EEE342 Feedback and Control Systems Preliminary Work 1

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1. Introduction

In this preliminary work, a method is suggested to determine a first order approximate transfer function between the angular velocity and input voltage. After the transfer functions is found, its pole is compared with the pole of the low-pass filter. In the second part, P and PI controllers' effects to system response (in terms of overshoot, settling time, peak time, etc.) are explained and its effect on steady-state error is shown. After that comments are made regarding the importance of integral controller for the steady-state error.

2. Laboratory Content

Part I

The Matlab workspace file prelab1_response.mat is downloaded from Moodle. It consists of time and velocity data having a sampling frequency of 10KHz and it is taken from a simulation of DC motor. 6V step change is given as an input to DC motor and its angular velocity data is recorded. In this part, first order approximate transfer function for our DC motor is required to be found. To achieve that, one needs to start with the motor torque:

$$T_m(s) = (K_1 K_f I_f) I_a(s) \text{ (Eq. 1)}$$

$$= K_m I_a(s) \text{ (Eq. 2)}$$

$$where\ K_m = K_1 K_f I_f \text{ (Eq. 3)}$$

K_m is a function of the permeability of the magnetic field. The armature current depends on the input voltage given to the armature as follows

$$V_a(s) = (R_a + L_a s) I_a(s) + V_b(s), \quad \text{(Eq. 4)}$$

where $V_b(s)$, L_a, and R_a are the back electromotive-force voltage proportional to the motor speed, inductance, and resistance respectively. Thus,

$$V_b(s) = K_b w(s), \quad (Eq. 5)$$

where w(s) is the Laplace transform of angular velocity. From physical unknowns, we know that

$$w(s) = \frac{T_m}{Js + b} = \frac{K_m I_a(s)}{Js + b} \quad (Eq. 6)$$

Where J is the inertia of the load and b is friction constant. Note that equations 1-4 are DC motor's Kirchhoff equations.

By combining equation 4, 5 and 6, following equation is acquired:

$$G_p(s) = \frac{w(s)}{V_a(s)} = \frac{\frac{K_m I_a(s)}{Js + b}}{I_a(s)(sL_a + R_a) + K_b \frac{K_m I_a(s)}{Js + b}} \quad (Eq. 7)$$

$$= \frac{K_m}{(sL_a + R_a)(Js + b) + K_b K_m} \quad (Eq. 8)$$

The reason for the name $G_p(s)$ is because it is process function. Note that before writing these equations, disturbance is ignored since it is negligible, and the purpose of this work is doing a first order approximation, so it is intentionally ignored. Additionally, to turn the equation 8 to first order approximation, inductance is assumed to be 0, $L_a = 0$. Hence,

$$G_p(s) \cong \frac{K_m}{R_a(Js + b) + K_b K_m} \quad (Eq. 9)$$

$$= \frac{K_m}{R_a Js + bR_a + K_b K_m} \quad (Eq. 10)$$

$$= \frac{\frac{K_m}{bR_a + K_b K_m}}{\frac{R_a Js}{bR_a + K_b K_m} + 1} \quad (Eq. 11)$$

$$= \frac{K_1}{\tau_1 s + 1} \quad (Eq. 12)$$

where $K_1 = \frac{K_m}{bR_a + K_b K_m}$, and $\tau_1 = \frac{R_a J}{bR_a + K_b K_m}$ (Eq. 13), (Eq. 14)

$$V_a(t) = 6u(t) \quad (Eq. 15)$$

As it is indicated in preliminary work, the input voltage to DC motor is 6V. Therefore, the output voltage is

$$y(t) = g_p(t) * v_a(t) \quad (Eq. 16)$$

Symbol * denotes convolution operation. In Laplace domain, it corresponds to multiplication.

$$Y(s) = G_p(s)V_a(s). \quad (Eq. 17)$$

Thus, by equation 12 and 15,

$$Y(s) = \frac{6K_1}{(\tau_1 s + 1)s} \quad (Eq. 18)$$

Because the Laplace transform of unit step is 1/s. To get the inverse Laplace transform of the equation 18, partial fraction expansion will be done.

$$Y(s) = \frac{6K_1}{(\tau_1 s + 1)s} = \frac{k_1}{s} + \frac{k_2}{(\tau_1 s + 1)} \quad (Eq. 19)$$

$$k_1 = sY(s)|_{s=0} = 6K_1, \quad (Eq. 20)$$

$$k_2 = (\tau_1 s + 1)Y(s)|_{s=-1/\tau_1} = -6K_1\tau_1 \quad (Eq. 21)$$

$$Y(s) = 6K_1\left(\frac{1}{s} - \frac{1}{\left(s + \frac{1}{\tau_1}\right)}\right). \quad (Eq. 22)$$

