1 三角函数相关公式

$\sin 2x = 2\sin x \cos x$

$\sin 3x = -4\sin^3 x + 3\sin x$

$\cos 3x = 4\cos^3 x - 3\cos x$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$\cot 2x = \frac{\cot^2 x - 1}{2\cot x}$

$\sin^2 \frac{x}{2} = \frac{1}{2}(1 - \cos x)$

$\cos^2 \frac{x}{2} = \frac{1}{2}(1 + \cos x)$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\sin \frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

$\cos \frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$\sin \alpha + \sin \beta = 2\sin \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2}$

$\sin \alpha - \sin \beta = 2\sin \frac{\alpha-\beta}{2} \cos \frac{\alpha+\beta}{2}$

$\cos \alpha + \cos \beta = 2\cos \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2}$

$\cos \alpha - \cos \beta = -2\sin \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2}$

$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$

$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$

$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$

$\cot(\alpha \pm \beta) = \frac{\cot \alpha \cot \beta \mp 1}{\cot \beta \pm \cot \alpha}$

$\tan \frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

$\cot \frac{x}{2} = \frac{\sin x}{1-\cos x} = \frac{\sin x}{1+\cos x} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

$\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

$\cos \alpha \sin \beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

$\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta)]$

$\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

万能公式: 若 $u = \tan \frac{x}{2}(-\pi < x < \pi)$, 则 $\sin x = \frac{2u}{1+u^2}, \cos x = \frac{1-u^2}{1+u^2}$

2 泰勒及其展开式

$e^x = 1 + x + \frac{1}{2!}x^2$ 　　　　　　$\arctan x = x - \frac{1}{3}x^3$

$\ln(1+x) = x - \frac{1}{2!}x^2$ 　　　　　$\cos x = 1 - \frac{1}{2!}x^2$

$\sin x = x - \frac{1}{3!}x^3$ 　　　　　　$\frac{1}{1-x} = 1 + x + x^2$

$\arcsin x = x + \frac{1}{3!}x^3$ 　　　　　$\frac{1}{1+x} = 1 - x + x^2$

$\tan x = x + \frac{1}{3}x^3$ 　　　　　　$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2$

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2}x^2$$
$$+ \cdots + \frac{f^n x_0}{n!}(x - x_0)^n + \frac{f^{n+1}(\theta)}{(n+1)!}(x - x_0)^{n+1}$$
$$f(x) = f(0) + f'(0)(x) + \frac{f''(0)}{2}x^2 + \cdots +$$
$$\frac{f^n(0)}{n!}(x)^n + \frac{f^{n+1}(\theta)}{(n+1)!}x^{n+1}$$

3 基本求导公式

$(x^k)' = kx^{k-1}$ 　　　　　　　　$(\tan x)' = \sec^2 x$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$ 　　　　$(\cot x)' = -\csc^2 x$

$(e^x)' = e^x$ 　　　　　　　　　　$(\sec x)' = \sec x \tan x$

$(a^x)' = a^x \ln a \quad (a > 0, a \neq 1)$ 　　$(\csc x)' = -\csc x \cot x$

$(\sin x)' = \cos x$ 　　　　　　　　$(\ln|\cos x|)' = -\tan x$

$(\cos x)' = -\sin x$ 　　　　　　　$(\ln|\sec x + \tan x|)' = \sec x$

$$(\ln|\csc x - \cot x|)' = -\csc x \qquad (\operatorname{arccot} x)' = \frac{-1}{1+x^2}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1+x^2}} \qquad [\ln(x+\sqrt{x^2+a^2})]' = \frac{1}{\sqrt{x^2+a^2}}$$

$$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}} \qquad [\ln(x+\sqrt{x^2-a^2})]' = \frac{1}{\sqrt{x^2-a^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

4 微分的几何应用

水平渐近线：$\begin{cases} \lim\limits_{x\to+\infty} f(x) = y_1 \\ \lim\limits_{x\to-\infty} f(x) = y_2 \end{cases} \Rightarrow \begin{cases} y = y_1 \\ y = y_2 \end{cases}$

斜渐近线：$\begin{cases} \lim\limits_{x\to+\infty} \dfrac{f(x)}{x} = k_1 \\ \lim\limits_{x\to+\infty} [f(x) - k_1 x] = b_1 \end{cases} \Rightarrow y = k_1 x + b_1$

