EEE-342 Feedback and Control Systems Preliminary Work 3

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1. Introduction

In this preliminary work, the aim was to predict the maximum applicable gain of the controller while keeping the closed-loop system stable [1]. In other words, the aim was to find the gain margin. This process consists of 5 steps. Unknown parameters in transfer function were calculated. Then root locus of the transfer function with PI controller was drawn. By looking at root locus plot, stability analysis was done. In the fourth part, due to the 10ms time delay, second order Pade approximation was done to model the delay [1]. Lastly, maximum available gain of the controller was found by using root locus.

2. Laboratory Content

Step 1

In lab 1, a PI controller was designed with first order approximation under some requirements and the open-loop transfer function was

$$G(s) = \left(\frac{K_p s + K_i}{s}\right) \times \left(\frac{K}{\tau s + 1}\right) \qquad (1)\ [1]$$

From lab 2, $K_p = 0.5$ and $K_i = 0.5$ [2] and from lab 1, $K = 14.78$ and $\tau = 0.191$ [1]. Equation 1 can be written as

$$G(s) = \left(\frac{K_c\left(\frac{s}{z} + 1\right)}{s}\right) \times \left(\frac{K}{\tau s + 1}\right) \qquad (2)$$

where $K_c = 0.5$ and z = 1.

Step 2

After putting the numerical values into equations, we get

$$G(s) = \left(\frac{0.5(s + 1)}{s}\right) \times \left(\frac{14.78}{0.191s + 1}\right) \qquad (3)$$

$$= \frac{7.39 \times (s + 1)}{s \times (0.191s + 1)} \qquad (4)$$

By writing following MATLAB command, root locus was drawn

```
q1 = [7.39 7.39]; %numerator
q2 = [0.191 1 0]; %denimator
sys = tf(q1,q2); %transfer function
rlocus(sys); %root locus
```

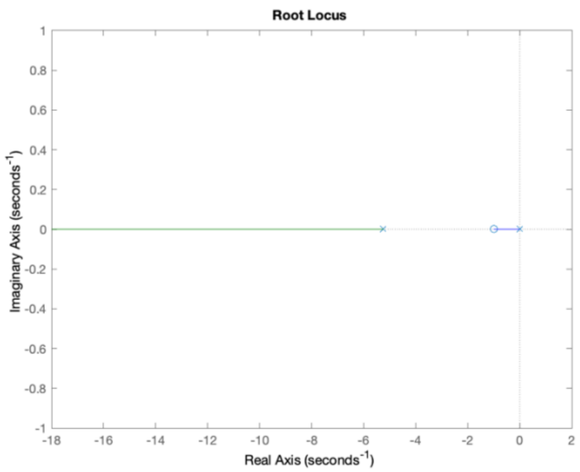

Figure 1: Root Locus of G(s)

Step 3

As it can be seen in Figure 1, the gain of the controller can be as large as desired without causing system to be unstable because all the roots are in the left half plane. If after some point, roots ere in right half plane, system would be unstable but in Figure 1, that is not the case. When the gain goes to infinity from zero, root locus is always on the left half plane.

Step 4

Since most of the physical systems have time delay due to actuators or communicators [1], it is an important concept and should not be ignored. Thus, our transfer function with first order approximation should be updated with second order Pade approximation to model 10ms time delay. It is

$$P(s) = \left(\frac{K}{\tau s + 1}\right) \times \left(\frac{s^2 - 600s + 120000}{s^2 + 600s + 120000}\right) \qquad (4)\ [1]$$

and by equation 2, we can write

$$G(s) = \left(\frac{K_c\left(\frac{s}{z} + 1\right)}{s}\right)\left(\frac{K}{\tau s + 1}\right)\left(\frac{s^2 - 600s + 120000}{s^2 + 600s + 120000}\right) \qquad (5)$$

By putting numerical values into the equation, we get

$$G(s) = \left(\frac{s+1}{s}\right)\left(\frac{7.39}{0.191s+1}\right)\left(\frac{s^2-600s+120000}{s^2+600s+120000}\right) \qquad (6)$$