Finally, by using the Laplace transformation in table [1],

$$y(t) = 6K_1\left(1 - e^{-\frac{t}{\tau_1}}\right)u(t) \quad (Eq. 23)$$

There are two unknowns. To find K_1, steady state response will be used, which is as time goes to infinity. Steady state can be found from the data given by writing the command round(mean(y)) to Matlab. It is 97 so

$$\lim_{t\to\infty} 6K_1\left(1 - e^{-\frac{t}{\tau_1}}\right) = 6K_1 = 97 \; (Eq. 24), (Eq. 25)$$

$$K_1 = \frac{97}{6} = 16.1667 \; (Eq. 26)$$

Then, using any point in output data, time constant can be found. Let's choose t = 0.05. To get the data corresponding to that time point, following Matlab code is written: y(.05 / .0001 + 1)

It is found to be 29.9436. The reason for division by 0.0001 is because of the sampling frequency (10kHz). From this, following equation can be written:

$$97\left(1 - e^{-\frac{0.05}{\tau_1}}\right) = 29.9436. \; (Eq. 27)$$

From that equation, time constant is found to be $\tau_1 = 0.1354\ s$.

Hence, first order approximation of DC motor's transfer function is

$$G_p(s) \cong \frac{16.1667}{0.1354s + 1} \quad (Eq. 28)$$

The pole of this transfer function is $p_{TF} = 7.3855 \times 10^{-4}$ and pole of the low-pass filter is $p_{LPF} = -1000$ because its transfer function is

$$H_{LPF}(s) = \frac{1}{0.001s + 1} \quad (Eq. 29)$$

Thus, the pole of the transfer function (p_{TF}) is dominant compared to pole of LPF (p_{LPF}), i.e.,

$$|7.3855 \times 10^{-4}| < \frac{|-1000|}{10} \quad (Eq. 30)$$

To get the filtered data and transfer functions' output, following design is made in Simulink:

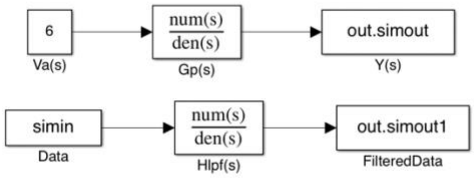

Figure 1: Block Diagrams the Output of Transfer Function and Filtered Data

Input data are taken from the workspace, and the outputs are sent to workspace so that their comparison in the same plot can be made by writing a Matlab code. The code written for these is given below

Matlab Code for Input Data:

```
simin = [t' y'];
```

after Simulink is run and outputs are sent to workspace.

Matlab Code for Output Data and Plot:

```
figure;
plot(out.simout, 'r');
hold on;
plot(out.simout1, 'b');
title("Time Series Plot (w(t) vs t) ");
xlabel("t (time s)");
ylabel("w(t) (angular velocity rad/s) ");
```

The comparison plot is given below

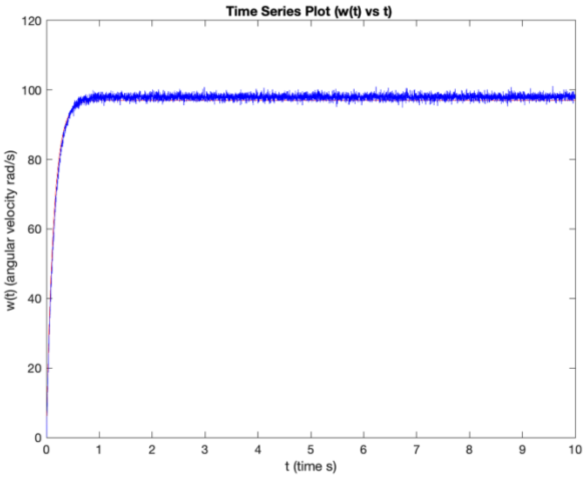

Figure 2: Angular Velocity Versus Time Plot

As you can see in Figure 2, red plot is filtered data and blue plot is noisy data.

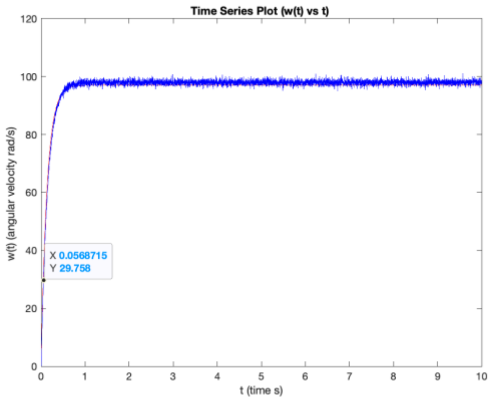

Figure 3: Angular Velocity Versus Time Plot with t = 0.05

As it can be seen in Figure 3, steady-state approximation is quite good because in the graph y(0.05) = 29.758, which is approximated as 29.9436. Another point is in steady-state region:

