Mehmet Berk Şahin
21703190
Section 1 Lab 2

Lab 2 Report

Part 1:

Matlab Code:

```
%---------------------------PART1--------------------------------
--
j = sqrt(-1);
t = [0:0.001:1];
n = mod(21703190,41);
A = rand(1,n)*3 + j*rand(1,n)*3;
omega = rand(1,n)*pi;


xs = SUMCS(A,t,omega);
realp = real(xs);
imagp = imag(xs);
mag = abs(xs);
phase = angle(xs);

plot(t,realp);
xlabel('time');
ylabel('real part');
title('real part vs time');
figure;
plot(t,imagp);
xlabel('time');
ylabel('imaginary part');
title('imaginary part vs time');
figure;
plot(t,mag);
xlabel('time');
ylabel('magnitude');
title('magnitude vs time');
figure;
plot(t,phase);
xlabel('time');
ylabel('phase');
title('phase vs time');
```

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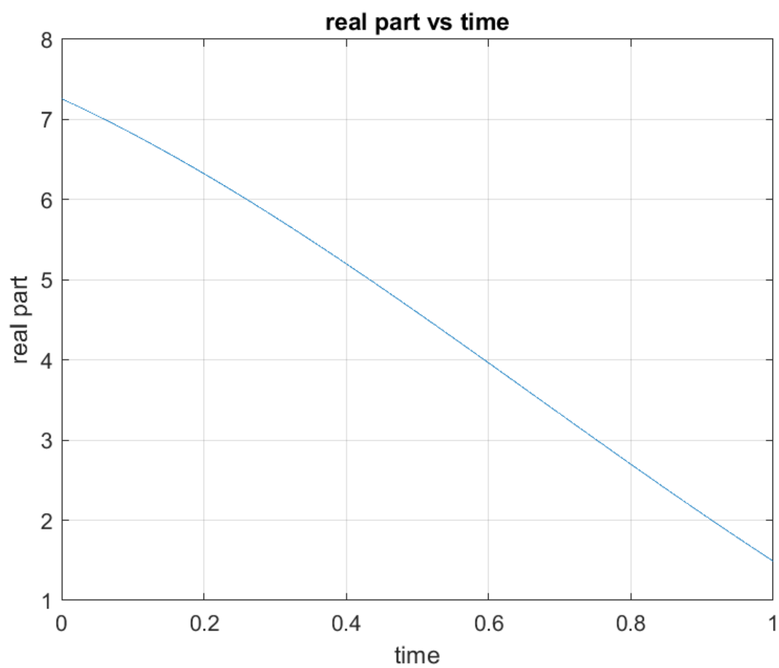
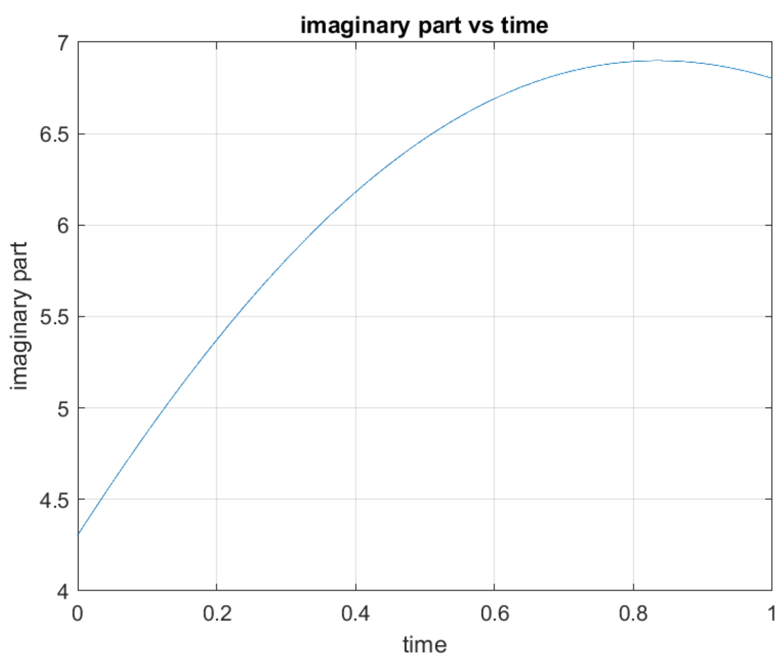
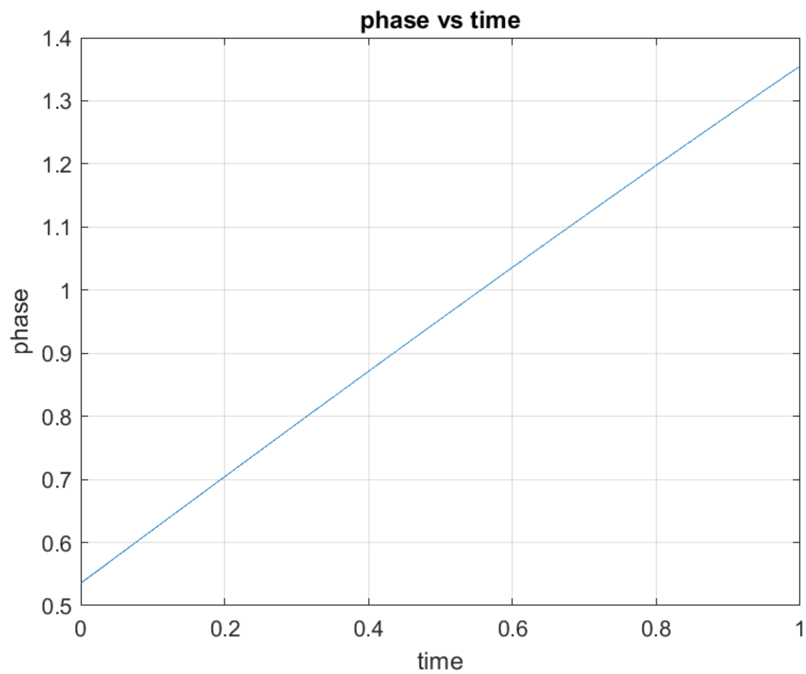
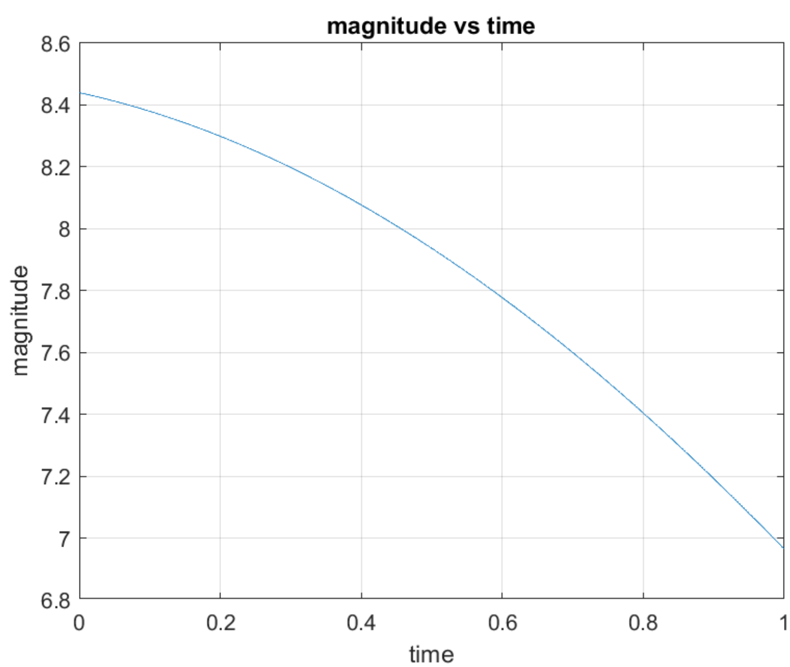
```
function [xs] = SUMCS(A,t,omega)
    xs = zeros(1,length(t));
    M = length(A);
    j = sqrt(-1);
    for i = 1:M
        xs = xs + A(i)*exp(j*t*omega(i)/2);
    end
end
```

