# Compressive Sensing Based Vehicle Information Recovery in Vehicular Networks

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Abstract—Vehicular ad hoc networks are expected to provide a reliable networking platform for cooperative safety communication systems. Those systems are of a broadcast nature and require to deliver both safety messages and vehicle tracking information while being interfered by other types of lower priority messages on the same channel. Vehicle tracking information are necessary to enable safety communication systems and intelligent transportation systems. Due to the large number of communicating vehicles and the amount of traffic exchanged in the broadcast mode, network congestion often occurs in vehicular communication systems. In this paper, a new methodology for avoiding such a network congestion is proposed. We identify the sparsity of the vehicle tracking information and propose a novel information recovery scheme. The proposed scheme reduces the amount of data exchanged due to vehicle tracking packets while providing a robust information reception at the receivers. Essentially, it utilizes compressive sensing to transmit a few measurements of the vehicles velocity vector that allows perfect recovery of the original vector at the receiver with a minimal error. Extensive simulation results demonstrate the effectiveness of applying compressive sensing in recovering vehicles tracking information, and relieving the network from unnecessary congestion due to the large amount of data exchanged.

Index Terms—Vehicular networks, Tracking information, Compressive Sensing (CS), Congestion avoidance.

# I. INTRODUCTION

Vehicular ad hoc networks (VANETs) are expected to provide a robust and reliable communication channel for a variety of applications that are currently supported by the intelligent transportation systems (ITS). Both industry [1] and academia [2] [3] are looking for solutions to provide the necessary quality-of-service for different network situations, especially when vehicles cooperate with each other without the involvement of any infrastructure. In North America, the Federal Communications Commission (FCC) [4] has allocated the frequency band 5.850-5.925 GHz spectrum for Dedicated Short Range Communications (DSRC). More recently, the IEEE 802.11p MAC and PHY amendment was finalized to support wireless access in vehicular environments (WAVE) [6]. Such developments have led to increased attention for safety communications, which refer to the exchange of packets to alert vehicles about an impending collision or dangerous situation [7].

Cooperative safety communication is a real-time broadcast system that is required to provide a complete distribution of messages within an area of at least 250m size in a very short deadline, usually 100ms [8]. The rate of safety messages is

specified to be 10Hz (i.e. 10 messages per second) [8]. This type of broadcast is normally difficult to implement and maintain high level of stability in a highly dynamic network such as VANETs. In a cooperative safety communication system, vehicles are required to keep track of each others to maintain the topology information. For example, a vehicle is required to provide updated information of its velocity, position, ID and other information when vehicles exchange their high priority safety and non-safety WAVE Short Message Protocol (WSMP) messages. This periodic vehicle-tracking broadcast is essential to maintain a stable networking performance for the cooperative safety applications. Considering the nature of frequent topology changes and the variable node densities in VANETs, congestion is more probable to occur due to the large amount of messages periodically broadcast through the control channel (CCH) of the DSRC spectrum. These messages are required to maintain the knowledge of the network.

Although a complex network protocol (which can be very difficult to implement) can handle network congestion to some extent, the significant amount of messages exchanged between the vehicles can easily degrade the protocol performance especially when vehicles move with a very high speed (which affects PHY performance) and/or when the node density is very high or suddenly changes (which affects MAC and network layer performance). The current IEEE 802.11p [6] protocol, included in the WAVE protocol stack, is the only standard for the MAC protocol in vehicular networks. The current protocol suffers from significant degradation in performance in terms of throughput and packet drop rate in both unicast [9] and broadcast [9] [10] communications. There has been some effort in the literature to handle congestion control in VANETs via power control [11] to reduce the traffic load in the channel. Other approaches are based on channel occupancy [12] and feedback control [13].

It is necessary to see the problem of congestion control not only from the perspective of enhancing the use of the current resources (as was done in the literature, e.g. reducing the frequency of transmission while sacrificing significant information or using power control), but by investigating the nature of VANETs and its applications and utilize that information to enhance broadcasting. Therefore, there should be a mechanism to specifically resolve the congestion due to the large amount of periodic vehicle tracking messages continuously exchanged via a limited channel capability.

