1　三角函数相关公式

$$\sin 2x = 2\sin x\cos x$$

$$\sin 3x = -4\sin^3 x + 3\sin x$$

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2\cot x}$$

$$\sin^2\frac{x}{2} = \frac{1}{2}(1-\cos x)$$

$$\cos^2\frac{x}{2} = \frac{1}{2}(1+\cos x)$$

$$\sin^2 x = \frac{1}{2}(1-\cos 2x)$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\sin\alpha+\sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha+\cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta = -2\sin\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta\pm\cos\alpha\sin\beta$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta\mp\sin\alpha\sin\beta$$

$$\tan(\alpha\pm\beta) = \frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$$

$$\cot(\alpha\pm\beta) = \frac{\cot\alpha\cot\beta\mp 1}{\cot\beta\pm\cot\alpha}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$\cot\frac{x}{2} = \frac{\sin x}{1-\cos x} = \frac{\sin x}{1+\cos x} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1-2\sin^2 x = 2\cos^2 x - 1$$

$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \tfrac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$$

$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$$

$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

万能公式：若 $u = \tan\frac{x}{2}(-\pi < x < \pi)$，则 $\sin x = \frac{2u}{1+u^2}, \cos x = \frac{1-u^2}{1+u^2}$

2　泰勒及其展开式

泰勒公式:

$$f(x)=f(x_0)+f'(x_0)(x-x_0)+\frac{f''(x_0)}{2}x^2$$
$$+\cdots+\frac{f^n x_0}{n!}(x-x_0)^n+\frac{f^{n+1}(\theta)}{(n+1)!}(x-x_0)^{n+1}$$

麦克劳林公式:

$$f(x)=f(0)+f'(0)(x)+\frac{f''(0)}{2}x^2+\cdots+$$
$$\frac{f^n(0)}{n!}(x)^n+\frac{f^{n+1}(\theta)}{(n+1)!}x^{n+1}$$

$e^x=1+x+\frac{1}{2!}x^2$

$\ln(1+x)=x-\frac{1}{2!}x^2$

$\sin x=x-\frac{1}{3!}x^3$

$\arcsin x=x+\frac{1}{3!}x^3$

$\tan x=x+\frac{1}{3}x^3$

$\arctan x=x-\frac{1}{3}x^3$

$\cos x=1-\frac{1}{2!}x^2$

$\frac{1}{1-x}=1+x+x^2$

$\frac{1}{1+x}=1-x+x^2$

$(1+x)^\alpha=1+\alpha x+\frac{\alpha(\alpha-1)}{2}x^2$

3　基本求导公式

$(x^k)'=kx^{k-1}$

$(\ln x)'=\frac{1}{x}\quad(x>0)$

$(e^x)'=e^x$

$(a^x)'=a^x\ln a\ (a>0,a\neq 1)$

$(\sin x)'=\cos x$

$(\cos x)'=-\sin x$

$(\tan x)'=\sec^2 x$

$(\cot x)'=-\csc^2 x$

$(\sec x)'=\sec x\tan x$

$(\csc x)'=-\csc x\cot x$

$(\ln|\cos x|)'=-\tan x$

$(\ln|\sec x+\tan x|)'=\sec x$

$$(\ln|\csc x-\cot x|)'=-\csc x$$

$$(\arcsin x)'=\frac{1}{\sqrt{1+x^2}}$$

$$(\arccos x)'=\frac{-1}{\sqrt{1-x^2}}$$

$$(\arctan x)'=\frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)'=\frac{-1}{1+x^2}$$

$$[\ln(x+\sqrt{x^2+a^2})]'=\frac{1}{\sqrt{x^2+a^2}}$$

$$[\ln(x+\sqrt{x^2-a^2})]'=\frac{1}{\sqrt{x^2-a^2}}$$

4 微分的几何应用

$$\text{水平渐近线}: \begin{cases}\lim\limits_{x\to+\infty} f(x)=y_1\\ \lim\limits_{x\to-\infty} f(x)=y_2\end{cases} \Rightarrow \begin{cases}y=y_1\\ y=y_2\end{cases}$$

$$\text{斜渐近线}: \begin{cases}\lim\limits_{x\to+\infty} \frac{f(x)}{x}=k_1\\ \lim\limits_{x\to+\infty} [f(x)-k_1x]=b_1\end{cases} \Rightarrow y=k_1x+b_1$$

$$\text{斜渐近线}: \begin{cases}\lim\limits_{x\to-\infty} \frac{f(x)}{x}=k_2\\ \lim\limits_{x\to-\infty} [f(x)-k_2x]=b_2\end{cases} \Rightarrow y=k_2x+b_2$$