Plots:






phase vs time
phase
time
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0
0.2
0.4
0.6
0.8
1


magnitude vs time
magnitude
time
8.6
8.4
8.2
8
7.8
7.6
7.4
7.2
7
6.8
0
0.2
0.4
0.6
0.8
1

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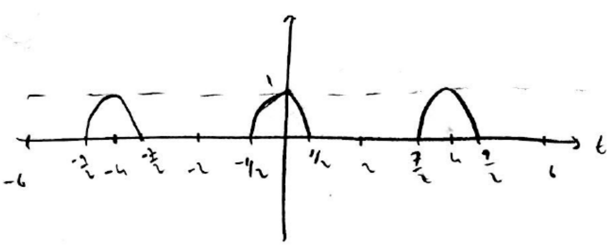
Part 2:-

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j\frac{2\pi k t}{T}} \quad , \quad X_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi k}{T}} dt$$

$$x(t) = \begin{cases} 1-4t^2 & \text{if } -\frac{w}{4} < t < \frac{w}{4}, \\ 0 & \text{otherwise}, \end{cases} \quad \text{where} \quad w < T$$

$$T = 4 \,,\, w = 2 \Rightarrow x(t) = \begin{cases} 1-4t^2 & \text{if } -\frac{1}{2} < t < \frac{1}{2}, \\ 0 & \text{otherwise}, \end{cases}$$

sketch $x(t)$ over $-6 < t < 6$



$$X_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi k}{T}t} dt = \frac{1}{T}\int_{-w/4}^{w/4} (1-4t^2)\, e^{-j\frac{2\pi k}{T}t} dt$$