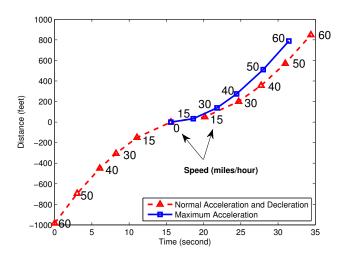


Fig. 1: A sample velocity trajectory based on traffic flow theory [14].

In this paper, we examine the periodic vehicle tracking messages and observe certain properties about the information included in these messages. From our observations, we reach an understanding that such messages can be simply broadcast to all neighbors without the need to exchange them at the same rate without losing significant information. This leads to a good amount of reduction in the network that results in solving part of the congestion problem. In particular, we observe that the kinematic messages are sparse in nature, and use compressive sensing to reduce the sampling and communication rate. To the best of our knowledge, we are the first to apply such a technique to reduce the congestion of the network via compressive sensing (CS). Our work shows the ability of CS to provide a near-original broadcast as will as to reduce network congestion. We claim that this approach can be used to recover the mobility trajectory in a vehicular network with few samples. This information can then be used for different ITS applications.

The rest of this paper is organized as follows. Section II discusses sparsity of the vehicle tracking information. Section III explains the system model including the mobility model and the proposed CS model. The proposed CS recovery scheme is evaluated and discussed in Section IV. Finally, Section V concludes the paper.

### II. SPARSITY OF VEHICLE TRACKING INFORMATION

The vehicle periodic broadcast is meant to provide nearly full geometry of the network for the mobile nodes disregarding the density of the network. This is difficult especially when the network is dense, or vehicles are moving with high speeds. However, there are two key issues that need to be considered. The first one is that vehicles usually change their velocity smoothly. In other words, a vehicle might change its speed by adding (reducing) few kilometers per hour per second. Based on the *traffic flow theory* [14], we plot a velocity trajectory

of a vehicle decreasing its speed to zero and increasing again for two cases: normal acceleration and deceleration, and maximum acceleration. Figure 1 shows an example velocity trajectory. It is clear in the figure that changing the acceleration to the maximum does not change the fact that the trajectory is smooth. Therefore, acceleration and deceleration imply the same idea of smooth change. This will be explained in more details in Section III-A.

By observing such a behavior of a vehicle velocity, it can be represented in a sparse domain. Taking the Fourier transform of such a velocity vector, it shows very few significant coefficients while the majority of the vector elements are zeros. A vector with such properties is considered sparse in the frequency domain. Hence, CS can be applied on this vector yielding a near-perfect recovery of the original vector via very few measurements of the main signal. The second issue is that, since vehicles move smoothly, their velocity shows a significant sparsity level in the frequency domain, their motion should reveal a higher level of sparsity in a millisecond scale. Thus, every millisecond (or hundreds of millisecond), a vehicle increases its velocity very smoothly. In this case, velocity shows a higher sparsity level in frequency, which is more beneficial from a vehicular communication perspective for vehicle tracking and periodic broadcasting.

To illustrate the idea, consider an example of a single vehicle with a velocity vector showing speed in km/h per second, as the one in Figure 2(a). Note that the speed vector increases over time since the vehicle is increasing its speed towards the maximum desired one. In this example, there are no obstacles in front of the vehicle that forces deceleration. After transforming the velocity vector into the frequency domain by taking the discreet Fourier transform (DFT), sparsity is depicted in Figure 2(c). Despite the fact that the vector is sparse, its sparsity level may differ based on its length and values of its elements. By applying compressive sensing and taking random measurements of the velocity vector (e.g. 60% of the size of the original vector in this example), we are able to achieve an excellent reconstruction of the original velocity vector in both time and frequency domain as Figures 2(b) and 2(d) show, respectively. This means that, a vehicle does not have to transmit a full-length vector; Instead, few random measurements are enough to reveal its speed at the receiver. More details in performance evaluation are given in Section IV.

