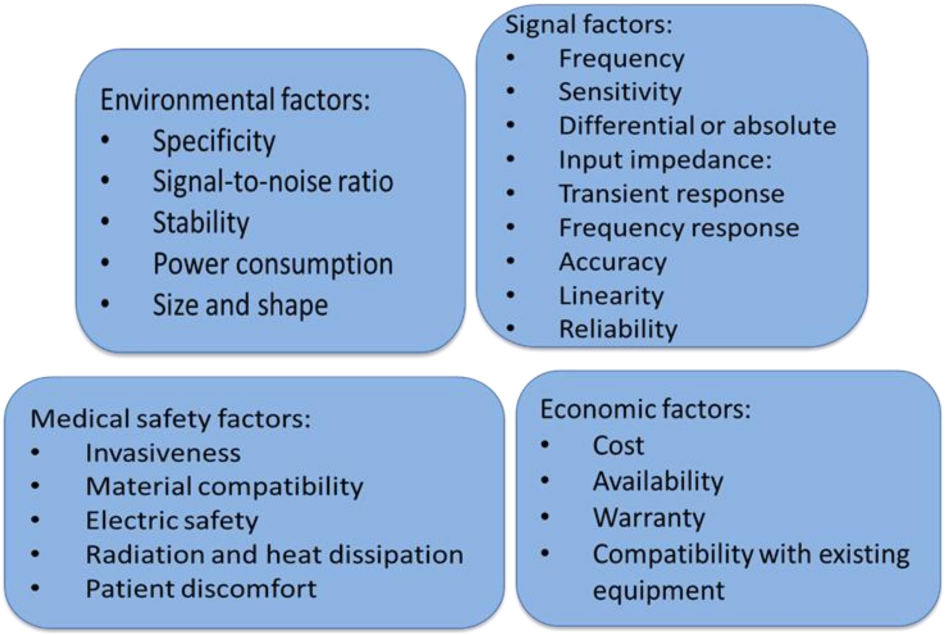
BIEN325: Homework #1 **Name:____Solutions____________**

1. **(From Lecture 1)** For each of the five categories of design criteria for a biomedical device (Environmental, Signal, Safety, Economics, Social), state one example that was not listed in the PowerPoint presentation.

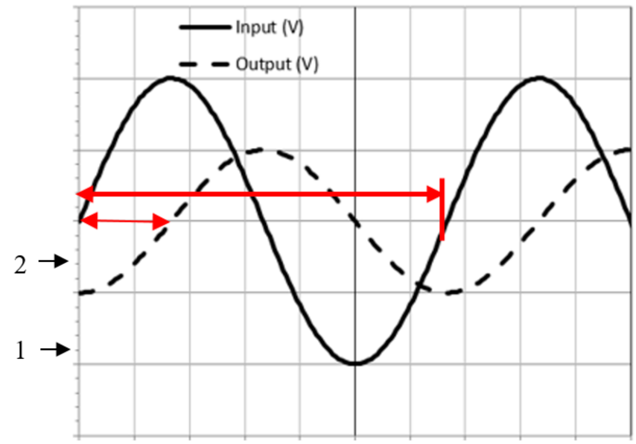
(Possible) Answer:

Signal: Quantitative or qualitative, ease of interpretation.
Environmental: Training of personnel, extent of use (e.g. expected hours of use per week).
Safety: Alarm(s), cleanability (to prevent contamination)
Economic factors: Training costs, storage, power consumption
Social factors: Disposal considerations, raw material sourcing.



2. **(From Lecture 1)** Explain why specificity is considered an "Environmental" factor, as opposed to a "Signal" factor.
Answer: Specificity is the ability to discern the signal of interest from signals emanating from other sources. The various other sources depend on the environment in which the device is being used.

3. **(From the Tutorial on Oscilloscopes)** Consider the oscilloscope trace shown in the figure below. The voltage gain for the input signal is 0.5 volts/division, and the voltage gain for the output signal is 0.1 volts/division. The time base is set on 0.2 msec/divison. The input is on Channel 1, and the output is on Channel 2.



a. What are the peak-to-peak voltages for the input and for the output?

Answer: For the input, it is 4 divisions×0.5 Volts/divison, which is 2 Volts. For the output it is 2 divisions×0.1 Volts/division, which is 0.2 Volts.

b. What is the gain for this system at this frequency?

Answer: Gain = $V_{\text{out}}/V_{\text{in:}} = (0.2 \text{ Volts})/(2 \text{ Volts}) = 0.1$.

c. What is the frequency of the two signals?

