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21703190
Lab 03

Part 1.1 Matlab Code:

```
%1.1 DTMF Transmitter
phonenumber = [0 5 3 1 6 7 3 8 4 1 9];
x = DTMFTRA(phonenumber);
soundsc(x,8192);


function [x] = DTMFTRA (number)
    %matrix of frequencies 1,2,3...9,0
    freqs =
[697,1209;697,1336;697,1477;770,1209;770,1336;770,1477;852,1209;
852,1336;852,1477;941,1336];
    x = [] %creating an empty array
    for index = 1:length(number)
        starttime = (index-1)*0.25;
        t = [starttime:1/8192:starttime+0.25]%time interval with
1/8192 sample freq
        digit = number(index); %digit needs to be transmitted
        if digit == 0
            a=freqs(10,1);b=freqs(10,2);
        else
            a=freqs(digit,1);b=freqs(digit,2);
        end
        xc = cos(2*pi*a*t) + cos(2*pi*b*t); %create array in the
given time interval
        x = cat(2,x,xc(2:length(xc)));%add the array for given
time interval to final array
    end
end
```

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Part 1

1.1 DTMF Transmitter:

Yep, it is what I hear when I dial my phone number in analog phones. Since every number is transmitted in different frequencies, as the transmitted digit changes, the sound I hear changes as well.

1.2 DTMF Receiver:

a) $x(t) = e^{j(2\pi f_0 t) + w_0}$

$$X(jw) = \int_{-\infty}^{+\infty} e^{j(2\pi f_0 t)} \cdot e^{w_0} e^{-jwt}\, dt = e^{w_0} \underbrace{\int_{-\infty}^{+\infty} e^{j(w_0 - w)t}\, dt}_{2\pi \delta(w - w_0)} = \boxed{e^{w_0} 2\pi \delta(w - w_0)}$$

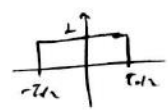
b) $x(t) = \cos(2\pi f_0 t) + \sin(2\pi f_0 t)$ $\qquad e^{j2\pi f_0} \xrightarrow{FS} 2\pi \delta(w - w_0)$

$$= \frac{e^{jw_0 t} + e^{-jw_0 t}}{2} + \frac{e^{+jw_0 t} - e^{-jw_0 t}}{j2} = \frac{e^{jw_0 t} + e^{-jw_0 t}}{2} + j\left(\frac{e^{-jw_0 t} - e^{jw_0 t}}{2}\right)$$

$$= \left(\frac{1-j}{2}\right) e^{jw_0 t} + \left(\frac{1+j}{2}\right) e^{-jw_0 t} \xrightarrow{FS} \frac{(1-j)}{2} 2\pi \delta(w - w_0) + \frac{(1+j)}{2} 2\pi \delta(w + w_0)$$

$$= \boxed{\pi\left(\delta(w - w_0) + \delta(w + w_0)\right) + j\pi\left(\delta(w + w_0) - \delta(w - w_0)\right)}$$

c) $x(t) = rect\left(\frac{t}{T_0}\right) \rightarrow$



$\downarrow FT$

$$X(jw) = \int_{-T_0/2}^{T_0/2} 1 \cdot e^{-jwt}\, dt = \left.\frac{e^{-jwt}}{-jw}\right|_{-T_0/2}^{T_0/2} = \frac{1}{-jw}\left(e^{-j\frac{wT_0}{2}} - e^{j\frac{wT_0}{2}}\right) = \frac{-j2\sin\left(\frac{wT_0}{2}\right)}{-jw} = \boxed{\frac{2}{w}\sin\left(\frac{wT_0}{2}\right)}$$

d) $x(t) = e^{j2\pi f_0 t} \underbrace{rect\left(\frac{t}{T_0}\right)}_{\frac{2}{w}\sin\left(\frac{wT_0}{2}\right)} \Rightarrow \boxed{X(jw) = \frac{2}{w - w_0}\sin\left(\frac{(w - w_0)T_0}{2}\right)}$

$\boxed{2\pi f_0 = w_0}$

Frequency shifting property

e) $x(t) = \cos(2\pi f_0 t)\,\mathrm{rect}\left(\frac{t}{T_0}\right) = \underbrace{\frac{e^{j\omega_0 t}}{2}\mathrm{rect}\left(\frac{t}{T_0}\right)}_{\downarrow FT} + \underbrace{\frac{e^{-j\omega_0 t}}{2}\mathrm{rect}\left(\frac{t}{T_0}\right)}_{\downarrow FT}$

$\omega_0 = 2\pi f_0$

$\cos(2\pi f_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

we know: $\mathrm{rect}\left(\frac{t}{T_0}\right) \xrightarrow{FT} \frac{2}{\omega}\sin\left(\frac{\omega T_0}{2}\right)$ $\quad \frac{1}{2}\left(\frac{2}{\omega-\omega_0}\right)\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right) \quad \frac{1}{2}\left(\frac{2}{\omega+\omega_0}\right)\sin\left(\frac{(\omega+\omega_0)T_0}{2}\right)$

$$\boxed{X(j\omega) = \left(\frac{1}{\omega-\omega_0}\right)\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right) + \frac{1}{(\omega+\omega_0)}\sin\left(\frac{(\omega+\omega_0)T_0}{2}\right)}$$

Note: Frequency Shifting Property is used.

