EEE 342 Feedback and Control Systems Lab 3

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1. Introduction

In this lab, margin analysis of a system was done. The purpose of this lab was understanding how gain, phase and delay margins could be estimated by using mathematical model and how these calculations can be verified [1]. Lab consists of two parts. In the first part, a controller was designed which embodies first order low pass filter and a PI controller. Then, for the given controller and the plant, which was found in lab 1 [2], bode plot was drawn. From that plot phase and gain margin were found. After that, delay margin was calculated by crossover frequency and phase margin. In the second part, unit-step input was applied to the system and by changing gain and delay, their margins were calculated by trial and error. This was done by observing the time response of the system.

2. Laboratory Content

Since this lab is related to gain, phase and delay margins, these concepts should be understood first. Gain margin is the maximum gain that can be applied to the system while the system remains stable. Thus, if the gain margin is large, system is more robust to changes in gain. To find the gain margin, first the frequency at which the phase is -180 degree should be found as follows

$$\angle G(j\omega_c) = -180^\circ \qquad (1)\ [3]$$

Then, gain margin is

$$GM = 20\log\left|\frac{1}{G(j\omega_c)}\right| \qquad (2)\ [3]$$

Phase margin is the maximum phase that can be added to the system while it remains stable. For the phase margin, first the frequency at which the gain is zero should be found as follows

$$|G(j\omega_c)| = 1 \quad (3)\ \ [3]$$

Name ω_c is crossover frequency. After that, phase margin can be found from

$$\angle G(j\omega_c) = -180^\circ + \phi_m \quad mod(360^\circ)\ (4)\ [3]$$

where ϕ_m is phase margin.

To find delay margin, first effect of time delay should be considered.

Let

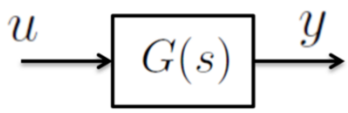

Figure 1: System with G(s) = e^{-sT}

and suppose G(s) = e^{-sT}. If s = jω, G(jω) = $e^{-j\omega T}$. Thus,

$$|G(j\omega)| = \left|e^{-j\omega T}\right| = 1 \qquad (5)$$

and

$$\angle G(j\omega) = -\omega T \qquad (6)$$

That means, time delay does not change the magnitude, but it decreases the phase linearly with $-\omega T$. Hence, if the delay is large, system may be destabilized so it is important to know the maximum delay that can be added to the system while the system is stable. To find the, consider an arbitrary transfer function G(s) and define new transfer function as

$$G'(s) = G(s)e^{-sT} \qquad (7)$$

By equation 5,

$$|G'(j\omega_c)| = |G(j\omega_c)| = 1 \quad (8)$$

By equation 4 and 6,

$$\angle G'(j\omega_c) = \angle G(j\omega_c) - \omega T = 180^\circ + \phi_m - \omega T \ \ (9)$$

So new phase margin is

$$\phi_m' = \phi_m - \omega T \qquad (10)$$

Since phase margin needs to be larger than 0, otherwise system becomes unstable,

$$\phi_m - \omega T > 0 \qquad (11)$$

we get

$$\frac{\phi_m}{\omega} > T \qquad (12)$$

That means, the largest delay that can be introduced to the system is

$$DM = \frac{\phi_m}{\omega} \qquad (13)$$

which is delay margin.

Part-1: Margin Estimation (Check-1)

Remember from lab 1, first order approximation of DC motor was found to be

$$G_p(s) = \frac{14.78}{0.191s + 1} \quad (14) \quad [2]$$

Controller given in the lab manual is

$$G_c(s) = \left(\frac{1}{s + \tau_{LPF}}\right)\left(\frac{K_c(s + 80)}{s}\right) \quad (15) \quad [1]$$

where K_c and τ_{LPF} are found by

$$K_c = \frac{2}{K_g}, \quad \tau_{LPF} = \frac{3}{\tau_p} \qquad (16)(17) \quad [1]$$

