

Part 1

- a) There are two different result. First one is 1×4 matrix, second one is 4×1 matrix.
- b) When the symbol ";" is put after the command, output is not displayed and as a output none of the program file is displayed.
- c) If we want our commands to be executed fast, ";" can be put because when I put ";" command is executed faster.
- d) I get an error: Error using \otimes . Incorrect dimensions for matrix multiplication. Check that... Since I tried matrix multiplication with the matrices 1×4 and 1×4 , it gives an error.
- e) Result is $-10 \ -6 \ 28 \ -24$.
By adding dots, element wise multiplication is performed. Result does not change.
- f) Result is -12 . Matlab performed matrix multiplication. 1×4 matrix is multiplied by 4×1 matrix and the result is 1×1 matrix.
- g) Matlab has done matrix multiplication. Matrix X (4×1) is multiplied by matrix Y (1×4).
Result is matrix Z (4×4) given below.
- | | | | |
|-----|-----|-----|-----|
| -10 | 12 | -14 | 16 |
| 5 | -6 | 7 | -8 |
| 20 | -24 | 28 | -32 |
| 15 | -18 | 21 | -24 |
- h) This command creates a matrix whose elements starts from 1, ends at 2 and incremented by 0.02.
In the example matrix is the following
- 1 1.0200 1.0400 ... 1.9800 2.0000
- i) Result is 0.000071 seconds
- j) Elapsed time is 0.000520 seconds. (This may vary!)
- k) Elapsed time is 0.000059 seconds. Results may change but generally i and k takes equal or close amount of time to execute and they are faster than j . Since i is fast and it requires just one line, order of efficiency is the following $i > k > j$

l) Every element of vector x goes through the function $y = \cos(x/2)$ and scalar result y is replaced by their old values. This new vector consisted of scalar values is y . Matlab add t to all corresponding values of $\cos(x/2)$.

m) $\text{plot}(x)$ function plots the columns of x versus their index. $\text{plot}(t, x)$ function plots the columns of x versus corresponding values. $\text{plot}(x, t)$ plots t versus x , which is the inverse function of $x = \cos(2\pi * t)$.

n) The line style of plot is changing. In the first plot, curve is drawn by "-" symbol. In second one, curve is drawn by "+" symbol.

o) $\frac{1-0}{0.01} + 1 = 101$ time points are included. Graph for the following parts are at the end of the report.

p) By using the following command: `linspace(0, 2, 101)`

(since these parts do not ask questions, I omit them.)

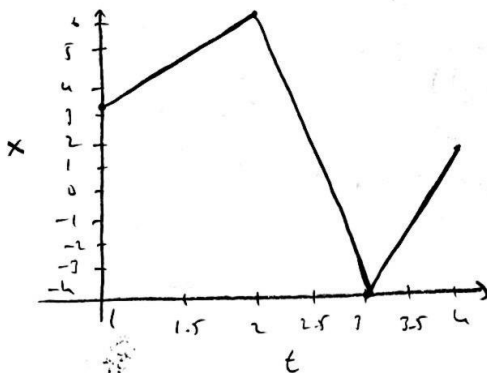
s) $\frac{1-0}{0.025} * 1 = 40 * 1 = 41$

(I omit the parts which does not ask question)

v) The choices of t given in part s is the best one because values of t are incremented by 0.025, which is the smallest increment among other choices of t , therefore, it gives the best approximation for continuous $x(t)$. Increment should be as small as possible because matlab calculates the function values with their corresponding t values. After doing that, matlab simply join the dots with straight lines. The more t values, the more curve seem smooth.

w) Plot command draw the points according to the function and the values of t . After that it fills the space between data points by drawing straight lines between them.

x)



y) The main difference between the two functions is the following: plot displays the continuous value of curve, stem displays the discrete values of the points on the curve.

Part 2

1)
a) Yes, I can hear both of them.

b, c, d) As the frequency increases, sound gets high-pitched and unpleasant.

2)
Matlab Code:

```
t = [0:1/8192:1];
```

```
f = 493.88;
```

```
a = 2;
```

```
x2 = (exp(-a*t)) .* cos(2*pi*f*t);
```

```
plot(t, x2);
```

```
sound(x2)
```

→ The effect of adding e^{-at} is damping. In the first case, signal goes continuously but in the second case, sound of the signal gets weaker and eventually it ends.

→ x_1 resembles piano more than x_2 and x_2 resembles flute more than x_1 .

→ As a increases, the sound gets shorter and after some point it becomes instantaneous and as a decreases, it lasts longer.

3)

→ Total frequency of the signal decreases and sound changes. After adding low-frequency cosine, signal sounds as if it is vibrating.

