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**Exercise 30, Linear Algebra: A Modern Introduction, 4th Edition**

NEXT QUESTION



Linear Algebra: A Modern Introduction, 4th Edition, Chapter 6: Vector Spaces

**Exercise 30, Page 514****Exercise 30 Answer****Step by step explanation**

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**Tip**

- $D$  is the differentiation operator.
- $D^{-1}$  is the integral as integral is anti-differentiation.

**Explanation**

- We will take  $W = \text{span}(\cos x, \sin x, x \cos x, x \sin x)$  as subspace of  $D$ .
- Consider  $\beta$  as basis of  $W$ .

- We will find out  $[D]_{\beta}$
- By theorem 6.28 , we get  $[D]_{\beta}^{-1}$
- By the method of theorem 6.26 we get the integral.

### step 1 of 2



Let,  $W = \text{span}(\cos x, \sin x, x \cos x, x \sin x)$  be the subspace of  $D$ .

$\beta = \{\cos x, \sin x, x \cos x, x \sin x\}$  is basis of  $W$ .

$$D(\cos x) = -\sin x$$

$$D(\sin x) = \cos x$$

$$D(x \cos x) = -x \sin x + \cos x$$

$$D(x \sin x) = x \cos x + \sin x$$

Thus,

$$[D(\cos x)]_{\beta} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$[D(\sin x)]_{\beta} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[D(x \cos x)]_{\beta} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$[D(x \sin x)]_{\beta} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Then,

$$[D]_{\beta} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

By theorem 6.28, linear transformation  $D$  is invertible

$$\begin{aligned} [D^{-1}]_{\beta} &= ([D]_{\beta})^{-1} \\ &= \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}^{-1} \end{aligned}$$

Step 2 of 2

$$= \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



$$[x \cos x + x \sin x]_{\beta} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

By theorem 6.26,

$$\begin{aligned} \left[ \int (x \cos x + x \sin x) dx \right]_{\beta} &= [D^{-1}(x \cos x + x \sin x)]_{\beta} \\ &= [D^{-1}]_{\beta} [x \cos x + x \sin x]_{\beta} \\ &= \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} \end{aligned}$$

therefore,

$$\int (x \cos x + x \sin x) dx = \cos x + \sin x - x \cos x + x \sin x + C$$

### ◆ Final Answer



$$\int (x \cos x + x \sin x) dx = \cos x + \sin x - x \cos x + x \sin x + C$$

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