f) $x(t) = \mathrm{rect}\left(\frac{t-t_0}{T_0}\right) \xrightarrow{FT} \boxed{X(j\omega) = \frac{2e^{-j\omega t_0}}{\omega}\sin\left(\frac{\omega T_0}{2}\right)}$

Time shifting

g) $x(t) = e^{j\omega_0 t}\,\mathrm{rect}\left(\frac{t-t_0}{T_0}\right) \xrightarrow{FT} \boxed{\frac{2e^{-j(\omega-\omega_0)t_0}}{(\omega-\omega_0)} \cdot \sin\left(\frac{(\omega-\omega_0)T_0}{2}\right)}$

Time Shifting and Frequency shifting!

h) $x(t) = \sin(2\pi f_0 t)\,\mathrm{rect}\left(\frac{t-t_0}{T_0}\right) \Rightarrow x(t) = \frac{e^{j\omega_0 t}}{j2}\mathrm{rect}\left(\frac{t-t_0}{T_0}\right) - \frac{e^{-j\omega_0 t}}{j2}\mathrm{rect}\left(\frac{t-t_0}{T_0}\right)$

$\sin(2\pi f_0 t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{j2}$

$$X(j\omega) = \frac{e^{-j\omega t_0}}{j2}\left[\frac{2}{(\omega-\omega_0)}\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right) - \frac{2}{(\omega+\omega_0)}\sin\left(\frac{(\omega+\omega_0)T_0}{2}\right)\right]$$

$$\boxed{X(j\omega) = \frac{e^{-j\omega t_0}}{j}\left[\frac{\sin\left(\frac{(\omega-\omega_0)T_0}{2}\right)}{(\omega-\omega_0)} - \frac{\sin\left(\frac{(\omega+\omega_0)T_0}{2}\right)}{(\omega+\omega_0)}\right]}$$

Peaks: $\frac{2\pi}{T} = w \Rightarrow f = \frac{w}{2\pi}$

x	Conversion to Hz	f(Hz)
±9290	→	1477
±8394	→	1336
±7596	→	1209
±5912	→	941
±5350	→	852
±4379	→	697

• Yes, they are the frequencies of the number used by the DTMF transreceivers.

• Only from this figure, it cannot be understood which number is dialed, because they are not in order, just their frequencies are seen.

$$x_1(t) \Rightarrow \begin{cases} x(t) & \text{for } 0 \le t \le 0.5 \\ 0 & \text{for } 0.5 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Let $r(t)$ be the rectangular signal such that

$r(t)\,x(t) = x_1(t)$

so $r(t) = \begin{cases} 1 & \text{for } 0 \le t \le 0.25 \\ 0 & \text{otherwise} \end{cases}$

Peaks for X_1:

x	f(Hz)
±8394 →	1336
±5912 →	941

$\Rightarrow$ First Digit is 0

Peaks for X_2:

x	f(Hz)
±9280 →	1477
±5350 →	852

$\Rightarrow$ Second Digit is 9

Peaks for X_3:

x	f(Hz)
±7596 →	1209
±4379 →	697

$\Rightarrow$ Third Digit is 1

Peaks for X_4:

x	f(Hz)
±9280 →	1477
±4379 →	697

$\Rightarrow$ Fourth Digit is 3

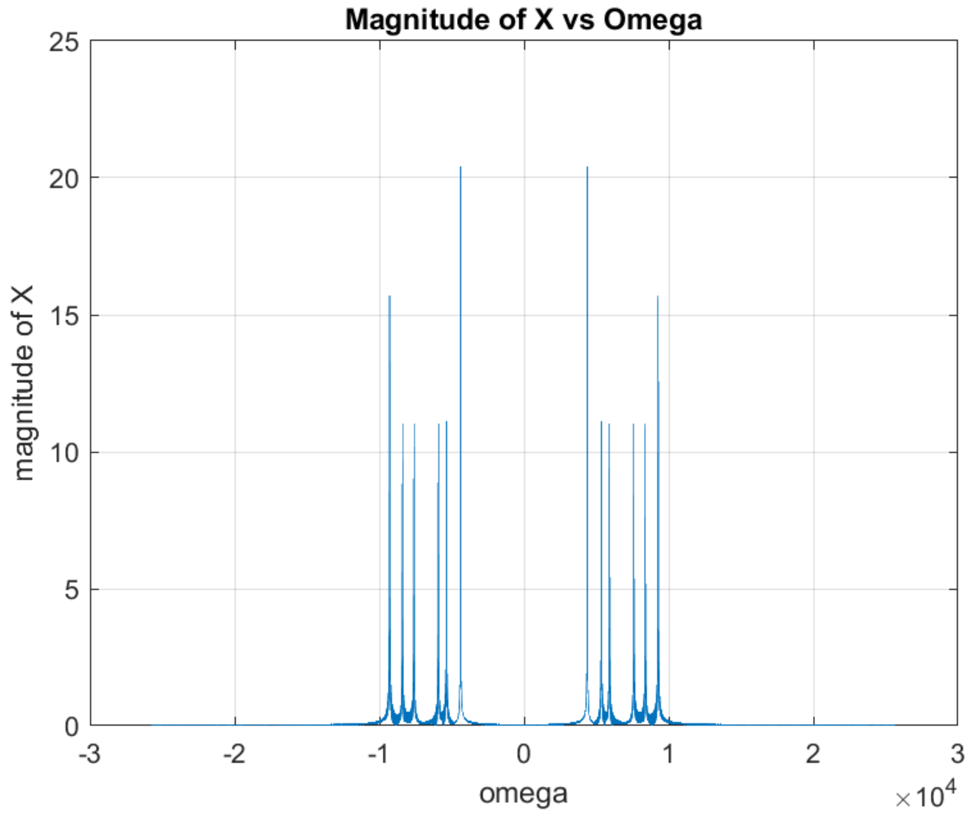
• This time I can understand which digit is dialed first, secon, third and the last

