1 三角函数相关公式

$$\sin 2x = 2\sin x\cos x$$

$$\sin 3x = -4\sin^3 x + 3\sin x$$

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2\cot x}$$

$$\sin^2\frac{x}{2} = \frac{1}{2}(1-\cos x)$$

$$\cos^2\frac{x}{2} = \frac{1}{2}(1+\cos x)$$

$$\sin^2 x = \frac{1}{2}(1-\cos 2x)$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\tfrac{\alpha+\beta}{2}\cos\tfrac{\alpha-\beta}{2}$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha\pm\beta) = \frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$$

$$\cot(\alpha\pm\beta) = \frac{\cot\alpha\cot\beta\mp 1}{\cot\beta\pm\cot\alpha}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$\cot\frac{x}{2} = \frac{\sin x}{1-\cos x} = \frac{\sin x}{1+\cos x} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \tfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

万能公式：若 $u = \tan\frac{x}{2}(-\pi < x < \pi)$，则 $\sin x = \frac{2u}{1+u^2}, \cos x = \frac{1-u^2}{1+u^2}$

2　泰勒及其展开式

泰勒公式：

$$
\begin{aligned}
f(x) = {} & f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2}x^2 \\
& + \cdots + \frac{f^n x_0}{n!}(x-x_0)^n + \frac{f^{n+1}(\theta)}{(n+1)!}(x-x_0)^{n+1}
\end{aligned}
$$

麦克劳林公式：

$$
\begin{aligned}
f(x) = {} & f(0) + f'(0)(x) + \frac{f''(0)}{2}x^2 + \cdots + \\
& \frac{f^n(0)}{n!}(x)^n + \frac{f^{n+1}(\theta)}{(n+1)!}x^{n+1}
\end{aligned}
$$

$e^x = 1 + x + \frac{1}{2!}x^2$

$\arctan x = x - \frac{1}{3}x^3$

$\ln(1+x) = x - \frac{1}{2!}x^2$

$\cos x = 1 - \frac{1}{2!}x^2$

$\sin x = x - \frac{1}{3!}x^3$

$\frac{1}{1-x} = 1 + x + x^2$

$\arcsin x = x + \frac{1}{3!}x^3$

$\frac{1}{1+x} = 1 - x + x^2$

$\tan x = x + \frac{1}{3}x^3$

$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2$

3　基本求导公式

$(x^k)' = kx^{k-1}$

$(a^x)' = a^x \ln a \ (a > 0, a \neq 1)$

$(\ln x)' = \dfrac{1}{x} \quad (x > 0)$

$(\sin x)' = \cos x$

$(e^x)' = e^x$

$(\cos x)' = -\sin x$

$$(\tan x)' = \sec^2 x$$

$$(\cot x)' = -\csc^2 x$$

$$(\sec x)' = \sec x \tan x$$

$$(\csc x)' = -\csc x \cot x$$

$$(\ln|\cos x|)' = -\tan x$$

$$(\ln|\sec x + \tan x|)' = \sec x$$

$$(\ln|\csc x - \cot x|)' = -\csc x$$

$$(\arcsin x)' = \frac{1}{\sqrt{1+x^2}}$$

$$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)' = \frac{-1}{1+x^2}$$

$$[\ln(x+\sqrt{x^2+a^2})]' = \frac{1}{\sqrt{x^2+a^2}}$$

$$[\ln(x+\sqrt{x^2-a^2})]' = \frac{1}{\sqrt{x^2-a^2}}$$

4　微分的几何应用

$$水平渐近线: \begin{cases} \lim\limits_{x\to+\infty} f(x) = y_1 \\ \lim\limits_{x\to-\infty} f(x) = y_2 \end{cases} \Rightarrow \begin{cases} y = y_1 \\ y = y_2 \end{cases}$$

$$斜渐近线: \begin{cases} \lim\limits_{x\to+\infty} \frac{f(x)}{x} = k_1 \\ \lim\limits_{x\to+\infty} [f(x) - k_1 x] = b_1 \end{cases} \Rightarrow y = k_1 x + b_1$$

$$斜渐近线: \begin{cases} \lim\limits_{x\to-\infty} \frac{f(x)}{x} = k_2 \\ \lim\limits_{x\to-\infty} [f(x) - k_2 x] = b_2 \end{cases} \Rightarrow y = k_2 x + b_2$$

旋转曲面的侧面积:

