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Compressive Mobile Sensing for Robotic Mapping

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Abstract— **Compressive sensing is an emerging research field based on the fact that a small number of linear measurements can recover a sparse signal under an orthogonal basis without losing any useful information. Using this approach, the signal can be recovered by a rate that is much lower than the requirement from the well-known Shannon sampling theory. In this paper, we propose a novel approach named compressive mobile sensing, which implements compressive sensing technique on a mobile sensor. This approach employs one mobile sensor or multiple sensors to reconstruct the sensing fields in an efficient way. Moreover, a special measurement process has been built under the constraint of the mobile sensors. It is also presented the simulation and experimental results of a robotic mapping problem using compressive mobile sensing.**

I. Introduction

Nowadays the sensor network provides novel approaches to reconstruct the sensing field. Basically, there are two types of sensors. One is classified as static sensors which can not move after they are deployed into the sensing field. They are low-consumption, low-cost, and have limited sensing capabilities. A lot of static sensors cooperate together to form a dense network to cover the sensing field. In contrast, another type is called mobile sensors. Unlike static sensors, they are equipped a more powerful sensors and can move to the specific points dynamically according to the different interests, so several mobile sensors are enough to cover a great scale of sensing field. However, the mobile sensors may get lost when they are not configured properly or motion error during the movement.

Most approaches of field recovery using static sensors lie on the prior information of the sensing field, such as the spatial and temporal correlations of the signal, to reduce computational load or other cost. However, the prior information in some applications is unavailable, making it difficult to apply those coorelation-based approaches. But Haupt [1] estimates sensed data from static sensors without any prior knowledge using an emerging compressive sensing framework [2] [3]. The theory of compressive sensing demonstrates that a sparse signal can be recovered by a non-adaptive linear measurement process under some conditions, so that no prior information is needed in advance. Unlike the traditional “sample and process” way, this approach acquires signal in the condensed way directly, even without the intermediate stage of signal sampling. In addition, compressive sensing can use random projection method to guarantee that most useful information is retained in linear measurements.

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However, the static sensors can not be well fitted into some scenarios. In some applications, different areas in the sensing field should be paid different attentions. A hot point should be sensed by a more powerful sensor, because the information from this point is more valuable. In this situation, the static sensor can do nothing about the diversity above. In contrast, the mobile sensor can move to the special point, use the powerful sensors to get more information. Combined with compressive sensing technique, compressive sensing in mobile sensors is more powerful to recovery the signal than the normal one. Therefore we propose a novel approach, compressive mobile sensing, by applying compressive sensing into a new paradigm of mobile sensing. Due to the constraint of the mobile sensor, the linear measurement process is carefully designed, so that it satisfies the sufficient condition by compressive sensing and works under the constraints of the sensors measurement model. In addition, the sensing field is reconstructed by only a few number of samplers because of the inherent merit of compressive sensing. Another desirable feature is that the coverage of the field is guaranteed by traversing the field in a random way.

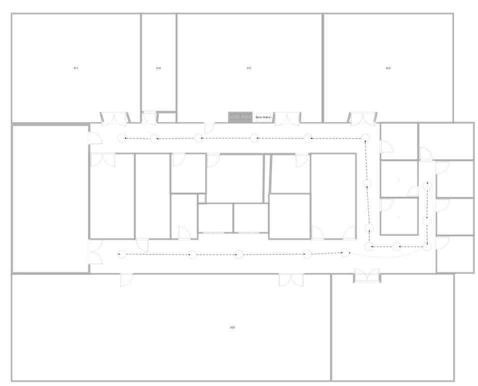

Fig. 1. A robotic mapping scenario using compressive mobile sensing: A mobile robot is driven to map a hallway on the 8th floor of the EERC Building at Michigan Tech. The circle represents a mobile robot, and the dash line means the trajectory of the robot.

