Exercise 28, Linear Algebra: A Modern Introduction, 4th Edition

Solved by experts

Exercise 28 Answer

Step by step explanation

HIDE ALL

Tip

- D is the differential operator.
- D^{-1} is the anti-differentiation that is integral.

Explanation

- We will take β as basis of W.
- With the help of basis we will find $[D]_\beta$.
- By using theorem 6.28 , we get inverse of $[D]_\beta$.

- With the help of method of theorem 6.26 we get the desired integral.

step 1 of 2

Let W $= \text{Span}(e^{2x}, e^{-2x})$ be the subspace of D.
Then, $\beta = \{e^{2x}, e^{-2x}\}$ is basis of W.

$$[D(e^{2x})]_\beta = 2e^{2x}$$

$$[D(e^{-2x})]_\beta = -2e^{-2x}$$

Then,

$$[D(e^{2x})]_\beta = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$[D(e^{-2x})]_\beta = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

therefore,

$$[D]_\beta = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

By theorem 6.28 , linear transformation D is invertible.

$$[D^{-1}]_\beta = ([D]_\beta)^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{-1}{2} \end{bmatrix}$$

Step 2 of 2

D^{-1} is the integration on W.

$$[5e^{-2x}]_\beta = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

By theorem 6.26,

$$[\int (5e^{-2x})dx]_\beta = [D^{-1}(5e^{-2x})]_\beta = [D^{-1}]_\beta[(5e^{-2x})]_\beta = \begin{bmatrix} 1/2 & 0 \\ 0 & -1/2 \end{bmatrix}\begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-5}{2} \end{bmatrix}$$

Therefore,

$$\int (5e^{-2x})dx = \frac{-5}{2}e^{-2x} + C$$