

### Optimal Control Problem, 3-State

$$\underset{F_m(t), F_b(t)}{\text{minimize}} J = \int_0^{t_f} P_{motor}(v(t), F_m(t)) dt \quad (1)$$

$$(2)$$

$$\text{subject to } \frac{ds(t)}{dt} = v(t) \quad (3)$$

$$\dot{v}(t) = \frac{F_m(t)}{m} + \frac{F_b(t)}{m} - \frac{\rho C_d A_F}{2m} v^2(t) - g(C_r \cos \alpha(s(t)) + \sin \alpha(s(t))) \quad (4)$$

$$= \frac{F_m(t)}{m} + \frac{F_b(t)}{m} - \frac{F_{air}}{m} - \frac{F_{\alpha}(t)}{m} \quad (5)$$

$$v_{min} \leq v(t) \leq v_{max} \quad (6)$$

$$P_{motor}(v(t), F_m(t)) = \theta(c_1 + c_2 F_m + c_3 F_m^2 + c_4 v^2 + c_5 v^2 F_m + c_6 v^4) \quad (7)$$

$$\theta \in [1, 0.1, 10^{-2}, \dots, 10^{-7}] \quad (8)$$

$$F_{m,min}(v(t)) \leq F_m(t) \leq F_{m,max}(v(t)) \quad (9)$$

$$F_{b,min} \leq F_b(t) \leq F_{b,max} \quad (10)$$

$$F_m(t) F_b(t) \geq 0 \quad (11)$$

$$s_{FV}(t) - 100 \leq s(t) \leq s_{FV}(t) \quad (12)$$

$$v(0) = 10[m/s] \quad (13)$$

$$v_{FV}(0) = 0[m/s] \quad (14)$$

$$v(t_f) = \text{free} \quad (15)$$

$$s(0) = 0[m] \quad (16)$$

$$s_{FV}(0) = 100[m] \quad (17)$$

$$s(t_f) = \text{free} \quad (18)$$