斜渐近线：$\begin{cases} \lim\limits_{x\to-\infty} \dfrac{f(x)}{x} = k_2 \\ \lim\limits_{x\to-\infty} [f(x) - k_2 x] = b_2 \end{cases} \Rightarrow y = k_2 x + b_2$

旋转曲面的侧面积，绕x轴：$S = 2\pi \int_a^b |y(x)| \sqrt{1 + [y'(x)]^2}\,\mathrm{d}x$

$\begin{cases} x = x(t) \\ y = y(t) \end{cases} \alpha \le t \le \beta \Rightarrow S = 2\pi \int_\alpha^\beta |y(t)| \sqrt{[x'(t)]^2 + [y'(t)]^2}\,\mathrm{d}t$

曲率及曲率半径：$k = \dfrac{|y''|}{[1+(y')^2]^{\frac{3}{2}}}, R = \dfrac{1}{k}$

5 基本积分公式

$$\int x^k \mathrm{d}x = \frac{1}{k+1}x^{k+1} + C \quad k \neq 1$$

$$\int \frac{1}{x^2} = -\frac{1}{x} + C$$

$$\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + C$$

$$\int \frac{1}{x} = \ln|x| + C$$

$$\int e^x \mathrm{d}x = e^x + C$$

$$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$$

$$\int \sin x \mathrm{d}x = -\cos x + C$$

$$\int \cos x \mathrm{d}x = \sin x + C$$

$$\int \tan x \mathrm{d}x = -\ln|\cos x| + C$$

$$\int \cot x \mathrm{d}x = \ln|\sin x| + C$$

$$\int \sec^2 x \mathrm{d}x = \tan x + C$$

$$\int \csc^2 x \mathrm{d}x = -\cot x + C$$

$$\int \sec x \tan x \mathrm{d}x = \sec x + C$$

$$\int \csc x \cot x \mathrm{d}x = -\csc x + C$$

$$\int \frac{1}{1+x^2}\mathrm{d}x = \arctan x + C$$

$$\int \frac{1}{a^2+x^2}\mathrm{d}x = \frac{1}{a}\operatorname{arccot}\frac{x}{a} + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\mathrm{d}x = \arcsin x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\mathrm{d}x = \arcsin\frac{x}{a} + C$$

$$\int \sin^2 x \mathrm{d}x = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$\int \cos^2 x \mathrm{d}x = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$\int \tan^2 x \mathrm{d}x = \tan x - x + C$$

$$\int \cot^2 x \mathrm{d}x = -\cot x + x + C$$

$$\int u\mathrm{d}v = uv - \int v\mathrm{d}u + C$$

$$\int_a^b f(x)\mathrm{d}x = \int_a^b f(a+b-x)\mathrm{d}x$$

$$\int \frac{\mathrm{d}x}{\cos x} = \int \sec x \mathrm{d}x = \ln|\sec x + \tan x| + C$$

$$\int \frac{\mathrm{d}x}{\sin x} = \int \csc x \mathrm{d}x = \ln|\csc x - \cot x| + C$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\mathrm{d}x = \ln\left(x + \sqrt{x^2+a^2}\right) + C$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\mathrm{d}x = \ln\left|x + \sqrt{x^2-a^2}\right| + C$$

$$\int \frac{1}{x^2-a^2}\mathrm{d}x = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{1}{a^2-x^2}\mathrm{d}x = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$$

$$\int \sqrt{a^2-x^2}\mathrm{d}x = \frac{a^2}{2}\arcsin\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} + C$$

$$\int_a^b f(x)\mathrm{d}x = \frac{1}{2}\int_a^b [f(x) + f(a+b-x)]\mathrm{d}x$$

$$\int_a^b f(x)\mathrm{d}x = \int_a^{\frac{a+b}{2}} [f(x) + f(a+b-x)]\mathrm{d}x$$

$$\int_0^\pi xf(\sin x)\mathrm{d}x = \frac{\pi}{2}\int_0^\pi f(\sin x)\mathrm{d}x$$

$$\int_0^\pi xf(\sin x)\mathrm{d}x = \pi\int_0^{\frac{\pi}{2}} f(\sin x)\mathrm{d}x$$

$$\int_0^{\frac{\pi}{2}} f(\sin x)\mathrm{d}x = \int_0^{\frac{\pi}{2}} f(\cos x)\mathrm{d}x$$

$$\int_0^{\frac{\pi}{2}} f(\sin x, \cos x)\mathrm{d}x = \int_0^{\frac{\pi}{2}} f(\sin x, \cos x)\mathrm{d}x$$