$u = 1-4t^2 \Rightarrow du = -8t\,dt$

$dv = e^{-j\frac{2\pi k}{T}t} dt$

$v = \frac{-T}{j2\pi k} e^{-j\frac{2\pi k}{T}t}$

$$= \frac{1}{T}\left\{ (1-4t^2)\left(\frac{-T}{j2\pi k} e^{-j\frac{2\pi k}{T}t}\right)\Bigg|_{-w/4}^{w/4} - \frac{T}{j2\pi k}\int_{-w/4}^{w/4} 8t \cdot e^{-j\frac{2\pi k}{T}t} dt \right\}$$

$u = 8t \quad du = 8\,dt \quad e^{-j\frac{2\pi k}{T}t} dt = dv$

$$\frac{(\frac{w^2}{4}-1)T}{j2\pi k}\left[e^{-j\frac{w\pi k}{2T}} - e^{j\frac{w\pi k}{2T}}\right] \qquad \frac{8t \cdot T e^{-j\frac{2\pi k}{T}t}}{-j2\pi k}\Bigg|_{-w/4}^{w/4} + \frac{8T}{j2\pi k}\int_{-w/4}^{w/4} e^{-j\frac{2\pi k}{T}t} dt \qquad \frac{T}{-j2\pi k} e^{-j\frac{2\pi k}{T}t} = v$$

$$\frac{wT e^{-j\frac{w\pi k}{2T}}}{-j\pi k} - \left(\frac{-wT e^{j\frac{w\pi k}{2T}}}{-j\pi k}\right) = \left(\frac{-wT}{j\pi k}\right)\left(e^{-j\frac{w\pi k}{2T}} + e^{j\frac{w\pi k}{2T}}\right) \qquad \frac{8T^2}{4\pi^2 k^2}\left(e^{-j\frac{w\pi k}{2T}} - e^{j\frac{w\pi k}{2T}}\right)$$

$$= \frac{1}{j2\pi k}\left[\left(\frac{w^2}{4}-1\right)\underbrace{\left[e^{-j\frac{w\pi k}{2T}} - e^{j\frac{w\pi k}{2T}}\right]}_{-j2\sin\left(\frac{w\pi k}{2T}\right)}\right] + \frac{wT}{j\pi k}\underbrace{\left(e^{-j\frac{w\pi k}{2T}} + e^{j\frac{w\pi k}{2T}}\right)}_{2\cos\left(\frac{w\pi k}{2T}\right)} - \frac{2T^2}{\pi^2 k^2}\underbrace{\left(e^{-j\frac{w\pi k}{2T}} - e^{j\frac{w\pi k}{2T}}\right)}_{-j2\sin\left(\frac{w\pi k}{2T}\right)}$$

$$\Rightarrow \boxed{X_k' = \left[-\frac{w^2}{4\pi k} + \frac{1}{\pi k} + \frac{2T^2}{\pi^3 k^3}\right]\left(\sin\left(\frac{w\pi k}{2T}\right)\right) + -\frac{wT}{\pi^2 k^2}\cos\left(\frac{w\pi k}{2T}\right)}$$

for $k \neq 0$

If $k = 0$, $x_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{T}\int_{-w/4}^{w/4}(1-4t^2)dt = \frac{1}{T}\left[t - \frac{4t^3}{3}\right]_{-\frac{w}{4}}^{\frac{w}{4}} = \left[\frac{w}{2} - \frac{4}{3}\frac{w^3}{32}\right]\frac{1}{T}$

2 Will be used in code! $\leftarrow$ $\boxed{X_0 = \frac{1}{T}\left(\frac{w}{2} - \frac{w^3}{24}\right)}$

Mehmet Berk Şahin
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Part 3:

This will be used in code:

$$\tilde{x}(t) = \sum_{k=-k}^{k} x_k e^{j\frac{2\pi k t}{T}} = \sum_{k=+1}^{k} x_k e^{j\frac{2\pi k t}{T}} + \sum_{k=1}^{+k} x_{-k} e^{-j\frac{2\pi k t}{T}} + x_0$$

Part 3:

Real Part:

Max Value: 1.0073

Min Value: −0.0728

Imaginary Part

Max Value: 5.5511×10^{-16}

Min Value: -5.5511×10^{-16}

* When they are compared, maximum value of real part is much greater than maximum value of imaginary part. And, minimum value of real part is much smaller than imaginary part. In fact, imaginary part can be ignored when it is compared to real part.

→ If I try $\sin(\frac{\pi}{6}) - 0.5$ in MATLAB, I get $\sin(\frac{\pi}{6}) - 0.5 = -5.5511 \times 10^{-17}$

Since matlab calculates sin(π/6) approximately, small error is generated.