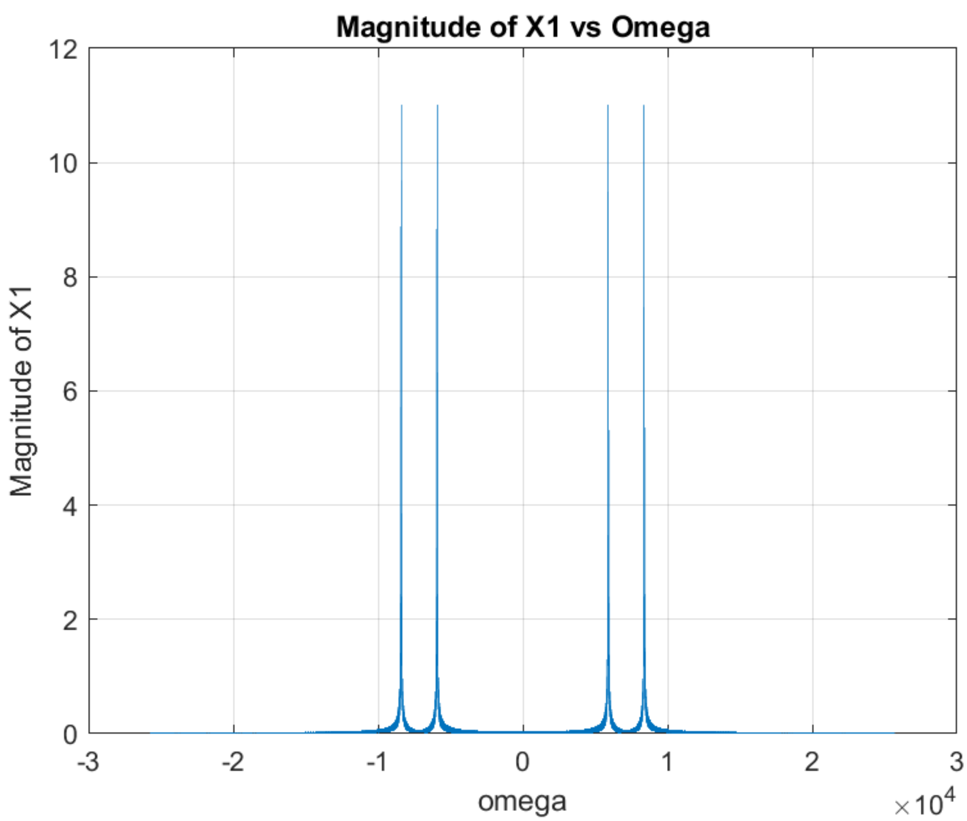
→ I shift the square wave +0.25 s right at each step so that I only take one digit of the Fourier Transform $X(j\omega)$ of the whole signal.

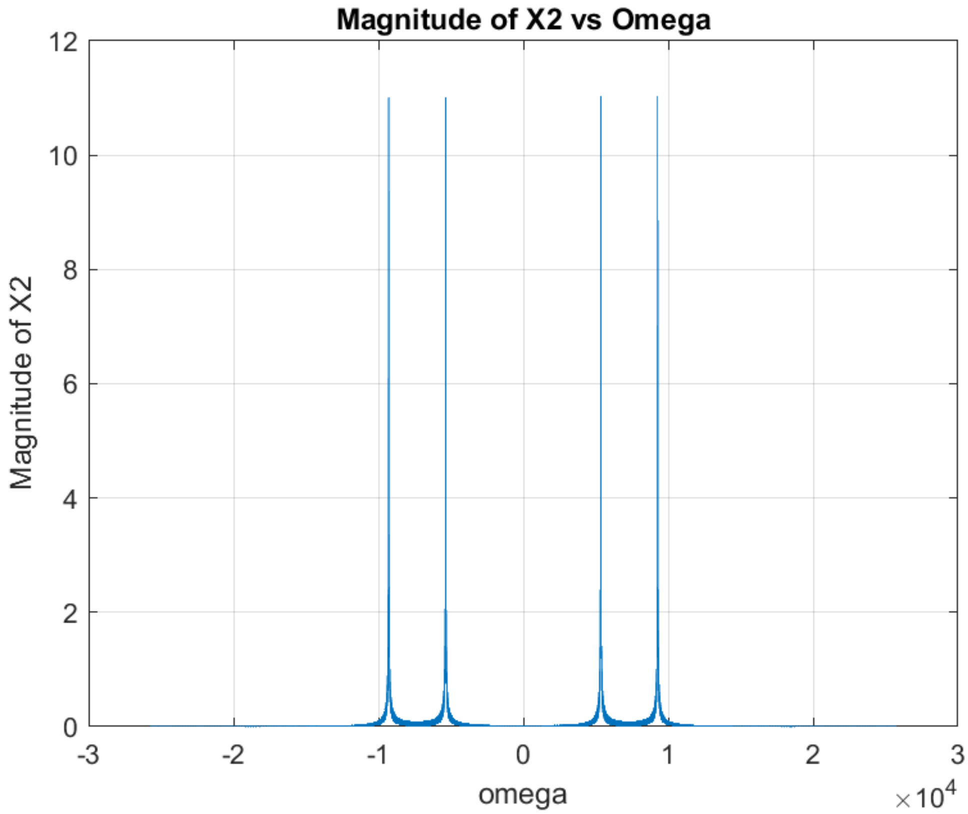
→ Since $X(\omega) = X_1(\omega) + X_2(\omega) + X_3(\omega) + X_4(\omega)$, when I looked at $X(\omega)$, all of the Fourier Transform of digit signals are inside X so they can not be distinguished as a digit. However, since X_1, X_2, X_3 and X_4 includes only two different frequencies, I was able to see the frequencies separately and decide which digit is dialed at the given t interval which is determined by r(t).

