Local Linearization and Linearized Control of a MIP

MIP track, week 5

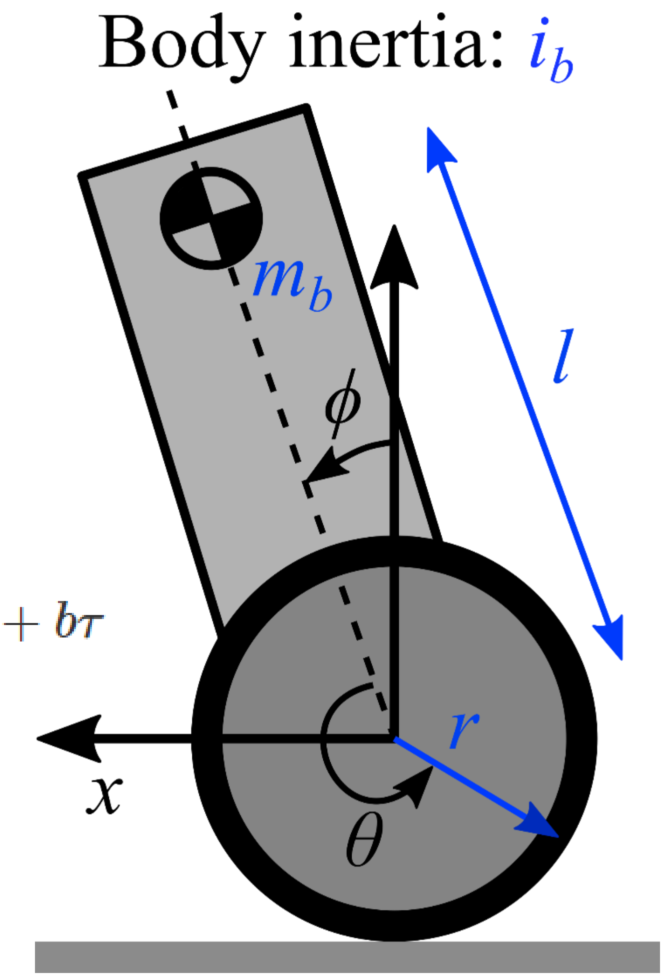
Linearization as a Truncated Taylor Expansion

- Define state $x := \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \implies \dot{x} = f(x, \tau)$

- Can locally linearize about "upright", $\bar{x} := 0$

- Linearized dynamics

$$\tilde{x} := x - \bar{x} \implies \dot{\tilde{x}} \approx D_x f(\bar{x}, 0)\tilde{x} + D_\tau f(\bar{x}, 0)\tau =: A\tilde{x} + b\tau$$



Linear Controllability and LQR

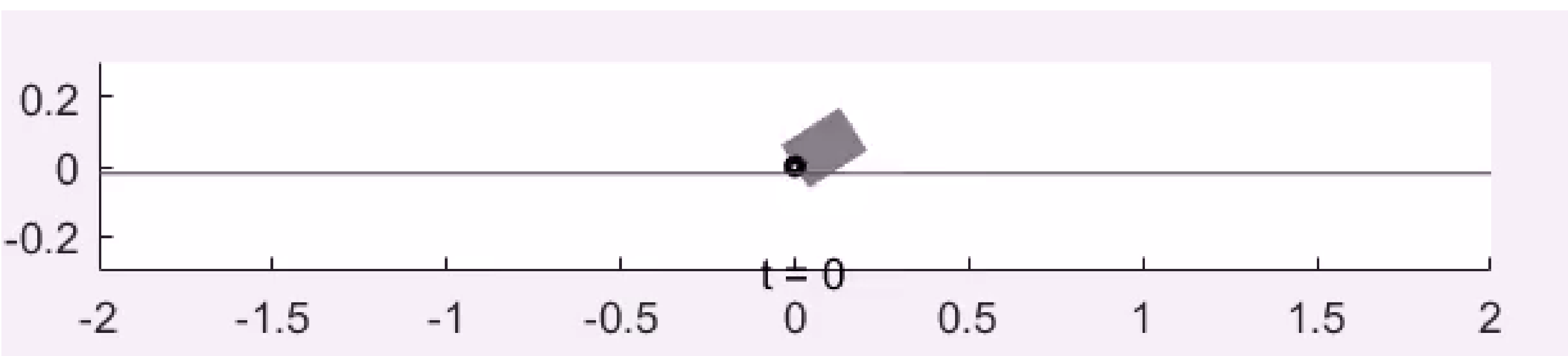
- Easy in MATLAB

```
Co=ctrb(A,b);
% Number of uncontrollable states
unco=length(A)-rank(Co)
```