Second Part (Dn Part):

* As k gets larger, approximation of $\tilde{x}(t)$ becomes better. Plot becomes more sharp rather than being smooth. This is because of the number of signals. In ideal case, when $k = \infty$, we acquire x(t) so as we increase k, $\tilde{x}(t)$ gets closer to ideal x(t) so $\tilde{x}(t)$'s success also increases.

Yes, I observe oscillations and irregularities in the neighborhoods of discontinuities. This is because of the Fourier series expansion. The value of discontinuities are average of its two side so oscillations arise at these points.

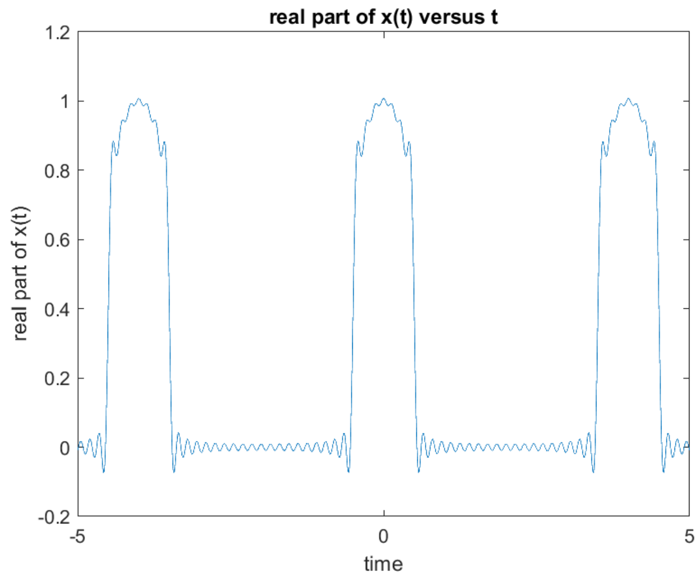
! Corresponding Plots for This ~~Part~~ are Given Below !

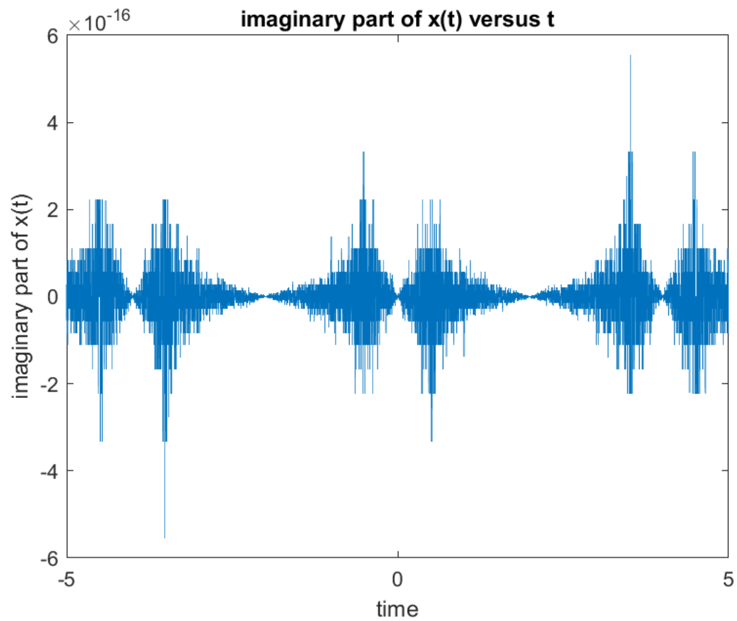
Matlab Code:

```
%------------------------------PART3------------------------------
D11 = mod(21703190,11);
D4 = mod(21703190,4);
T=2;W=1;t=[-5:0.001:5];
%--------------------------First Part--------------------------
K=23+D11
```

```
xt = FSWave(t,K,T,W);
realpart = real(xt);
imaginaryp = imag(xt);
max(realpart)
min(realpart)
max(imaginaryp)
min(imaginaryp)
figure;
plot(t,realpart);
title('real part of x(t) versus t');
xlabel('time');
ylabel('real part of x(t)');
figure;
plot(t,imaginaryp)
title('imaginary part of x(t) versus t');
xlabel('time');
ylabel('imaginary part of x(t)');

function [xs] = SUMCS(A,t,omega)
    xs = zeros(1,length(t));
    M = length(A);
    j = sqrt(-1);
    for i = 1:M
        xs = xs + A(i)*exp(j*t*omega(i)/2);
    end
end

function [xt] = FSWave(t,K,T,W)
    Xk = zeros(1,2*K+1);j=sqrt(-1);
    omegas = [-K:K]*2*pi/T;
    X0 = (1/T)*(W/2 -W^3/24); %mean value of the function in
given period
    for k = -K:K
        if k == 0
            Xk(K+1)= X0;
        else
            Xk(k + K+1) = (-W^2/(4*pi*k) + 1/(pi*k) +
2*T^2/(pi^3 * k^3))*sin((W*pi*k)/(2*T)) -(W*T/(pi^2 *
k^2))*cos(W*pi*k/(2*T));
        end
    end
    xt = SUMCS(Xk(K+2:2*K+1),t,omegas(K+2:2*K+1)) +
SUMCS(Xk(1:K),t,omegas(1:K)) + X0;
end
```