Note that K_g is DC gain of first order approximation and τ_p is the reciprocal of the distance of pole of the plant to the imaginary axis; therefore, $K_g = 14.78$ and $\tau_p = 0.191$. For $G_p(s)G_c(s)$, bode plot was drawn by the following MATLAB code

```
K = 14.78;
tau = 0.191;
Kc = 2/K;
tauLPF = 3/tau;
q1 = [1];
q2 = [1 tauLPF];
q3 = [Kc 80*Kc];
q4 = [1 0];
sys1 = tf(q1,q2);
sys2 = tf(q3,q4);
Gc = series(sys1,sys2);
Gp = tf(K,[tau 1]);
L = series(Gc,Gp);
bode(L)
grid on;
hold on;
```

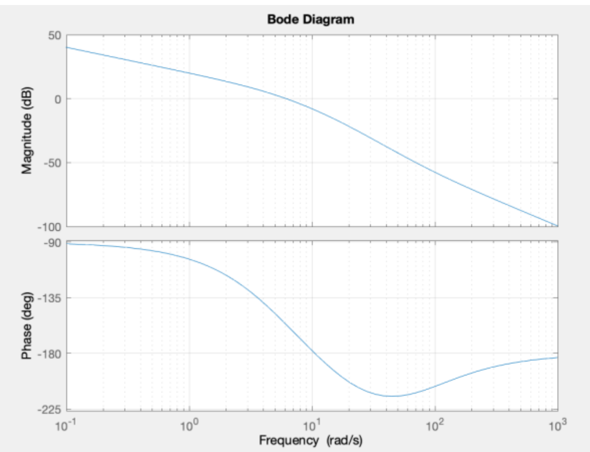

Figure 2: Bode Plot of $G_c(s)G_p(s)$ for Magnitude and Phase

To be exact, desired margins were calculated via "margins = allmargin(L)" MATLAB command and results are

Gain Margin: 8.8962 (dB)
GM Frequency: 10.5544
Phase Margin: 23.3762 (degree)
PM Frequency: 6.1564
Delay Margin: 0.0663 (second)
DM Frequency: 6.1564
Stable: 1

Additionally, gain and phase margin can also be found by using the bode plot and delay margin can be calculated by using phase margin by using equation 13.