→ As f_1 increases, signal sounds more as if it is vibrating and its frequency increases.

$$x_3(t) = \cos(2\pi f_1 t) \cos(2\pi f_0 t)$$

$$= \frac{1}{2} \left[\cos(2\pi(f_0 + f_1)t) + \cos(2\pi(f_0 - f_1)t) \right]$$

$$\Rightarrow x_3(t) = \frac{1}{2} \cos(2\pi(f_0 + f_1)t) + \frac{1}{2} \cos(2\pi(f_0 - f_1)t)$$

Matlab Code:

```
t = [0:1/8192:1];
```

```
f0 = 880;
```

```
f1 = 2;
```

```
x3 = cos(2*pi*f1*t) .* cos(2*pi*f0*t);
```

```
plot(t, x3);
```

```
sound(x3);
```

Part 3

1 - $x_1(t) = \cos(2\pi f_0 t) \Rightarrow \phi(t) = f_0 t$

$\Rightarrow \frac{d\phi(t)}{dt} = f_0$ for all t .

2 - $x_2(t) = \cos(\pi a t^2) \Rightarrow \phi(t) = \frac{a t^2}{2}$

$\Rightarrow \frac{d\phi(t)}{dt} = a t$ for all t .

3 - $t=0, f_{\text{ins}}=0$. When $t=t_0$, $f_{\text{ins}}=60a$

4 - Random number generator $a = 1816$.

$0 \leq t \leq 1$ so

$0 \leq f_{\text{ins}} \leq 1816$

5 - Computation of x_2 :

$x_2 = \cos(\pi * 1816 * t * t);$

→ As a increases, signal gets high pitched and becomes more unpleasant because frequency of the signal increases.

→ As a decreases, since frequency decreases, signal gets low-pitched.

6 - Matlab Code:

$x_5 = \cos(2 * \pi * (-250 * t - 4000 * t + 800 * t + 6000));$

→ As time goes on frequency decreases because signal sounds low-pitched. This can be observed in the plot as well.

→ Instantaneous frequency is the following:

$\phi(t) = -250t^2 + 800t + 4000$ so $\frac{d\phi(t)}{dt} = f_{\text{ins}}(t) = -500t$

$f_{\text{ins}}(0) = 800\text{Hz}$, $f_{\text{ins}}(1) = 300\text{Hz}$, $f_{\text{ins}}(2) = -200\text{Hz}$

Part 4

Neither the volume nor the pitch of the sound changes as it is expected because frequency and the amplitude remains the same, only the phase is changing, which does not have an effect on the volume and the pitch of the sound.

Part 5:

$$x_1(t) = A_1 \sin(2\pi f_0 t + \phi_1)$$

$$A_1 > 0, A_2 > 0, A_3 > 0$$

$$x_2(t) = A_2 \sin(2\pi f_0 t + \phi_2)$$

$$x_3(t) = A_3 \cos(2\pi f_3 t + \phi_3)$$

Since the frequency of $x_1(t)$ and $x_2(t)$ are the same, phasor domain can be used to compute $x_3(t)$. The frequency of $x_3(t)$ should be equal to the frequency of $x_1(t)$ and $x_2(t)$ based on phasor analysis $\Rightarrow \boxed{f_0 = f_3}$

$$\Rightarrow \left. \begin{aligned} x_1(t) &= A_1 \cos(2\pi f_0 t + \phi_1 - \pi/2) \\ x_2(t) &= A_2 \cos(2\pi f_0 t + \phi_2 - \pi/2) \end{aligned} \right\} \xrightarrow{\text{phasor domain}} \begin{aligned} x_1 &= A_1 e^{j(\phi_1 - \pi/2)} \\ x_2 &= A_2 e^{j(\phi_2 - \pi/2)} \end{aligned}$$

$$x_3 = A_3 e^{j\phi_3}$$

$$\Rightarrow A_3 \cos \phi_3 + j A_3 \sin \phi_3 = \underbrace{A_1 \cos(\phi_1 - \pi/2)}_{\sin \phi_1} + \underbrace{j A_1 \sin(\phi_1 - \pi/2)}_{- \cos \phi_1} + \underbrace{A_2 \cos(\phi_2 - \pi/2)}_{\sin \phi_2} + \underbrace{j A_2 \sin(\phi_2 - \pi/2)}_{- \cos \phi_2}$$

$$\Rightarrow A_3 \cos \phi_3 + j A_3 \sin \phi_3 = (A_1 \sin \phi_1 + A_2 \sin \phi_2) - j(A_1 \cos \phi_1 + A_2 \cos \phi_2)$$

$$\Rightarrow \begin{cases} \text{① } A_3 \cos \phi_3 = A_1 \sin \phi_1 + A_2 \sin \phi_2 \\ \text{② } A_3 \sin \phi_3 = -(A_1 \cos \phi_1 + A_2 \cos \phi_2) \end{cases}$$