Figure 1 shows a typical application of compressive mobile sensing in which a mobile robot is employed to reconstruct the map of an indoor environment. Since the structure indoor environment is unknown, the static sensor is difficult to be deployed in advance to cover the entire environment. Therefore, a mobile robot equipped a laser scanner is applied to reconstruct the map of this environment using compressive mobile sensing. The compressive mobile sensing is motivated by several observations: the traditional approaches for robotic mapping need to sample each point in the environment, which has a significantly computational load; it is impossible to deploy the static sensors in an indoor environment without its floor plan, because it can not guarantee its coverage of the environment; unlike other application of compressive sensing, an unique measurement process is designed for robotic mapping.

II. Related Work

Most state-of-the-art approaches aiming at the robotic mapping problem model the robot and the environment in a probabilistic way. If the robot knows its pose, a Bayesian updating model [4], [5] is the typical one for grid-based map building methods. Bayesian filters are used to calculate the posterior over the map. Otherwise, the mapping problem becomes a SLAM (Simultaneous Localization and Mapping) problem, which builds a consistent map of an unknown environment by a mobile robot while at the same time determining its own location using the map. One cluster of solutions is Expectation Maximization approaches. EM algorithms can solve the correspondence problem, also known as data association problem perfectly. However, EM algorithms do not retain a full notion of uncertainty and cannot build maps incrementally.

Fortunately, compressive sensing gives a good solution to overcome this drawback, using the down-sampling method to precisely recover the signal. Candes et al [2] and Donoho [3] proposes that compressive sensing can directly sample linear measurements to recovery the signal without the intermediate stage, hence it reduce the load of sampling the signal. The Restricted Isometry Property(RIP) [2] should be satisfied [3], as a sufficient condition to ensure compressive sensing works. Based on the RIP, much work focus on different pairs of the sparsity basis and the measurement basis. Baraniuk gives a theoretical proof of RIP on random matrices [6]. Berinde gives the performance of the sparse binary matrix [7]. In this way, the random linear measurements can be applied to many transform-based orthogonal bases, such as Fourier, wavelet, discrete cosinusoid [8]. Moreover, the variations of compressive sensing are also discussed. Duarte proposes a distributed form of compressive sensing [9]; Ji considers compressive sensing from a Bayesian perspective [10].

Although compressive sensing guarantees that the sparse signal can be recovered with the requirement of the RIP, there is still an open problem to find a fast recovery algorithm. According to compressive sensing, the sparse signal recovery problem is a l_0 norm minimization problem, can be approximately converted to a convex l_1 norm optimization problem [3]. Unfortunately, l_0 norm minimization problem is both numerically unstable and NP-complete, and even a linear program algorithm for l_1 norm optimization, known as Basis Pursuit [11], has computational complexity $O(N^3)$. Hence many novel recovery algorithms are proposed to reduce the computational complexity, and still meet the requirement of the recovery performance at the same time. Reweighted l_1 norm minimization [12] outperforms Basis Pursuit in the sense that substantially fewer measurements are needed for exact recovery. The approach in [13] is an effort to solve the nonconvex problem in a efficient way. Moreover, Orthogonal Matching Pursuit (OMP) [14] and Tree-Based Orthogonal Matching Pursuit(TOMP) [15] belong to Matching Pursuit algorithm, which are much faster based on the greedy algorithms.

III. Compressive Mobile Sensing

In order to implement compressive sensing, three requirements should be met [16]: the signal has a sparse representation in one orthogonal basis; in addition, the measurements are the linear combinations of the desired signal; and a nonlinear reconstruction is used to recover the data.

Consider a 2D discrete signal, such as an image signal, $\mathbf{x} = \{x_{i,j}\},\ i,j = 1,2\ldots\sqrt{N}$, where N is a square number and represents the total number of points in **x**. For notation simplification, **x** is vectorized into an $N \times 1$ column vector,$\{x_i\},\ i = 1,2,\ldots,N$, in $\Re^N$.

