BILKENT UNIVERSITY ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

EEE - 321: Signals and Systems
Lab Assignment 7

Please carefully study this assignment before coming to the laboratory. You may begin working on it or even complete it if you wish, but you do not have to. There will be short quizzes both at the beginning and end of the lab session; these may contain conceptual, analytical, and Matlab-based questions. Within one week, complete the assignment in the form of a report and turn it in to the assistant. Some of the exercises will be performed by hand and others by using Matlab. What you should include in your report is indicated within the exercises.

Note: Along with this pdf file, you will find a .rar file containing two Matlab functions and a .mat file. Unzip the archive and place all files contained in it under the current directory of Matlab.

1 Part 1

In this lab, you will see an application of the concepts you learned in the signals and systems course to analog telecommunications. You will make some exercises about a common analog communication technique that is called **amplitude modulation**.

Amplitude modulation is one of the oldest techniques used for analog signal communication. In the EEE-211 Analog Electronics course that most of you took, you actually constructed a transceiver that transmits signals using amplitude modulation.

In amplitude modulation, we start with a so-called message signal $m(t)$, which is assumed to be a low frequency (sometimes called baseband) signal which as the name implies carries the information that we wish to transmit to the receiver side. For instance, when we listen to a radio station, the speech or music that we hear is the message signal. If $m(t)$ represents a speech signal, usually it has a bandwidth of about $8kHz$, that is, $M(\omega) \approx 0$ for $|\omega| > 2\pi 4000$ rad/sec where $M(\omega)$ denotes the Fourier transform of $m(t)$.

Almost every of us at some time asked ourselves why we do not transmit $m(t)$ directly but use schemes such as amplitude modulation. Direct transmission of $m(t)$ to the receiver side is impractical for two main reasons:

- Mostly the communication takes place over the air through electromagnetic waves propagating from the transmitter to the receiver. Both the transmitter and the receiver should be using an antenna to emit or sense radiation. A commonly known rule for antenna design says that if λ represents the wavelength of the central frequency of radiation, the antenna length should be about $\lambda/10$. If we wished to radiate $m(t)$ directly, assuming that the central frequency is 2 kHz (which corresponds to a wavelength of $\lambda = 150$ km), we would need an antenna whose length is approximately 15 km!

- Suppose $m_1(t)$ represents the message signal of RADIO STATION I, $m_2(t)$ represents the message signal of RADIO STATION II, etc. All these message signals will occupy the same frequency band ($|\omega| < 2\pi 4000$ rad/sec). Even if all the radio stations had the equipment to directly transmit their message signals through air, and we had a receiver to sense that radiation, it would be very hard (or impossible) for us to listen only to $m_1(t)$ but do not listen to $m_2(t)$. And obviously we would not want to listen to all the radio stations at the same time!

Due to these main and several other reasons, the usual practice in analog communications is to prepare a new signal $x(t)$ which CARRIES ALL THE INFORMATION within $m(t)$, but which is easier to transmit and select out of other signals. ONE TECHNIQUE of preparing that $x(t)$ is AMPLITUDE MODULATION. In the most basic form of amplitude modulation, $x(t)$ is prepared as:

$$x(t) = m(t)\cos(2\pi f_c t) \tag{1}$$

where f_c is much larger compared to $8kHz$ (bandwidth of the message signal). For instance, while constructing your TRC-10s, you took $f_c \approx 30$ MHz, which corresponds to a wavelength of 10 meters (so your antennas were about 1 meter). In general, f_c is chosen sufficiently high that $x(t)$ can be transmitted into air with a reasonable sized antenna. Regarding the issue of multiple message signals, different radio stations are assigned different carrier frequencies (for instance, RADIO STATION I can be assigned $f_{c_1} = 29$ MHz while RADIO STATION II can be assigned $f_{c_2} = 29.5$ MHz, etc). Using appropriate TUNING on the receiver side, it becomes quite easy to select between different radio channels. Indeed, in Part 2, you are going to implement an AM radio receiver in software. But let us first understand the basics of the amplitude modulation technique from a signals and systems perspective. Answer the following questions and include your answers to your report.

- a) Suppose $x(t) = \exp(j2\pi f_0 t)$. What is $X(\omega)$? ($X(\omega)$ is the Fourier transform of $x(t)$)
- b) Suppose $x(t) = \cos(2\pi f_0 t)$. What is $X(\omega)$?
- c) Suppose $x(t) = \exp(j2\pi f_0 t)$ and $y(t) = x(t)\exp(j2\pi f_1 t)$. What is $Y(\omega)$? Also write $Y(\omega)$ in terms of $X(\omega)$.
- d) Suppose $y(t) = x(t)\exp(j2\pi f_1 t)$ where $x(t)$ is an arbitrary signal. What is $Y(\omega)$ in terms of $X(\omega)$? (Hint: Consider $x(t)$ as a superposition of complex exponentials at different frequencies.)
- e) Suppose $y(t) = m(t)\cos(2\pi f_c t)$ as in amplitude modulation. What is $Y(\omega)$ in terms of $M(\omega)$?
- f) Suppose we have a radio station called STATION I. Let $m_1(t)$ denote the message signal of this station. Assume that the Fourier transform of $m_1(t)$ is denoted by $M_1(\omega)$ and it is given as:

  $$M_1(\omega) = \begin{cases} \frac{|\omega|}{2\pi 2500} & \text{if } |\omega| \le 2\pi 2500 \text{ rad/sec} \\ 0 & \text{otherwise} \end{cases}$$

