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Listex 6

3)a- $\begin{cases} T(1) = 1 \\ T(n) = 3T(n-1) + 1 \end{cases}$

1. $T(n) = 3T(n-1) + 1$
2. passo: $T(n-1)$
3. $T(n-1) \rightarrow 3T(n-2) + 1$
   $T(n-2) \rightarrow 3T(n-3) + 1$
4. $T(n) = 3(3(n-2)+1)+1$
   $= 3^2(n-2)+3+1$
   $= 3^2(n-2)+4$
   $T(n) = 3^2(3(n-3)+1)+4$
   $3^3(n-3)+9+4$
   $3^3(n-3)+13$
5. $3^i(n-i) + \sum_{k=0}^{k} 3^i$ / $3^i(n-i) + \frac{3^{i-1+1}-1}{2} = 3^i(n-i) + \frac{3^i-1}{2}$ //
   $3^i(n-i) + \frac{3^{k+1}-1}{3-1}$
6. $T(n-i) = T(1)$
   $n-i = 1$
   $i = n-1$
7. $3^{n-1}(n-1) + \frac{3^{n-1}-1}{2}$ | $3^{n-1} + \frac{3^{n-1}-1}{2}$ | $\frac{3^n-1}{2}$ /
   $T(1)$
8. $T(n) \in \Theta(3^n)$
9. base: $\frac{3^n-1}{2} = 1 \rightarrow \frac{3^1-1}{2} = \frac{2}{2} = 1$ // correto

indutiva: $3(3^{n-1}-1/2)+1$

$\frac{3 \cdot 3^n \cdot 1}{2} - 3 + 1$ | $\frac{3^n-3}{2} + 1$ | $\frac{3^n-3+2}{2}$ | $\frac{3^n-1}{2}$ // correto

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Listex 6

3) b- $\begin{cases} T(1)=1 \\ T(n)=4T(n/2)+n \end{cases}$

1. $T(n)=4T(n/2)+n$
2. passo $i=1$ $(n/2)$

3. $T(n/2)=4T(n/4)+n$
   $T(n/4)=4T(n/8)+n$
4. $T(n)=4(4(n/4)+n)+n$
   $= 4^2(n/4)+2n+n$
   $4^2(n/4)+3n \Rightarrow 4^2(n/2^2)+3n$
   $T(n)=4^2(4(n/8)+n)+3n$
   $= 4^3(n/8)+4n+3n$
   $4^3(n/8)+7n \Rightarrow 4^3(n/2^3)+7n$
5. $4^i(n/2^i)+\sum_{i=0}^{k} 2^i \cdot n$
   $4^i(n/2^i)+(2^{(i-1)+1}-1)\cdot n$
   $4^i(n/2^i)+(2^i-1)\cdot n$
6. $T(n/2^i)=T(1)$
   $n/2^i=1 \Rightarrow \frac{n}{2^i}=1 \Rightarrow 2^i=n \Rightarrow i=\lg n$
7. $4^i(n/2^i)+(2^i-1)\cdot n$
   $\Downarrow$
   $T(1)=(2^i-1)$

   $4^{\lg n}(n/2^{\lg n})+(2^{\lg n}-1)\cdot n$
   $4^{\lg n}+(2^{\lg n}-1)\cdot n$
   $2^{2\lg n}+(n-1)\cdot n \Rightarrow 2^{\lg n^2}+(n-1)\cdot n$
   $\Rightarrow n^2+n^2-n \Rightarrow 2n^2-n$
8. $T(n) \in O(n^2)$
9. base: $T(2n^2-n)=T(1) \Rightarrow 2.1^2-1 \Rightarrow 2-1=1$ correto
   indutivo: $4(2n^2/2-n/2)+n$
   $\frac{4(2n^2-2n)}{4}+n$
   $2n^2-2n+n$
   $2n^2-n$ correto