绕x轴: $S = 2\pi \int_a^b |y(x)| \sqrt{1+[y'(x)]^2} \mathrm{d}x$

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases} \quad \alpha \leq t \leq \beta \Rightarrow S = 2\pi \int_{\alpha}^{\beta} |y(t)| \sqrt{[x'(t)]^2 + [y'(t)]^2} \mathrm{d}t$$

曲率及曲率半径: $k = \frac{|y''|}{[1+(y')^2]^{\frac{3}{2}}}, R = \frac{1}{k}$

5 基本积分公式

$$\int x^k \mathrm{d}x = \frac{1}{k+1} x^{k+1} + C \quad k \neq 1$$

$$\int \frac{1}{x^2} = -\frac{1}{x} + C$$

$$\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + C$$

$$\int \frac{1}{x} = \ln|x| + C$$

$$\int e^x \mathrm{d}x = e^x + C$$

$$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$$

$$\int \sin x \mathrm{d}x = -\cos x + C$$

$$\int \cos x \mathrm{d}x = \sin x + C$$

$$\int \tan x \mathrm{d}x = -\ln|\cos x| + C$$

$$\int \cot x \mathrm{d}x = \ln|\sin x| + C$$

$$\int \sec^2 x \mathrm{d}x = \tan x + C$$

$$\int \csc^2 x \mathrm{d}x = -\cot x + C$$

$$\int \sec x \tan x \mathrm{d}x = \sec x + C$$

$$\int \csc x \cot x \mathrm{d}x = -\csc x + C$$

$$\int \frac{1}{1+x^2} \mathrm{d}x = \arctan x + C$$

$$\int \frac{1}{a^2+x^2} \mathrm{d}x = \frac{1}{a} \operatorname{arccot} \frac{x}{a} + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \mathrm{d}x = \arcsin x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \mathrm{d}x = \arcsin \frac{x}{a} + C$$

$$\int \sin^2 x \mathrm{d}x = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$\int \cos^2 x \mathrm{d}x = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$\int \tan^2 x \mathrm{d}x = \tan x - x + C$$

$$\int \cot^2 x \mathrm{d}x = -\cot x + x + C$$

$$\int u \mathrm{d}v = uv - \int v \mathrm{d}u + C$$

$$\int_a^b f(x) \mathrm{d}x = \int_a^b f(a+b-x) \mathrm{d}x$$

$$\int \frac{\mathrm{d}x}{\cos x} = \int \sec x \mathrm{d}x = \ln|\sec x + \tan x| + C$$

$$\int \frac{\mathrm{d}x}{\sin x} = \int \csc x \mathrm{d}x = \ln|\csc x - \cot x| + C$$

$$\int \frac{1}{\sqrt{x^2+a^2}} \mathrm{d}x = \ln(x + \sqrt{x^2+a^2}) + C$$

$$\int \frac{1}{\sqrt{x^2-a^2}} \mathrm{d}x = \ln\left|x + \sqrt{x^2-a^2}\right| + C$$

$$\int \frac{1}{x^2-a^2} \mathrm{d}x = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{1}{a^2-x^2} \mathrm{d}x = \frac{1}{2a} \ln\left|\frac{x+a}{x-a}\right| + C$$

$$\int \sqrt{a^2-x^2} \mathrm{d}x = \frac{a^2}{2} \arcsin\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} + C$$

$$\int_a^b f(x)\mathrm{d}x = \frac{1}{2}\int_a^b [f(x) + f(a+b-x)]\mathrm{d}x$$

$$\int_a^b f(x)\mathrm{d}x = \int_a^{\frac{a+b}{2}} [f(x) + f(a+b-x)]\mathrm{d}x$$

$$\int_0^\pi x f(\sin x)\mathrm{d}x = \frac{\pi}{2}\int_0^\pi f(\sin x)\mathrm{d}x$$

$$\int_0^\pi x f(\sin x)\mathrm{d}x = \pi\int_0^{\frac{\pi}{2}} f(\sin x)\mathrm{d}x$$

$$\int_0^{\frac{\pi}{2}} f(\sin x)\mathrm{d}x = \int_0^{\frac{\pi}{2}} f(\cos x)\mathrm{d}x$$

$$\int_0^{\frac{\pi}{2}} f(\sin x, \cos x)\mathrm{d}x = \int_0^{\frac{\pi}{2}} f(\sin x, \cos x)\mathrm{d}x$$