Plots:





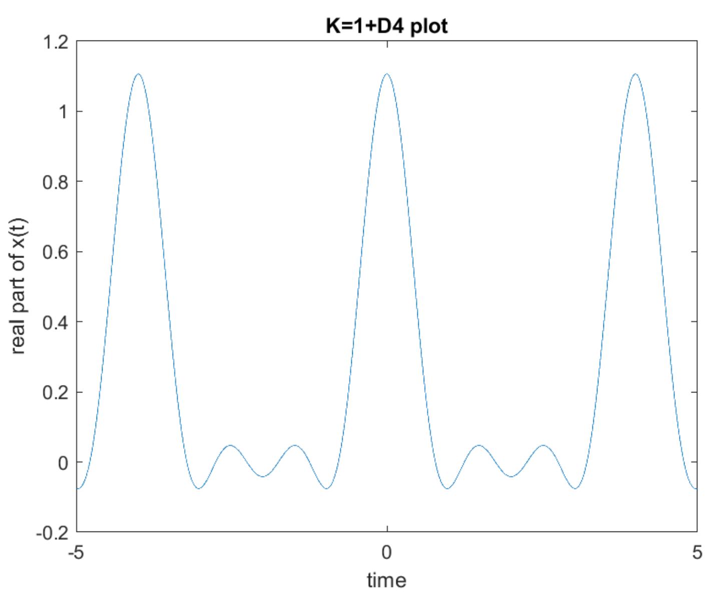
Matlab Code:

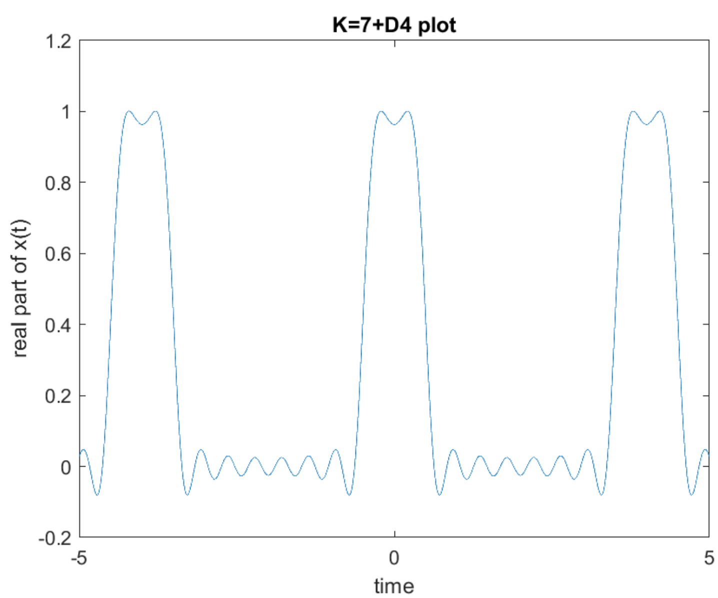
```
%----------------------Second Part-----------------
K= 1+D4;
xt = FSWave(t,K,T,W);
figure;
plot(t,real(xt));
title('K=1+D4 plot');
xlabel('time');
```

