Experimental Design

Part 3: Statistical Design of Experiments

Today’s Quote

“Learning without thinking is useless. Thinking without learning is dangerous.”

— Confucius

Statistical Design of Experiments

- Statistical Design of Experiments
  - Broad field – you could take multiple classes in this
  - Today: a brief overview
- In any general process or system
  - Input
  - Controllable factors
  - Uncontrollable factors
  - Outputs
- Goals
  - ____
  - _________________________________

Statistical Design of Experiments

- Approaches
  - ***<u>Best Guess Approach</u>***
    - Can work with great deal of knowledge about system or if you get very lucky
    - Disadvantages
      - May require many “best guesses” if at first you don’t succeed
      - May get acceptable result, but not likely ______________________
  - ***<u>One Factor at a Time Approach</u>***
    - Set baseline levels for each controllable factor and vary one at a time
    - Problem – ___________________________________
  - ***<u>Factorial Approach</u>***
    - Factors varied together, systematically

Statistical Design of Experiments

- Usually abbreviated as “DoE”
- Typical applications of DoE
  - Screen to determine important factors in a process
  - Process development
  - Process optimization
- Limitations of DoE approach
  - ____
    ____

Basic Principles of DoE

- ***Replication***
  - Repetition of basic experiment
  - Necessary for ______________________________
- ***Randomization***
  - Allocation of experimental material and order of experiments performed randomly
- ***Blocking***
  - A portion of experimental material that should be more homogeneous than the entire set of material
  - [E.g., comparing the effect of two diets on N persons – Block 1: All females, Block 2: All males; Block 1 + Block 2 = N.]

Factorial Design

- Typically, most efficient for effect of 2 or more factors on output variable
- Complete trial or replicant of experiment at all levels of the factors
- Example: 2 factors *A* and *B*
    - *a* levels of A and *b* levels of B
    - *ab* total treatment levels
    - Factors said to be crossed
    - For 2 factors at 2 levels (high +; low -) 2^2 factorial

EXPERIMENT	A	B
1	-	-
2	+	-
3	-	+
4	+	+

Factorial Design

- Factorial design leads to a regression model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon$$

  - Parameters to be determined are β's
    - *Note the interaction* β_{12}
      - Interaction occurs when the effect of one variable depends on the level of another variable
  - x_1 represents *A*, x_2 represents *B*
  - Random, unexplained error is ε
  - Model accuracy from ____________________
    - Residual is difference between model prediction and measured data point
    - Other statistical measures of error can also be calculated

Sample Size

- How many replicants?
  - As # or replicants increases, likelihood of error decreases
  - Decrease in error probability is nonlinear
  - Dependent on
    - _________________
    - _______________________
    - ______________________________

Blocking

Blocking provides some advantages although experiments may be designed to be randomized.

- Example
  - Clinical trials for a vaccine on 10,000 randomly selected volunteers
  - Separate total sample into two blocks of equal size – male and female
  - Each becomes a block; choose members randomly within each block
  - Advantage: May provide data on whether vaccine efficacy is the same for males and females.

- Other common blocks
  - Time – e.g., run N experiment per day for M days – how do the results change (or do they)?
  - Individuals – e.g., you run one set (replicant), someone else runs another set –do results change?

- Regression model with blocking term

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \delta + \varepsilon$$

  - Residual error associated with the blocking is δ

See https://en.wikipedia.org/wiki/Blocking_(statistics)

