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**Exercise 27, Linear Algebra: A Modern Introduction, 4th Edition**

Solved by experts



NEXT QUESTION

**Exercise 27 Answer****Step by step explanation**

HIDE ALL

Tip

- D is the differential operator.
- Integration is the antiderivative that is D^{-1} .

Explanation

- We will first take the β as basis of W .
- With the help of basis we will find out $[D]_{\beta}$.
- By theorem 6.28 we will find out inverse of $[D]_{\beta}$.

- With the help of method of theorem 6.26, we get the desired integral.

Step 1 of 2

Let $W = \text{Span}(\sin x, \cos x)$ be the subspace of D .

Then, $\beta = \{\sin x, \cos x\}$ is the basis of W .

$$D(\sin x) = \cos x$$

$$D(\cos x) = -\sin x$$

Then,

$$[D(\sin x)]_{\beta} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$[D(\cos x)]_{\beta} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

We get ,

$$[D]_{\beta} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

By theorem 6.28, linear transformation D is invertible.

$$\begin{aligned} [D^{-1}]_{\beta} &= ([D]_{\beta})^{-1} \\ &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \end{aligned}$$

Step 2 of 2

D^{-1} is the matrix of integration on W .

$$[\sin x - 3\cos x]_{\beta} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

By theorem 6.26, we have

$$\begin{aligned} \left[\int (\sin x - 3\cos x) dx \right]_{\beta} &= [D^{-1}(\sin x - 3\cos x)]_{\beta} \\ &= [D^{-1}]_{\beta} [(\sin x - 3\cos x)]_{\beta} \\ &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix} \\ &= \begin{bmatrix} -3 \\ -1 \end{bmatrix} \end{aligned}$$

$$\therefore \int (\sin x - 3\cos x) dx = -3\sin x - \cos x + C$$