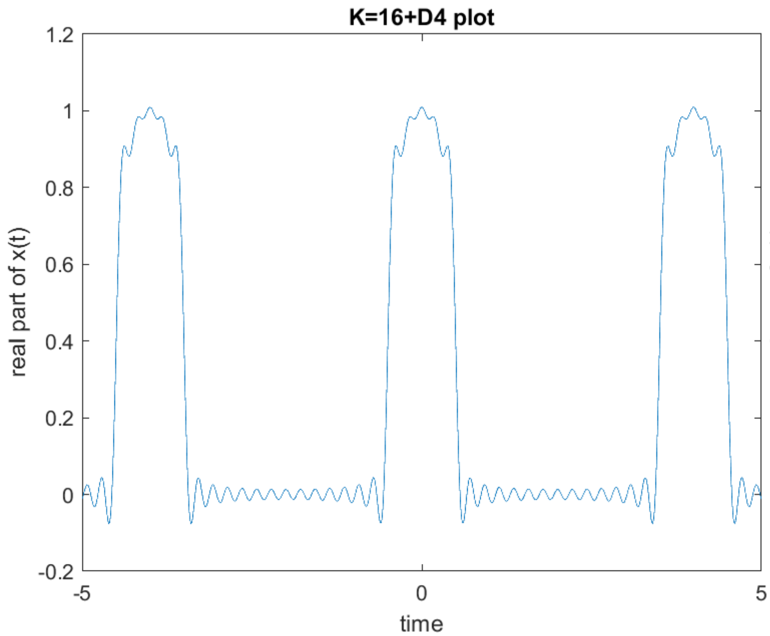
```
ylabel('real part of x(t)');
K = 7+D4;
xt = FSWave(t,K,T,W);
figure;
plot(t,real(xt));
title('K=7+D4 plot');
xlabel('time');
ylabel('real part of x(t)');
K = 16+D4;
xt = FSWave(t,K,T,W);
figure;
plot(t,real(xt));
title('K=16+D4 plot');
xlabel('time');
ylabel('real part of x(t)');
K = 100 + D4;
xt = FSWave(t,K,T,W);
figure;
plot(t,real(xt));
title('K=100+D4 plot');
xlabel('time');
ylabel('real part of x(t)');
K = 200 + D4;
xt = FSWave(t,K,T,W);
figure;
plot(t,real(xt));
title('K=200+D4 plot');
xlabel('time');
ylabel('real part of x(t)');


function [xs] = SUMCS(A,t,omega)
    xs = zeros(1,length(t));
    M = length(A);
    j = sqrt(-1);
    for i = 1:M
        xs = xs + A(i)*exp(j*t*omega(i)/2);
    end
end

function [xt] = FSWave(t,K,T,W)
    Xk = zeros(1,2*K+1);j=sqrt(-1);
    omegas = [-K:K]*2*pi/T;
    X0 = (1/T)*(W/2 -W^3/24); %mean value of the function in
given period
    for k = -K:K
```

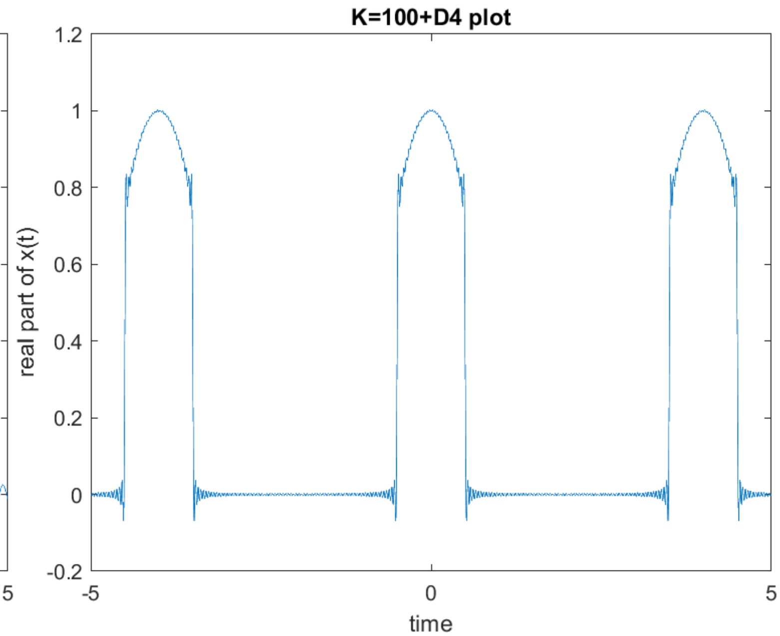
```
        if k == 0
            Xk(K+1)= X0;
        else
            Xk(k + K+1) = (-W^2/(4*pi*k) + 1/(pi*k) +
2*T^2/(pi^3 * k^3))*sin((W*pi*k)/(2*T)) -(W*T/(pi^2 *
k^2))*cos(W*pi*k/(2*T));
        end
    end
    xt = SUMCS(Xk(K+2:2*K+1),t,omegas(K+2:2*K+1)) +
SUMCS(Xk(1:K),t,omegas(1:K)) + X0;
end
```