$$\int_a^b f(x)\mathrm{d}x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(\frac{a+b}{2} + \frac{b-a}{2}\sin t) \cdot \frac{b-a}{2}\cos x \mathrm{d}x$$

$$\int_a^b f(x)\mathrm{d}x = \int_0^1 (b-a) f[a + (b-a)t]\mathrm{d}t$$

$$\int_{-a}^a f(x)\mathrm{d}x = \int_0^a [f(x) + f(-x)]\mathrm{d}x (a > 0)$$

$$\int_0^{\frac{\pi}{2}} \sin^n x \mathrm{d}x = \int_0^{\frac{\pi}{2}} \sin^n x \mathrm{d}x = \begin{cases} \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \cdots \cdot \frac{2}{3} \cdot 1 \\ \\ \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \cdots \cdot \frac{1}{2} \cdot \frac{\pi}{2} \end{cases}$$

$$\int_0^{\pi} \sin^n x \mathrm{d}x = \begin{cases} 2 \cdot \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} \cdot 1 \\ \\ 2 \cdot \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} \cdot \frac{\pi}{2} \end{cases}$$

$$\int_0^{\pi} \cos^n x \mathrm{d}x = \begin{cases} 0 \\ \\ 2 \cdot \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} \cdot \frac{\pi}{2} \end{cases}$$

$$\int_0^{2\pi} \sin^n x \mathrm{d}x = \int_0^{2\pi} \cos^n x \mathrm{d}x = \begin{cases} 0 \\ \\ 4 \cdot \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} \cdot \frac{\pi}{2} \end{cases}$$

6 积分的简单应用

面积:

直角坐标系下: $S = \int_a^b |f(x) - g(x))| \mathrm{d}x$

极坐标下: $S = \int_a^b \frac{1}{2} |r_2^2(\theta) - r_1^2(\theta))| \mathrm{d}\theta$

参数方程下: $S = \int_a^b f(x) \mathrm{d}x = \int_\alpha^\beta y(t) \mathrm{d}x(t)$

旋转体体积:

绕x轴: $V_x = \int_a^b \pi y^2(x) \mathrm{d}x$

绕y轴: $V_y = \int_a^b 2\pi x|y(x)|\mathrm{d}x$(柱壳法)

平面曲线的弧长:

直角坐标系下: $s = \int_a^b \sqrt{1+[y'(x)]^2}\mathrm{d}x$

参数方程下: $s = \int_\alpha^\beta \sqrt{[x'(t)]^2+[y'(t)]^2}\mathrm{d}t$

极坐标方程下: $s = \int_\alpha^\beta \sqrt{[r(\theta)]^2+[r'(\theta)]^2}\mathrm{d}\theta$

旋转曲面的面积 (侧面积):

直角坐标系下，绕x轴: $S = 2\pi \int_a^b |y(x)|\sqrt{1+[y'(x)]^2}\mathrm{d}x$

参数方程下，绕x轴: $S = 2\pi \int_\alpha^\beta |y(t)|\sqrt{[x'(t)]^2+[y'(t)]^2}\mathrm{d}t$

曲边梯形的形心公式:

$$\bar{x} = \frac{\iint\limits_D x\mathrm{d}\sigma}{\iint\limits_D \mathrm{d}\sigma} = \frac{\int_a^b \mathrm{d}x \int_0^{f(x)} x\mathrm{d}y}{\int_a^b \mathrm{d}x \int_0^{f(x)} \mathrm{d}y} = \frac{\int_a^b xf(x)\mathrm{d}x}{\int_a^b f(x)\mathrm{d}x}$$

$$\bar{y} = \frac{\iint\limits_D y\mathrm{d}\sigma}{\iint\limits_D \mathrm{d}\sigma} = \frac{\int_a^b \mathrm{d}x \int_0^{f(x)} y\mathrm{d}y}{\int_a^b \mathrm{d}x \int_0^{f(x)} \mathrm{d}y} = \frac{\frac{1}{2}\int_a^b f^2(x)\mathrm{d}x}{\int_a^b f(x)\mathrm{d}x}$$

平行截面面积已知的立体体积: $V = \int_a^b S(x)\mathrm{d}x$

总路程: $S = \int_{t_1}^{t_2} v(t)\mathrm{d}x$

变力沿直线做功: $W = \int_a^b F(x)\mathrm{d}x$

提取物体做功: $W = \rho g \int_a^b sA(x)\mathrm{d}x$

静水压力: $P = \int_a^b \rho g x \cdot [f(x)-h(x)]\mathrm{d}x$

细杆质心: $\bar{x}=\dfrac{\int_a^b x\rho(x)\mathrm{d}x}{\int_a^b \rho(x)\mathrm{d}x}$