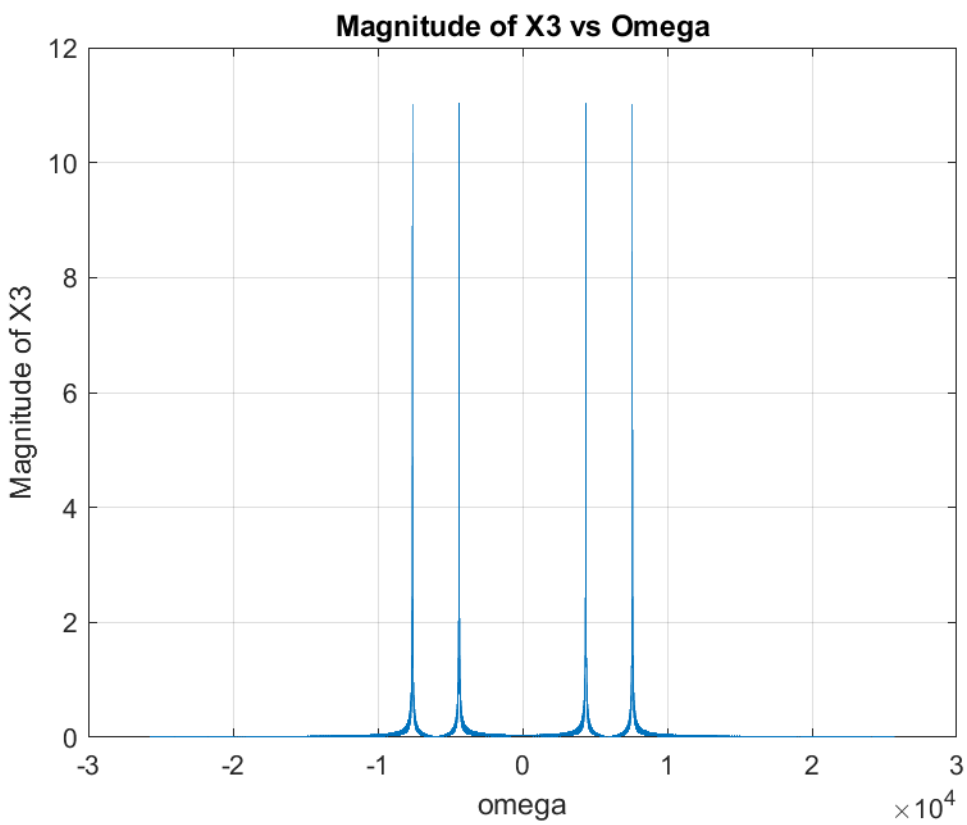
Part 1.2:

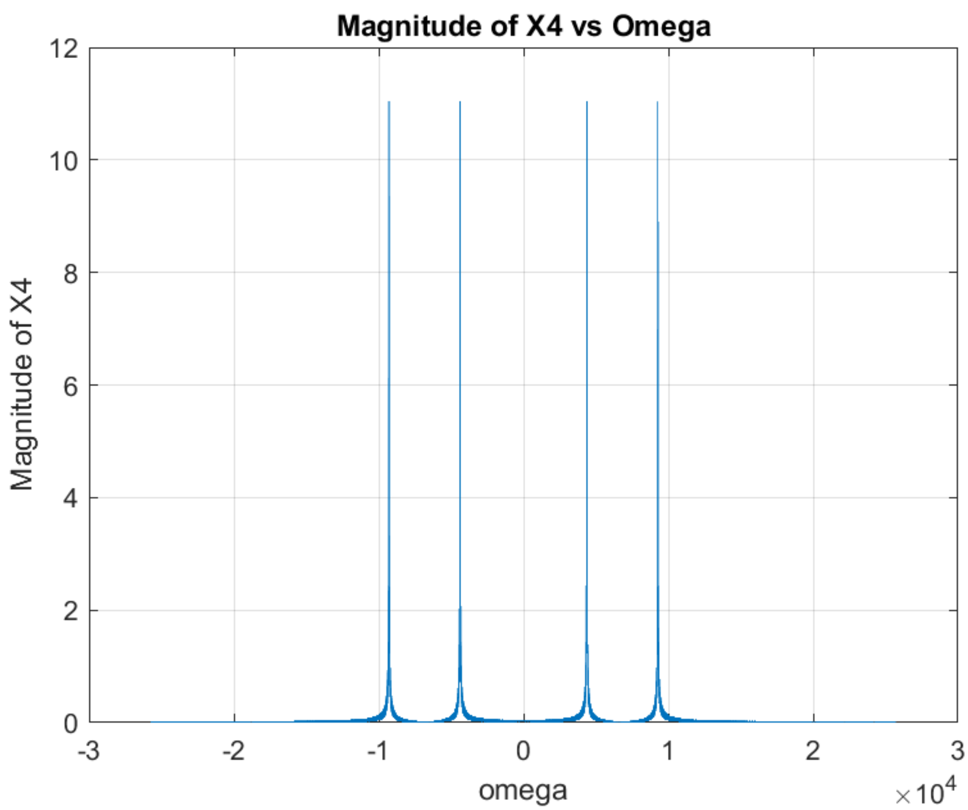
```
% %1.2 DTMF Receiver
Number = [0 9 1 3]; %ID: 21703190
x = DTMFTRA(Number);
orgx = x;
soundsc(x,8192);

%-----------------first part of 1.2 DTMF Receiver--------------------------
X = FT(x);
omega=linspace(-8192*pi,8192*pi,8193);
omega=omega(1:8192);
figure;
plot(omega,abs(X));
xlabel('omega');
ylabel('magnitude of X');
title('Magnitude of X vs Omega');
grid ON;

%----------------second part of 1.2 DTMF Receiver--------------------------
%-----First Digit---------------
figure
r = [ones(1,2048),zeros(1,6144)];
x=x.*r;
X1 = FT(x);
plot(omega,abs(X1));
xlabel('omega');
ylabel('Magnitude of X1');
title('Magnitude of X1 vs Omega');
grid ON;
% %-----Second Digit--------------
x = orgx;
figure
r = [zeros(1,2048),ones(1,2048),zeros(1,4096)];
x=x.*r;
X2 = FT(x);
plot(omega,abs(X2));
xlabel('omega');
ylabel('Magnitude of X2');
title('Magnitude of X2 vs Omega');
grid ON;
% %-----Third Digit--------------
x = orgx;
figure
r = [zeros(1,4096),ones(1,2048),zeros(1,2048)];
x=x.*r;
X3 = FT(x);
plot(omega,abs(X3));
xlabel('omega');
ylabel('Magnitude of X3');
title('Magnitude of X3 vs Omega');
grid ON;
% %-----Fourth Digit--------------
x = orgx;
figure
r = [zeros(1,6144),ones(1,2048)];
x=x.*r;
X4 = FT(x);
plot(omega,abs(X4));
xlabel('omega');
ylabel('Magnitude of X4');
title('Magnitude of X4 vs Omega');
grid ON;
```


Magnitude of X vs Omega
magnitude of X
omega
25
20
15
10
5
0
-3
-2
-1
0
1
2
3
$\times 10^4$


Magnitude of X1 vs Omega
Magnitude of X1
omega
12
10
8
6
4
2
0
-3
-2
-1
0
1
2
3
$\times 10^4$


Magnitude of X2 vs Omega
Magnitude of X2
omega
$\times 10^4$
0
2
4
6
8
10
12
-3
-2
-1
0
1
2
3


Magnitude of X3 vs Omega
Magnitude of X3
omega
$\times 10^4$
0
2
4
6
8
10
12
-3
-2
-1
0
1
2
3


Magnitude of X4 vs Omega
Magnitude of X4
omega
12
10
8
6
4
2
0
-3
-2
-1
0
1
2
3
×10^4

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Part 2:

My recording is " Hello guys, my name is Berk, my ID is 21703190 and this is signals and systems course; good luck! "

Matlab Code:

```
recObj = audiorecorder (8192, 8, 1);
disp (" Start Recording ... ");
recordblocking (recObj, 10);
disp (" Stop Recording ... ");
x = transpose (getaudiodata (recObj));
soundsc (x, 8192);
```

$$y(t) = x(t) + \sum_{i=1}^{M} A_i \, x(t-t_i)$$

a) $y(t) = x(t) * h(t) \xrightarrow{FS} Y(j\omega) = X(j\omega) H(j\omega) \Rightarrow H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)}$

$$\Rightarrow y(t) \xrightarrow{FS} Y(j\omega) = X(j\omega) + \sum_{i=1}^{M} A_i e^{-j\omega t_i} \cdot X(j\omega)$$

$$\Rightarrow H(j\omega) = 1 + \sum_{i=1}^{M} A_i e^{-j\omega t_i} \Rightarrow \boxed{h(t) = \delta(t) + \sum_{i=1}^{M} A_i \, \delta(t-t_i)}$$

$1 \xrightarrow{FS^{-1}} \delta(t)$, $\quad e^{-j\omega t_i} \rightarrow \delta(t-t_i)$

b) As found in part a, $\boxed{H(j\omega) = 1 + \sum_{i=1}^{M} A_i e^{-j\omega t_i}}$

c) Relation between $X(\omega)$, $Y(\omega)$ and $H(\omega)$ is $\boxed{Y(\omega) = X(\omega) H(\omega)}$ → convolution property

d) $X(\omega) = \dfrac{Y(\omega)}{H(\omega)}$ by the convolution property!

- The sound I hear is the sound I orginally heard but with additional delayed echoes. There were 5 delayed signals with scaled amplitudes A_i. Sound was so crowded that after a while, I couldn't understand what I said.