Take the square of both equation!

$$A_3^2 \cos^2 \phi_3 = A_1^2 \sin^2 \phi_1 + A_2^2 \sin^2 \phi_2 + 2 A_1 A_2 \sin \phi_1 \sin \phi_2$$

$$+ A_3^2 \sin^2 \phi_3 = A_1^2 \cos^2 \phi_1 + A_2^2 \cos^2 \phi_2 + 2 A_1 A_2 \cos \phi_1 \cos \phi_2$$

$$A_3^2 = A_1^2 + A_2^2 + 2 A_1 A_2 \cos(\phi_1 - \phi_2) \Rightarrow \boxed{A_3 = \sqrt{A_1^2 + A_2^2 + 2 A_1 A_2 \cos(\phi_1 - \phi_2)}} \quad A_3 > 0 \text{ - given in the question.}$$

$$\frac{\text{②}}{\text{①}} \rightarrow \tan \phi_3 = \frac{-(A_1 \cos \phi_1 + A_2 \cos \phi_2)}{A_1 \sin \phi_1 + A_2 \sin \phi_2}$$

For A_3 to be maximum,

$$\boxed{\phi_1 = \phi_2 + 2\pi k} \quad \text{where } k \in \mathbb{Z}$$

$$\boxed{\max \{A_3\} = A_1 + A_2}$$

For A_3 to be minimum,

$$\text{③ } \phi_1 - \phi_2 = \pi + 2\pi k$$

$$\Rightarrow \boxed{\phi_1 = \phi_2 + \pi + 2\pi k} \quad \text{where } k \in \mathbb{Z}$$

$$\boxed{\min \{A_3\} = A_1 - A_2}$$



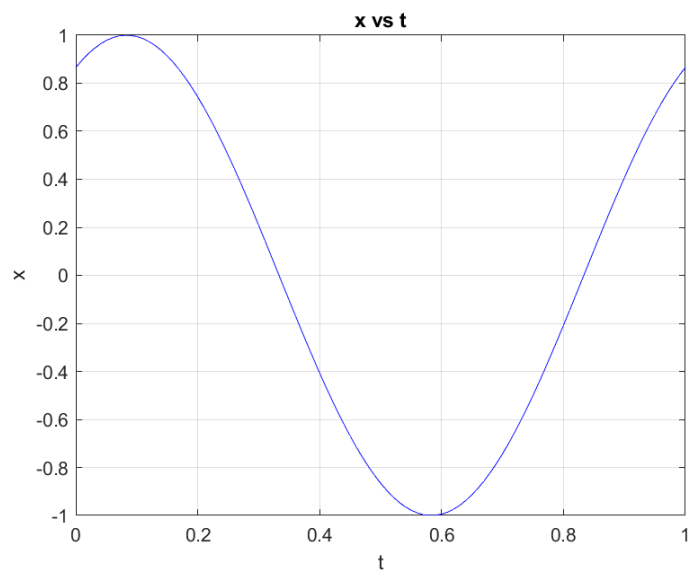
Scanned with CamScanner

(5)

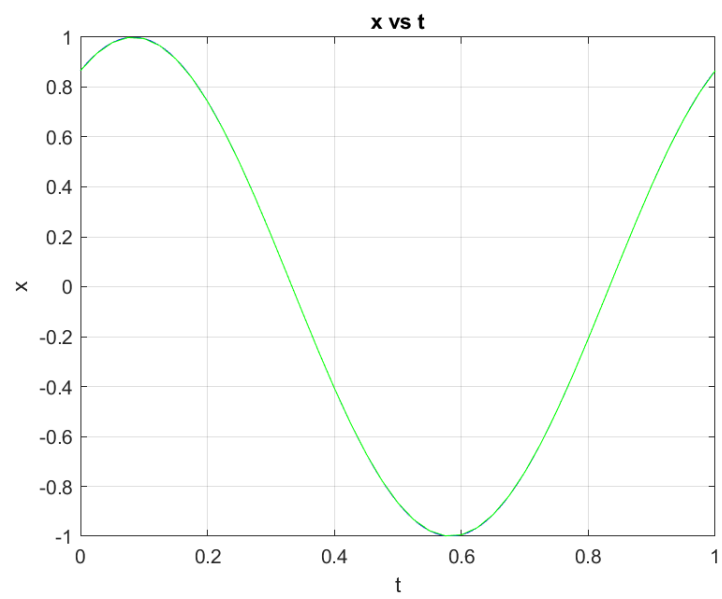
Graphs:

Part1:

