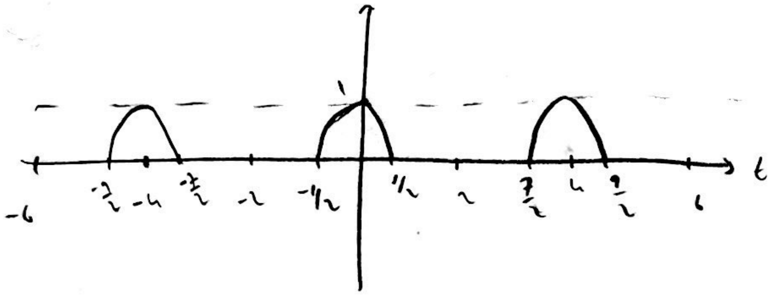
Part 2:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j\frac{2\pi k t}{T}} \quad , \quad X_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi k}{T}}\, dt$$

$$x(t) = \begin{cases} 1-4t^2 & \text{if } -\frac{w}{4} < t < \frac{w}{4}, \\ 0 & \text{otherwise}, \end{cases} \quad \text{where } w < T$$

$$T=4,\ w=2 \Rightarrow x(t) = \begin{cases} 1-4t^2 & \text{if } -\frac{1}{2} < t < \frac{1}{2}, \\ 0 & \text{otherwise}, \end{cases} \quad \text{sketch } x(t) \text{ over } -6 < t < 6$$



$$X_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi k}{T}t}\, dt = \frac{1}{T}\int_{-w/4}^{w/4} (1-4t^2)\, e^{-j\frac{2\pi k}{T}t}\, dt$$

$u = 1-4t^2 \Rightarrow du = -8t\,dt$

$dv = e^{-j\frac{2\pi k}{T}t}\,dt$

$v = \frac{-T}{j2\pi k} e^{-j\frac{2\pi k}{T}t}$

$$= \frac{1}{T}\left\{ (1-4t^2)\left(\frac{-T}{j2\pi k} e^{-j\frac{2\pi k}{T}t}\right)\Bigg|_{-w/4}^{w/4} - \frac{T}{j2\pi k}\int_{-w/4}^{w/4} 8t \cdot e^{-j\frac{2\pi k}{T}t}\, dt \right\}$$

$u = 8t \quad du = 8dt \quad e^{-j\frac{2\pi k}{T}t}dt = dv$

$$\frac{(\frac{w^2}{4}-1)T}{j2\pi k}\left[e^{-j\frac{w\pi k}{2T}} - e^{j\frac{w\pi k}{2T}}\right] \qquad \frac{8t \cdot T e^{-j\frac{2\pi k}{T}t}}{-j2\pi k}\Bigg|_{-w/4}^{w/4} + \frac{8T}{j2\pi k}\int_{-w/4}^{w/4} e^{-j\frac{2\pi k}{T}t}\, dt \qquad \frac{T}{-j2\pi k} e^{-j\frac{2\pi k}{T}t} = v$$

$$\frac{wT e^{-j\frac{w\pi k}{2T}}}{-j\pi k} - \left(\frac{-wT e^{j\frac{w\pi k}{2T}}}{-j\pi k}\right) = \left(\frac{-wT}{j\pi k}\right)\left(e^{-j\frac{w\pi k}{2T}} + e^{j\frac{w\pi k}{2T}}\right) \qquad \frac{8T^2}{4\pi^2 k^2}\left(e^{-j\frac{w\pi k}{2T}} - e^{j\frac{w\pi k}{2T}}\right)$$

$$= \frac{1}{j2\pi k}\left[\left(\frac{w^2}{4}-1\right)\underbrace{\left[e^{-j\frac{w\pi k}{2T}} - e^{j\frac{w\pi k}{2T}}\right]}_{-j2\sin\left(\frac{w\pi k}{2T}\right)} + \frac{wT}{j\pi k}\underbrace{\left(e^{-j\frac{w\pi k}{2T}} + e^{j\frac{w\pi k}{2T}}\right)}_{2\cos\left(\frac{w\pi k}{2T}\right)} - \frac{2T^2}{\pi^2 k^2}\underbrace{\left(e^{-j\frac{w\pi k}{2T}} - e^{j\frac{w\pi k}{2T}}\right)}_{-j2\sin\left(\frac{w\pi k}{2T}\right)}\right]$$

$$\Rightarrow \boxed{X_k = \left[-\frac{w^2}{4\pi k} + \frac{1}{\pi k} + \frac{2T^2}{\pi^3 k^3}\right]\sin\left(\frac{w\pi k}{2T}\right) + -\frac{wT}{\pi^2 k^2}\cos\left(\frac{w\pi k}{2T}\right)} \quad \text{for } k \neq 0$$

If $k=0$, $X_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{T}\int_{-w/4}^{w/4}(1-4t^2)\,dt = \frac{1}{T}\left[t - \frac{4t^3}{3}\right]_{-\frac{w}{4}}^{\frac{w}{4}} = \left[\frac{w}{2} - \frac{4}{3}\frac{w^3}{32}\right]\frac{1}{T}$

(2) will be used in code! $\leftarrow$ $\boxed{X_0 = \frac{1}{T}\left(\frac{w}{2} - \frac{w^3}{24}\right)}$

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This will be used in code :

$$\tilde{x}(t) = \sum_{k=-k}^{k} x_k e^{j\frac{2\pi k t}{T}} = \sum_{k=+1}^{k} x_k e^{j\frac{2\pi k t}{T}} + \sum_{k=1}^{+k} x_{-k} e^{-j\frac{2\pi k t}{T}} + x_0$$

Part 3:

Real Part:

Max Value : 1.0073

Min Value : -0.0728

Imaginary Part

Max Value: 5.5511×10^{-16}

Min Value: -5.5511×10^{-16}

* When they are compared, maximum value of real part is much greater than maximum value of imaginary part. And, minimum value of real part is much smaller than imaginary part. In fact, imaginary part can be ignored when it is compared to real part.

→ If I try $\sin(\frac{\pi}{6}) - 0.5$ in MATLAB, I get $\sin\left(\frac{\pi}{6}\right) - 0.5 = -5.5511 \times 10^{-17}$

since matlab calculates $\sin(\pi/6)$ approximately, small error is generated.

Second Part (Dn Part) :

* As K gets larger, approximation of $\tilde{x}(t)$ becomes better. Plot becomes more sharp rather than being smooth. This is because of the number of signals. In ideal case, when $k = \infty$, we acquire $x(t)$ so as we increase k, $\tilde{x}(t)$ gets closer to ideal $x(t)$ so $\tilde{x}(t)$'s success also increases.

Yes, I observe oscillations and irregularities in the neighborhoods of discontinuities. This is because of the Fourier series expansion. The value of discontinuities are average of its two side so oscillations arise at these points.

! Corresponding Plots for This Part are Given Below !

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