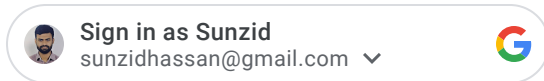




Sign up with:



SIGN UP WITH APPLE

SIGN UP WITH FACEBOOK

By creating an account, you accept the **Brainly Terms of Service** & **Privacy Policy**.

ASK QUESTION

LOG IN

JOIN FOR FREE



**Textbook Solutions** > Linear Algebra: A Modern Introducti... > Chapter 6: Vector Spaces > Exercise 28



## Exercise 28. Linear Algebra: A Modern Introduction, 4th Edition



NEXT QUESTION



## Exercise 28 Answer

### Step by step explanation

HIDE ALL

#### Tip



- $D$  is the differential operator.
- $D^{-1}$  is the anti-differentiation that is integral.

#### Explanation



- We will take  $\beta$  as basis of  $W$ .
- With the help of basis we will find  $[D]_{\beta}$ .
- By using theorem 6.28, we get inverse of  $[D]_{\beta}$ .

- With the help of method of theorem 6.26 we get the desired integral.

### step 1 of 2

Let  $W = \text{Span}(e^{2x}, e^{-2x})$  be the subspace of  $D$ .

Then,  $\beta = \{e^{2x}, e^{-2x}\}$  is basis of  $W$ .

$$[D(e^{2x})]_{\beta} = 2e^{2x}$$

$$[D(e^{-2x})]_{\beta} = -2e^{-2x}$$

Then,

$$[D(e^{2x})]_{\beta} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$[D(e^{-2x})]_{\beta} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

therefore,

$$[D]_{\beta} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

By theorem 6.28, linear transformation  $D$  is invertible.

$$[D^{-1}]_{\beta} = ([D]_{\beta})^{-1}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$$

### Step 2 of 2

$D^{-1}$  is the integration on  $W$ .

$$[5e^{-2x}]_{\beta} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

By theorem 6.26,

$$\begin{aligned} \left[ \int (5e^{-2x}) dx \right]_{\beta} &= [D^{-1}(5e^{-2x})]_{\beta} \\ &= [D^{-1}]_{\beta} [(5e^{-2x})]_{\beta} \\ &= \begin{bmatrix} 1/2 & 0 \\ 0 & -1/2 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ -5/2 \end{bmatrix} \end{aligned}$$

Therefore,

$$\int (5e^{-2x}) dx = -\frac{5}{2} e^{-2x} + C$$