- When I listen $x_e(t)$, there were still some delays but it was closer to the original record $x(t)$ than $y(t)$. I was able to understand what I said. $x_e(t)$ is different than $x(t)$, which is expected because $x_e(t)$ is estimation of $x(t)$. In fact, from the plots, how $x_e(t)$ reduce some of the noises (additional echoes) can be seen clearly. $y(t)$ and $x_e(t)$

Part 2 Matlab Code:

```
recObj = audiorecorder(8192,8,1);
disp("Start Recording...");
recordblocking(recObj,10);
disp("Stop Recording...");
x = transpose(getaudiodata(recObj));
%soundsc(x,8192); %listen the original sound


M=5;
Ai=[0.35 0.5 0.65 0.05 0.15];
time = [0:1/8192:10-1/8192];
ti=[0.5 1.25 2.5 3 2.75];
echo = zeros(1,81920);

for i = 1:M
    for t = 0:1/8192:10-1/8192
        if t>=ti(i) %croping the signal
            echo(t*8192+1) = echo(t*8192+1) + Ai(i)*x((t-ti(i))*8192 +1); % summation operation
        end
    end
end
y = x + echo; %creating the final y(t) signal
%soundsc(y,8192); %listen echoed sound

%plots
figure
plot(time,x);
xlabel("time (second)");
ylabel("x(t)");
title("x(t) versus time");
figure
plot(time,y);
xlabel("time (seconds)");
ylabel("y(t)");
title("y(t) versus time");

Y=FT(y);
omega=linspace(-8192*pi,8192*pi,81921);
omega=omega(1:81920);
H=ones(1,81920);
for i = 1:M
    H = H + Ai(i)*exp(-1.0i*omega*ti(i)); %formula found in part b
end

h = IFT(H);
figure
plot(time,h);
title("h(t) versus time");
xlabel("time (second)");
ylabel("h(t)");
figure
plot(omega,abs(H));
title("|H(jw)| versus omega");
xlabel("omega (rad/s)");
ylabel("|H(jw)|");

Xe = Y./H; %estimated X(jw)
xe = IFT(Xe);
soundsc(xe,8192);%listen estimated x(t)

figure
plot(time,xe);
title("xe(t) versus time");
xlabel("time (second)");
ylabel("xe(t)");
```