# III. SYSTEM MODEL

Consider vehicles traveling through a highway. Each vehicle is equipped with a GPS device and records its velocity in a vector and periodically broadcasts that vector through a DSRC device to its neighbours with a specific frequency via a single-hop ad hoc network. Assume that vehicles operate according to the standard IEEE 802.11p protocol at the MAC layer. The proposed system model is causal. In other words, the random measurements are taken from the velocity vector at the current time having information from past contributing in the compressive sensing model.

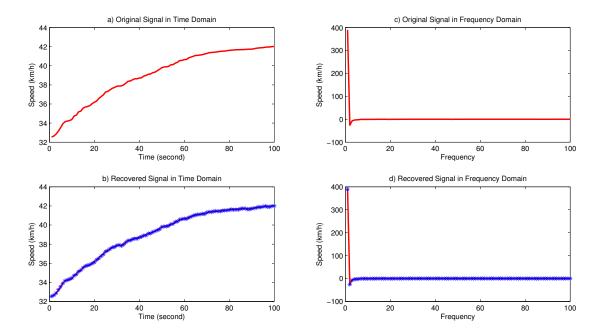


Fig. 2: An example of a sparse vehicle speed recovery via compressive sensing. Figure a) shows the variation of a vehicle speed over a period of 100 seconds. In b), the recovered signal is compared to the original speed. Figures c) and d) reflect the same operation in the frequency domain. The number of measurements used is 60.

Let  ${\bf x}$  be the  $N \times 1$  actual measurement vector, where for simplicity in our simulations we will consider to be the velocity vector measured over an interval of interest. In our system model, the sampling is performed by a random matrix

$$\bar{\mathbf{x}} = \Lambda \mathbf{x} \tag{1}$$

where  $\Lambda$  is a  $N \times N$  random sampling matrix. The IEEE 802.11p standard requires that each vehicle transmit its reading periodically. Hence, if N is the number of samples in the interval of interest, the sampling matrix in IEEE802.11p is the identity matrix and the transmitted vector of each vehicle is  $\bar{\mathbf{x}}$ .

In this paper, we suggest that at each transmission instant, each vehicle transmit a linear combination of all the past measurements in the observation window. With this assumption the sampling matrix  $\boldsymbol{\Lambda}$  will be lower-triangular, and the transmitted vector will be

$$\mathbf{y} = \Gamma \bar{\mathbf{x}} = \Gamma \Lambda \mathbf{x} \tag{2}$$

where  $\Gamma$  is the  $M \times N$  (where  $M \ll N$ ) selection matrix with the elements 0 and 1 and with at most one 1 in each row and column. With this assumption, each vehicle is only required to transmit M samples of its kinematic vector. The samples can be transmitted at any time instant and do not necessarily be transmitted periodically. Using the theory of compressive sensing, we will show that if  $M \approx c \log(N)$ , where c is constant, then the whole velocity vector can be thoroughly reconstructed. With defining the  $M \times N$  sampling

matrix  $\Phi = \Gamma \Lambda$ , the transmitted vector is

$$\mathbf{y} = \Phi \mathbf{x} \tag{3}$$

In practice, each vehicle might be required to send  $\alpha M$  samples, where  $\alpha>1$ , to account for the loss in the wireless channel. It is important to note that any subset of received packets with dimensionality M is sufficient to reconstruct the velocity vector. This is a very powerful tool since it removes the need for receiving all transmitted samples. In the sequel, for the simplicity of discussion we assume that  $\alpha=1$ .

Assume that there are L vehicles transmitting their measurements periodically. The total number of measurements to be transmitted in our approach is reduced to  $L \times M$ , where the measurements are sampled by the sensing matrix. Compared to the standard IEEE 802.11p where the total number of transmissions is  $L \times N$ , our approach preserves the information in the reduced vector and recovers that information via the compressive sensing model. In a typical IEEE 802.11p MAC, our approach can be implemented as a software patch with no need for any change in the hardware.