旋转曲面的侧面积:

绕x轴: $S=2\pi\int_a^b|y(x)|\sqrt{1+[y'(x)]^2}\mathrm{d}x$

$$\begin{cases}x=x(t)\\ y=y(t)\end{cases} \alpha\le t\le\beta \Rightarrow S=2\pi\int_\alpha^\beta|y(t)|\sqrt{[x'(t)]^2+[y'(t)]^2}\mathrm{d}t$$

曲率及曲率半径: $k=\frac{|y''|}{[1+(y')^2]^{\frac{3}{2}}}, R=\frac{1}{k}$

5　基本积分公式

$$\int x^k \mathrm{d}x = \frac{1}{k+1}x^{k+1} + C \quad k \neq 1$$

$$\int \frac{1}{x^2} = -\frac{1}{x} + C$$

$$\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + C$$

$$\int \frac{1}{x} = \ln|x| + C$$

$$\int e^x \mathrm{d}x = e^x + C$$

$$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$$

$$\int \sin x \mathrm{d}x = -\cos x + C$$

$$\int \cos x \mathrm{d}x = \sin x + C$$

$$\int \tan x \mathrm{d}x = -\ln|\cos x| + C$$

$$\int \cot x \mathrm{d}x = \ln|\sin x| + C$$

$$\int \sec^2 x \mathrm{d}x = \tan x + C$$

$$\int \csc^2 x \mathrm{d}x = -\cot x + C$$

$$\int \sec x \tan x \mathrm{d}x = \sec x + C$$

$$\int \csc x \cot x \mathrm{d}x = -\csc x + C$$

$$\int \frac{1}{1+x^2} \mathrm{d}x = \arctan x + C$$

$$\int \frac{1}{a^2+x^2} \mathrm{d}x = \frac{1}{a} \operatorname{arccot} \frac{x}{a} + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \mathrm{d}x = \arcsin x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \mathrm{d}x = \arcsin \frac{x}{a} + C$$

$$\int \sin^2 x \mathrm{d}x = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$\int \cos^2 x \mathrm{d}x = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$\int \tan^2 x \mathrm{d}x = \tan x - x + C$$

$$\int \cot^2 x \mathrm{d}x = -\cot x + x + C$$

$$\int u \mathrm{d}v = uv - \int v \mathrm{d}u + C$$

$$\int_a^b f(x) \mathrm{d}x = \int_a^b f(a+b-x) \mathrm{d}x$$

$$\int \frac{\mathrm{d}x}{\cos x} = \int \sec x \mathrm{d}x = \ln|\sec x + \tan x| + C$$

$$\int \frac{\mathrm{d}x}{\sin x} = \int \csc x \mathrm{d}x = \ln|\csc x - \cot x| + C$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\mathrm{d}x = \ln\left(x + \sqrt{x^2+a^2}\right) + C$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\mathrm{d}x = \ln\left|x + \sqrt{x^2-a^2}\right| + C$$

$$\int \frac{1}{x^2-a^2}\mathrm{d}x = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{1}{a^2-x^2}\mathrm{d}x = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$$

$$\int \sqrt{a^2-x^2}\mathrm{d}x = \frac{a^2}{2}\arcsin\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} + C$$

$$\int_a^b f(x)\mathrm{d}x = \frac{1}{2}\int_a^b [f(x) + f(a+b-x)]\mathrm{d}x$$

$$\int_a^b f(x)\mathrm{d}x = \int_a^{\frac{a+b}{2}} [f(x) + f(a+b-x)]\mathrm{d}x$$

$$\int_0^\pi x f(\sin x)\mathrm{d}x = \frac{\pi}{2}\int_0^\pi f(\sin x)\mathrm{d}x$$

$$\int_0^\pi x f(\sin x)\mathrm{d}x = \pi\int_0^{\frac{\pi}{2}} f(\sin x)\mathrm{d}x$$

$$\int_0^{\frac{\pi}{2}} f(\sin x)\mathrm{d}x = \int_0^{\frac{\pi}{2}} f(\cos x)\mathrm{d}x$$

$$\int_0^{\frac{\pi}{2}} f(\sin x, \cos x)\mathrm{d}x = \int_0^{\frac{\pi}{2}} f(\sin x, \cos x)\mathrm{d}x$$