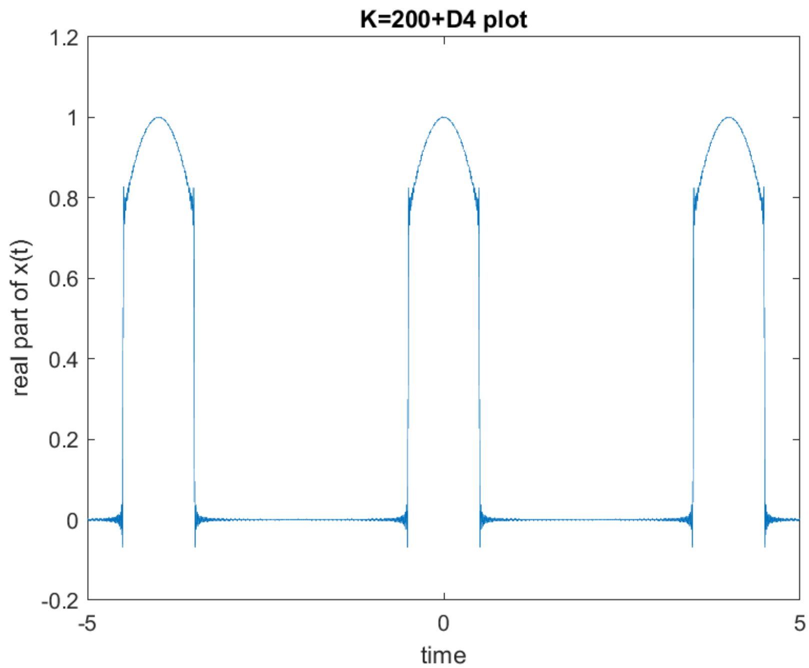
Plots:






K=16+D4 plot
real part of x(t)
time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-5
0
5


K=100+D4 plot
real part of x(t)
time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-5
0
5


K=200+D4 plot
real part of x(t)
time
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-5
0
5

Part 4:

Part 4: All required changes are thought with respect to original FSWave function.

Part a:

→ Required change is I need to reverse the X_k array. As an example:

$$x_k = [3, 1, 2] \xrightarrow{\text{reverse}} y_k = x_{-k} = [2, 1, 3]$$

To achieve this, flip( ) function is used. The effect of the operation is time reversal, because

$y_k = x_{-k}$

Let $x(t) = \sum_{k=-K}^{K} x_k e^{+jk\omega t}$

$$y(t) = \sum_{k=-K}^{K} x_{-k} e^{+jk\omega t} = \sum_{k=-K}^{K} x_k e^{-jk\omega t} = x(-t)$$

Part b:

→ Required change is I need to multiply X_k values with $e^{-j\frac{2\pi k t_0}{T}}$ when I calculate X_k.

• I need to multiply the formula derived in part 2 with $e^{-j\frac{2\pi k t_0}{T}}$.

• I also create another parameter for FSWave, which is t_0, so that I can calculate Y_k with respect to t_0.

The effect of $e^{-j\frac{2\pi k t_0}{T}}$ factor is it shifts the signal $+t_0$ right so

$$y(t) = x(t-t_0)$$

Part c:

→ Required change is I need to multiply the formula derived in Part 2 with $jk\frac{2\pi}{T}$.

• By doing this operation, I take the derivative of $x(t)$ so $y(t) = \frac{dx(t)}{dt}$

Part d:

→ By defining Y_k, the complex conjugate of X_k and symmetry of x_k is taken with respect to the y-axis simultaneously.

In order to achieve this, following code should be written in the for loop which is iterating through $-K$ to K.

```
if k > 0
    xk(2*K + 2 - k) = - - -
elseif k < 0
    xk(-k) = - -
else
    xk(:,K+1) = 0
```

! Corresponding plots to these parts are given below!