7 二重积分

普通对称性:

若D关于y轴对称

$$\iint\limits_D f(x,y)\mathrm{d}\sigma=\begin{cases}2\iint\limits_{D_1} f(x,y)\mathrm{d}\sigma & f(x,y)=f(-x,y)\\ \\ 0 & f(x,y)=-f(-x,y)\end{cases}$$

若D关于x轴对称

$$\iint\limits_D f(x,y)\mathrm{d}\sigma=\begin{cases}2\iint\limits_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y)=f(x,-y)\\ \\ 0 & f(x,y)=-f(x,-y)\end{cases}$$

若D关于原点对称

$$\iint\limits_D f(x,y)\mathrm{d}\sigma=\begin{cases}2\iint\limits_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y)=f(-x,-y)\\ \\ 0 & f(x,y)=-f(-x,-y)\end{cases}$$

若D关于$y=x$对称

$$\iint\limits_D f(x,y)\mathrm{d}\sigma=\begin{cases}2\iint\limits_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y)=f(y,x)\\ \\ 0 & f(x,y)=-f(y,x)\end{cases}$$

若D关于$y=a$对称

$$\iint_D f(x,y)\mathrm{d}\sigma=\begin{cases}2\iint_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y)=f(x,2a-y)\\ 0 & f(x,y)=-f(x,2a-y)\end{cases}$$

若D关于$x=a$轴对称

$$\iint_D f(x,y)\mathrm{d}\sigma=\begin{cases}2\iint_{D_1} f(x,y)\mathrm{d}\sigma, & f(x,y)=f(2a-x,y)\\ 0 & f(x,y)=-f(2a-x,y)\end{cases}$$

轮换对称性

若将D中的x,y对调后,D不变，则有

$$I=\iint_D f(x,y)\mathrm{d}x\mathrm{d}y=\iint_D f(y,x)\mathrm{d}x\mathrm{d}y$$

若$f(x,y)+f(y,x)\underset{>}{=}a$则

$$I=\frac{1}{2}\iint_D [f(x,y)+f(y,x)]\mathrm{d}x\mathrm{d}y\underset{>}{=}\frac{1}{2}\iint_D a\mathrm{d}x\mathrm{d}y=\frac{a}{2}S_D$$

8 微分方程

$$y'=f(x)\cdot g(y)\text{ 型：}\Rightarrow\frac{\mathrm{d}y}{g(y)}=f(x)\mathrm{d}x\Rightarrow\int\frac{\mathrm{d}y}{g(y)}=\int f(x)\mathrm{d}x$$

$y'=f(ax+by+c)$ 型： 令 $u=ax+by+c\Rightarrow u'=a+bf'(u)\Rightarrow$

$$\frac{\mathrm{d}x}{a+bf(u)}=\mathrm{d}x\Rightarrow\int\frac{\mathrm{d}x}{a+bf(u)}=\int\mathrm{d}x$$

$y'=f(\frac{y}{x})$ 型： 令 $\frac{y}{x}=u\Rightarrow y=ux\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=u+x\frac{\mathrm{d}u}{\mathrm{d}x}$ 原方程

$x\frac{\mathrm{d}u}{\mathrm{d}x}+u=f(u)\Rightarrow\frac{\mathrm{d}u}{f(u)-u}=\frac{\mathrm{d}x}{x}\Rightarrow\int\frac{\mathrm{d}u}{f(u)-u}=\int\frac{\mathrm{d}x}{x}$

$\frac{1}{y'}=f(\frac{x}{y})$ 型：令 $\frac{x}{y}=u\Rightarrow x=uy\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=u+y\frac{\mathrm{d}u}{\mathrm{d}y}$ 原方程

$y\frac{\mathrm{d}u}{\mathrm{d}y}+u=f(u)\Rightarrow\frac{\mathrm{d}u}{f(u)-u}=\frac{\mathrm{d}y}{y}\Rightarrow\int\frac{\mathrm{d}u}{f(u)-u}=\int\frac{\mathrm{d}y}{y}$