$$\int_a^b f(x)\mathrm{d}x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(\frac{a+b}{2} + \frac{b-a}{2}\sin t) \cdot \frac{b-a}{2}\cos x \mathrm{d}x$$

$$\int_a^b f(x)\mathrm{d}x = \int_0^1 (b-a)f[a+(b-a)t]\mathrm{d}t$$

$$\int_{-a}^a f(x)\mathrm{d}x = \int_0^a [f(x)+f(-x)]\mathrm{d}x(a>0)$$

$$\int_0^{\frac{\pi}{2}} \sin^n x\mathrm{d}x = \int_0^{\frac{\pi}{2}} \sin^n x\mathrm{d}x = \begin{cases} \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\cdots\cdot\frac{2}{3}\cdot 1 \\ \\ \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\cdots\cdot\frac{1}{2}\cdot\frac{\pi}{2} \end{cases}$$

$$\int_0^{\pi} \sin^n x\mathrm{d}x = \begin{cases} 2\cdot\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{2}{3}\cdot 1 \\ \\ 2\cdot\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{2}{3}\cdot\frac{\pi}{2} \end{cases}$$

$$\int_0^{\pi} \cos^n x\mathrm{d}x = \begin{cases} 0 \\ \\ 2\cdot\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{2}{3}\cdot\frac{\pi}{2} \end{cases}$$

$$\int_0^{2\pi} \sin^n x\mathrm{d}x = \int_0^{2\pi} \cos^n x\mathrm{d}x = \begin{cases} 0 \\ \\ 4\cdot\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{2}{3}\cdot\frac{\pi}{2} \end{cases}$$

面积:

直角坐标系下: $S=\int_a^b |f(x)-g(x))|\mathrm{d}x$

极坐标下: $S=\int_a^b \frac{1}{2}|r_2^2(\theta)-r_1^2(\theta))|\mathrm{d}\theta$

参数方程下: $S=\int_a^b f(x)\mathrm{d}x=\int_\alpha^\beta y(t)\mathrm{d}x(t)$

旋转体体积:

绕x轴: $V_x=\int_a^b \pi y^2(x)\mathrm{d}x$

绕y轴: $V_y=\int_a^b 2\pi x|y(x)|\mathrm{d}x$(柱壳法)

平面曲线的弧长:

直角坐标系下: $s=\int_a^b \sqrt{1+[y'(x)]^2}\mathrm{d}x$

参数方程下: $s=\int_\alpha^\beta \sqrt{[x'(t)]^2+[y'(t)]^2}\mathrm{d}t$

极坐标方程下: $s=\int_\alpha^\beta \sqrt{[r(\theta)]^2+[r'(\theta)]^2}\mathrm{d}\theta$

旋转曲面的面积 (侧面积):

直角坐标系下，绕x轴: $S=2\pi\int_a^b |y(x)|\sqrt{1+[y'(x)]^2}\mathrm{d}x$

参数方程下，绕x轴: $S=2\pi\int_\alpha^\beta |y(t)|\sqrt{[x'(t)]^2+[y'(t)]^2}\mathrm{d}t$

曲边梯形的形心公式:

$$\bar{x}=\frac{\iint\limits_D x\mathrm{d}\sigma}{\iint\limits_D \mathrm{d}\sigma}=\frac{\int_a^b \mathrm{d}x\int_0^{f(x)} x\mathrm{d}y}{\int_a^b \mathrm{d}x\int_0^{f(x)} \mathrm{d}y}=\frac{\int_a^b xf(x)\mathrm{d}x}{\int_a^b f(x)\mathrm{d}x}$$

$$\bar{y} = \frac{\iint\limits_D y\mathrm{d}\sigma}{\iint\limits_D \mathrm{d}\sigma} = \frac{\int_a^b \mathrm{d}x \int_0^{f(x)} y\mathrm{d}y}{\int_a^b \mathrm{d}x \int_0^{f(x)} \mathrm{d}y} = \frac{\frac{1}{2}\int_a^b f^2(x)\mathrm{d}x}{\int_a^b f(x)\mathrm{d}x}$$

平行截面面积已知的立体体积: $V = \int_a^b S(x)\mathrm{d}x$

总路程: $S = \int_{t_1}^{t_2} v(t)\mathrm{d}x$

变力沿直线做功: $W = \int_a^b F(x)\mathrm{d}x$

提取物体做功: $W = \rho g \int_a^b sA(x)\mathrm{d}x$

静水压力: $P = \int_a^b \rho g x \cdot [f(x) - h(x)]\mathrm{d}x$

细杆质心: $\bar{x} = \frac{\int_a^b x\rho(x)\mathrm{d}x}{\int_a^b \rho(x)\mathrm{d}x}$