  Suppose STATION I uses amplitude modulation as in Eq. 1. Let f_{c_1} denote the carrier frequency of STATION I. Suppose $f_{c_1} = 10$ KHz (kilohertz). Sketch with your hand $X_1(\omega)$ where $x_1(t) = m_1(t)\cos(2\pi f_{c_1} t)$. (In your sketch, do not forget to include the negative frequency components.)

- g) Suppose we have another radio station called STATION II which also uses amplitude modulation. Let $m_2(t)$ denote the message signal of this station. Assume that the Fourier transform of $m_2(t)$ is denoted by $M_2(\omega)$ and it is given as:

  $$M_2(\omega) = \begin{cases} 1 - \frac{|\omega|}{2\pi 3500} & \text{if } 0 < |\omega| \le 2\pi 3500 \text{ rad/sec} \\ 0 & \text{otherwise} \end{cases}$$

  Let f_{c_2} denote the carrier frequency of STATION II. Suppose $f_{c_2} = 30$ KHz. On the drawing you made in item f, sketch with your hand $X_2(\omega)$ where $x_2(t) = m_2(t)\cos(2\pi f_{c_2} t)$. Note that the final form of your drawing illustrates the Fourier transform of the signal received by a radio which has access to both of the stations. Let us call this signal as $r(t)$ so that $r(t) = x_1(t) + x_2(t)$

- h) Suppose your radio receives the signal $r(t) = x_1(t) + x_2(t)$ that you drew in item g. Suppose we modulate this signal with $\cos(2\pi f_{c_r} t)$. Let $d(t)$ denote the resulting signal such that $d(t) = r(t)\cos(2\pi f_{c_r} t)$. Assume that $f_{c_r} = 30$ KHz. Sketch the Fourier transform of $d(t)$.

- i) Suppose we have an ideal band pass filter that fully passes frequencies for which $0.10B_1 < |\omega| < 0.80B_2$ but fully blocks the frequencies outside this range. Suppose we apply this filter to $d(t)$. What value can we select for B_1 and B_2 so that we can listen to STATION II?

- j) Now suppose we wish to listen to STATION 1. Starting from $r(t)$, what should we do?

- k) Assuming that $f_{c_1} < f_{c_2}$, can we make $f_{c_2} - f_{c_1}$ smaller? What is the smallest value that this difference can take so that we can listen to any radio station we wish without interference from the other station?

An AM receiver basically implements the items h and i above. First, the received signal is modulated with a cosine signal. This cosine signal must have the frequency of the radio station we wish to listen to. The output of this stage is then low pass filtered so that only the original message signal remains.

2 Part 2

Unzip the file named **lab_7.rar**. You will see that it contains three files named **Part2.mat**, **FT.m** and **IFT.m**. Place all these three files under the current directory of Matlab. Clear everything in the Matlab workspace issuing the command **clear all**. Then type **load Part2**. Three vectors will appear in the workspace. **r** represents the densely taken samples of a continuous signal $r(t)$ that is received by an AM radio that operates in a region where there are three different stations. Hence, we have

$$r(t) = m_1(t)\cos(2\pi f_{c_1} t) + m_2(t)\cos(2\pi f_{c_2} t) + m_3(t)\cos(2\pi f_{c_3} t).$$

For instance, if we wish to listen to the first station, we should extract $m_1(t)$ from $r(t)$.

The samples of $r(t)$ are taken over the time grid specified by **t**. Plot $r(t)$ against t and put it in the report. You can also compute the Fourier transform $R(\omega)$ of $r(t)$ over the frequency grid specified by **omega** if you type **R=FT(r)**. Plot magnitude of $R(\omega)$ against ω and put it in the report. Examine this magnitude plot and find and write the center frequencies of each radio station. Call these frequencies as f_{c1}, f_{c2} and f_{c3}.

Also, try to listen to the signal $r(t)$ by typing **soundsc(r,262144)**. Can you hear anything? If you can't, give the reason why you can't hear anything.

Now you will write a function of the following form:

function []= ListenToMyRadio(r,fc,t,omega)

where

- **r**, **t** and **omega** are the already explained vectors that are contained in **Part2.mat**.
- **fc** is the central frequency of the radio station that we wish to listen to.

In your function, you will essentially do the following operations:

- Modulate $r(t)$ with $\cos(2\pi f_c t)$. Let $d(t) = r(t)\cos(2\pi f_c t)$.
- Apply an ideal low pass filter to $d(t)$. Take the bandwidth as $B = 2\pi 8200$ rad/sec. You will implement the low pass filter in the frequency domain. Do it as follows: Compute the Fourier transform $D(\omega)$ of $d(t)$ typing **D=FT(d)**. Recall that **FT** computes the Fourier transform over the grid specified by **omega**. After computing $D(\omega)$, set the parts outside $|\omega| < 2\pi 4100$ rad/sec to 0. Call the resulting function as $M(\omega)$. Then, compute the inverse Fourier transform of $M(\omega)$ typing **m=IFT(M)**.
- Finally, type **soundsc(m,262144)** so that you can listen to the message signal.