Any **x** in $\Re^N$ is *K-sparse* if there exists an orthonormal basis $\{\psi_i\}_{i=1}^N$, called *sparsity basis*[8], such that **x** can be expressed by

$$\mathbf{x} = \sum_{i=1}^{N} s_i \psi_i = \sum_{l=1}^{K} \theta\left(i_l\right) \psi_{i_l} \tag{1}$$

where $\{s_i\}$ are the weighting coefficients, and $\{i_l\}$ are the indices where the corresponding points do not equal to zero. Here, only K of the s_i in (1) are nonzero and other $N - K$ are zero. And By forming a sparsity basis matrix $\Psi = [\psi_1|\ldots|\psi_N]$ by stacking the vectors $\{\psi_i\}_{i=1}^N$ as columns, **x** can be expressed as $\mathbf{x} = \sum_{i=1}^N s_i\psi_i \ or \ \mathbf{x} = \Psi\mathbf{s}$, where $\mathbf{s} = \left\{s_i = \langle X, \psi_i\rangle = \psi_i^T X\right\}$ is the $N \times 1$ column vector of weighting coefficients , where ψ_i^T denotes the transpose of ψ_i and $\langle .,. \rangle$ denotes the inner product. Clearly, **x** and **s** are equivalent representations of the same signal, with **x** under the spatial domain and **s** under the sparsity basis domain. Sparse expression is motivated by the fact that many natural and manmade signals are compressible in the sense that there exists a basis where the representation (1) has just a few large coefficients and many small coefficients, and compressible signals can be well approximated by K-sparse representations.

Unlike the sample-then-compress framework [16], compressive sensing recovers the signal directly into a compressed representation without taking a large number of samples under the Nyquist rate. Suppose sampling is a linear measurement process, such that the i^{th} measurement, $y_i = \sum x_i\phi_i^T$, is a linear combination of $\{x_i\}$, where $\left\{\phi_i^T\right\}$, $i = 1,\ldots,M$ is a second set of vectors. In a matrix notation,

$$\mathbf{y} = \Phi\mathbf{x} = \Phi\Psi\mathbf{s} = \Theta\mathbf{s} \tag{2}$$

where Φ is a $M \times N$ *Measurement Basis* and M is not greater than N.

The recovered signal $\hat{\mathbf{s}}$ can be obtained by a l_0 norm minimization problem with objective function$\hat{\mathbf{s}}$ =

$arg\min_{\mathbf{s}'} \|\mathbf{s}'\|_1$ s.t. $\Theta = \mathbf{s}'\mathbf{y}$ This optimization problem, also known as Basis Pursuit [11], can be solved with traditional linear programming techniques.

In this paper, we propose a novel approach which apply compressive sensing to a new paradigm of the mobile sensing. The objective of compressive mobile sensing is to use one mobile sensor or multiple mobile sensors to estimate the parameters of a sensing field or reconstruct the sensing field, Figure 1 shows a scenario in which a mobile robot is applied to reconstruct an indoor environment using compressive mobile sensing. Unlike static sensors, compressive mobile sensing owns several unique challenges. First, it is easy for static sensors to implement a linear measurement process. In a static sensor network, the sensor readings are transmitted to a fusion center via one hop or multiple hops, and in the fusion center, those readings are merged into a linear measurement by $\mathbf{y} = \left\{ y_i = \sum_j \Phi_{i,j} x_j \right\}$ where $\Phi = \{\Phi_{i,j}\}$ is the measurement basis in (2) and x_j is the jth sensor's reading. However, for a single mobile sensor, it should sample the entire environment in an iterative way due to its limit insight of the environment. Therefore, to apply compressive sensing, a proper measurement basis is designed which not only has the RIP, but also is executable under constraints of the sensor's measurement model. Second, the spatial information corresponding to one sensor reading is also needed for reconstruction. The static sensors can be deployed manually with locations known in advance or localized by many existed algorithms. However, those algorithms based on Markov Chain and Mento Carlo all lay on the assumption that the noises at each time instances are independent. Unfortunately, it is not true in most situations of mobile robot localization. In our approach, we propose an approach based on optical flow to estimate the correlated error of the robot motion. In this section, a robotic mapping problem is presented to prove the efficiency and robustness of compressive mobile sensing.