Blocking: Example

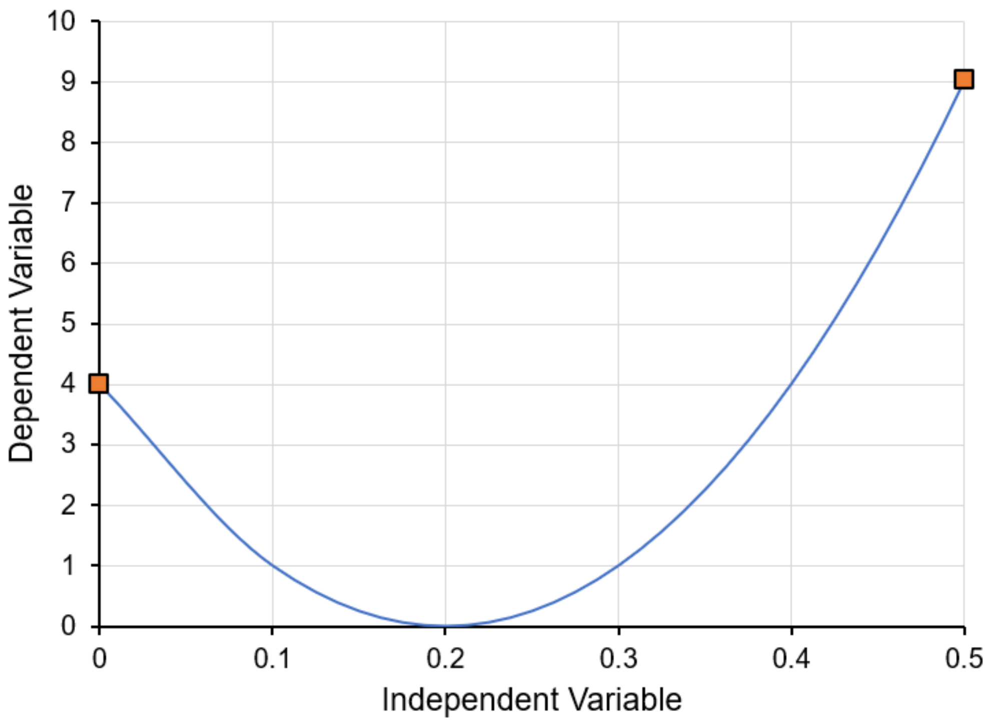
- The results from a randomized experiment conducted at the American Economic Review on the effects of double-blind versus single-blind peer reviewing on acceptance rates and referee rating indicate that **acceptance rates are lower and referees are more critical** **when the reviewer is unaware of the author's identity**.

- These patterns are not significantly different between female and male authors. *(What are the blocks?)*

- Authors at top-ranked universities and at colleges and low-ranked universities are largely unaffected by the different reviewing practices. *(What are the blocks?)*

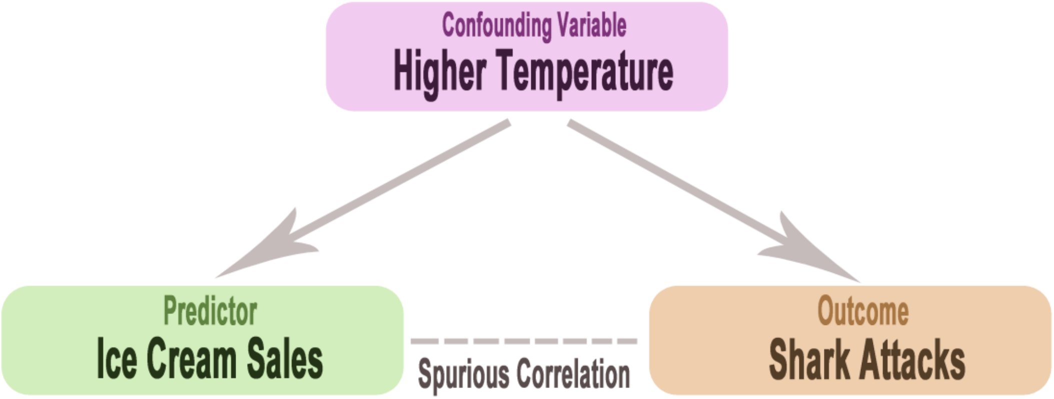
- The authors at near-top-ranked universities and at nonacademic institutions have lower acceptance rates under double-blind reviewing. *(What are the blocks?)*

Design Considerations



- ***<u>Curvature</u>***
    - Linearity of factor effects assumed in 2^k design
    - *<u>Add center points – high, medium, low</u>*
        - Check for curvature
        - Minimize additional experiments

- ***<u>Confounding variables</u>***
    - Observed correlation between A and B could be caused by a confounding variable C



Statistical Design of Experiments

- **Full factorial design**
  - 2^2 design: 4 experiments per replicate
  - 2^6 design: 64 experiments per replicate
  - ***<u>Numbers increase rapidly</u>***

- **Fractional factorial Design**
  - Assume main effects and low order interactions most important
  - Design such that fraction can be projected onto specific subset of factors
  - Design such that combination of runs of 2 or more fractions can be assembled into a larger design

Useful as ____

Statistical Design of Experiments

- **Conclusions**
  - Several software packages available to set up and analyze statistically designed experiments
  - Not widely done in academia – little fundamental insights
  - Extremely important in industry
  - You may want to consider taking a class in this area if you haven't already!

- **For further reading:**
  - Douglas C. Montgomery, *Design and Analysis of Experiments, 10th ed*., John Wiley & Sons, Inc., New York, 2020
  - Robert L. Mason, Richard F. Gunst, and James L. Hess, *Statistical Design and Analysis of Experiments - With Applications to Engineering and Science 2nd ed.,* John Wiley & Sons, Inc., 2003 (Available through Knovel)

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