$$\int_a^b f(x)\mathrm{d}x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(\frac{a+b}{2} + \frac{b-a}{2}\sin t)\cdot \frac{b-a}{2}\cos x\mathrm{d}x$$

$$\int_a^b f(x)\mathrm{d}x = \int_0^1 (b-a)f[a + (b-a)t]\mathrm{d}t$$

$$\int_{-a}^a f(x)\mathrm{d}x = \int_0^a [f(x) + f(-x)]\mathrm{d}x \quad (a > 0)$$

$$\int_0^{\frac{\pi}{2}} \sin^n x\mathrm{d}x = \int_0^{\frac{\pi}{2}} \sin^n x\mathrm{d}x = \begin{cases} \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\cdots\frac{2}{3}\cdot 1 \\[2mm] \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\cdots\frac{1}{2}\cdot\frac{\pi}{2} \end{cases}$$

$$\int_0^\pi \sin^n x\mathrm{d}x = \begin{cases} 2\cdot\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{2}{3}\cdot 1 \\[2mm] 2\cdot\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{2}{3}\cdot\frac{\pi}{2} \end{cases}$$

$$\int_0^\pi \cos^n x\mathrm{d}x = \begin{cases} 0 \\[2mm] 2\cdot\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{2}{3}\cdot\frac{\pi}{2} \end{cases}$$

$$\int_0^{2\pi} \sin^n x\mathrm{d}x = \int_0^{2\pi} \cos^n x\mathrm{d}x = \begin{cases} 0 \\[2mm] 4\cdot\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{2}{3}\cdot\frac{\pi}{2} \end{cases}$$

6 积分的简单应用

面积：

直角坐标系下：$S = \int_a^b |f(x) - g(x))|\mathrm{d}x$

极坐标下：$S = \int_a^b \frac{1}{2}|r_2^2(\theta) - r_1^2(\theta))|\mathrm{d}\theta$

参数方程下：$S = \int_a^b f(x)\mathrm{d}x = \int_\alpha^\beta y(t)\mathrm{d}x(t)$

旋转体体积：

绕x轴：$V_x = \int_a^b \pi y^2(x)\mathrm{d}x$

绕y轴：$V_y = \int_a^b 2\pi x|y(x)|\mathrm{d}x$(柱壳法)

平面曲线的弧长：

直角坐标系下：$s = \int_a^b \sqrt{1 + [y'(x)]^2}\mathrm{d}x$

参数方程下：$s = \int_\alpha^\beta \sqrt{[x'(t)]^2 + [y'(t)]^2}\mathrm{d}t$

极坐标方程下：$s = \int_\alpha^\beta \sqrt{[r(\theta)]^2 + [r'(\theta)]^2}\mathrm{d}\theta$

旋转曲面的面积 (侧面积)：

直角坐标系下，绕x轴：$S = 2\pi \int_a^b |y(x)|\sqrt{1 + [y'(x)]^2}\mathrm{d}x$

参数方程下，绕x轴：$S = 2\pi \int_\alpha^\beta |y(t)|\sqrt{[x'(t)]^2 + [y'(t)]^2}\mathrm{d}t$

曲边梯形的形心公式：

$$\bar{x} = \frac{\iint\limits_D x\mathrm{d}\sigma}{\iint\limits_D \mathrm{d}\sigma} = \frac{\int_a^b \mathrm{d}x \int_0^{f(x)} x\mathrm{d}y}{\int_a^b \mathrm{d}x \int_0^{f(x)} \mathrm{d}y} = \frac{\int_a^b xf(x)\mathrm{d}x}{\int_a^b f(x)\mathrm{d}x}$$

$$\bar{y} = \frac{\iint\limits_D y\mathrm{d}\sigma}{\iint\limits_D \mathrm{d}\sigma} = \frac{\int_a^b \mathrm{d}x \int_0^{f(x)} y\mathrm{d}y}{\int_a^b \mathrm{d}x \int_0^{f(x)} \mathrm{d}y} = \frac{\frac{1}{2}\int_a^b f^2(x)\mathrm{d}x}{\int_a^b f(x)\mathrm{d}x}$$

平行截面面积已知的立体体积: $V = \int_a^b S(x)\mathrm{d}x$

总路程: $S = \int_{t_1}^{t_2} v(t)\mathrm{d}x$

变力沿直线做功: $W = \int_a^b F(x)\mathrm{d}x$

提取物体做功: $W = \rho g \int_a^b sA(x)\mathrm{d}x$

静水压力: $P = \int_a^b \rho g x \cdot [f(x) - h(x)]\mathrm{d}x$

细杆质心: $\bar{x} = \dfrac{\int_a^b x\rho(x)\mathrm{d}x}{\int_a^b \rho(x)\mathrm{d}x}$