Include your code to the report.

Now, let fc=f_{c2}. Run your receiver for this center frequency and listen to it. Do you hear the message clearly? What is the message? Write it down in your report.

Now, try the other frequencies from f_c = 20 kHz up to 60 kHz. At which frequencies are the remaining two channels located? What are their messages? What happens when you do not tune properly into a channel (at f_c = 41 kHz, f_c = 42 kHz for instance.) Does the degradation in sound quality a familiar one, do you remember experiencing it before? Include your answers and comments to your report.

For the report, include plot statements in your function, and for f_c = 55 kHz, produce and include the plots of

- $r(t)$ and $|R(\omega)|$
- $d(t)$ and $|D(\omega)|$
- $m(t)$ and $|M(\omega)|$

3 Part 3

In Part 1 and Part 2, we explored the basics of amplitude modulation. In those parts, we considered the most basic form of amplitude modulation that is given in Eq. 1. That form is named **Double Side Band Amplitude Modulation**. It is abbreviated as **DSB AM** for short. Now, explain that in the DSB AM technique, to transmit a message signal that has a bandwidth B_m, we have to use an AM signal whose bandwidth is $2B_m$.

Thus, the DSB AM technique possesses an inefficiency regarding the usage of the frequency spectrum. In this part, we will explore a new amplitude modulation scheme, which is named **Single Side Band Amplitude Modulation**, and abbreviated by **SSB AM**. We will see that the SSB AM scheme uses the frequency spectrum more efficiently.

First perform the following exercises in Matlab.

- a) Reload the contents of **Part2.mat** to the Matlab workspace. Recall that **r** represents the received signal $r(t)$.
- b) Apply an ideal bandpass filter to $r(t)$ to select out the AM signal transmitted by the third station. Note that the bandpass filter should pass the frequencies for which $||\omega| - 2\pi 40000| \leq 2\pi 4100$ and block the other frequencies. You can easily implement this band pass filter in the frequency domain by generating the appropriate rectangular function. Name the resulting signal as $w(t)$. Include your code to your report.
- c) Plot the magnitude of the Fourier transform of $w(t)$. Include the plot to the report.
- d) Input $w(t)$ to your function **ListenToMyRadio** taking $f_c = 40000$ Hz. Is there any difference compared to what you heard in Part 2?
- e) Now apply a low pass filter to $w(t)$. The low pass filter should block the frequencies for which $|\omega| < 2\pi 40000$. Name the resulting signal as $q(t)$. Include the code to your report.
- f) Plot the magnitude of the Fourier transform of $q(t)$. Include the plot to the report. In this operation, did we distort the signal transmitted by the radio station?
- g) Input $q(t)$ to your function **ListenToMyRadio** taking $f_c = 40000$ Hz. Is there any difference compared to what you heard in Part 2 and in item d?
- h) Would there be any difference if the radio station transmitted $q(t)$ instead of $w(t)$?
- i) Compare the bandwidth of $q(t)$ to that of $w(t)$. How much bandwidth do we save when we use $q(t)$ instead of $w(t)$?

If instead of $w(t)$, the radio station transmitted $q(t)$, we would name the technique as **single side band amplitude modulation**. Now you should have some idea about the superiority of the SSB AM technique over DSB AM technique regarding the efficient usage of available frequency spectrum. Next, carry out the following exercises to see the reasoning behind SSB AM technique more clearly. Answer the following questions and include the answers to your report.

- j) Let $x(t)$ denote a signal whose Fourier transform is $X(\omega)$. Let $y(t) = x^*(t)$. Show that $Y(\omega) = X^*(-\omega)$.
- k) Using the result in item j, show that $x(t)$ is real valued **if and only if** $X(\omega) = X^*(-\omega)$.

- l) Suppose we are told that $x(t)$ is a real valued signal. Also suppose that we are given the values of $X(\omega)$ for $\omega \geq 0$. Is this information sufficient to determine $x(t)$?
- m) Examine the sketch you made in item g of Part 1 using the message signal $m_2(t)$. Based on the property in item k, is $m_2(t)$ a real valued signal?
- n) As in item g of Part 1, let $x_2(t) = m_2(t)\cos(2\pi f_{c_2} t)$ where $f_{c_2} = 30000$ Hz as before. Suppose to $x_2(t)$, we apply an ideal low pass filter which blocks the frequencies for which $|\omega| < 2\pi 25000$ rad/sec. Call the resulting output as $q_2(t)$. Sketch $Q_2(\omega)$ with your hand.
- o) Does $q_2(t)$ contain all the information contained within $m_2(t)$?
- p) What is the bandwidth of $q_2(t)$?
- q) Suppose a radio receives the signal $q_2(t)$. Devise a method so that we can extract $m_2(t)$ out of $q_2(t)$.