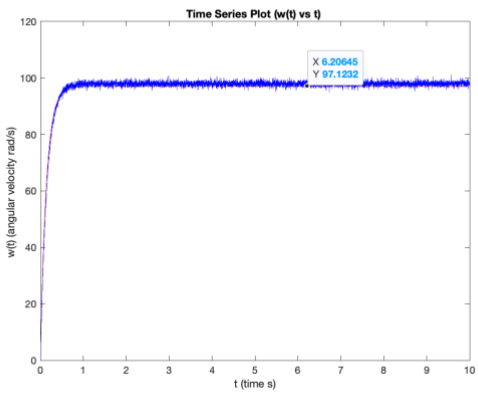

Figure 4: Angular Velocity Versus Time Plot with A Point In Steady-State

As it can be seen in Figure 4, steady-state on the plot is 97.1232, which is approximated as 97.

Part II

In this part, the effect of Proportional (P) and Proportional-Integral (PI) controller will be explained and showed. Let $G_c(s)$ be the transfer function of controller, which is added before $G_p(s)$ to the system. Then for the unity feedback gain, following equations can be written:

$$G(s) = G_c(s)G_p(s) \quad (Eq.28)$$

And error between the desired output and input is defined as

$$E(s) = R(s) - Y(s) = \frac{R(s)}{1 + G_c G_p(s)} \quad (Eq.29), (Eq.30)$$

Let's define the steady-state error as

$$e_{ss}(t) = \lim_{t \to \infty} e(t) \quad (Eq.31)$$

or in Laplace domain, its equivalent is

$$E_{ss}(s) = \lim_{s \to 0} sE(s) \quad (Eq.32)$$

by the final value theorem. Considering equation 12 and R(s) = 6/s, following cases can be written

If the controller is Proportional Controller (P),

$$G_c(s) = K_p. \quad (Eq.33)$$

$$E_{ss}(s) = \lim_{s \to 0} \frac{sR(s)}{1 + G_p(s)G_c(s)} = \lim_{s \to 0} \frac{6}{1 + K_p \frac{K_1}{\tau_1 s + 1}}$$
$$= \frac{6}{1 + K_p K_1} \neq 0. \quad (Eq.34)$$

If the controller is Proportional-Integral Controller (PI),

$$G_c(s) = K_p + \frac{K_i}{s} \quad (Eq.35)$$

$$E_{ss}(s) = \lim_{s \to 0} \frac{sR(s)}{1 + G_p(s)G_c(s)}$$
$$= \lim_{s \to 0} \frac{6}{1 + (K_p + \frac{K_i}{s}) \frac{K_1}{\tau_1 s + 1}}$$
$$= 0. \quad (Eq.36)$$

Hence, we need to add integral controller to compensate for the steady-state error in the system. Otherwise, error in steady-state response can never be 0, which we do not want. Other effects (overshoot, settling time and rise time) of P and I controllers are observed after some trials, which can be seen below

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate

Figure 5: Effects of P and I controller [2]

Thus, while designing PI controller, P controller can be added to improve the overshoot, and integral is added to eliminate the steady-state error. Furthermore, applying P controller decreases the rise time and after a certain reduction value for steady-state error, increasing K_p only leads to overshoot of the system response [2]. If P controller is sufficiently aggressive it causes oscillation [2].

Although PI controller eliminate the steady-state error, interms of the stability of the system, it creates negative impact [3]. As it can be seen, there is always a trade-off between the controllers. There is nothing perfect.

3. Conclusion

The purpose of this preliminary work was to find an approximate first order transfer function and understand the importance of P and PI controllers. In the first part, two unknowns of transfer function were found by using the steady-state response's value and any t in the range of 0 to 0.1 s. Then, two poles were compared to see the dominance of low-pass filter's pole. After that filtered data and noisy data are plotted and compared in Matlab. In the second part, by doing necessary calculations, it was seen that P controller cannot eliminate the steady-state error, but PI controller can do so. Other effects of P and PI controllers were shown in Figure 5 and comments were made.

REFERENCES

1. R. H. Bishop and R. C. Dorf, *Modern Control Systems: International Edition*, 10th ed. Upper Saddle River, NJ: Pearson, 2005.

2. *Researchgate.net*. [Online]. Available: https://www.researchgate.net/publication/269764111_Effects_of_PID_Controller_on_a_Closed_Loop_Feedback_System. [Accessed: 28-Feb-2021].

3. L. C. Westphal, "What are the separate and combined effects of increasing proportional and derivative values of P I controller in a feedback loop?," *Researchgate.net*, 01-Jan-2001. [Online]. Available: https://www.researchgate.net/post/What-are-the-separate-and-combined-effects-of-increasing-proportional-and-derivative-values-of-P-I-controller-in-a-feedback-loop. [Accessed: 28-Feb-2021].