7 二重积分

普通对称性:

若D关于y轴对称

$$\iint\limits_D f(x,y)\mathrm{d}\sigma = \begin{cases} 2\iint\limits_{D_1} f(x,y)\mathrm{d}\sigma & f(x,y) = f(-x,y) \\ 0 & f(x,y) = -f(-x,y) \end{cases}$$

若D关于x轴对称

$$\iint\limits_D f(x,y)\mathrm{d}\sigma = \begin{cases} 2\iint\limits_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y) = f(x,-y) \\ 0 & f(x,y) = -f(x,-y) \end{cases}$$

若D关于原点对称

$$\iint_D f(x,y)\mathrm{d}\sigma=\begin{cases}2\iint_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y)=f(-x,-y)\\ 0 & f(x,y)=-f(-x,-y)\end{cases}$$

若D关于$y=x$对称

$$\iint_D f(x,y)\mathrm{d}\sigma=\begin{cases}2\iint_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y)=f(y,x)\\ 0 & f(x,y)=-f(y,x)\end{cases}$$

若D关于$y=a$对称

$$\iint_D f(x,y)\mathrm{d}\sigma=\begin{cases}2\iint_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y)=f(x,2a-y)\\ 0 & f(x,y)=-f(x,2a-y)\end{cases}$$

若D关于$x=a$轴对称

$$\iint_D f(x,y)\mathrm{d}\sigma=\begin{cases}2\iint_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y)=f(2a-x,y)\\ 0 & f(x,y)=-f(2a-x,y)\end{cases}$$

轮换对称性

若将D中的x,y对调后,D不变，则有

$$I=\iint_D f(x,y)\mathrm{d}x\mathrm{d}y=\iint_D f(y,x)\mathrm{d}x\mathrm{d}y$$

若$f(x,y)+f(y,x)\underset{>}{=}a$则

$$I=\frac{1}{2}\iint_D [f(x,y)+f(y,x)]\mathrm{d}x\mathrm{d}y\underset{>}{=}\frac{1}{2}\iint_D a\mathrm{d}x\mathrm{d}y=\frac{a}{2}S_D$$

8 微分方程

$$y'=f(x)\cdot g(y)\text{ 型：}\Rightarrow\frac{\mathrm{d}y}{g(y)}=f(x)\mathrm{d}x\Rightarrow\int\frac{\mathrm{d}y}{g(y)}=\int f(x)\mathrm{d}x$$

$y' = f(ax+by+c)$ 型：令 $u = ax+by+c \Rightarrow u' = a + bf'(u) \Rightarrow$ $\dfrac{\mathrm{d}x}{a+bf(u)} = \mathrm{d}x \Rightarrow \displaystyle\int \frac{\mathrm{d}x}{a+bf(u)} = \int \mathrm{d}x$

$y' = f(\frac{y}{x})$ 型：令 $\frac{y}{x} = u \Rightarrow y = ux \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = u + x\frac{\mathrm{d}u}{\mathrm{d}x}$ 原方程 $x\dfrac{\mathrm{d}u}{\mathrm{d}x} + u = f(u) \Rightarrow \dfrac{\mathrm{d}u}{f(u)-u} = \dfrac{\mathrm{d}x}{x} \Rightarrow \displaystyle\int \frac{\mathrm{d}u}{f(u)-u} = \int \frac{\mathrm{d}x}{x}$

$\frac{1}{y'} = f(\frac{x}{y})$ 型：令 $\frac{x}{y} = u \Rightarrow x = uy \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = u + y\frac{\mathrm{d}u}{\mathrm{d}y}$ 原方程 $y\dfrac{\mathrm{d}u}{\mathrm{d}y} + u = f(u) \Rightarrow \dfrac{\mathrm{d}u}{f(u)-u} = \dfrac{\mathrm{d}y}{y} \Rightarrow \displaystyle\int \frac{\mathrm{d}u}{f(u)-u} = \int \frac{\mathrm{d}y}{y}$

