

## Graded quiz on Sets, Number Line, Inequalities, Simplification, and Sigma Notation

Оценка последней работы: 100%

1. Let  $B = \{3, 5, 10, 11, 14\}$ . Is the following statement true or false:  $3 \notin B$

1 / 1 балл

☐ True

☒ False

Правильно

The symbol  $\notin$  stands for “is not an element of.” Since 3 is in an element of the set  $B$ , the given statement is not true.
2. Let  $A = \{1, 3, 5\}$  and  $B = \{3, 5, 10, 11, 14\}$ . Which of the following sets is equal to the union  $A \cup B$ ?

1 / 1 балл

☐  $\{1, 10, 18\}$

☐  $\{3, 5, 10, 11, 14\}$

☒  $\{1, 3, 5, 10, 11, 14\}$

☐  $\{1, 3, 5, 3, 5, 10, 11, 14\}$

Правильно

The union of two sets consists precisely of the elements that are in at least one of the two sets. That is precisely what is listed here.
3. How many real numbers are there between the integers 1 and 4?

1 / 1 балл

☐ None

☐ 2

☐ 4

☒ Infinitely many

Правильно

There are in fact infinitely many real numbers between any pair of distinct integers, or indeed any pair of distinct real numbers!
4. Suppose I tell you that  $x$  and  $y$  are two real numbers which make the statement  $x \geq y$  true. Which pair of numbers cannot be values for  $x$  and  $y$ ?

1 / 1 балл

☐  $x = 2$  and  $y = 1$

☐  $x = 10$  and  $y = 10$

☒  $x = -1$  and  $y = 0$

☐  $x = 5$  and  $y = 3.3$

Правильно

Recall that the statement  $x \geq y$  means that  $x$  is either equal to  $y$  or  $x$  is to the right of  $y$  on the real number line. Since  $-1$  is actually to the left of  $0$ , these cannot be values for  $x$  and  $y$ .
5. Suppose that  $z$  and  $w$  are two positive numbers with  $z < w$ . Which of the following inequalities is false?

1 / 1 балл

☐  $w - 7 > z - 7$

☐  $z + 3 < w + 3$

☒  $-5z < -5w$

☐  $-z > -w$

Правильно

If we start with  $z < w$  and multiply both sides by  $-5$ , we need to flip the less-than sign, which would give  $-5z > -5w$ . For an example, try  $z = 1$  and  $y = 2$  and see what happens!
6. Find the set of all  $x$  which solve the inequality  $-2x + 5 \leq 7$

1 / 1 балл

☐  $x \geq -6$

☐  $x = -1$

☐  $x \leq -1$

☒  $x \geq -1$

Правильно

Subtracting 5 from both sides of the given inequality gives  $-2x \leq 2$ . Then we divide both sides by  $-2$ , remembering to flip the inequality sign, and we obtain this answer
7. Which of the following real numbers is not in the closed interval  $[2, 3]$

1 / 1 балл

☒ 1

☐ 2.1

☐ 2

☐ 3

Правильно

Recall that the closed interval  $[2, 3]$  consists of all real numbers  $x$  which satisfy  $2 \leq x \leq 3$ . Since  $2 \leq 1$  is false,  $1 \notin [2, 3]$
8. Which of the following intervals represents the set of all solutions to:

1 / 1 балл

$-5 \leq x + 2 < 10?$

☐  $[-7, 8]$

☐  $[-5, 10)$

☒  $[-7, 8)$

☐  $(7, 8)$

Правильно

Subtracting 2 from all sides of the inequalities gives  $-7 \leq x < 8$ , and the set of all real numbers  $x$  which make that true is exactly the half-open interval  $[-7, 8)$ .
9. Which of the numbers below is equal to the following summation:  $\sum_{k=2}^5 2k$ ?

1 / 1 балл

☐ 14

☐ 10

☒ 28

☐ 4

Правильно

We compute  $\sum_{k=2}^5 2k = 4 + 6 + 8 + 10 = 28$ .
10. Suppose we already know that  $\sum_{k=1}^{20} k = 210$ . Which of the numbers below is equal to  $\sum_{k=1}^{20} 2k$ ?

1 / 1 балл

☐ 210

☐ 2

☐ 40

☒ 420

Правильно

By applying one of our Sigma notation simplification rules, we can rewrite the summation in question as  $2 \left( \sum_{k=1}^{20} k \right) = 2 \times 210 = 420$ .
11. Which of the numbers below is equal to the summation  $\sum_{i=2}^{10} 7$ ?

1 / 1 балл

☐ 7

☐ 48

☐ 70

☒ 63

Правильно

According to one of our Sigma notation simplification rules, this summation is just equal to 9 copies of the number 7 all added together, and so we get  $9 \cdot 7 = 63$ .
12. Which of the following numbers is the variance of the set  $Z = \{-2, 4, 7\}$ ?

1 / 1 балл

☐ 69

☒ 14

☐  $\sqrt{14}$

☐ 42

Правильно

To get the variance of a set of numbers, you need to perform four steps:  
  
First compute the mean (which is 3)  
  
Then calculate all the squared differences between the numbers in the set and this mean (here you get 25, 1, 16)  
  
Then add all these up (here you get 42)  
  
Then divide by the number of elements in the set (which is 3).  
  
Therefore, the variance of  $Z$   
  
$$= \frac{1}{3} [(-2 - 3)^2 + (4 - 3)^2 + (7 - 3)^2]$$
$$= \frac{1}{3} [25 + 1 + 16] = \frac{42}{3} = 14$$
13. Which of the following sets does *not* have zero variance? (hint: don't do any calculation here, just think!)

1 / 1 балл

☐  $\{1, 1, 1, 1\}$

☒  $\{2, 5, 9, 13\}$

☐  $\{5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5\}$

☐  $\{0, 0, 0, 0, 0, 0\}$

Правильно

Intuitively, the numbers in this set are spread out.