### A. Mobility Model

The mobility model used in this paper is based on the traffic flow theory [14], which has been selected due to its accuracy in modeling vehicles traveling in both highway and city scenarios. Each vehicle has a uniformly distributed random starting speed  $S \sim U(s_{min}, s_{max})$ , where  $s_{min}$  and  $s_{max}$  are the minimum and maximum allowed speeds in the highway, respectively. When possible, each vehicle increases

its speed by a random acceleration a that is randomly selected in  $[a_{min}, a_{max}]$ ; otherwise, it holds a minimum safe distance to the vehicle ahead. If the vehicle has to reduce its speed, it chooses a random deceleration value  $d \sim U(d_{min}, d_{max})$ , where  $d_{min}$  and  $d_{max}$  are the minimum and maximum deceleration values, respectively. Optionally, a and d can take the same distribution and/or values (i.e. d=-a). It can be seen from the mobility model that once a vehicle reaches its desired speed (e.g.  $s_{max}$ ), it maintains that speed until it has to reduce it as needed (e.g. the vehicle ahead reduced its speed). During the periodic broadcast, we focus on the speed transitions and avoid transmitting the constant speed vector (which can be easily compressed to a single speed value). Sample path of the speed vector is shown in Figures 3(a), 4(a), and 5(a).

#### B. Compressive Sensing Model

Compressive sensing is a technique that has been used to recover original signals from a few measurements in a noisy environment only if the signals are sparse [15]. As discussed in Section II, a vehicle velocity vector shows a sparsity level that varies based on the time scale at which the velocity vector is captured. The sparsity nature of the velocity vector is the key to reduce the number of collected samples. The idea here is to apply CS to the broadcast information, and to send a few measurements that can result in a precise reconstruction of the original vector. In this paper, we show that CS can be used to reduce the amount of data exchanged in a vehicular broadcast and recover the complete mobility trajectory with minimal error.

A periodic broadcast of velocity can be formulated as follows. Consider a  $N \times 1$  velocity vector  $\mathbf{x}$  in the time domain and its corresponding sparse vector in the frequency domain,  $\mathbf{x}_f$ . Let us assume that  $\mathbf{x}_f$  is K-sparse.  $\mathbf{x}_f$  elements are all zeros except for few indexes  $n_i$ ,  $\forall i \in \{1,...,k\}$ , and  $K \ll N$ . These indexes indicate a few significant frequency coefficients of the vehicle velocity vector, namely,

$$\mathbf{x}_f = [0, \dots, 0, x_1, 0, \dots, 0, x_M, 0, \dots, 0]^T,$$

where  $(\cdot)^T$  denotes the transposition operation, and  $x_i$  denotes the ith non-zero component in the sparse domain. Therefore, the vector  $\mathbf{x}$  can be represented as  $\mathbf{x} = \Psi \mathbf{x}_f$ , where  $\Psi$  is the basis at which  $\mathbf{x}$  is sparse. By CS, we can randomly take M measurements to be transmitted instead of complete vector elements. Then, our problem can be expressed as

$$\mathbf{y} = \Phi \Psi \mathbf{x}_f \tag{4}$$

where  $\mathbf{y}$  is an  $M \times 1$  vector,  $\Phi$  is the random sensing matrix (taken from the time domain). This vector  $\mathbf{y}$  contains the data to be transmitted and then recovered with a minimal error.

Following CS recovery procedure via  $\ell 1$  norm minimization, we can enable perfect reconstruction of the original velocity vector  $\mathbf{x}$ . Therefore  $\mathbf{x}_f$  can be recovered with a very high probability by

$$\hat{\mathbf{x}}_f = \operatorname{argmin} \quad \|\mathbf{x}_f\|_1$$
  
subject to  $y = \Phi \Psi \mathbf{x}_f$  (5)

Finally, the time domain vector **x** can be recovered by taking the inverse Fourier transform. Practically, the velocity vector is not purely K-sparse; rather it contains noisy measurements.