A. Sparsity Basis

An indoor environment can be modeled using a 2D grid-based map, $\mathbf{m} = \{m_{x,y}\}$, where (x, y) denotes the coordinates of a grid and a binary variable $m_{x,y}$ denotes its occupancy, either the grid is occupied or it is free. Using probabilistic state belief, the mapping problem is converted to calculate a posterior distribution of the state belief, $p\left(m_{x,y}|z^{1:t}, \mathbf{p}^{1:t}\right)$, over $\mathbf{m} = \{m_{x,y}\}$, where the state belief means the probability of occupancy of the grid on (x, y), $z^{1:t}$ is the set of observations of the sensor from time instance 1 to t and $\mathbf{p}^{1:t}$ represents the sequential poses of the robot from time instance 1 to t.

One necessary condition of compressive sensing is that the desired signal $\mathbf{m}$ is sparse or compressible in an orthogonal basis. Obviously, it can be found that most of the indoor environment is piece-wise continuous, the most area is flat and the uncontinuities only exist along the walls and other boundaries, therefore it can be envisioned that $\mathbf{m}$ is compressible under discrete Fourier basis and discrete wavelet basis. Figure 2 is a grid-based map corresponding to the floor plan shown in Figure 1.

Fig. 2. Grid-based map corresponding to the floor plan shown in Fig. 1.

Among the popular transform-based bases, discrete Haar wavelet basis outperforms other options due to its two unique characteristics. First, Haar wavelet can express piece-wise continuous signal better than discrete Fourier basis and cosine basis, since it describes the jumping rather the smoothness of the signal. In this example, it is obvious that the number of points with zero-value or small quantity are much greater than the other two bases, so it indicates that this 2D signal $\mathbf{m}$ is more compressible in discrete Haar wavelet basis than others. Second, Haar wavelet transforms has lower time complexity with $O(N)$, while FFT as the fast algorithm of discrete Fourier transform has the time complexity with $O(N \log N)$.Therefore, discrete Haar wavelet basis is taken as the sparsity basis in compressive mobile sensing.

Besides sparsity, another requirement that compressive sensing can be applied to robotic mapping problem is the implementation of a linear measurement process by the sensor equipped on the mobile robot. Candes [17] introduces an approach of random projection in discrete Fourier basis, when the signal is sparse in time domain. It has been proved that this random projection method is incoherent with discrete Fourier basis. We are inspired by the similarity of wavelet and Fourier transform and the symmetry of wavlet transform and inverse wavelet transform. The measurement basis is constructed by random sampling $\mathbf{m}$ under the spatial domain, when the grid-based map is compressible under Haar Wavelet basis. It can be generated as follows:

1) Vectorize $\mathbf{m}$ into a 1D vector, $\mathbf{m} = \{m_i\}$, $i = 1, 2, \ldots, N$, where N is the total number of points;

2) Draw M samples uniformly from $\mathbf{m}$ mutually excluded, denoted as $(s) = \{s_i = \mathbf{m}_{l_i}\}$, $i = 1, 2, \ldots, M$ where l_i is the ith samples index in $\mathbf{m}$;

3) Generate a $M \times N$ matrix, where for each entry in Φ,

$$\begin{cases} \Phi_{i,j} = 1, & j = l_i \\ \Phi_{i,j} = 0, & j \neq l_i \end{cases} \tag{3}$$

Therefore, Suppose $\mathbf{m}$ is sparse in a discrete Haar wavelet basis Ψ, that is $\mathbf{x} = \Psi\mathbf{m}$. The compressive sensing measurement can be expressed by $\mathbf{y} = \Psi\mathbf{m} = \Phi\Psi^{-1}\mathbf{x}$. If Φ is a random matrix which satisfies the RIP or incoherence with Ψ^{-1}, the sparse coefficient vector $\mathbf{x}$ can be recovered from $\mathbf{y}$ by solving