To find gain and phase margin, following plots can be observed

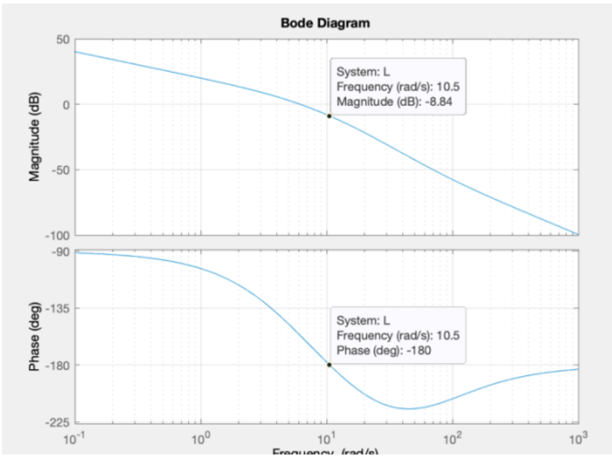

Figure 3: Gain Margin Measurement

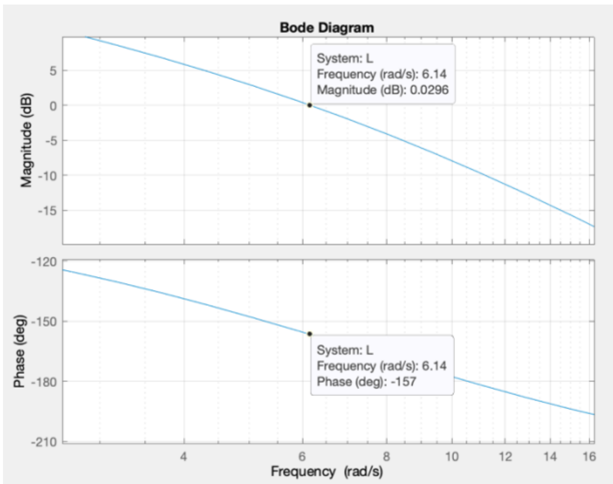

Figure 4: Phase Margin Measurement

From equation 2, it can be understood that gain margin is the dB value between 0 dB and $20\log|G(j\omega_c)|$ where at ω_c, phase is -180°. Thus, by Figure 3, gain margin was found to be 8.84 dB. From equation 4, it can be understood that phase margin is the phase value between the phase of the transfer function and -180° for the frequency ω_c at which the gain is 0 dB. Hence, phase margin was found to be 23°. When these results are compared to the results of the "allmargin()" function of MATLAB, values found from plots are pretty accurate. Of course, they cannot be exactly accurate due to the lack of precision of plots, but they are reasonably accurate.

In order to calculate the delay margin by the values acquired from plots, using equation 13 is enough

$$DM = \frac{\pi\phi_m}{180\omega} = \frac{\pi \times 23}{180 \times 6.14} = 0.065 \quad (18)$$

When this value is compared with the exact value, it can be seen that they are close to each other. By zooming in to plots, this precision can be increased to some level. To make the location of points easy to comprehend, I prefer to not zoom in too much.

In this part, it can be understood that phase, gain and delay margins can be calculated through bode plot. Provided an exact plot, these values can be found accurately.

Part-2: Margin Verification (Check- 2&3)

In this part, gain and phase margins were estimated by trial and error. From these estimations, delay margin was calculated, and results were compared with the calculation in previous part.

First 'lab3_DCmotor.slx' was downloaded from moodle to use the DC motor model and 'lab3_step_GM.slx' was downloaded for the estimation of gain margin. Then, controller and input were set to Gc(s) and 40u(t) respectively and the gain in the Simulink was adjusted to workspace parameter so that it can be changed through MATLAB.

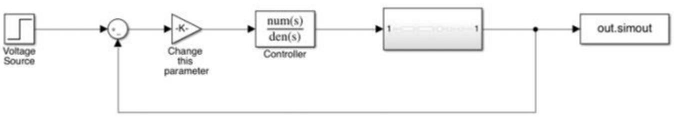

Figure 5: Model for Estimating Gain Margin

To make the estimation, following MATLAB code was written

```
dc_gain = 2.7849;
model = 'lab3_step_GM';
load_system(model);
simOutput = sim(model);
output = simOutput.simout;
plot(output,'LineWidth',1.5);
title('Output Signal for Gain =
2.7489');
ylim([min(output) max(output)])
```

As it was suggested in lab manual, I started with the calculated gain which is 8.8962 dB

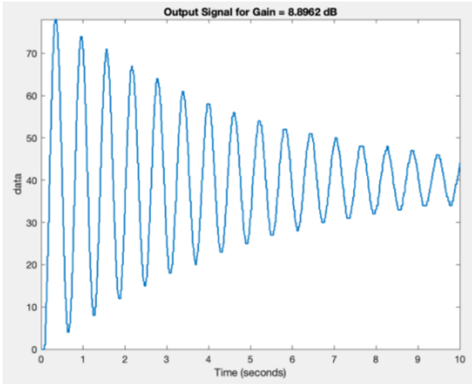

Figure 6: Time Response When Gain = 8.8962

System is stable as it is seen in Figure 6 so gain was increased to find the boundary after which system becomes unstable. After some trial and error, I found gain margin as

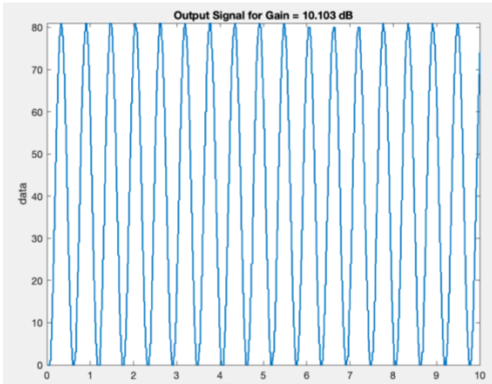

Figure 7: Time Response When Gain = 10.103 dB

As it can be seen above, when the gain is 10.103 dB, I got pure oscillation so at that gain, system is marginally stable, which is unstable. If I decrease the gain slightly, I get

