Lagrangian Modeling of a MIP

MIP track, week 4

Modeling a Mobile Inverted Pendulum (MIP)

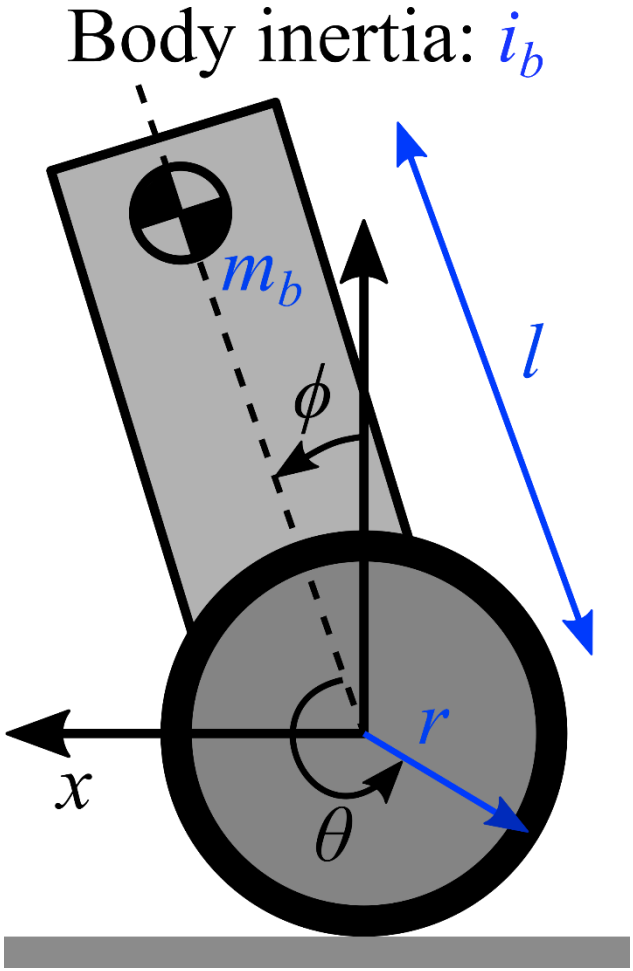
- Generalized coordinates

$$q := \begin{bmatrix} \theta \\ \phi \end{bmatrix}$$

Wheel/body angle

Body roll angle

- In the absence of slip $x = (\theta + \phi)r$
- Assume massless wheel



Lagrangian Formulation

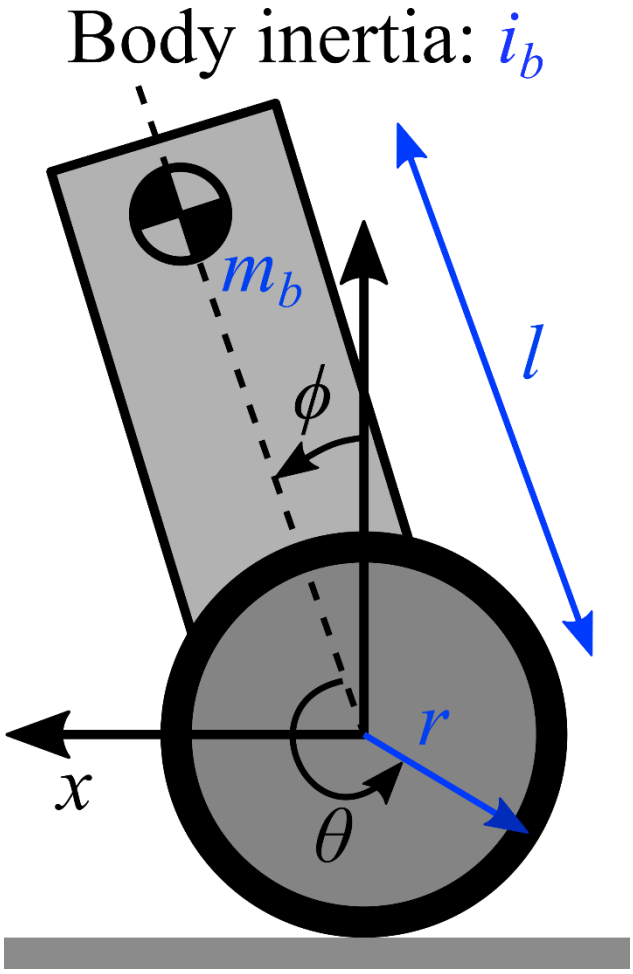
Center of Mass

- Position of COM relative to drawn coord frame

$$p = r \begin{bmatrix} \theta + \phi \\ 0 \end{bmatrix} + l \begin{bmatrix} \sin\phi \\ \cos\phi \end{bmatrix}$$

- Kinetic energy $T := \frac{1}{2} m_b \dot{p}^T \dot{p} + \frac{1}{2} i_b \dot{\phi}^2$
- Potential energy $V := m_b g l \cos\phi$
- Lagrangian $L := T - V$
- Equations of motion Equation of Motion

$$(D_q - D_t D_{\dot{q}}) L = \begin{bmatrix} \tau \\ 0 \end{bmatrix}$$



Simulation

- Should get “natural” motions
- Ignore ground intersection