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}'} \|\mathbf{x}'\|_1 \quad \text{s.t.} \quad \mathbf{y} = \Phi\Psi^{-1}\mathbf{x}'. \tag{4}$$

Actually, more practical situation is coefficient vector $\mathbf{x}$ is compressible in discrete Haar wavelet basis. In this case, compressive sensing recovery algorithm can still be applied to obtain an approximation a $\mathbf{x}$ by [1]

$$\hat{\mathbf{x}} = arg\min_{\mathbf{x}'} \left\{ \|y - \Phi\Psi^{-1}\mathbf{x}'\|_2^2 + \lambda\|\mathbf{x}'\|_0 \right\}. \tag{5}$$

And then the approximation of $\mathbf{m}$ can be calculated by $\hat{\mathbf{m}} = \Psi^{-1}\hat{\mathbf{x}}$.

B. Measurement Model and Uncertainty Consideration

Occupancy of an indoor environment is often percepted by a laser scanner which detects the occupancy by calculating the distance between the center of the laser scanner and the nearest obstacle along the laser beam. Therefore, a proper measurement model is proposed and the linear measurements required by (3) can be obtained by the laser scanner. Suppose that the pose of the mobile robot at time t is denoted by $\mathbf{p}^t$ with locating at $\left(p_x^t, p_y^t\right)$ and heading atp_θ^t, the laser scanner at the same location can percept the area ahead itself. If the reading of one laser scan is $z^t\left(r_\alpha\right)$, where α is the relative angle to the robots orientation; r_α is the reading which represents the distance between the end point and the center of the laser scanner $\left(p_x^t, p_y^t\right)$, we can calculate the end point, denoted as $q\left(q_x, q_y\right)$ of a laser beam, by projecting the laser scanner's reading to a global coordinate. According to the geometrical relation, the ideal end point of the laser beam can be obtained by:

$$\begin{cases} q_x = p_x^t + r_\alpha \cos\left(p_\theta^t - \frac{\pi}{2} + \alpha\right) \\ q_y = p_y^t + r_\alpha \sin\left(p_\theta^t - \frac{\pi}{2} + \alpha\right) \end{cases} \tag{6}$$

Unfortunately, noise results from both the laser sensing and the robot motion. For the sensing uncertainty, the noise can be modeled as a zero-mean Gaussian random variable with $\Sigma = \varepsilon$. Therefore, the coordinate of the end point from the laser's reading , $\tilde{q}\left(\tilde{q}_x, \tilde{q}_y\right)$, is Gaussian random vector with $\mu = (q_x, q_y)'$ and $\Sigma = \varepsilon I$, where I is a 2×2 identical matrix. Since the laser scanner only detects the edge of an opaque obstacle, 1) we mark the points with $p(m|z^t, \mathbf{p}^t) = p_{free}$ which lay on the line segment from the (p_x, p_y) to $\tilde{q}\left(\tilde{q}_x, \tilde{q}_y\right)$; 2) the end point on $\tilde{q}\left(\tilde{q}_x, \tilde{q}_y\right)$ is marked by $p(m|z^t, \mathbf{p}^t) = p_{occ}$; 3) the occupancy of points around $q\left(q_x, q_y\right)$ is according to the 2d Gaussian distribution with $\mu = (q_x, q_y)$ and $\Sigma = \varepsilon I$; and 4) the points from $q\left(q_x, q_y\right)$ to the limit of laser beam keep unchanged. According to the Bayesian Filter, we update the posterior map can be written as:

$$\begin{aligned} \log\frac{p\left(m_{x,y}|z^{1:t}, \mathbf{p}^{1:t}\right)}{1 - p\left(m_{x,y}|z^{1:t}, \mathbf{p}^{1:t}\right)} &= \log\frac{p\left(m_{x,y}|z^t, \mathbf{p}^t\right)}{1 - p\left(m_{x,y}|z^t, \mathbf{p}^t\right)} + \\ \log\frac{1 - p\left(m_{x,y}\right)}{p\left(m_{x,y}\right)} &+ \log\frac{p\left(m_{x,y}|z^{1:t-1}, \mathbf{p}^{1:t-1}\right)}{1 - p\left(m_{x,y}|z^{1:t-1}, \mathbf{p}^{1:t-1}\right)} \end{aligned} \tag{7}$$

The grid-based map is the result of discreting the continuous environment. It is defined that the occupancy of the grid equals to

$$\bar{p} = \frac{\iint_{grid} p\left(m_{x,y}|z^{1:t}, \mathbf{p}^{1:t}\right) dxdy}{\iint_{grid} dxdy}. \tag{8}$$

Another uncertainty comes from the robot's motion. Unlike the noise which may be modeled as an independent random variable, this kind of error from the motor is systematic and correlated. Due to this noise, pieces of partial map recovered at each step can not be projected into the same global coordinates.

In compressive mobile sensing, the information obtained by the laser scanner is employed to estimate the offset between the actual pose of the robot and the belief pose which is calculated from the odometer. We assume the pose of the robot at time $t-1$ is $\left(p_x^{t-1}, p_y^{t-1}, p_\theta^{t-1}\right)$ and the recovered area is denoted R^{t-1}. In addition, the pose we obtained from the odometer can written by $\left(\tilde{p}_x^t, \tilde{p}_y^t, \tilde{p}_\theta^t\right)$, and the area recovered should be $\tilde{R}^t$. Since the noise of the robot motion, the actual of the robot is $\left(p_x^t, p_y^t, p_\theta^t\right)$ and the recovered area should be R^t, which is different from $\tilde{R}^t$. Consider the overlap of R^{t-1} and R^t, it should be offsetted from the overlap of R^{t-1} and $\tilde{R}^t$. By comparing with these two overlap areas, it can be estimated the offset between $\mathbf{p}^t$ and $\tilde{\mathbf{p}}^t$, and update the robot's pose further. If it is defined that $\mathbf{p}^t = W(\hat{\mathbf{p}}^t; P)$. In matrix notation,

$$\begin{bmatrix} p_x^t \\ p_y^t \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Delta\theta & -\sin\Delta\theta & \Delta x \\ \sin\Delta\theta & \cos\Delta\theta & \Delta y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{p}_x^t \\ \hat{p}_y^t \\ 1 \end{bmatrix} \tag{9}$$

where the parameter set $P = (\Delta x, \Delta y, \Delta\theta = p_\theta^t - \tilde{p}_\theta^t)$. If $\mathbf{m}_0$ denotes the intersection of R^{t-1} and R^t, and $\mathbf{m}_1$ denotes the intersection of R^{t-1} and $\tilde{R}^t$. P can be estimated by minimizing the *sum of squared differences* (SSD) function

$$E_{SSD}(P) = \sum_i \left[m_1\left(W\left(x_i, P\right)\right) - m_0\left(x_i\right)\right]^2 \tag{10}$$

Figure 3 shows the difference of $\mathbf{m}_0$ and $\mathbf{m}_1$ and Figure 4 represents the offset estimation result using Lucas-Kanade algorithm [18]. The flow field represents the virtual motion of each point on Figure 4, if it is supposed that the points in the map moves rather than the offset of the mobile robot. We estimate the parameter P which are identical in most points, and consider this value as the robot's offset.

