# Exercise 34, Linear Algebra: A Modern Introduction, 4th Edition

Solved by experts



Linear Algebra: A Modern Introduction, 4th Edition, Chapter 6: Vector Spaces

**Exercise 34, Page 514**

## Exercise 34 Answer

### Step by step explanation

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#### Tip

- In this question, we need to find eigenvalues and eigenvectors.

#### Explanation

- We will take $\beta$ as the standard basis.
- We will find eigenvalues and eigenvectors.

- As geometric and algebraic multiplicity of eigenvalue is different. Basis C does not exist.

**Step 1 of 2**

Let, $\beta$ = $\{1, x, x, ^2\}$ be the standard basis of $P_2$.
Then,

$T(1)$ = 1
$T(x) = 1 +$ x
$T(x^2) = (1 + x)^2$

Thus,

$$[T]_\beta = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Eigenvalues of $[T]_\beta$ :

$$\det([T]_\beta - \lambda I) = \det(\begin{bmatrix} 1-\lambda & 1 & 1 \\ 0 & 1-\lambda & 2 \\ 0 & 0 & 1-\lambda \end{bmatrix}$$

$$= (1-\lambda)^3$$

Eigenvalue is 1 with multiplicity 3.

**Step 2 of 2**

Eigenvector corresponding to eigenvalue 1 :
Let,

$v_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ be the vector such that

$$[T]_\beta v_1 = 1.v_1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} a+b+c \\ b+2c \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

a $\in \mathbb{R}$ , b,c = 0

$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be the eigenvector corresponding to eigenvalue 1.

Geometric multiplicity of $\lambda$ = 1 is 1 and algebraic multiplicity is 3.
Therefore, $[T]_\beta$ is not diagonalizable and there is no basis C such that T is diagonalizable.