7　二重积分

普通对称性:

若D关于y轴对称

$$\iint\limits_D f(x,y)\mathrm{d}\sigma = \begin{cases} 2\iint\limits_{D_1} f(x,y)\mathrm{d}\sigma & f(x,y) = f(-x,y) \\ 0 & f(x,y) = -f(-x,y) \end{cases}$$

若D关于x轴对称

$$\iint\limits_D f(x,y)\mathrm{d}\sigma = \begin{cases} 2\iint\limits_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y) = f(x,-y) \\ 0 & f(x,y) = -f(x,-y) \end{cases}$$

若D关于原点对称

$$\iint\limits_D f(x,y)\mathrm{d}\sigma = \begin{cases} 2\iint\limits_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y) = f(-x,-y) \\ 0 & f(x,y) = -f(-x,-y) \end{cases}$$

若D关于$y = x$对称

$$\iint\limits_{D} f(x,y)\mathrm{d}\sigma = \begin{cases} 2\iint\limits_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y) = f(y,x) \\ 0 & f(x,y) = -f(y,x) \end{cases}$$

若D关于$y = a$对称

$$\iint\limits_{D} f(x,y)\mathrm{d}\sigma = \begin{cases} 2\iint\limits_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y) = f(x,2a-y) \\ 0 & f(x,y) = -f(x,2a-y) \end{cases}$$

若D关于$x = a$轴对称

$$\iint\limits_{D} f(x,y)\mathrm{d}\sigma = \begin{cases} 2\iint\limits_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y) = f(2a-x,y) \\ 0 & f(x,y) = -f(2a-x,y) \end{cases}$$

轮换对称性

若将D中的x,y对调后,D不变,则有

$$I = \iint\limits_{D} f(x,y)\mathrm{d}x\mathrm{d}y = \iint\limits_{D} f(y,x)\mathrm{d}x\mathrm{d}y$$

若$f(x,y) + f(y,x) \underset{>}{=} a$则

$$I = \frac{1}{2}\iint\limits_{D}[f(x,y)+f(y,x)]\mathrm{d}x\mathrm{d}y \underset{>}{=} \frac{1}{2}\iint\limits_{D} a\mathrm{d}x\mathrm{d}y = \frac{a}{2}S_D$$

8 微分方程

$$y' = f(x) \cdot g(y) \text{ 型}: \Rightarrow \frac{\mathrm{d}x}{g(y)} = f(x)\mathrm{d}x \Rightarrow \int \frac{\mathrm{d}x}{g(y)} = \int f(x)\mathrm{d}x$$

$$y' = f(ax+by+c) \text{ 型}: \text{令 } u = ax+by+c \Rightarrow u' = a + bf'(u) \Rightarrow$$

$$\frac{\mathrm{d}x}{a+bf(u)} = \mathrm{d}x \Rightarrow \int \frac{\mathrm{d}x}{a+bf(u)} = \int \mathrm{d}x$$

$$y' = f(\frac{y}{x}) \text{ 型}: \text{令 } \frac{y}{x} = u \Rightarrow y = ux \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = u + x\frac{\mathrm{d}u}{\mathrm{d}x} \text{ 原方程}$$

$$x\frac{\mathrm{d}u}{\mathrm{d}x} + u = f(u) \Rightarrow \frac{\mathrm{d}u}{f(u)-u} = \frac{\mathrm{d}x}{x} \Rightarrow \int \frac{\mathrm{d}u}{f(u)-u} = \int \frac{\mathrm{d}x}{x}$$

$\frac{1}{y'} = f(\frac{x}{y})$ 型： 令 $\frac{x}{y} = u \Rightarrow x = uy \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = u + y\frac{\mathrm{d}u}{\mathrm{d}y}$ 原方程

$$y\frac{\mathrm{d}u}{\mathrm{d}y} + u = f(u) \Rightarrow \frac{\mathrm{d}u}{f(u)-u} = \frac{\mathrm{d}y}{y} \Rightarrow \int \frac{\mathrm{d}u}{f(u)-u} = \int \frac{\mathrm{d}y}{y}$$

