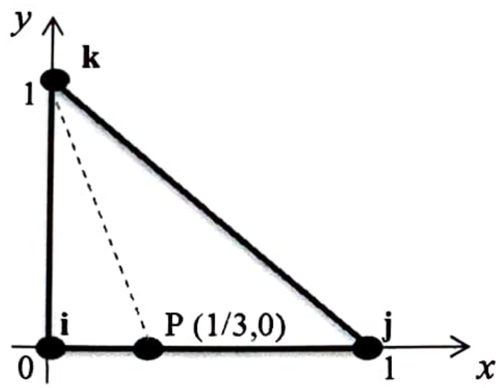
Math 574 Final 5/21/20

NAME:____________________

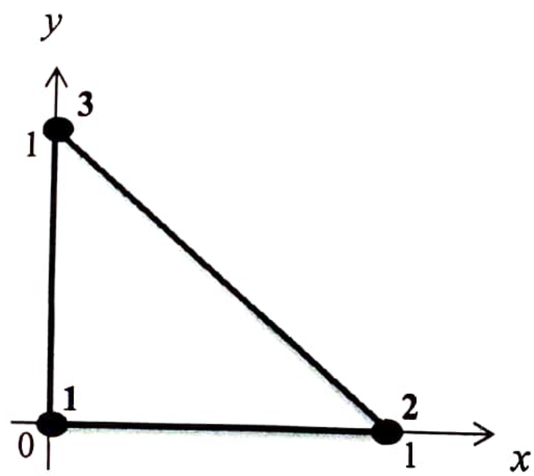
ok 1 (6 pts). A right triangular element is shown in Fig. 1, where node P is at $\frac{1}{3}$ length of $\boldsymbol{l_{ij}}$ from node $\boldsymbol{i}$. Evaluate $\boldsymbol{L_i(P), L_j(P), L_k(P)}$.

Fig. 1



ok 2 (24 pts). A triangle is given in Fig. 2. Calculate $\boldsymbol{K} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$, where $k_{ij} = \frac{1}{4S}(b_i b_j + c_i c_j)$, i,j=1,2,3, and $b_1 = y_2 - y_3, b_2 = y_3 - y_1, b_3 = y_1 - y_2, c_1 = x_3 - x_2, c_2 = x_1 - x_3, c_3 = x_2 - x_1$.

Fig. 2



3 (20 pts). Node **11** is located at a rectangular region as shown in Fig. 3.
Choose linear basis function as

$$\varphi_{ij}(x,y)=\begin{cases}\left(1-\dfrac{|x-x_i|}{\Delta x}\right)\left(1-\dfrac{|y-y_j|}{\Delta y}\right), & (x,y)\in R_\mu,\\[2ex] 0, & \text{Others}\end{cases}$$

Evaluate $\iint_R \left[\frac{\partial \varphi_{11}}{\partial x}\frac{\partial \varphi_{11}}{\partial x}+\frac{\partial \varphi_{11}}{\partial y}\frac{\partial \varphi_{11}}{\partial y}\right] dxdy.$

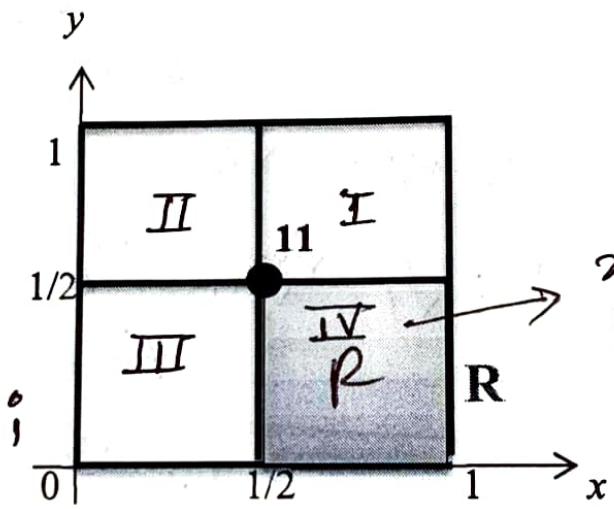


Fig. 3

$x > x_i$
$y < y_j$

Zone R limit

$x : x_i \to x_{i+1}$

$y : y_{j-1} \to y_j$

4 (50 pts). Consider the problem

$$-2u'' + u = 1,\ 0 < x < 1,$$
$$u(0) = 2, u(1) = 0.$$

Let $x_i = ih, i = 0, 1, 2, 3$, and $h = \frac{1}{3}$. Choose linear shape functions as basis functions:

$$\varphi_i(x) = \begin{cases} 1 + \dfrac{x - x_i}{h}, & x_{i-1} \le x \le x_i, \\ 1 - \dfrac{x - x_i}{h}, & x_i \le x \le x_{i+1}, \\ 0, & \text{Others} \end{cases} \quad i = 1, 2, \text{ and } \varphi_0 = \begin{cases} 1 - \frac{x - x_0}{h}, & x_0 \le x \le x_1, \\ 0, & \text{others.} \end{cases}$$

Use the Galerkin method $\sum_{i=1}^{n-1} a(\varphi_i, \varphi_j) u_j = (f, \varphi_j) - 2a(\varphi_0, \varphi_j)$ to find u_1, u_2 and the solution $u(x)$. (Choose four digits after decimal point if rational numbers are not used)