Exercise 29, Linear Algebra: A Modern Introduction, 4th Edition

Solved by experts

Linear Algebra: A Modern Introduction, 4th Edition, Chapter 6: Vector Spaces

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Exercise 29 Answer

Step by step explanation

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Tip

- D is differential operator.
- D^{-1} is anti-differentiation that is integral.

Explanation

- We will take W = span($e^{2x}\cos x$, $e^{2x}\sin x$) be the subspace of D.
- β = $\left\{e^{2x}\cos x\right.$, $\left.e^{2x}\sin x\right\}$ be the basis of W.

- With the help of basis we will find $[D]_\beta$.
- By the use of theorem 6.28, we get $[D]_\beta^{-1}$.
- With the help of theorem 6.26 , we get the desired integral

Step 1 of 2

Let , W = span$(e^{2x}\cos x$, $e^{2x}\sin x)$ be the subspace of D

$\beta = \{e^{2x}\cos x , e^{2x}\sin x\}$ is basis of W.

$D(e^{2x}\cos x) = 2e^{2x}\cos x - e^{2x}\sin x$

$D(e^{2x}\sin x) = 2e^{2x}\sin x + e^{2x}\cos x$

Thus,

$[D(e^{2x}\cos x)]_\beta = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$[D(e^{2x}\sin x)]_\beta = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Then,

$$[D]_\beta = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$$

By theorem 6.28, linear transformation D is invertible

$$[D^{-1}]_\beta = ([D]_\beta)^{-1}$$
$$= \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}^{-1}$$
$$= \begin{bmatrix} \frac{2}{5} & \frac{-1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

Step 2 of 2

$[e^{2x}\cos x - 2e^{2x}\sin x]_\beta = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Then, By theorem 6.26,

$$[\int (e^{2x}\cos x - 2e^{2x}\sin x)dx]_\beta = [D^{-1}(e^{2x}\cos x - 2e^{2x}\sin x)]_\beta$$
$$= [D^{-1}]_\beta[(e^{2x}\cos x - 2e^{2x}\sin x)]_\beta$$
$$= \begin{bmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{4}{5} \\ \frac{-3}{5} \end{bmatrix}$$

$$\int (e^{2x} \cos x - 2e^{2x} \sin x)dx = \frac{4}{5}e^{2x} \cos - \frac{3}{5}e^{2x} \sin x + C$$

Final Answer

$$\int (e^{2x} \cos x - 2e^{2x} \sin x)dx = \frac{4}{5}e^{2x} \cos - \frac{3}{5}e^{2x} \sin x + C$$