Solved by experts

Exercise 27, Linear Algebra: A Modern Introduction, 4th Edition

NEXT QUESTION

Exercise 27 Answer

Step by step explanation

HIDE ALL

Tip

- D is the differential operator.
- Integration is the antiderivative that is D^{-1} .

Explanation

- We will first take the β as basis of W .
- With the help of basis we will find out $[D]_{\beta}$.
- By theorem 6.28 we will find out inverse of $[D]_{\beta}$.

- With the help of method of theorem 6.26, we get the desired integral.

Step 1 of 2

Let W = Span(Sinx , Cosx) be the subspace of D.

Then, $\beta = \{\sin x, \cos x\}$ is the basis of W.

$D(\sin x) = \cos x$

$D(\cos x) = -\sin x$

Then,

$$[D(\sin x)]_\beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$[D(\cos x)]_\beta = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

We get ,

$$[D]_\beta = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

By theorem 6.28, linear transformation D is invertible.

$$[D^{-1}]_\beta = ([D]_\beta)^{-1}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Step 2 of 2

D^{-1} is the matrix of integration on W.

$$[\sin x - 3\cos x]_\beta = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

By theorem 6.26, we have

$$\left[\int (\sin x - 3\cos x)dx\right]_\beta = [D^{-1}(\sin x - 3\cos x)]_\beta$$

$$= [D^{-1}]_\beta[(\sin x - 3\cos x)]_\beta$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ -1 \end{bmatrix}$$

$$\therefore \int (\sin x - 3\cos x)dx = -3\sin x - \cos x + C$$