C. Recovery Approach

Total Variation(TV) minimization algorithm is applied to recover the grid-based map. Let $\|\mathbf{x}\|_{TV}$ be the TV norm of a 2D object $\mathbf{x}$.

$$\|\mathbf{x}\|_{TV} = \sum_{i,j} \sqrt{(m_{i,j} - m_{i-1,j})^2 + (m_{i,j} - m_{i,j-1})^2} \tag{11}$$

To recover $\mathbf{x}$ which is equivalent $\mathbf{m}$ from the random samplings, we can obtain the solution by solving

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}'} \|\mathbf{x}'\|_{TV} \quad s \cdot t \cdot \quad \Theta = \mathbf{y} = \Phi\Psi^{-1}\mathbf{x}' \tag{12}$$

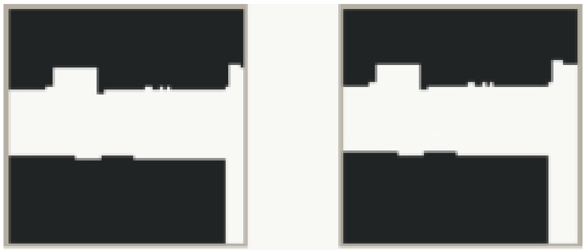

Fig. 3. (a) is the overlap of R^{t-1} and $\tilde{R}^t$. (b) is the corresponding part in R^t.

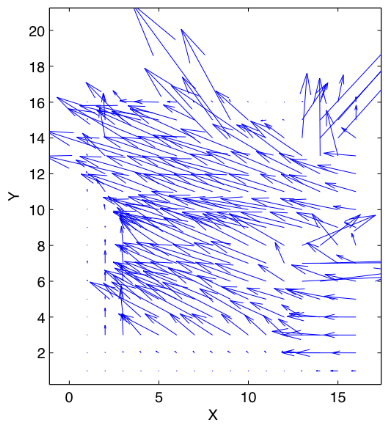

Fig. 4. Virtual optical flow of each point. Every point can be considered moving the inverse direction of the offset of the robot.

The advantage of TV norm minimization exists that it can partially diminish some of the artifacts by suppress spurious high frequency features. This approach is an iterative scheme with two steps. If the algorithm starts with an initial guess of $\mathbf{x} = \Psi\Phi^T\mathbf{y}$, and each descent iteration can be viewed as an improvement on the last round, and the process can be terminated till the user's requirement is satisfied.

IV. Simulation and Experimental Results

In this section, the compressive mobile sensing is implemented both in the simulation environment and the experiment. The results from both simulation and the experiment provide the evidence that the approach of compressive mobile sensing can recover the desired map, and reduce the required number of the samples significantly.

A. Simulation Results

Figure 2 is the grid-based map, which is reconstructed by sampling all the grids in this hallway environment without compressive sensing. This map contains $154 \times 249 = 38364$ grids, and each grid corresponds to the square with $250mm \times 250mm$ in reality. Suppose K numbers of samples are drawn randomly among the entire environment. Figure 5 is the result using the TV norm minimization algorithm to recover this map. with (a) corresponding to K=600, (b) corresponding to K=1500, (c) corresponding to K=3000, (d) corresponding to K=8000. Intuitively, it can be found that using K=600 samples, the recovered map only has sparse points, it can not represent the original map; when K=1500 or above, most information of the environment has been gathered by the samples, so the recovered map can represent the original floor plan well. Obviously, the more samples are drawn, the better result would be reconstructed.

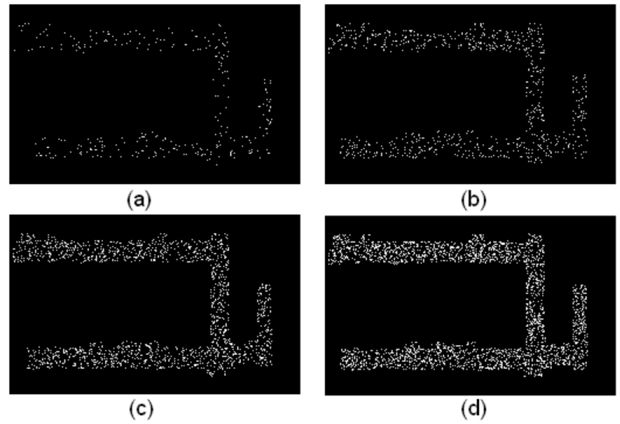

Fig. 5. Comparison of the compessive mobile sensing under different number of samples. (a) K=8000; (b) K=3000; (c) K=1500; (d) K=600.