$y'+p(x)y=q(x)$ 型：方程两边同时乘上 $e^{\int p(x)\mathrm{d}x}\Rightarrow e^{\int p(x)\mathrm{d}x}\cdot y'+e^{\int p(x)\mathrm{d}x}p(x)\cdot y=e^{\int p(x)\mathrm{d}x}\cdot q(x)\Rightarrow\left[e^{\int p(x)\mathrm{d}x}\cdot y\right]'=e^{\int p(x)\mathrm{d}x}\cdot q(x)\Rightarrow$

$e^{\int p(x)\mathrm{d}x}\cdot y=\int e^{\int p(x)\mathrm{d}x}\cdot q(x)\mathrm{d}x+C$

得 $y=e^{-\int p(x)\mathrm{d}x}\left[\int e^{\int p(x)\mathrm{d}x}\cdot q(x)\mathrm{d}x+C\right]$

$y'+p(x)y=q(x)y^n$ 型：先变形到 $y^{-n}\cdot y'+p(x)y^{1-n}=q(x)\overset{z=y^{1-n}}{\Longrightarrow}$ 得 $\frac{\mathrm{d}z}{\mathrm{d}x}=(1-n)y^{-n}\frac{\mathrm{d}y}{\mathrm{d}x}$，则 $\int\frac{1}{1-n}\frac{\mathrm{d}z}{\mathrm{d}x}+p(x)z=q(x)$

$y''=f(x,y')$ 型：令 $y'=p\Rightarrow y''=p'\Rightarrow\frac{\mathrm{d}p}{\mathrm{d}x}=f(x,p)$ 若解得 $p=\varphi(x,C_1)$ 即 $y'=\varphi(x,C_1)$ 则通解为 $y=\int\varphi(x,C_1)\mathrm{d}x+C_2$

$y''=f(y',y'')$ 型：令 $y'=p\Rightarrow y''=\frac{dp}{\mathrm{d}x}=\frac{\mathrm{d}p}{\mathrm{d}y}\cdot\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}p}{\mathrm{d}y}p$ 得 $p\frac{\mathrm{d}p}{\mathrm{d}y}=f(y,p)$ 若解得 $p=\varphi(y,C_1)$ 则由 $p=\frac{\mathrm{d}y}{\mathrm{d}x}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\varphi(y,C_1)$ 分离变量得

$\frac{\mathrm{d}y}{\varphi(y,C_1)}=\mathrm{d}x\Rightarrow\int\frac{\mathrm{d}y}{\varphi(y,C_1)}=x+C_2$

$y''+py'+qy=f(x)$ 型：

$$\left\{\begin{array}{l}\lambda^2+p\lambda+q=0\Rightarrow\lambda_1,\lambda_2\Rightarrow\text{写出齐次方程的通解}\\ \\ \text{设特解}y''\Rightarrow\text{回代，求待定系数}\Rightarrow\text{特解}\end{array}\right.\Rightarrow\text{写出通解}$$

$y''+py'+qy=f_1(x)+f_2(x)$ 型：

$$\left\{\begin{array}{l}
写\lambda^2+px+q=0\Rightarrow 齐次方程的通解 \\
\\
y''+py'+q=f_1(x)写特解y_1^* \\
\\
y''+py'+qy=f_2(x)写特解y_2^*
\end{array}\right. \begin{array}{l} \\ \\ \Rightarrow 故y_1^*+y_2^*为特解 \end{array} \Rightarrow 通解$$

(1) 齐次方程的通解

$p^2-4q>0,$即$\lambda_1\neq\lambda_2$ 通解为$y=C_1e^{\lambda_1 x}+C_2e^{\lambda_2 x}$

$p^2-4q=0,$即$\lambda_1=\lambda_2=\lambda$ 通解为$y=(C_1+C_2x)e^{\lambda x}$

$p^2-4q<0,$共轭复根为$\alpha\pm\beta\mathrm{i}$ 通解为$y=e^{\alpha x}(C_1\cos\beta x+C_2\sin\beta x)$

(2) 非齐次方程的特解

自由项$f(x)=P_n(x)e^{ax}$时, 特解$y^*=e^{ax}Q_n(x)x^k$

$$\left\{\begin{array}{l}
e^{ax}照抄 \\
\\
Q_n(x)为x的n次一般多项式 \\
\\
k=\left\{\begin{array}{l}
0\ \ \alpha\neq\lambda_1,\alpha\neq\lambda_2 \\
\\
1\ \ \alpha\neq\lambda_1或\alpha\neq\lambda_2 \\
\\
2\ \ \alpha=\lambda_1=\lambda_2
\end{array}\right.
\end{array}\right.$$