Using an EKF to get scalar orientation from an IMU

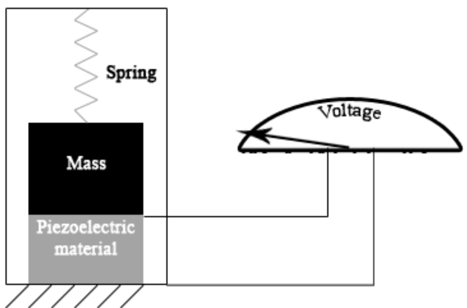
MIP track, week 3

A 1-DOF Rotating Rigid Body

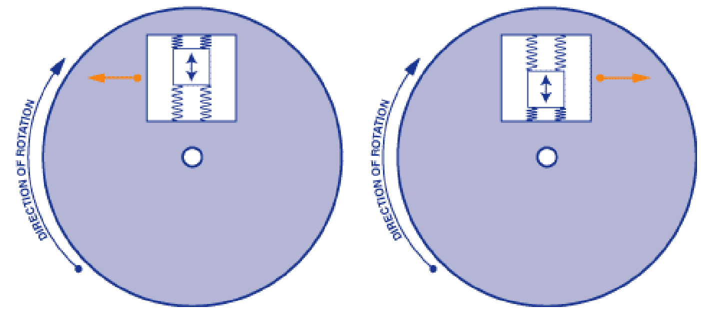
- Define body's "roll angle" $\phi \in S^1$
- Model motion: $\ddot{\phi} = 0$
- Define state:
$$x_k := \begin{bmatrix} \phi(t_k) \\ \dot{\phi}(t_k) \end{bmatrix}$$
- State update (discrete) $x_{k+1} = \begin{bmatrix} 1 & dt \\ 0 & 1 \end{bmatrix} x_k =: A_k x_k$

- "Zero order hold" (Mobility 1.2)
- Note: is linear

Measurements from an IMU



https://learn.sparkfun.com/tutorials/accelerometer-basics



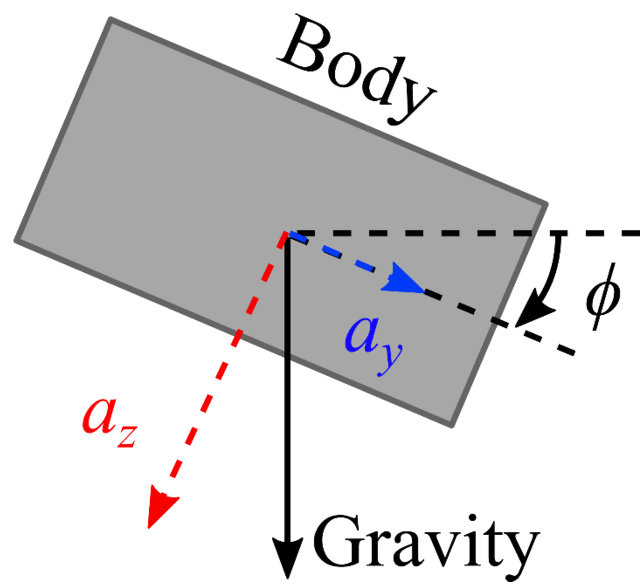
https://learn.sparkfun.com/tutorials/gyroscope

- MEMS accelerometer and gyroscope

- Projection of acceleration vector on body frame
- Rotation rate

- Measurement vector $z_k = \begin{bmatrix} \sin\phi(t_k) \\ \cos\phi(t_k) \\ \dot{\phi}(t_k) \end{bmatrix} =: h(x_k)$

- Need to calculate Jacobian $H := Dh$



Extended Kalman Filter

- State update linear, measurement non-linear
- EKF idea (review): use *local linearization*, i.e.

$$h(x_{k+1}) \approx h(x_k) + H(x_k)(x_{k+1} - x_k)$$

- The rest is the same as a KF; summarized here
- New state: $\boxed{(\hat{x}, P)}$
- Predict:
$$\boxed{\begin{aligned} \hat{x}_{k|k-1} &= A_{k-1}\hat{x}_{k-1} \\ P_{k|k-1} &= A_{k-1}P_{k-1}A_{k-1}^T + Q \end{aligned}}$$
- Optimal Kalman gain
$$\boxed{K_k = P_{k|k-1}H_k^T(H_kP_{k|k-1}H_k^T + R)^{-1}}$$
- Update:
$$\boxed{\begin{aligned} \hat{x}_k &= \hat{x}_{k|k-1} + K_k(z_k - h(\hat{x}_{k|k-1})) \\ P_k &= (I - K_kH_k)P_{k|k-1} \end{aligned}}$$
- Tuning params: Q and R