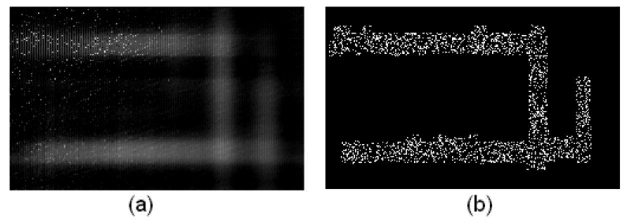

Fig. 6. Comparison of the compessive mobile sensing under different sparsity bases. (a) discrete cosine basis; (b) discrete Haar wavelet basis.

Figure 6 is the comparison between different sparsity bases with the same K=8000, where (a) is the recovery result based on discrete cosine basis, and (b) is based on discrete Haar wavelet basis. In Figure 6(a), the recovered map is blurry due to the inherent characteristic of cosinoid which express the smooth signal better. The result also supports the analysis in previous section.

Based on the simulation results, we find that the grid-based map can be recovered by using compressive sensing, and discrete Haar wavelet basis is a good candidate of sparsity basis. In addition, from Figure 6(b), it is enough good to choose the ratio of the number of samples and total points at $\frac{K}{N} \approx 10\%$.

B. Experimental Results

A Pioneer II mobile robot equipped with a LMS200 laser scanner is used in this experiment. The laser scanner is configured to cover $180°$ range for one time scanning with 181 readings and the limit of sensing range is set to $8000mm$. And to simplify the algorithm, the valid reading is restricted into a 64×32 square, in which each grid corresponds to the square with $250mm \times 250mm$ in reality.

Fig. 7. The upper hallway recovered with manually calibrating the pose of the robot.

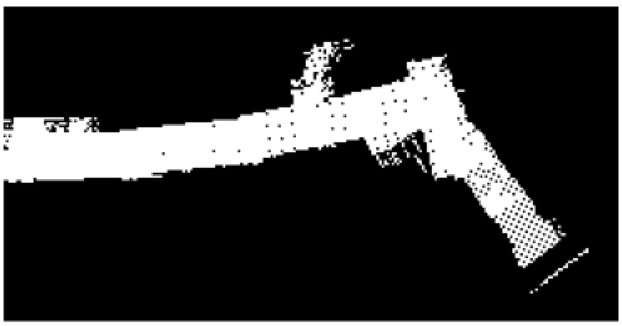

Fig. 8. The map stitched according of the pose calculated by odometer.

The data beyond this square is discarded. Therefore, the partial map recovered at each step is a square-like shape. The Pioneer II robot is driven along the hallway to cover this indoor environment. The interval of each step is at most $4m$, so that we have the enough overlap to calibrate the pose of the robot. Figure 7 is the upper hallway recovered with manually calibrating the pose of the robot. Compared to the floor plan in Figure 1, most detailed information has been retained by using the compressive mobile sensing. However, using the odometer readings to calculate the pose of robot and we can project the partial maps in to a global coordinate. Figure 8 shows the results that the straight hall way is distorted due to the offset of the robot pose. Figure 9 shows the final recovered map to which the optical flow approach is applied.

V. Conclusion

We have proposed a new compressive mobile sensing approach for robotic mapping which use much fewer samples to recover an indoor environment. It addresses the challenges of an unique measurement model for the laser scanner and consideration of correlated uncertainty caused by robot motion. The compressive mobile sensing proves to be an efficient approach to reduce the number of samples when recovering the grid-based map.

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Fig. 9. The map stitched according the pose after calibration.

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