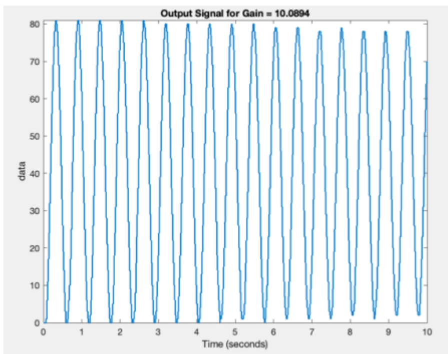

Figure 8: Time Response When Gain = 10.0894 dB

As it can be seen in Figure 8, system is stable when I decrease the gain slightly. If I increase the gain slightly, I get

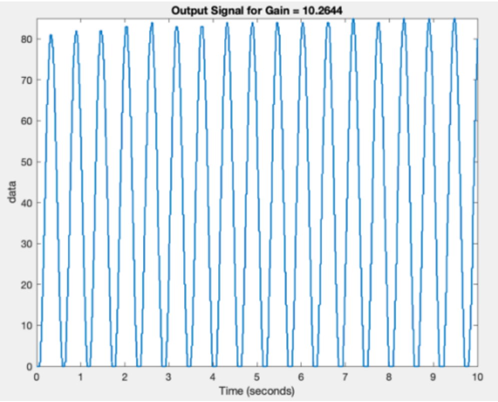

Figure 9: Time Response When Gain = 10.2644 dB

As it can be seen in above figure, when the gain is slightly increased, time response becomes larger and larger as times passes.

Similar process was done for delay margin as well. ‘lab3_step_DM.slx’ was downloaded for the estimation of phase margin from moodle. Then, controller and input were set to Gc(s) and 40u(t) respectively and the time delay in Simulink was adjusted to workspace parameter so that it can be adjusted through MATLAB.

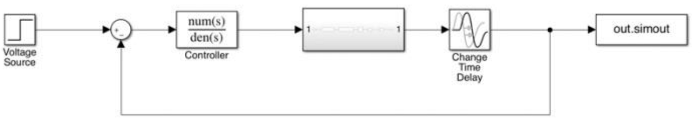

Figure 10: Model for Estimating Delay Margin

To make the estimation by trial and error, following MATLAB code was written

```
h = 0.0663;
model2 = 'lab3_step_DM';
load_system(model2);
simOutput2 = sim(model2);
output2 = simOutput2.simout;
plot(output2,'LineWidth',1.5);
title('Output Signal for Delay = 
0.0663s');
ylim([min(output2) max(output2)])
```

By changing variable h, time response of the system was observed. As it is suggested in lab manual, first, calculated DM margin in previous part was plotted.

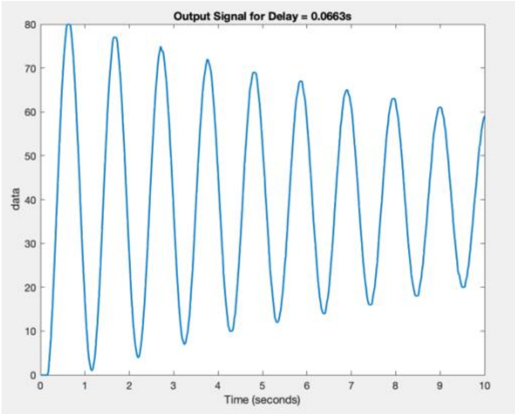

Figure 11: Time Response When Delay = 0.0663s

As it can be seen in above figure, system is stable in calculated DM because time response is converging, it does not become larger and larger. After some trial and error, delay margin was found to be

