LAB 03

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Part 1

1.1 DTMF Transmitter:

Yep, it is what I hear when I dial my phone number in analog phones. As the transmitted digit changes, since every number is transmitted in different frequencies, the sound I hear changes as well.

1.2 DTMF Receiver:

a) $x(t) = e^{j(2\pi f_0 t) + w_0}$

$$X(jw) = \int_{-\infty}^{+\infty} e^{j(2\pi f_0 t)} \cdot e^{w_0} e^{-jwt}\, dt = e^{w_0} \underbrace{\int_{-\infty}^{+\infty} e^{j(w_0 - w)t}\, dt}_{2\pi\delta(w - w_0)} = \boxed{e^{w_0}\, 2\pi \delta(w - w_0)}$$

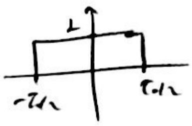
b) $x(t) = \cos(2\pi f_0 t) + \sin(2\pi f_0 t)$ $\qquad e^{j2\pi f_0} \xrightarrow{FS} 2\pi\delta(w - w_0)$

$$= \frac{e^{jw_0 t} + e^{-jw_0 t}}{2} + \frac{e^{+jw_0 t} - e^{-jw_0 t}}{j2} = \frac{e^{jw_0 t} + e^{-jw_0 t}}{2} + j\left(\frac{e^{-jw_0 t} - e^{jw_0 t}}{2}\right)$$

$$= \left(\frac{1-j}{2}\right) e^{jw_0 t} + \left(\frac{1+j}{2}\right) e^{-jw_0 t} \xrightarrow{FS} \frac{(1-j)}{2} 2\pi\delta(w - w_0) + \frac{(1+j)}{2} 2\pi\delta(w + w_0)$$

$$= \boxed{\pi\left(\delta(w - w_0) + \delta(w + w_0)\right) + j\pi\left(\delta(w + w_0) - \delta(w - w_0)\right)}$$

c) $x(t) = \text{rect}\left(\frac{t}{T_0}\right) \rightarrow$



$\downarrow FS$

$$X(jw) = \int_{-T_0/2}^{T_0/2} 1 \cdot e^{-jwt}\, dt = \left.\frac{e^{-jwt}}{-jw}\right|_{-T_0/2}^{T_0/2} = \frac{1}{-jw}\left(e^{-j\frac{wT_0}{2}} - e^{j\frac{wT_0}{2}}\right) = \frac{-j2\sin\left(\frac{wT_0}{2}\right)}{-jw} = \boxed{\frac{2}{w}\sin\left(\frac{wT_0}{2}\right)}$$

d) $x(t) = e^{j2\pi f_0 t} \underbrace{\text{rect}\left(\frac{t}{T_0}\right)}_{\frac{2}{w}\sin\left(\frac{wT_0}{2}\right)} \Rightarrow \boxed{X(jw) = \frac{2}{w - w_0}\sin\left(\frac{(w - w_0)T_0}{2}\right)}$

$\boxed{2\pi f_0 = w_0}$

$\uparrow$ Frequency shifting property

e) $x(t) = \cos(2\pi f_0 t)\,\text{rect}\left(\frac{t}{T_0}\right) = \underbrace{\frac{e^{j\omega_0 t}}{2}\text{rect}\left(\frac{t}{T_0}\right)}_{\downarrow FS} + \underbrace{\frac{e^{-j\omega_0 t}}{2}\text{rect}\left(\frac{t}{T_0}\right)}_{\downarrow FS}$

$\omega_0 = 2\pi f_0$

$\cos(2\pi f_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

we know: $\text{rect}\left(\frac{t}{T_0}\right) \xrightarrow{FS} \frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right)$ $\quad \frac{1}{2}\left(\frac{2}{\omega-\omega_0}\right)\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right)$ $\quad \frac{1}{2}\left(\frac{2}{\omega+\omega_0}\right)\sin\left(\frac{(\omega+\omega_0)T_0}{2}\right)$

$$\boxed{X(j\omega) = \left(\frac{1}{\omega-\omega_0}\right)\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right) + \frac{1}{(\omega+\omega_0)}\sin\left(\frac{(\omega+\omega_0)T_0}{2}\right)}$$

Note: Frequency Shifting Property is used.

f) $x(t) = \text{rect}\left(\frac{t-t_0}{T_0}\right) \xrightarrow{FS}$ $\boxed{X(j\omega) = \frac{2e^{-j\omega t_0}}{\omega}\sin\left(\frac{\omega T_0}{2}\right)}$

Time shifting

g) $x(t) = e^{j\omega_0 t}\,\text{rect}\left(\frac{t-t_0}{T_0}\right) \xrightarrow{FS}$ $\boxed{\frac{2e^{-j(\omega-\omega_0)t_0}}{(\omega-\omega_0)} \cdot \sin\left(\frac{(\omega-\omega_0)T_0}{2}\right)}$

Time Shifting and Frequency shifting!

h) $x(t) = \sin(2\pi f_0 t)\,\text{rect}\left(\frac{t-t_0}{T_0}\right) \Rightarrow x(t) = \frac{e^{j\omega_0 t}}{j2}\text{rect}\left(\frac{t-t_0}{T_0}\right) - \frac{e^{-j\omega_0 t}}{j2}\text{rect}\left(\frac{t-t_0}{T_0}\right)$

