would we find $5+6+7+\cdots+40$? Or $2+4+6+8+\ldots 50$?

Problem 0.5. Find the following:

$$\begin{array}{ccccccccc} 1 & + & 3 & + & 5 & & & & \\ 1 & + & 3 & + & 5 & + & 7 & & \\ 1 & + & 3 & + & 5 & + & 7 & + & 9 \end{array}$$

Do you see a pattern? Can you prove why this is the case?

Problem 0.6. Evaluate the following as fast as possible.

- $37 \cdot 7 + 7 \cdot 13$
- 1111111111^2
- $30303 \cdot 50505 \cdot 80808$
- $13^2 - 3^2 - 12^2$
- $73 \cdot 77 - 58 \cdot 62$

Problem 0.7. Consider the numbers

$$\begin{aligned} 1 \cdot 8 + 1 \\ 12 \cdot 8 + 2 \\ 123 \cdot 8 + 3 \\ 1234 \cdot 8 + 4 \\ \cdots \end{aligned}$$

Notice a pattern? Now find $123456789 \cdot 8 + 9$.

Problem 0.8. Find the following: $1 \cdot 2$, $2 \cdot 3$, $3 \cdot 4$, $4 \cdot 5$. Then, find 15^2, 25^2, 35^2, and 45^2. Look familiar? Now, find 405^2 in your head. Do you remember how to prove this pattern?

Problem 0.9. We found a formula for the sum of the first n positive integers: $1 + 2 + \cdots + (n-1) + n = \frac{n(n+1)}{2}$. Can you do the same for the sum of the first n perfect squares? How about perfect cubes? Hints: for sum of squares, notice that $n^2 = \underbrace{n + n + \cdots + n}_{n \text{ times}}$. Then, try to use our previous formula after expanding this out. For sum of cubes, the formula is pretty easy to spot. Once you have it, can you prove that it is true? Ask for help: we haven't taught how to do this yet!