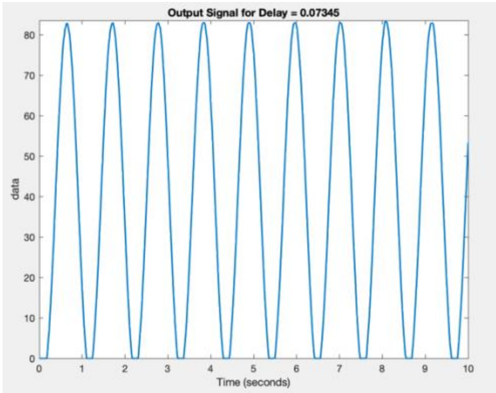

Figure 12: Time Response When Delay = 0.07345s

In above figure, signal is purely oscillatory, so system is marginally stable when the given time delay was applied to the system. Hence, the maximum time delay that can be applied to the system is 0.07345 s, which is delay margin. If the time delay is slightly decreased, I get

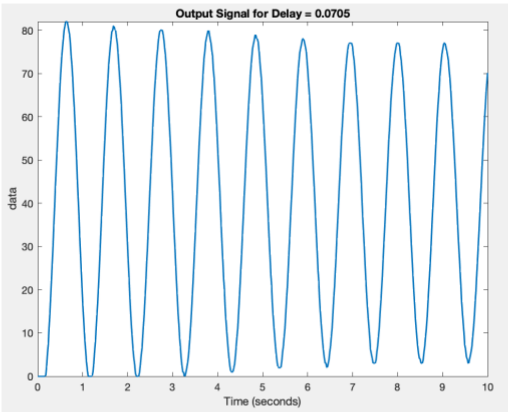

Figure 13: Time Response When Delay = 0.0705s

In Figure 13, when time delay was slightly decreased, system becomes stable because time response becomes lower as times passes. If I increase the time delay slightly, I get

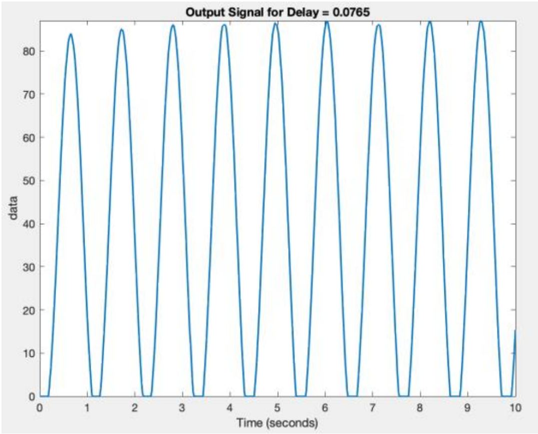

Figure 14: Time Response When Delay = 0.0765s

As it can be seen in above figure, when the delay is increased slightly from 0.07345 s, system becomes unstable because time response output becomes larger and larger as times passes. To see the comparison of GM and DM, I gather the data in one table

	GM (dB)	DM (second)
First Order Approximation Model	8.8962	0.0663
Nonlinear DC Motor Model	10.103	0.07345

Table 1: GM and DM Comparison Table

As it can be seen in above table, there is a little difference between two models' gain and delay margin. The reason for that is because the plant function of the system in part 1 was the model of nonlinear DC model's first order approximation. However, the plant function in second system, which can be seen in Figure 5 and 10, is nonlinear DC motor's model without first order approximation; therefore, gain and delay margins were found to be different.

3. Conclusion

The main purpose of this lab was to comprehend how gain, phase and delay margins can be estimated by using mathematical model and to verify the calculations via estimations [1]. In the first part, DC motor's first order approximated model was used with the given controller to find gain, phase and delay margins. In the second part, nonlinear DC motor model was used without approximation and its gain and delay margins were found by applying unit step input with various gains and delays. By trial and error, gain and delay margins were found. Results of two models were compared and they were found to be little different due to the approximation reasons, but they were close. In this lab, I achieved the goal and I have learnt how to find gain and phase margin through bode plot and MATLAB function "allmargin()". Also, I have learnt how to estimate the gain and delay margin experimentally.

REFERENCES

1. Bilkent University, "eee342_lab3" 19 April 2021. [Online]. Available: moodle.bilkent.edu.tr. [Accessed 24 April 2021].
2. [2] M. B. Şahin, "MehmetBerkŞahin_EEE342_LAB01Report," 2021.
3. Dorf, R. C., & Bishop, R. H. (2022). *Modern control systems*. Hoboken, NJ: Pearson Education.