To draw the root locus of this transfer function following MATLAB code was written:

```
q1 = [7.39 7.39];
q2 = [0.191 1 0];
q3 = [1 -600 120000];
q4 = [1 600 120000];
numerator = conv(q1,q3);
denimerator = conv(q2,q4);
sys = tf(numerator,denimerator);
rlocus(sys);
```

Following plot was obtained

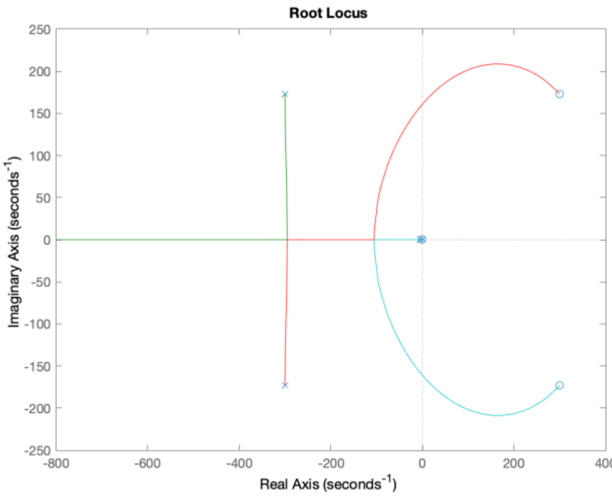

Figure 2: Root Locus of Second Transfer Function with Pade Approximation

Step 5

In this step, maximum available gain was found by analyzing root locus of the updated transfer function. When the root locus passes to right half plane, system become unstable so imaginary axis crossing points are the last point for stability. At these points system is marginally stable, which is unstable because system gives unbounded output when bounded sinusoidal input with resonance frequency is given to system. In short, to see the maximum available gain, the gain value at the imaginary axis crossing points should be measured as follows

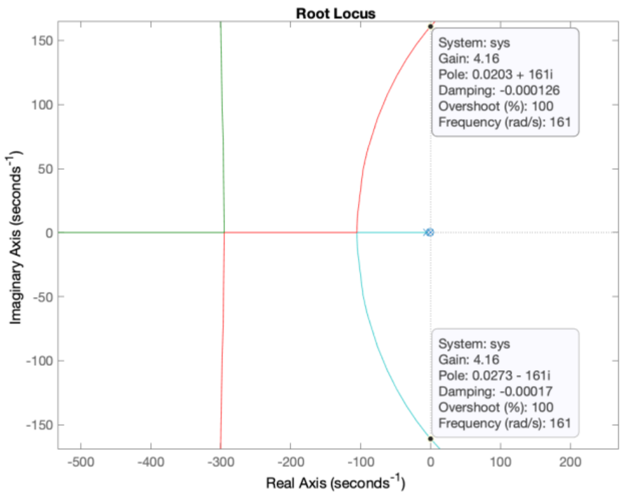

Figure 4: Root Locus of Second Transfer Function with Measurement of Imaginary Axis Crossing Points

As it can be seen in above figure, maximum available gain of the controller is 4.16. After that gain, system becomes unstable.

3. Conclusion

The purpose of this preliminary work was estimating the maximum applicable gain of the controller while keeping the close-loop transfer function stable [1]. For the transfer function PI controller with first order approximation function were used. In addition to these, second order Pade approximation was applied to transfer function due to the 10ms time delay, which is generated from communicator or actuators in physical systems. Lastly, root locus of the final transfer function was drawn via MATLAB and maximum available gain was measured from the plot. To see the maximum available gain, imaginary axis crossing points were marked and gain at these points were observed. In this preliminary work, I have learnt how to see the gain margin by looking at the root locus of the transfer function and I have learnt how to draw root locus in MATLAB.

REFERENCES

[1] Bilkent University, "eee342_lab3_preliminary_work_manual.pdf," 19 April 2021. [Online]. Available: moodle.bilkent.edu.tr. [Accessed 8 April 2021].

[2] M. B. Şahin, "MehmetBerkŞahin_EEE342_Lab02Report," 2021.