Answer: One period is about 6.7 divisions at 0.2 msec/divison, or T =1.34 msec. Frequency is $1/T = 1/(1.34 \text{ msec}) = 746$ Hz.

d. What is the phase lag in degrees between the input signal and the output signal?

Answer: The time lag is the short red double-arrow (1.7 divisions times 0.2 msec/div) and the period is the long red double-arrow (6.7 divisions times 0.2 msec/div). Since 0.2 msec/div is in both the numerator and denominator, it cancels out and we do not need to include it.

$$\phi = \frac{1.7}{6.7} 360 = 91.3 \text{ degrees}$$

e. What are the offsets of the two signals?

Answer: The input signal is centered on the middle horizontal line of the oscilloscope, which is 1.8 divisions above the arrow marked "1." The offset is therefore (1.8 divisions)(0.5 volts/division) = 0.9 volts. The output signal is centered on the same line, which is 0.6 divisions above the arrow marked "2." The offset is therefore (0.6 divisions)(0.1 volts/division) = 0.06 volts.

4. **(From Lecture 1, "Mean and RMS")** Consider the signal $s(t) = B + A\sin(\omega t)$.

a. Find the mean and RMS values of this signal, where T is exactly one period of the signal, i.e. $T = \frac{2\pi}{\omega}$. (For the RMS, you can use the integral $\int \sin^2(\omega t)\, dt = \frac{t}{2} - \frac{1}{4\omega}\sin(2\omega t)$.)

Answer: The mean is

$$\bar{s} = \frac{1}{T}\int_0^T B + \mathrm{A}\sin(\omega t)\ \ dt = \frac{B(T-0)}{T} + \frac{A}{T\omega}\cos(\omega t)|_0^T = B - \frac{A}{T\omega}(\cos(\omega T) - 1)$$

With $T = 2\pi/\omega$

$$\bar{s} = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} B + \mathrm{A}\sin(\omega t)\ \ dt = B - \frac{\mathrm{A}}{2\pi}(1-1)$$

$$\bar{s} = B - \frac{A}{2\pi}(\cos(2\pi) - A\cos(0)) = B - \frac{A}{2\pi}(1-1) = B$$

The root mean squared (RMS) value of a time signal over time T is defined as

$$s_{\mathrm{rms}} = \left(\frac{1}{T}\int_0^T (s(t) - \bar{s})^2\, dt\right)^{\frac{1}{2}}$$

Since $\bar{s} = B$, $s(t) - \bar{s} = A\sin(\omega t)$

$$s_{\mathrm{rms}} = \left(\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} A^2\sin^2(\omega t)\ \ dt\right)^{\frac{1}{2}}$$

With the given integral

$$s_{\mathrm{rms}} = \left(\frac{A^2\omega}{2\pi}\left(\frac{t}{2} - \underbrace{\frac{1}{4\omega}\sin(2\omega t)}_{\text{Goes to zero}}\right)\Bigg|_0^{\frac{2\pi}{\omega}}\right)^{\frac{1}{2}} = \left(\frac{A^2}{2}\right)^{\frac{1}{2}} = \frac{A}{\sqrt{2}}.$$

The sine term of the integral is zero because sin is 0 at both 0 and 4π.

b. Find the mean value of this signal, where T is one and one half period of the signal, i.e. $T = \frac{3\pi}{\omega}$. (For the RMS, you can use the integral $\int \sin^2(\omega t)\, dt = \frac{t}{2} - \frac{1}{4\omega}\sin(2\omega t)$.)

Answer: From the previous result for $\bar{s}$, we need to evaluate $B - \frac{A}{T\omega}(\cos(\omega T) - 1)$ at $T = 3\pi/\omega$.

$$\bar{s} = B - \frac{A}{T\omega}(\cos(\omega T) - 1)\Big|_{T=\frac{3\pi}{\omega}} = B - \frac{A}{3\pi}(-1-1) = B + \frac{2A}{3\pi}$$

The extra half cycle adds to the mean an amount a little over 1/5th the amplitude.

5. **(From Lecture 1, "Magnitude and Phase")** What are the magnitude and phase shift for a system that has the following transfer function

$$\frac{V_{\text{out}}}{V_{\text{in}}} = K\frac{\tau_1 j\omega}{(\tau_1 j\omega + 1)}$$

Answer:

$$\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = |K|\frac{|\tau_1 j\omega|}{|\tau_1 j\omega + 1|} = K\frac{\tau_1 \omega}{\sqrt{\tau_1^2\omega^2 + 1^2}}$$

$$\angle\left[K\frac{\tau_1 j\omega}{(\tau_1 j\omega + 1)}\right] = \angle K + \angle \tau_1 j\omega - \angle(\tau_1 j\omega + 1)$$

