Exercise 30, Linear Algebra: A Modern Introduction, 4th Edition

Solved by experts

Linear Algebra: A Modern Introduction, 4th Edition, Chapter 6: Vector Spaces

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Exercise 30 Answer

Step by step explanation

HIDE ALL

Tip

- D is the differentiation operator.
- D^{-1} is the integral as integral is anti-differentiation.

Explanation

- We will take W = $\operatorname{span}(\cos x, \sin x, x\cos x, x\sin x)$ as subspace of D.
- Consider β as basis of W.

- We will find out $[D]_\beta$
- By theorem 6.28 , we get $[D]_\beta^{-1}$
- By the method of theorem 6.26 we get the integral.

step 1 of 2

Let, W = span$(\cos x, \sin x, x\cos x, x\sin x)$ be the subspace of D.
$\beta = \{\cos x, \sin x, x\cos x, x\sin x\}$ is basis of W.
$D(\cos x) = -\sin x$
$D(\sin x) = \cos x$
$D(x\cos x) = -x\sin x + \cos x$
$D(x\sin x) = x\cos x + \sin x$
Thus,

$$[D(\cos x)]_\beta = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$[D(\sin x)]_\beta = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[D(x\cos x)]_\beta = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$[D(x\sin x)]_\beta = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Then,

$$[D]_\beta = \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

By theorem 6.28, linear transformation D is invertible

$$[D^{-1}]_\beta = ([D]_\beta)^{-1}$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Step 2 of 2

$[x\cos x + x\sin x]_\beta = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

By theorem 6.26,

$[\int (x\cos x + x\sin x)dx]_\beta = [D^{-1}(x\cos x + x\sin x)]_\beta$

$= [D^{-1}]_\beta[(x\cos x + x\sin x)]_\beta$

$$= \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

therefore,

$\int (x \cos x + x\sin x)dx = \cos x + \sin x - x \cos x + x\sin x + C$

Final Answer

$\int (x \cos x + x\sin x)dx = \cos x + \sin x - x \cos x + x\sin x + C$

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