$\sin(2\pi f_0 t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{j2}$

$$X(j\omega) = \frac{e^{-j\omega t_0}}{j2}\left[\frac{2}{(\omega-\omega_0)}\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right) - \frac{2}{(\omega+\omega_0)}\sin\left(\frac{(\omega+\omega_0)T_0}{2}\right)\right]$$

$$\boxed{X(j\omega) = \frac{e^{-j\omega t_0}}{j}\left[\frac{\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right)}{(\omega-\omega_0)} - \frac{\sin\left(\frac{(\omega+\omega_0)T_0}{2}\right)}{(\omega+\omega_0)}\right]}$$

Scanned with CamScanner

Peaks: $\frac{2\pi}{T} = w \Rightarrow f = \frac{w}{2\pi}$

x	Conversion to Hz	f (Hz)
± 9290	$\longrightarrow$	1477
± 8394	$\longrightarrow$	1336
± 7596	$\longrightarrow$	1209
± 5912	$\longrightarrow$	941
± 5350	$\longrightarrow$	852
± 4379	$\longrightarrow$	697

- Yes, they are the frequencies of the number used by the DTMF tranreceivers.
- Only from this figure, it cannot be understood which number is dialed, because they are not in order, just their frequencies are seen.

$$x_1(t) \Rightarrow \begin{cases} x(t) & \text{for } 0 \le t \le 0.5 \\ 0 & \text{for } 0.5 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Let $r(t)$ be the rectangular signal such that

$r(t)\,x(t) = x_1(t)$

so $r(t) = \begin{cases} 1 & \text{for } 0 \le t \le 0.25 \\ 0 & \text{otherwise} \end{cases}$

Peaks for X_1:

x		f (Hz)
± 8394	$\longrightarrow$	1336
± 5912	$\longrightarrow$	941

$\Rightarrow$ First Digit is 0

Peaks for X_2:

x		f(Hz)
± 9280	$\longrightarrow$	1477
± 5350	$\longrightarrow$	852

$\Rightarrow$ Second Digit is 9

Peaks for X_3:

x		f(Hz)
± 7596	$\longrightarrow$	1209
± 4379	$\longrightarrow$	697

$\Rightarrow$ Third Digit is 1

Peaks for X_4:

x		f(Hz)
± 9280	$\longrightarrow$	1477
± 4379	$\longrightarrow$	697

$\Rightarrow$ Fourth Digit is 3

- This time I can understand which digit is dialed first, secon, third and the last

$\rightarrow$ I shift the square wave $+0.25$ s right at each step so that I only take one digit of the Fourier Transform $X(jw)$ of the whole signal.

$\rightarrow$ Since $X(w) = X_1(w) + X_2(w) + X_3(w) + X_4(w)$, when I looked at $X(w)$, all of the Fourier Transform of digit signals are inside X so they can not be distinguished as a digit. However, since X_1, X_2, X_3 and X_4 includes only two different frequencies, I was able to see the frequencies separately and decide which digit is dialed at the given t interval which is determined by $r(t)$.

Part 2:

My recording is "Hello guys, my name is Berk, my ID is 2170310 and this is signals and systems course; good luck!"

Matlab Code:

```
recObj = audiorecorder(8192,8,1);
disp("Start Recording...");
recordblocking(recObj,10);
disp("Stop Recording...");
x = transpose(getaudiodata(recObj));
sound(x,8192);
```

$$y(t) = x(t) + \sum_{i=1}^{m} A_i\, x(t-t_i)$$

a) $y(t) = x(t) * h(t) \xrightarrow{FS} Y(j\omega) = X(j\omega)H(j\omega) \Rightarrow H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)}$

$\Rightarrow y(t) \xrightarrow{FS} Y(j\omega) = X(j\omega) + \sum_{i=1}^{m} A_i e^{-j\omega t_i} \cdot X(j\omega)$

$\Rightarrow H(j\omega) = 1 + \sum_{i=1}^{m} A_i e^{-j\omega t_i} \Rightarrow \boxed{h(t) = \delta(t) + \sum_{i=1}^{m} A_i\, \delta(t-t_i)}$

$1 \xrightarrow{FS^{-1}} \delta(t)$, $\quad e^{-j\omega t_i} \rightarrow \delta(t-t_i)$

b) As found in part a, $\boxed{H(j\omega) = 1 + \sum_{i=1}^{m} A_i e^{-j\omega t_i}}$

c) Relation between $X(\omega)$, $Y(\omega)$ and $H(\omega)$ is $\boxed{Y(\omega) = X(\omega)H(\omega)}$ + convolution property

d) $X(\omega) = \dfrac{Y(\omega)}{H(\omega)}$ by the convolution property!

- The sound I hear is the sound I originally heard but with additional delayed echos. There were 5 delayed signals with scaled amplitudes A_i. Sound was so crowded that after a while, I couldn't understand what I said.

- When I listen $x_e(t)$, there were still some delays but it was closer to the original record $x(t)$ than $y(t)$. I was able to understand what I said. $x_e(t)$ is different than $x(t)$, which is expected because $x_e(t)$ is estimation of $x(t)$. In fact, from the plots of $y(t)$ and $x_e(t)$, how $x_e(t)$ reduce some of the noises (additional echoes) can be seen clearly.