For a complex number, the phase angle is $\text{atan}\left(\frac{\Im}{\Re}\right)$, where $\Im$ is the imaginary part and $\Re$ is the real part.

$$\angle\left[K\frac{\tau_1 j\omega}{(\tau_1 j\omega + 1)}\right] = \text{atan}\frac{0}{K} + \text{atan}\left(\frac{\tau_1\omega}{0}\right) - \text{atan}\left(\frac{\tau_1\omega}{1}\right)$$

$$= 0° + 90° - \text{atan}(\tau_1\omega)$$

6. **(From Lecture 1, "Bode Plot")** Find expressions for the magnitude and phase of the following transfer function, and use Excel to plot these quantities on a Bode plot, with frequency ranging from 0.1 to 10000 Hz. (Use separate plots, with the magnitude plot above the phase plot.)$\tau_1 j\omega$

$$T(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{s+3}{s^2 + 10s + 100}$$

Answer:

First, set $s = j\omega$.

$$T(j\omega) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{j\omega + 3}{(j\omega)^2 + 10j\omega + 100}$$

Take the magnitude

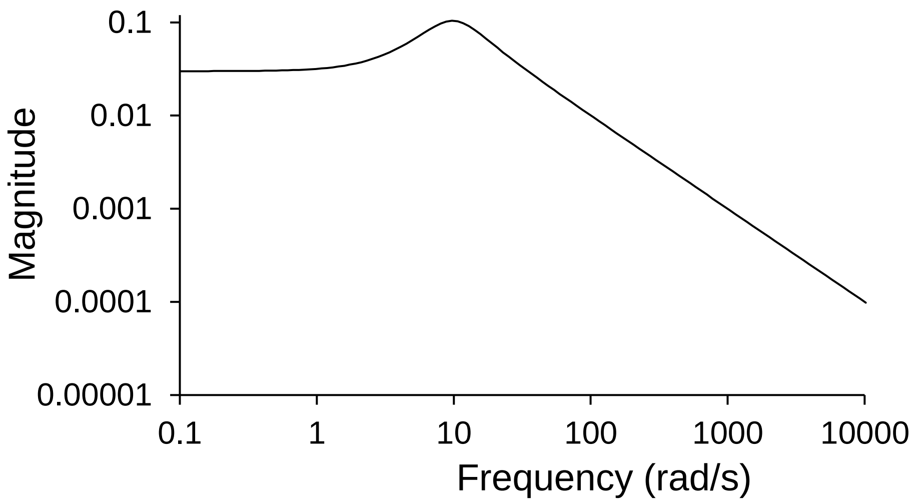
$$|T(j\omega)| = \frac{\sqrt{\omega^2 + 3^2}}{\sqrt{(100 - \omega^2)^2 + 10^2\omega^2}}$$

Notice that $(j\omega)^2$ in the denominator became $j^2\omega^2 = -1\omega^2$, which is real and therefore is combined with the other real term, 100.

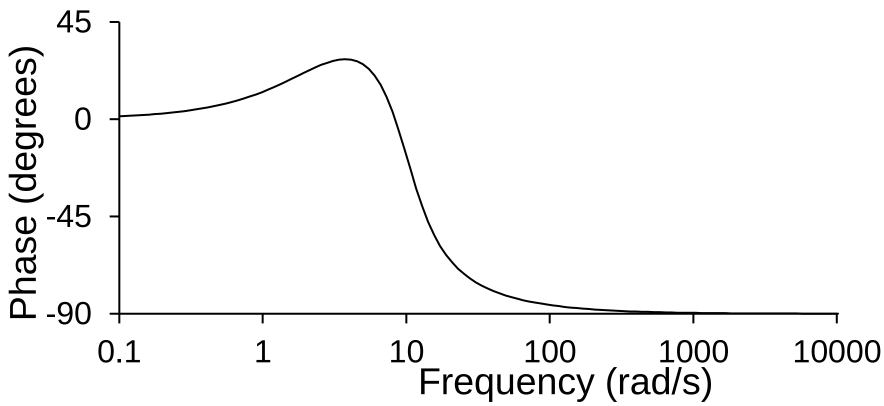
Take the phase

$$\angle T(j\omega) = \text{atan}\left(\frac{\omega}{3}\right) - \text{atan}\left(\frac{10\omega}{100 - \omega^2}\right)$$

The Bode plot is



Magnitude
0.1
0.01
0.001
0.0001
0.00001
0.1
1
10
100
1000
10000
Frequency (rad/s)



Phase (degrees)
45
0
-45
-90
0.1
1
10
100
1000
10000
Frequency (rad/s)