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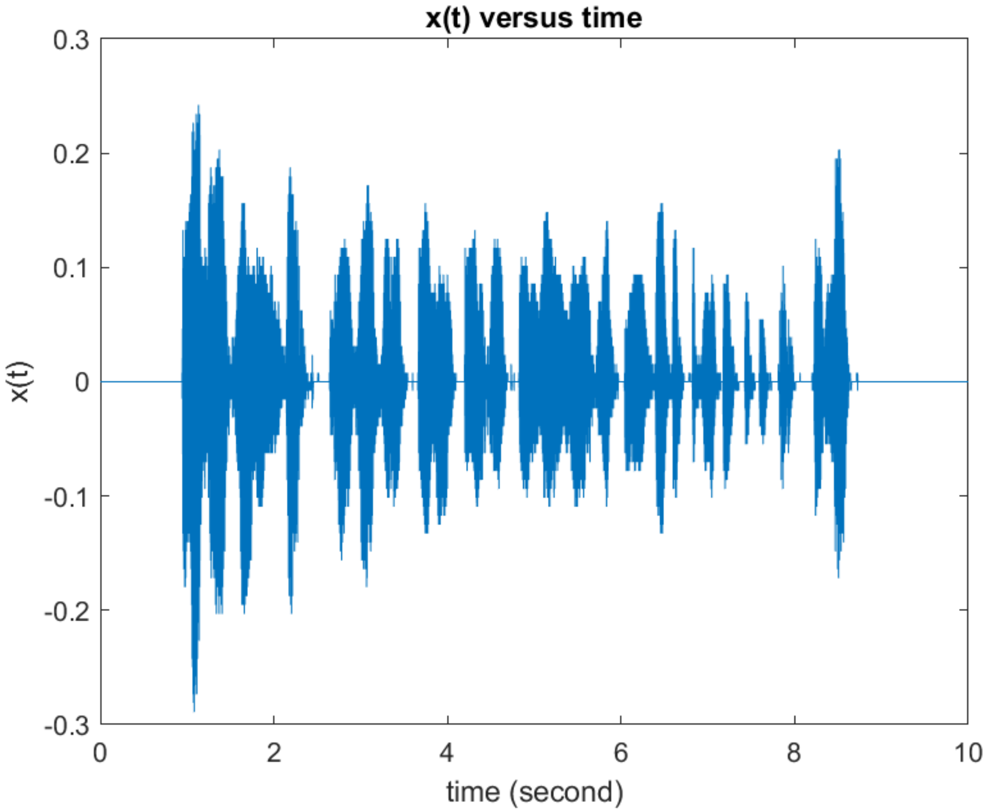
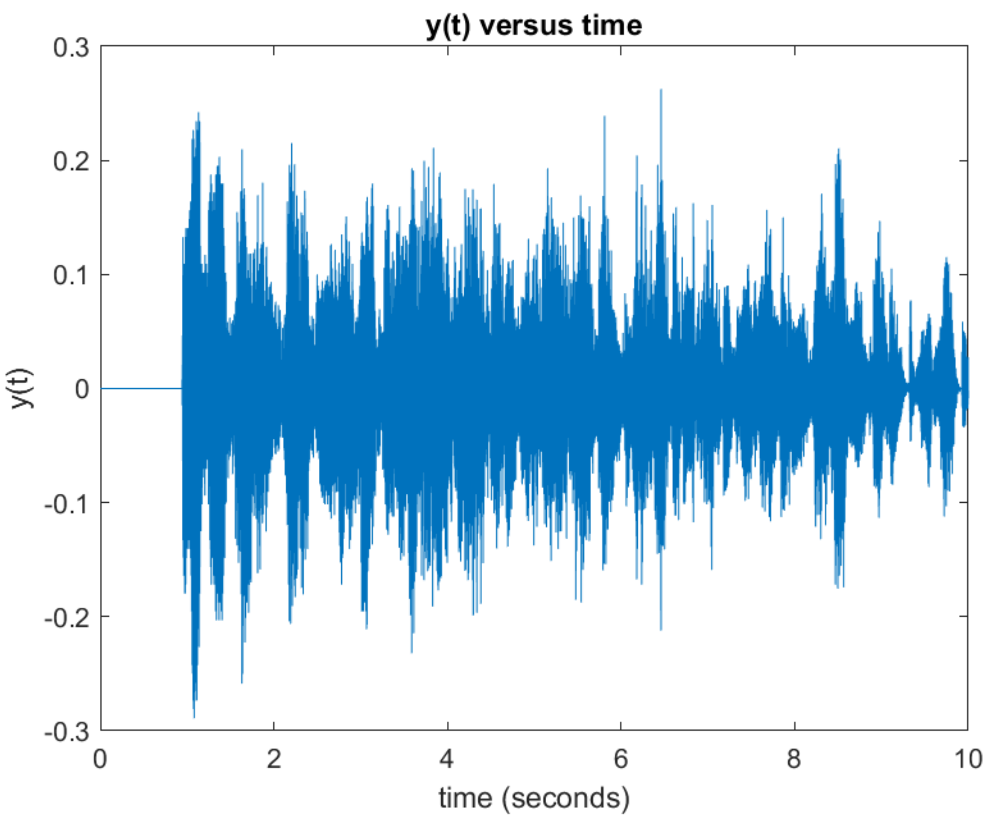
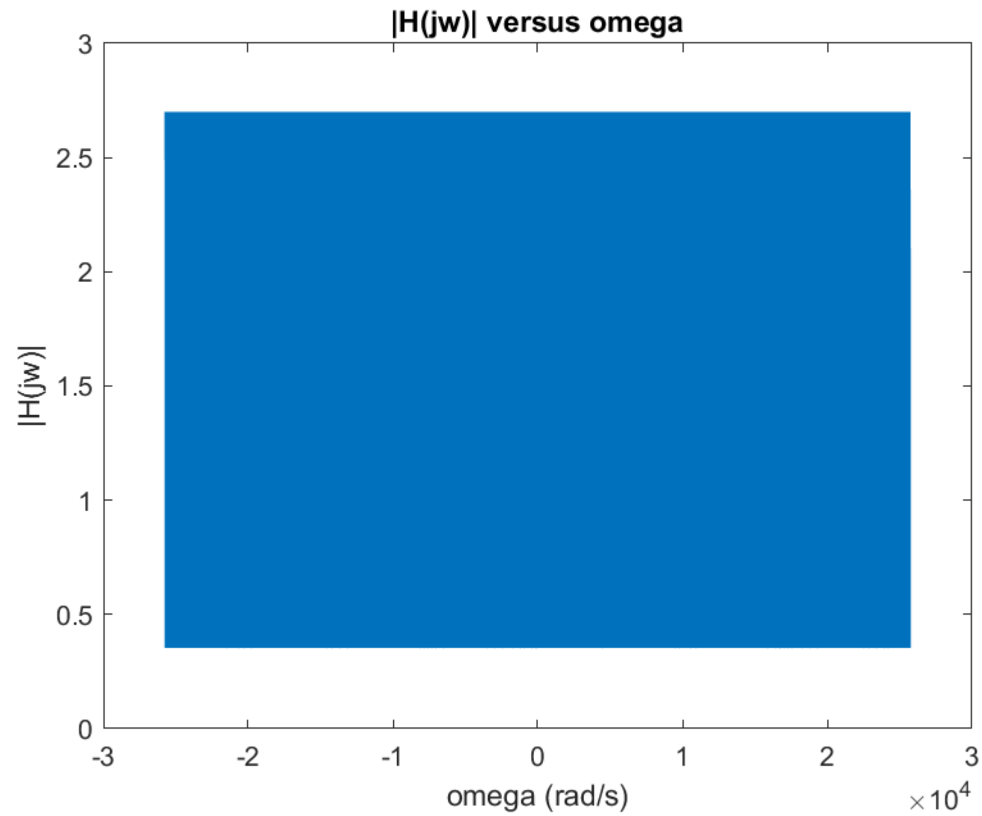
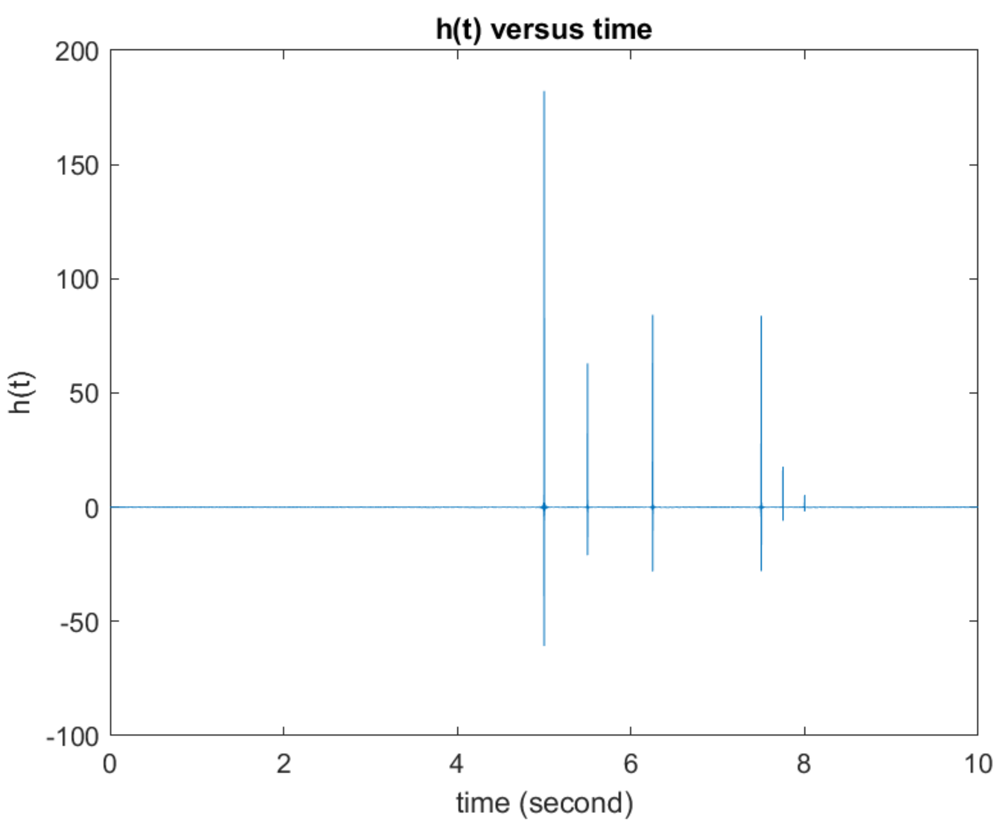
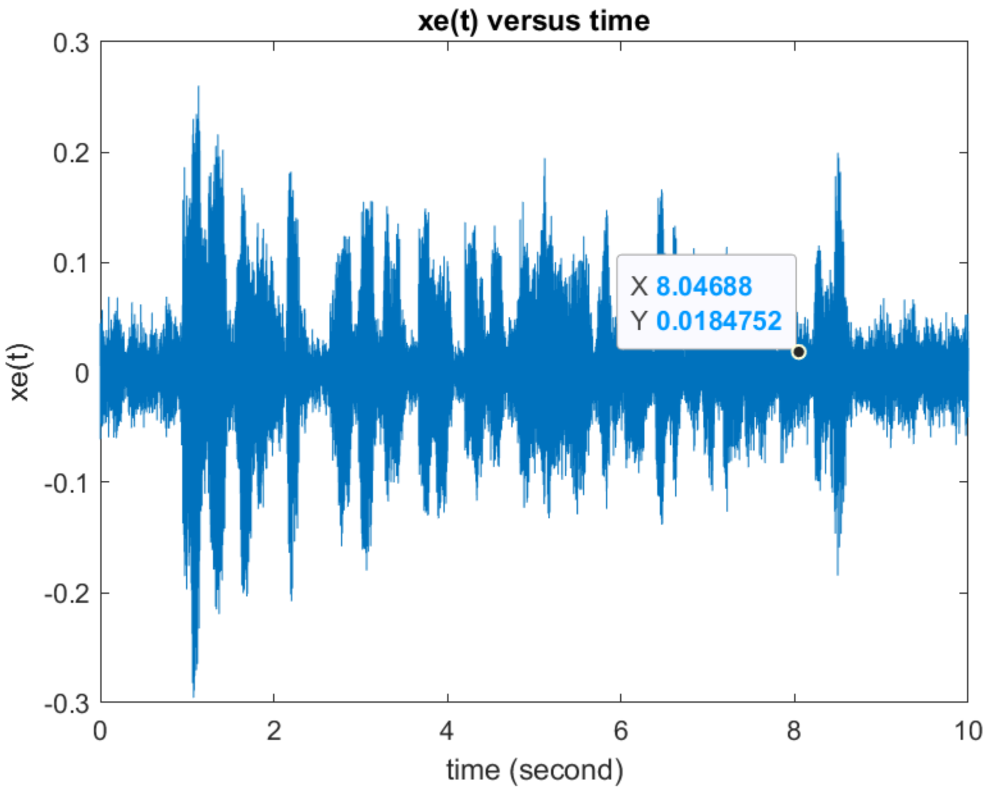
Lab 03






|H(jw)| versus omega
|H(jw)|
omega (rad/s)
$\times 10^4$
3
2.5
2
1.5
1
0.5
0
-3
-2
-1
0
1
2
3


h(t) versus time
h(t)
time (second)
200
150
100
50
0
-50
-100
0
2
4
6
8
10


xe(t) versus time
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
xe(t)
0
2
4
6
8
10
time (second)
X 8.04688
Y 0.0184752