$y' + p(x)y = q(x)$ 型： 方程两边同时乘上 $e^{\int p(x)\mathrm{d}x} \Rightarrow e^{\int p(x)\mathrm{d}x} \cdot y' +$

$e^{\int p(x)\mathrm{d}x}p(x) \cdot y = e^{\int p(x)\mathrm{d}x} \cdot q(x) \Rightarrow \left[e^{\int p(x)\mathrm{d}x} \cdot y\right]' = e^{\int p(x)\mathrm{d}x} \cdot q(x) \Rightarrow$

$e^{\int p(x)\mathrm{d}x} \cdot y = \int e^{\int p(x)\mathrm{d}x} \cdot q(x)\mathrm{d}x + C$

得 $y = e^{-\int p(x)\mathrm{d}x}\left[\int e^{\int p(x)\mathrm{d}x} \cdot q(x)\mathrm{d}x + C\right]$

$y' + p(x)y = q(x)y^n$ 型： 先变形到 $y^{-n} \cdot y' + p(x)y^{1-n} = q(x) \overset{z=y^{1-n}}{\Longrightarrow}$ 得

$\frac{\mathrm{d}z}{\mathrm{d}x} = (1-n)y^{-n}\frac{\mathrm{d}y}{\mathrm{d}x}$, 则 $\frac{1}{1-n}\frac{\mathrm{d}z}{\mathrm{d}x} + p(x)z = q(x)$

$y'' = f(x, y')$ 型： 令 $y' = p \Rightarrow y'' = p' \Rightarrow \frac{\mathrm{d}p}{\mathrm{d}x} = f(x, p)$ 若解得

$p = \varphi(x, C_1)$ 即 $y' = \varphi(x, C_1)$ 则通解为 $y = \int \varphi(x, C_1)\mathrm{d}x + C_2$

$y'' = f(y', y'')$ 型： 令 $y' = p \Rightarrow y'' = \frac{\mathrm{d}p}{\mathrm{d}x} = \frac{\mathrm{d}p}{\mathrm{d}y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}p}{\mathrm{d}y}p$ 得 $p\frac{\mathrm{d}p}{\mathrm{d}y} =$

$f(y, p)$ 若解得 $p = \varphi(y, C_1)$ 则由 $p = \frac{\mathrm{d}y}{\mathrm{d}x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \varphi(y, C_1)$ 分离变量得

$\frac{\mathrm{d}y}{\varphi(y, C_1)} = \mathrm{d}x \Rightarrow \int \frac{\mathrm{d}y}{\varphi(y, C_1)} = x + C_2$

$y'' + py' + qy = f(x)$ 型：

$$\begin{cases} \lambda^2 + p\lambda + q = 0 \Rightarrow \lambda_1, \lambda_2 \Rightarrow \text{写出齐次方程的通解} \\ \text{设特解}y'' \Rightarrow \text{回代，求待定系数} \Rightarrow \text{特解} \end{cases} \Rightarrow \text{写出通解}$$

$y'' + py' + qy = f_1(x) + f_2(x)$ 型：

$$\begin{cases} \text{写} \lambda^2 + px + q = 0 \Rightarrow \text{齐次方程的通解} \\ y'' + py' + q = f_1(x) \text{写特解} y_1^* \\ y'' + py' + qy = f_2(x) \text{写特解} y_2^* \end{cases} \Rightarrow \text{故} y_1^* + y_2^* \text{为特解} \Rightarrow \text{通解}$$

(1) 齐次方程的通解

$p^2 - 4q > 0, \text{即} \lambda_1 \neq \lambda_2$ 　　　通解为 $y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$

$p^2 - 4q = 0, \text{即} \lambda_1 = \lambda_2 = \lambda$ 　　通解为 $y = (C_1 + C_2 x) e^{\lambda x}$

$p^2 - 4q < 0, \text{共轭复根为} \alpha \pm \beta \mathrm{i}$ 　通解为 $y = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$

(2) 非齐次方程的特解

自由项 $f(x) = P_n(x) e^{ax}$ 时, 特解 $y^* = e^{ax} Q_n(x) x^k$

$$\begin{cases} e^{ax} \text{照抄} \\ Q_n(x) \text{为} x \text{的} n \text{次一般多项式} \\ k = \begin{cases} 0 & \alpha \neq \lambda_1, \alpha \neq \lambda_2 \\ 1 & \alpha \neq \lambda_1 \text{或} \alpha \neq \lambda_2 \\ 2 & \alpha = \lambda_1 = \lambda_2 \end{cases} \end{cases}$$