- If controllable

```
K=lqr(A,b,eye(4),1,0)
```

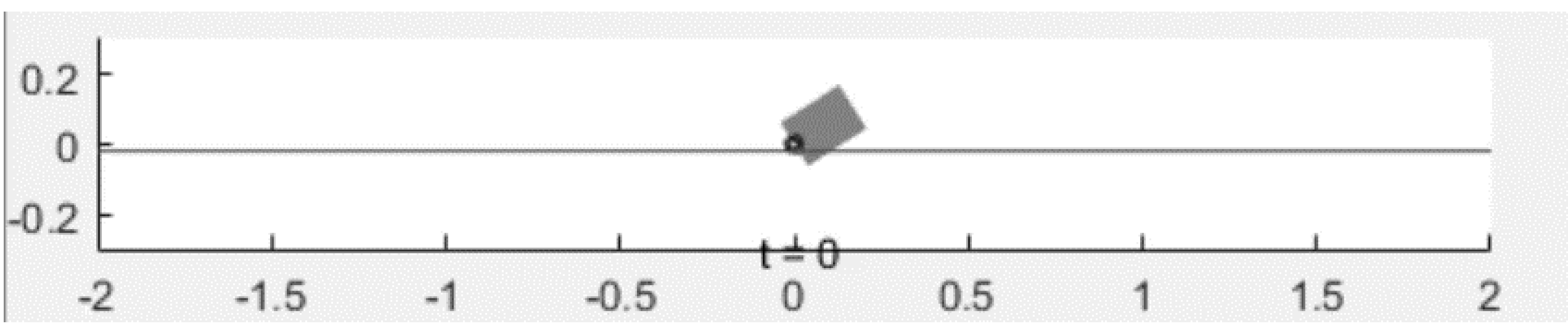
- Now the feedback control $\tau := -Kx$ stabilizes to the goal state



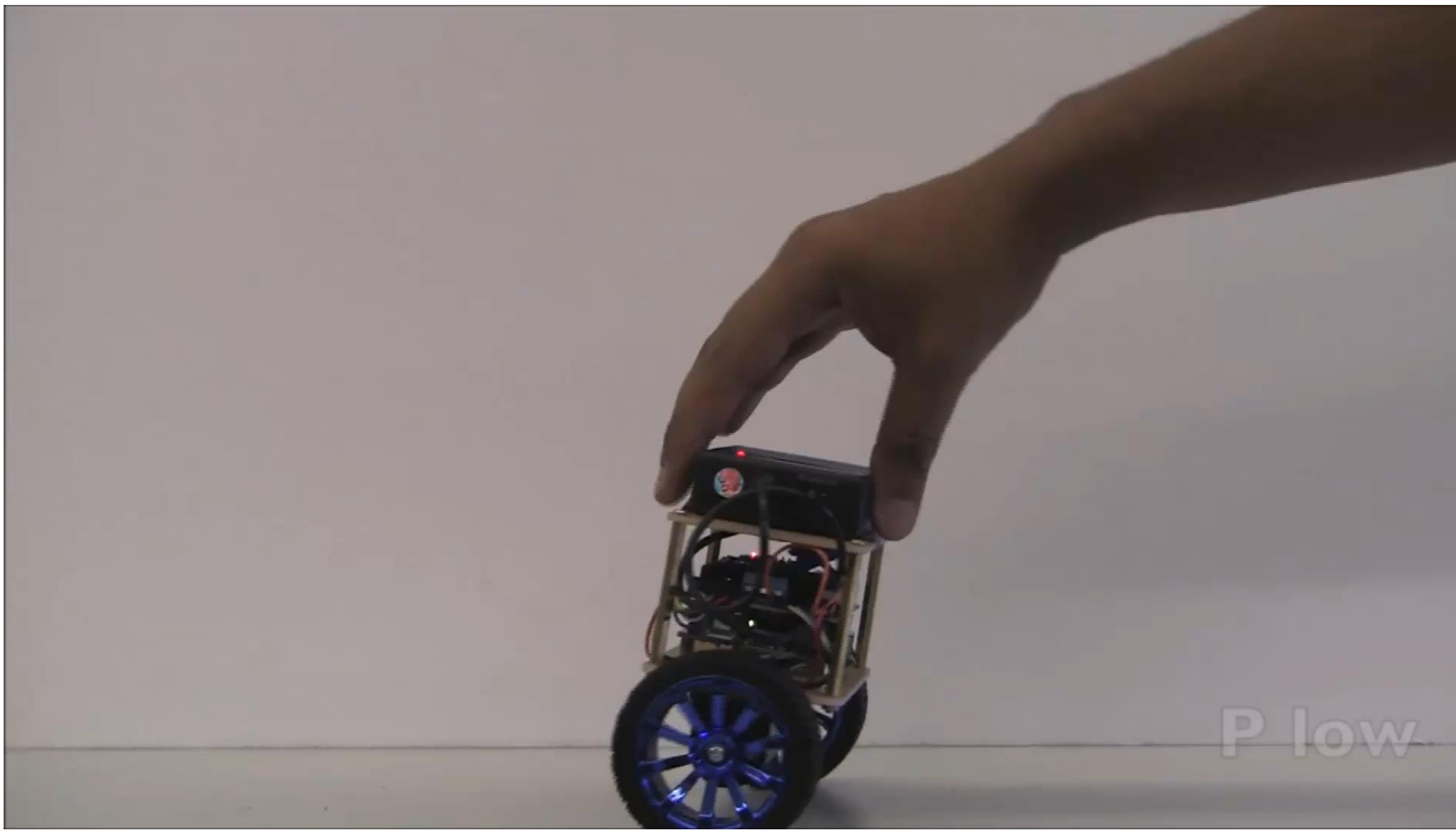
Balancing Using a Non-optimal PD Controller

- Expand eqns of motion $\ddot{\phi}$
- Get $\ddot{\phi} = f(\phi, \dot{\phi}) + g(\phi, \dot{\phi})\tau$
- Assume (as approximation) $\phi = 0, \dot{\phi} = 0$
  - Then f vanishes, g is constant

- Try yourself: should get $\ddot{\phi}|_{\phi=0,\dot{\phi}=0} = \alpha\tau$
- Recall week 2—can be stabilized by PD
- Not controlling wheel angle
- Any wheel damping will slow to stop anyway



Gain Tuning on a Physical Platform



Once Tuned