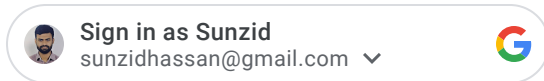




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Exercise 33, Linear Algebra: A Modern Introduction, 4th Edition

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Linear Algebra: A Modern Introduction, 4th Edition, Chapter 6: Vector Spaces

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Exercise 33 Answer

Step by step explanation

HIDE ALL

Tip



- In this type of questions, we need to find eigenvectors and eigenvalues.

Explanation



- We will take β as the standard basis.
- We will then find eigenvalues of $[T]_{\beta}$
- After eigenvalues we will find eigenvector corresponding to eigenvalues.

- We will find that \mathcal{C} is linearly independent. Therefore, $[T]_{\mathcal{C}}$ exists.

Step 1 of 3

Let, $\beta = \{1, x\}$ be the standard basis of P_1

Then,

$$T(1) = 4 + x$$

$$T(x) = 2 + 3x$$

Thus,

$$[T]_{\beta} = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

Eigenvectors of $[T]_{\beta}$:

$$\begin{aligned} \det([T]_{\beta} - \lambda I) &= \det\left(\begin{bmatrix} 4 - \lambda & 2 \\ 1 & 3 - \lambda \end{bmatrix}\right) \\ &= \lambda^2 - 7\lambda + 10 \\ &= (\lambda - 5)(\lambda - 2) \end{aligned}$$

The eigenvector of $[T]_{\beta}$ is 5 and 2.

Step 2 of 3

Eigenvector corresponding to eigenvalue 2:

Let,

$v_1 = \begin{bmatrix} x \\ y \end{bmatrix}$ be the vector such that

$$\begin{aligned} [T]_{\beta} v_1 &= 2 \cdot v_1 \\ \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 2x \\ 2y \end{bmatrix} \\ \begin{bmatrix} 4x + 2y \\ x + 3y \end{bmatrix} &= \begin{bmatrix} 2x \\ 2y \end{bmatrix} \end{aligned}$$

$$x = -y$$

Thus,

$$v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ is an eigenvector corresponding to the eigenvalue 2.}$$

Eigenvector corresponding to the eigenvalue 5:

Let,

$v_2 = \begin{bmatrix} x \\ y \end{bmatrix}$ be vector such that,

$$\begin{aligned} [T]_{\beta} v_2 &= 5 \cdot v_2 \\ \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 5x \\ 5y \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} 4x + 2y \\ x + 3y \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

$$x = 2y$$

Thus,

$v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is an eigenvector corresponding to the eigenvalue 5.

Step 3 of 3

We get, $V_1 = [1 - x]_{\beta}$

$$v_2 = [2 + x]_{\beta}$$

Then, $C = \{1 - x, 2 + x\}$

Let, α, β be scalar such that

$$\alpha(1 - x) + \beta(2 + x) = 0$$

$$\alpha, \beta = 0$$

Therefore C is linearly independent.

It implies that T is diagonalizable and

$$[T]_c = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$$

◆ Final Answer

$$C = \{1 - x, 2 + x\}$$

$$[T]_c = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$$

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