The reconstruction above is suitable for sparse and incoherent signals. We use the same preprocessing procedure in [16] to ensure incoherence required by the CS theory. Let us define

$$R = \Phi \Psi$$

Since  $\Psi$  and  $\Phi$  are generally coherent in the spatial domain, R is not in general unitary. Let

$$Q = orth(R^T)^T$$

where  $\operatorname{orth}(R)$  is an orthogonal basis for R. It is possible to show that

$$QR^{\dagger}R = Q$$

where  $(\cdot)^{\dagger}$  is the pseudo-inverse operator. Hence if we define

$$T = QR^{\dagger},$$

then

$$\hat{\mathbf{y}} = T\mathbf{y} = Q\mathbf{x}_f$$

will have incoherent sampling matrix. The velocity vector in the frequency domain,  $\mathbf{x}_f$ , can be reconstructed with high probability from  $\hat{\mathbf{y}}$  by solving the  $\ell_1$  minimization problem

$$\hat{\mathbf{x}}_f = \operatorname{argmin} \quad \|\mathbf{x}_f\|_1$$
  
subject to  $\hat{\mathbf{y}} = Q\mathbf{x}_f$  (6)

Again, we can take the inverse Fourier transform of  $\mathbf{x}_f$  to recover  $\mathbf{x}$  in the time domain.

## IV. PERFORMANCE EVALUATION

### A. Simulation Setup

Simulation of the proposed scheme is performed in two phases; mobility generation and the CS recovery. We implemented the vehicle mobility model as described in Section III-A. The mobility change occurs every 100ms and is meant to simulate the exchange of periodic messages with a frequency (update rate) of 10Hz to be aligned with the broadcast requirement as stated in [8]. The length of the highway is left unspecified. Therefore, vehicles start at the origin of the highway and keep moving without a restriction of a maximum road length. However, system simulation time is limited, and therefore, velocity vectors created during system simulation are also limited. In all of the simulations, velocity vectors do not include any values that are constant over time (i.e. we exclude the part when a vehicle reaches the desired speed and maintains it, if founded). The driver behavior that we are simulating is that the driver increases speed for some time then decreases the speed. This type of oscillation might occur several times. In the simulations, we consider the cases where the velocity vectors has one, three, and four peaks. We do not consider the edge as a peak. For the sake of simplicity, we will refer to the three types of vectors as type-1, 2, and 3, respectively.

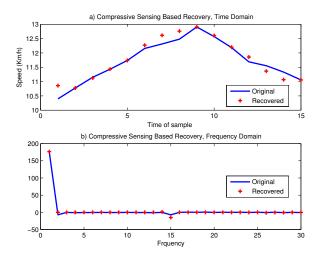


Fig. 3: CS Recovery of a type-1 velocity vector with M=10 and N=15

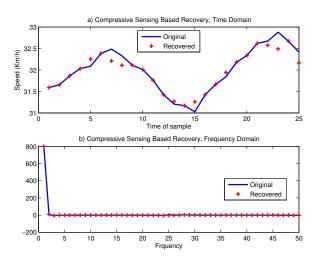


Fig. 4: CS Recovery of a type-2 velocity vector with M=15, and N=25

# B. Simulation Results

To study the performance of CS on the recovery of the vehicle tracking vector, we first show the mean error of the recovery process in time domain of the vector at the receiver in Figure 3(a) for type-1 velocity vector. We observe that the original velocity vector is recovered with minimal error based on few samples of the original signal. By looking at the frequency domain in Figure 3(b), we notice the near-perfect recovery of the vector. Both figures suggest that the idea of using CS in the recovery of mobility information with few samples is feasible.

We consider a type-2 velocity vector with more variations as illustrated in Figure 4. We can see from the figure that it is still possible to recover the original vector with minimal

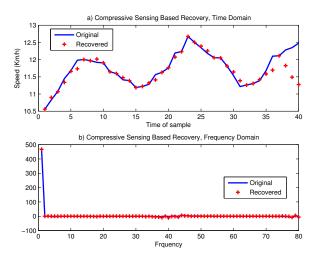
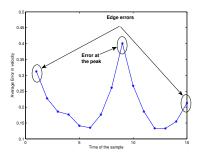


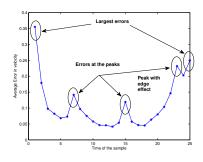
Fig. 5: CS Recovery of a type-3 velocity vector with M=20, and N=40

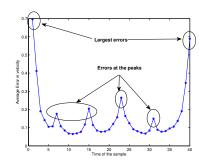
error. Although the signal representation in the frequency is slightly different from the previous example, that does not change the fact that the signal is sparse in the frequency domain. Preserving sparsity allows recovery via CS. However, we note that the recovered vector has slightly higher errors towards edges of the vector. It seems that a small change in the frequency domain recovery leads to an error at the right edge of Figure 4(a). We will see later that both edges of the vector suffer from such an error. The other obvious errors are the ones that are at the peaks. We can see that CS approximations become more clear at those positions. Figure 5 shows similar observations for a type-3 velocity vector. The figure shows a very good approximation of the whole signal except at the right most edge.

We repeated the first experiment over 100 velocity vectors and performed CS recovery 20 different times. For type-1 vectors, the mean error of the results is plotted in Figure 6(a). The figure shows a maximum mean error of 0.4 km/h at the velocity sample that the driver changes the speed. Moreover, the figure clearly shows the edge effect which causes larger mean errors. In this figure, we can see that the edge effect is not significant. We repeat the same experiment for 100 type-2 velocity vectors, and the mean error is plotted in Figure 6(b). It is obvious that the edge effect is significant in this figure, and the error is larger than the ones at the peaks of the vector. Figure 6(c) shows the results for type-3 vectors. The mean error in the recovery process is large at the edges and the number of errors at the peaks is increased compared to type-2 vectors.

Finally, we repeat the experiment over 50 vectors of each type for different number of random measurements and plot the results in Figure 7. It is obvious that increasing the number of measurements results in a lower mean error in recovery. This figure shows the importance of determining the number of measurements for an acceptable mean error in the CS recovery.







(a) Mean error in CS recovery for M=5 and (b) Mean error in CS recovery for M=15 and (c) Mean error in CS recovery for M=20 and N=15 for 100 type-1 vectors. N=25 for 100 type-2 vectors. N=40 for 100 type-3 vectors.

Fig. 6: Mean error for different types of velocity vectors.

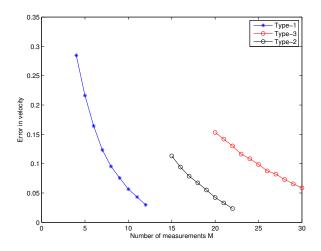


Fig. 7: Mean error in CS recovery for different values of M for 50 velocity vectors of each type.

For a specific ITS application, the number of measurements depends on the acceptable mean error in the estimation of the mobility trajectory.

## V. CONCLUSIONS

In this paper, we proposed a novel compressive sensing based vehicle tracking recovery scheme in vehicular networks. First, we identified that vehicle tracking information are of a sparse nature in the frequency domain. Specifically, the velocity of a vehicle can be modeled as a sparse vector with a few significant measurements. Therefore, compressive sensing is used to reduce the number of data in the packets exchanged between cooperating vehicles. We showed that by using few samples of the velocity vector, a complete recovery of the original vector could be performed with minimal error. We tested three different types of velocity vectors that have different peaks in the velocity vector. We found that the proposed CS recovery scheme performs near-perfect recovery for the three types of vectors, and we identified the edge effect

in the recovery process. We found that the error was minimal and reached its maximum at the peak or the edges of the velocity vector.

#### VI. ACKNOWLEDGMENT

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