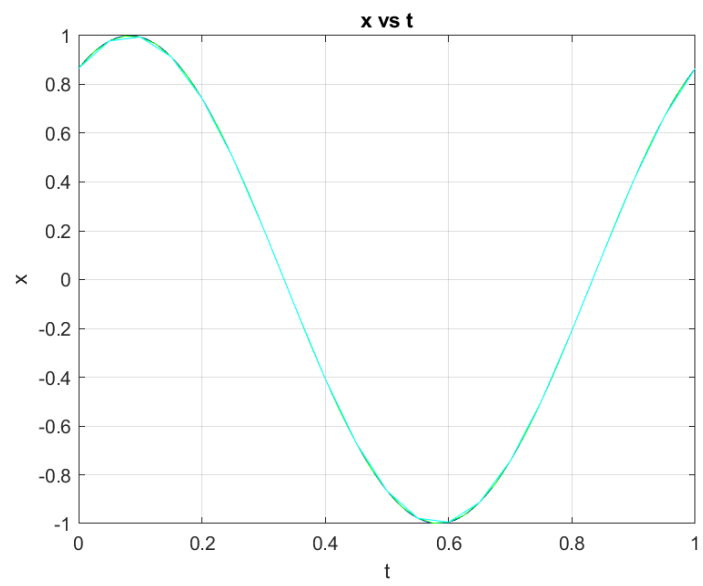
r)



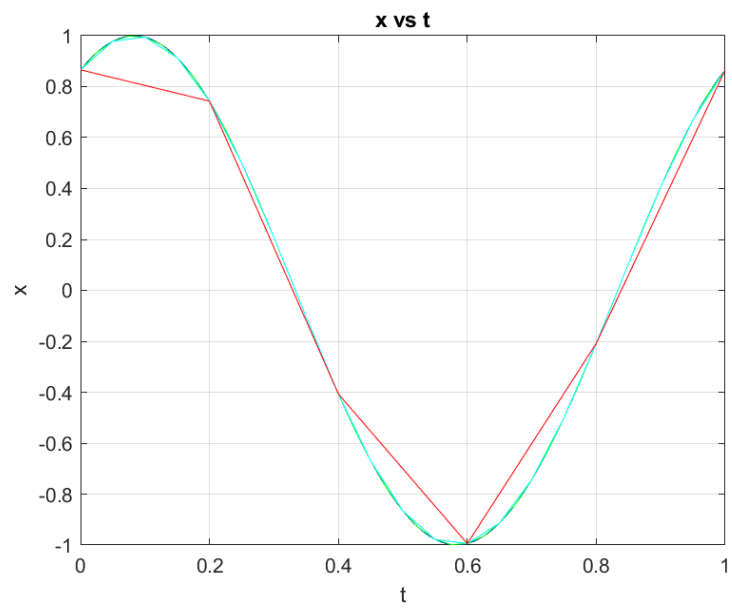
s)



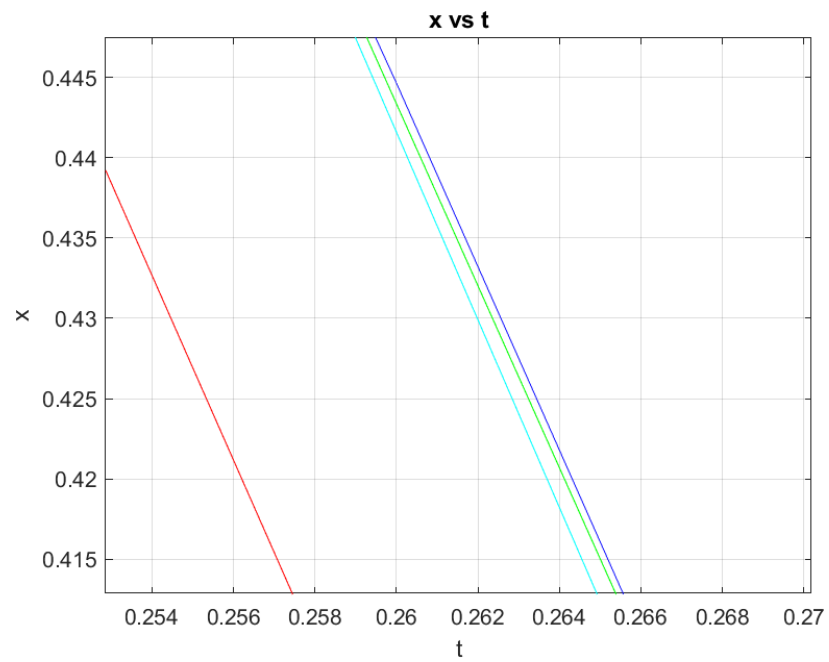
t)



u)



v) Close examination of the figure:



Each function of x can be seen separately.

Part 2:

b) The plot of the signal, which is listened.

