Optimal Control Problem, 3-State

$$\underset{F_m(t),F_b(t)}{\text{minimize}}\ J = \mathbf{\Phi}(s(t_f)) + \int_0^{t_f} P_{motor}(v(t), F_m(t))dt \tag{1}$$

$$\tag{2}$$

$$\text{subject to } \frac{ds(t)}{dt} = v(t) \tag{3}$$

$$\dot{v}(t) = \frac{F_m(t)}{m} + \frac{F_b(t)}{m} - \frac{\rho C_d A_F}{2m} v^2(t) - g(C_r \cos\alpha(s(t)) + \sin\alpha(s(t))) \tag{4}$$

$$= \frac{F_m(t)}{m} + \frac{F_b(t)}{m} - \frac{F_{air}}{m} - \frac{F_\alpha(t)}{m} \tag{5}$$

$$v_{min} \leq v(t) \leq v_{max} \tag{6}$$

$$P_{motor}(v(t), F_m(t)) = \theta(c_1 + c_2 F_m + c_3 F_m^2 + c_4 v^2 + c_5 v^2 F_m + c_6 v^4) \tag{7}$$

$$\theta \in [1, 0.1, 10^{-2}, \cdots, 10^{-7}] \tag{8}$$

$$F_{m,min}(v(t)) \leq F_m(t) \leq F_{m,max}(v(t)) \tag{9}$$

$$F_{b,min} \leq F_b(t) \leq F_{b,max} \tag{10}$$

$$F_m(t) F_b(t) \geq 0 \tag{11}$$

$$s(t) \leq s_{FV}(t) \tag{12}$$

$$v(0) = 10[m/s] \tag{13}$$

$$v_{FV}(0) = 0[m/s] \tag{14}$$

$$v(t_f) = \text{free} \tag{15}$$

$$s(0) = 0[m] \tag{16}$$

$$s_{FV}(0) = 100[m] \tag{17}$$

$$s(t_f) = \text{free} \tag{18}$$

$$\mathbf{\Phi}(s(t_f)) = \begin{cases} -s(t_f) & (s(t_f) \leq s_{FV}(t_f)) \\ \inf & (s(t_f) \geq s_{FV}(t_f)) \end{cases} \tag{19}$$