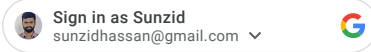




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Exercise 31, Linear Algebra: A Modern Introduction, 4th Edition

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Exercise 31 Answer

Step by step explanation

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Tip

- To solve this question we need to find eigenvalues and their eigenvectors.

Explanation

- We will consider β as standard basis of \mathbb{R}^2 .
- we will find eigenvalues and their eigenvectors.
- Eigenvectors will be basis of C for V .
- with the help of C we get $[T]_C$

Step 1 of 3

Let, $\beta = \{e_1, e_2\}$ be the standard basis of \mathbb{R}^2

$$T(e_1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T(e_2) = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$$

$$\text{Then, } [T]_\beta = \begin{bmatrix} 0 & -4 \\ 1 & 5 \end{bmatrix}$$

eigenvalues of $[T]_\beta$:

$$\begin{aligned} \det([T]_\beta - \lambda I) &= \det \begin{bmatrix} -\lambda & -4 \\ 1 & 5-\lambda \end{bmatrix} \\ &= \lambda^2 - 5\lambda + 4 \\ &= (\lambda - 4)(\lambda - 1) \end{aligned}$$

Eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = 4$

step 2 of 3

Eigenvectors:

Let,

$$v_1 = \begin{bmatrix} x \\ y \end{bmatrix}$$

Then, Eigenvector corresponding to eigenvalue 1 :

$$\begin{aligned} [T]_{\beta} v_1 &= 1 \cdot v_1 \\ \begin{bmatrix} -4y \\ x + 5y \end{bmatrix} &= \begin{bmatrix} x \\ y \end{bmatrix} \\ &\Rightarrow -4y = x \end{aligned}$$

 $v_1 = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$ be the eigenvector corresponding to eigenvalue 1.

Let,

$$v_2 = \begin{bmatrix} z \\ w \end{bmatrix}$$

Then, Eigenvector corresponding to eigenvalue 4 :

$$\begin{aligned} [T]_{\beta} v_2 &= 4 \cdot v_2 \\ \begin{bmatrix} -4w \\ z + 5w \end{bmatrix} &= \begin{bmatrix} 4z \\ 4w \end{bmatrix} \\ &\Rightarrow z = -w \end{aligned}$$

 $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ be the eigenvector corresponding to 4.

Step 3 of 3

$$C = \left\{ \begin{bmatrix} -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

Let, α, β be scalars such that

$$\alpha v_1 + \beta v_2 = 0$$

Then ,

$$\begin{bmatrix} -4\alpha \\ \alpha \end{bmatrix} + \begin{bmatrix} \beta \\ -\beta \end{bmatrix} = 0$$

Thus, $\alpha, \beta = 0$ Therefore C is linearly independent, T is diagonalizable,

$$[T]_C = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$

◆ Final Answer

$$C = \left\{ \begin{bmatrix} -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

$$[T]_C = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$

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