$y' + p(x)y = q(x)$ 型：方程两边同时乘上 $e^{\int p(x)\mathrm{d}x} \Rightarrow e^{\int p(x)\mathrm{d}x} \cdot y' +$ $e^{\int p(x)\mathrm{d}x} p(x) \cdot y = e^{\int p(x)\mathrm{d}x} \cdot q(x) \Rightarrow \left[e^{\int p(x)\mathrm{d}x} \cdot y\right]' = e^{\int p(x)\mathrm{d}x} \cdot q(x) \Rightarrow$ $e^{\int p(x)\mathrm{d}x} \cdot y = \displaystyle\int e^{\int p(x)\mathrm{d}x} \cdot q(x)\mathrm{d}x + C$

得 $y = e^{-\int p(x)\mathrm{d}x}\left[\displaystyle\int e^{\int p(x)\mathrm{d}x} \cdot q(x)\mathrm{d}x + C\right]$

$y' + p(x)y = q(x)y^n$ 型：先变形到 $y^{-n} \cdot y' + p(x)y^{1-n} = q(x) \overset{z=y^{1-n}}{\Longrightarrow}$ 得 $\frac{\mathrm{d}z}{\mathrm{d}x} = (1-n)y^{-n}\frac{\mathrm{d}y}{\mathrm{d}x}$，则 $\displaystyle\int \frac{1}{1-n}\frac{\mathrm{d}z}{\mathrm{d}x} + p(x)z = q(x)$

$y'' = f(x, y')$ 型：令 $y' = p \Rightarrow y'' = p' \Rightarrow \frac{\mathrm{d}p}{\mathrm{d}x} = f(x,p)$ 若解得 $p = \varphi(x, C_1)$ 即 $y' = \varphi(x, C_1)$ 则通解为 $y = \displaystyle\int \varphi(x, C_1)\mathrm{d}x + C_2$

$y'' = f(y', y'')$ 型：令 $y' = p \Rightarrow y'' = \frac{dp}{dx} = \frac{\mathrm{d}p}{\mathrm{d}y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}p}{\mathrm{d}y}p$ 得 $p\frac{\mathrm{d}p}{\mathrm{d}y} =$ $f(y,p)$ 若解得 $p = \varphi(y, C_1)$ 则由 $p = \frac{\mathrm{d}y}{\mathrm{d}x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \varphi(y, C_1)$ 分离变量得 $\dfrac{\mathrm{d}y}{\varphi(y, C_1)} = \mathrm{d}x \Rightarrow \displaystyle\int \frac{\mathrm{d}y}{\varphi(y, C_1)} = x + C_2$

$y'' + py' + qy = f(x)$ 型：

$$\left\{\begin{array}{l}\lambda^2+p\lambda+q=0\Rightarrow\lambda_1,\lambda_2\Rightarrow\text{写出齐次方程的通解}\\ \\ \text{设特解}y''\Rightarrow\text{回代，求待定系数}\Rightarrow\text{特解}\end{array}\right.\Rightarrow\text{写出通解}$$

$y''+py'+qy=f_1(x)+f_2(x)$ 型：

$$\left\{\begin{array}{l}\text{写}\lambda^2+px+q=0\Rightarrow\text{齐次方程的通解}\\ \\ \left.\begin{array}{l}y''+py'+q=f_1(x)\text{写特解}y_1^*\\ \\ y''+py'+qy=f_2(x)\text{写特解}y_2^*\end{array}\right\}\Rightarrow\text{故}y_1^*+y_2^*\text{为特解}\end{array}\right.\Rightarrow\text{通解}$$

(1) 齐次方程的通解

$p^2-4q>0,$ 即$\lambda_1\neq\lambda_2$ 通解为$y=C_1e^{\lambda_1x}+C_2e^{\lambda_2x}$

$p^2-4q=0,$ 即$\lambda_1=\lambda_2=\lambda$ 通解为$y=(C_1+C_2x)e^{\lambda x}$

$p^2-4q<0,$ 共轭复根为$\alpha\pm\beta\mathrm{i}$ 通解为$y=e^{\alpha x}(C_1\cos\beta x+C_2\sin\beta x)$

(2) 非齐次方程的特解

自由项$f(x)=P_n(x)e^{ax}$时, 特解$y^*=e^{ax}Q_n(x)x^k$

$$\left\{\begin{array}{l}e^{ax}\text{照抄}\\ \\ Q_n(x)\text{为}x\text{的}n\text{次一般多项式}\\ \\ k=\left\{\begin{array}{ll}0 & \alpha\neq\lambda_1,\alpha\neq\lambda_2\\ \\ 1 & \alpha\neq\lambda_1\text{或}\alpha\neq\lambda_2\\ \\ 2 & \alpha=\lambda_1=\lambda_2\end{array}\right.\end{array}\right.$$