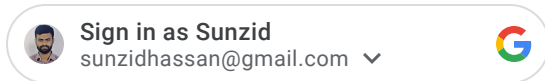




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Exercise 34, Linear Algebra: A Modern Introduction, 4th Edition

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Linear Algebra: A Modern Introduction, 4th Edition, Chapter 6: Vector Spaces

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Exercise 34 Answer

Step by step explanation

HIDE ALL

Tip



- In this question, we need to find eigenvalues and eigenvectors.

Explanation



- We will take β as the standard basis.
- We will find eigenvalues and eigenvectors.

- As geometric and algebraic multiplicity of eigenvalue is different. Basis C does not exist.

Step 1 of 2

Let, $\beta = \{1, x, x^2\}$ be the standard basis of P_2 .

Then,

$$T(1) = 1$$

$$T(x) = 1 + x$$

$$T(x^2) = (1 + x)^2$$

Thus,

$$[T]_{\beta} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Eigenvalues of $[T]_{\beta}$:

$$\begin{aligned} \det([T]_{\beta} - \lambda I) &= \det \begin{bmatrix} 1-\lambda & 1 & 1 \\ 0 & 1-\lambda & 2 \\ 0 & 0 & 1-\lambda \end{bmatrix} \\ &= (1-\lambda)^3 \end{aligned}$$

Eigenvalue is 1 with multiplicity 3.

Step 2 of 2

Eigenvector corresponding to eigenvalue 1:

Let,

$$v_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ be the vector such that}$$

$$[T]_{\beta} v_1 = 1 \cdot v_1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} a + b + c \\ b + 2c \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$a \in \mathbb{R}, b, c = 0$$

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ be the eigenvector corresponding to eigenvalue 1.}$$

Geometric multiplicity of $\lambda = 1$ is 1 and algebraic multiplicity is 3.

Therefore, $[T]_{\beta}$ is not diagonalizable and there is no basis C such that T is diagonalizable.