Mehmet Berk Şahin
21703190
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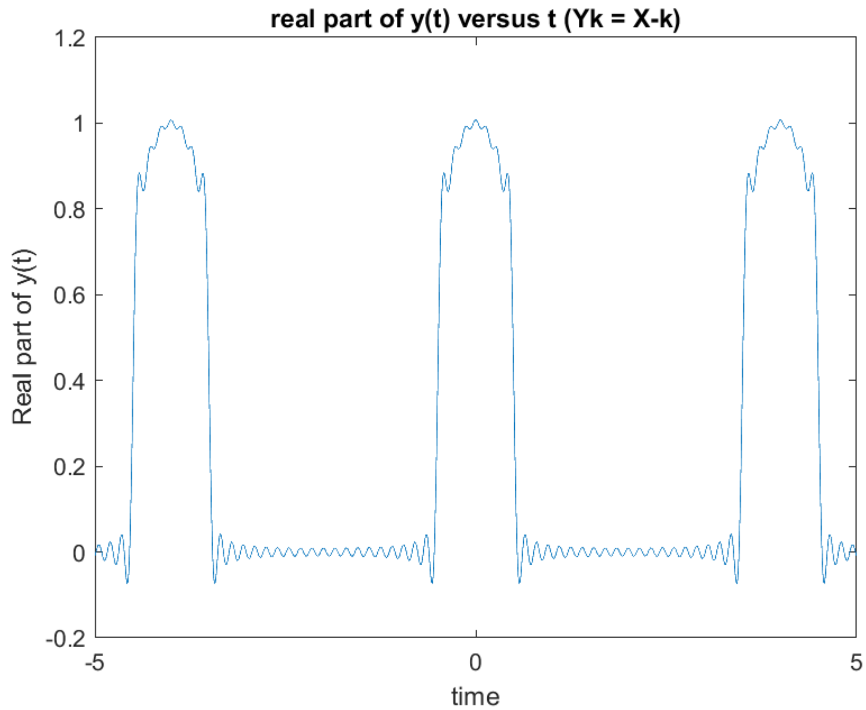
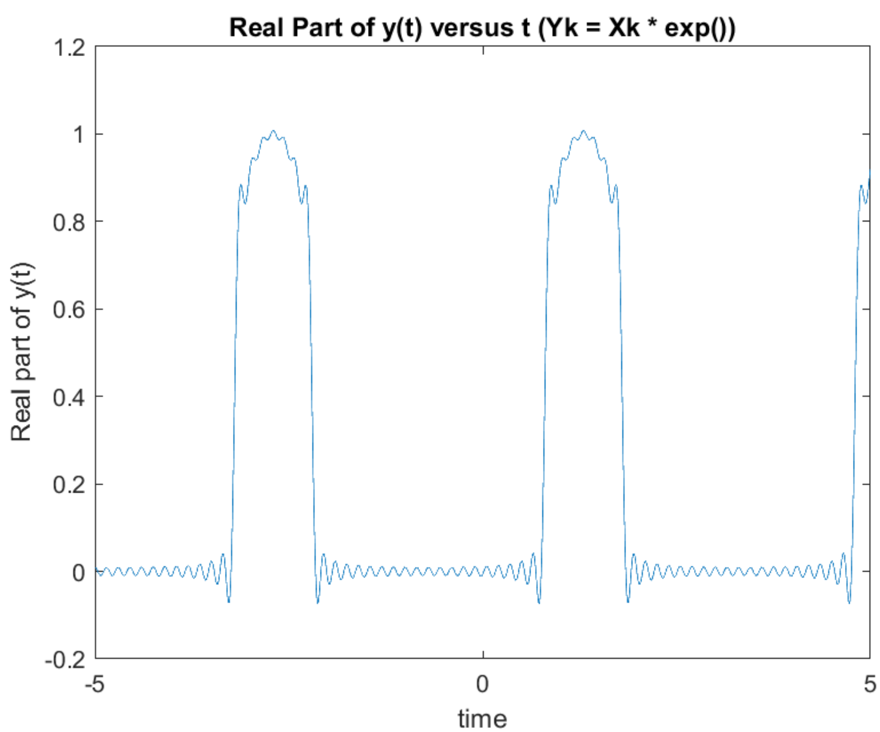
Part A:



Part B:



Mehmet Berk Şahin
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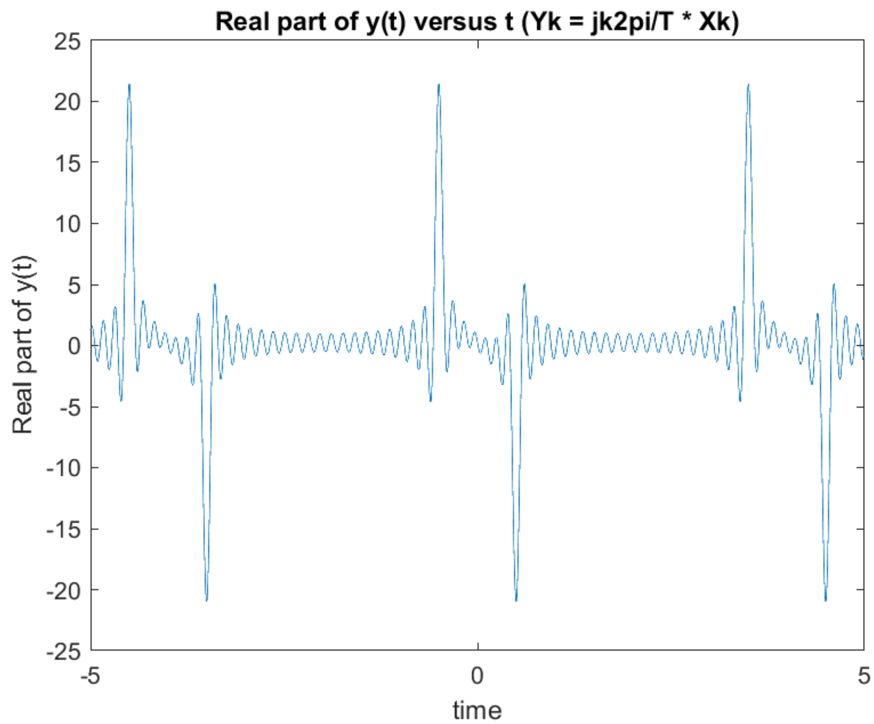
Part C:



Part D: