Lesson 3: Trees

CSC325 – ADVANCED DATA STRUCTURES & ALGORITHMS | SPRING 2022

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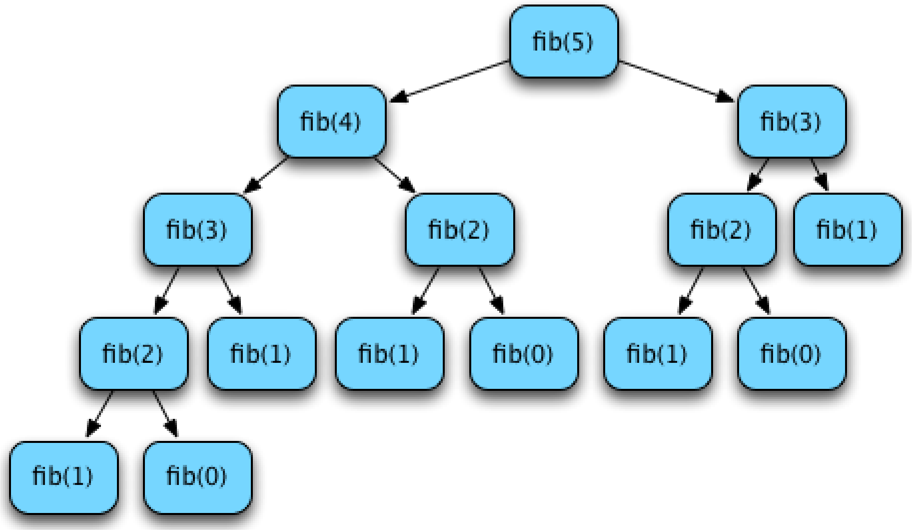
OUTLINE

- Introduction.
- Binary search trees.
- Search space.
- AVL trees.
- Splay trees.

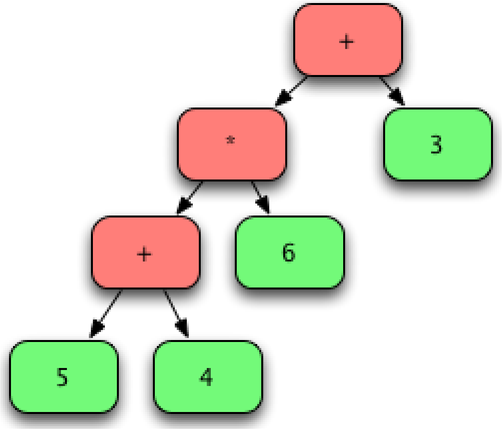
INTRODUCTION

- **Trees** data structure.
  - **Non-sequential**.
  - Data stored in **nodes** with **links** to **two** or **more** other nodes.
    - Root, parents, children, siblings, subtrees.
  - **Abstract Syntax Trees (ASTs)** used by programming languages to convert source code to a tree-like structure for evaluation.



Fib(5) call Tree

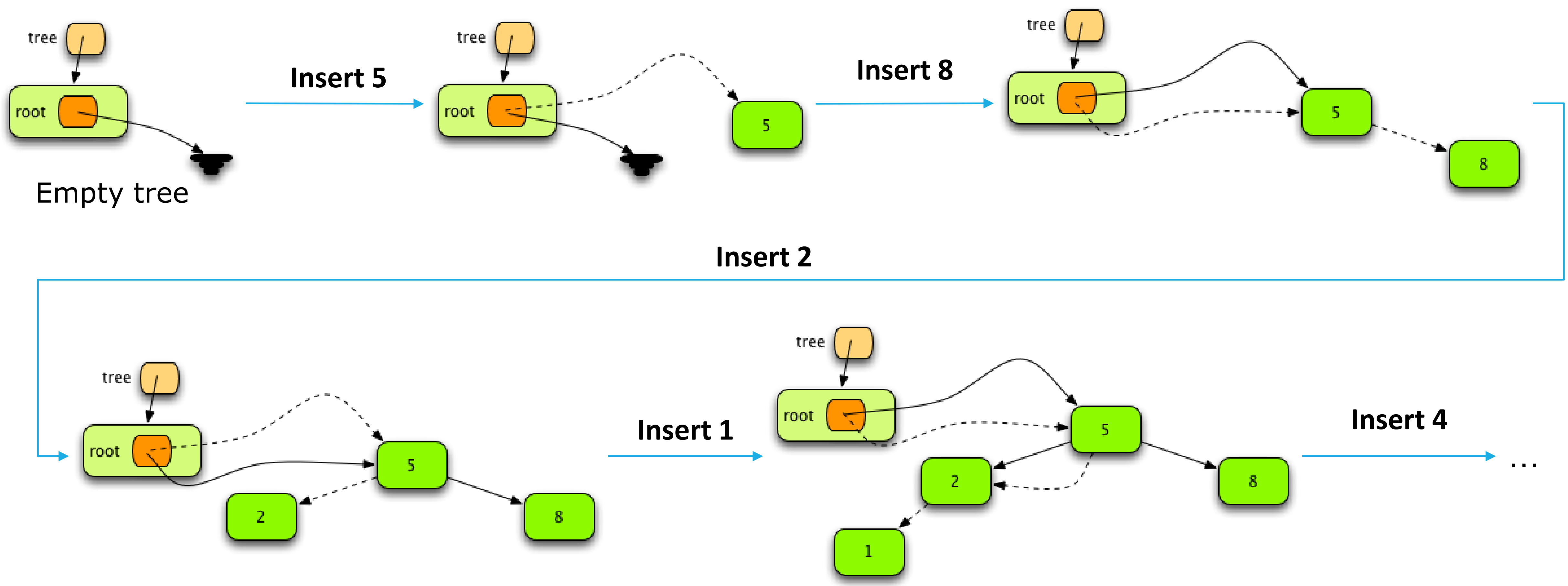
- **Binary search trees.**
  - Each node has **up to two child nodes**.
  - Values in **left subtree < root** value **<** values in **right subtree**.
  - **Left** & **right** subtrees are **binary search trees**.
  - **Insertion** is *O(nlogn)* but takes **more** space than lists.
    - If items sorted, then insertion is $O(n^2)$.
      - Tree -> stick -> linked list.
  - **Improved performance:** AVL trees & splay trees.



Abstract syntax tree for (5 + 4) * 6 + 3

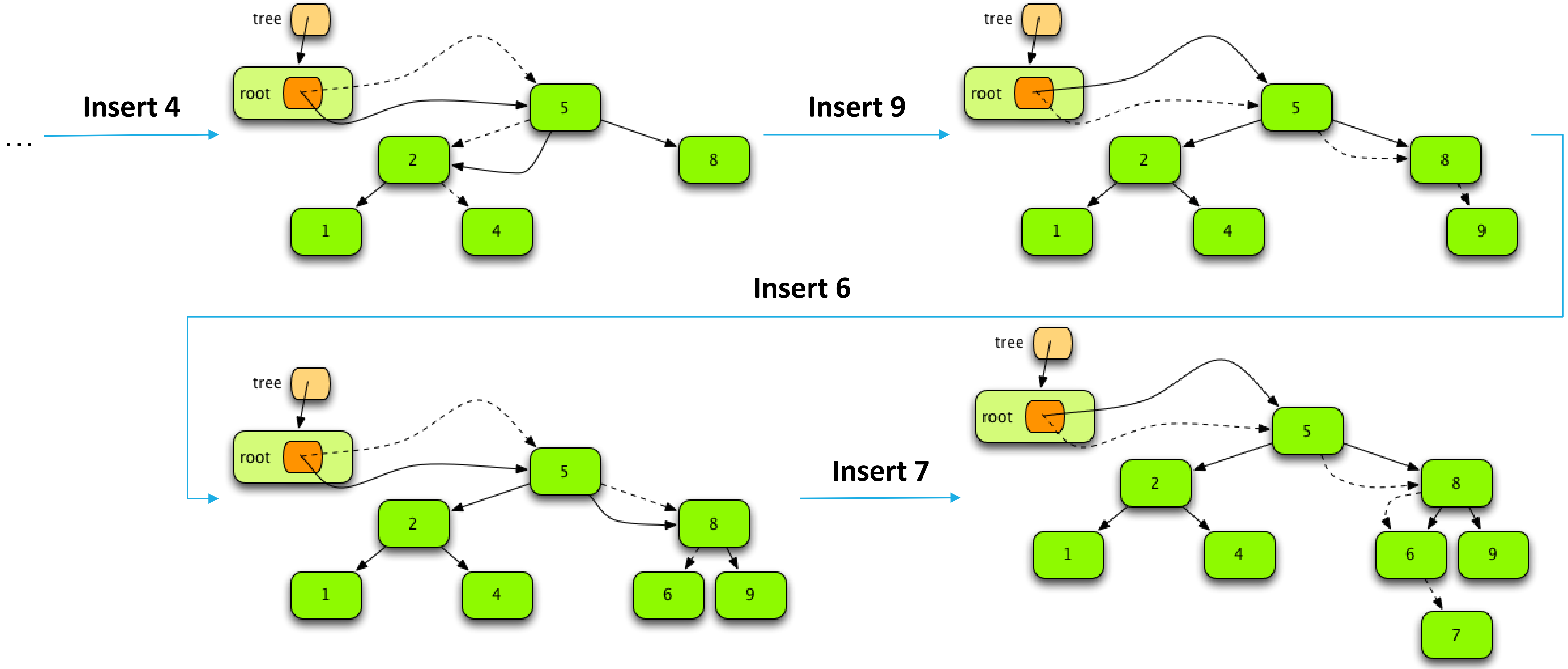
BINARY SEARCH TREES: INSERTION (1)

- **Inserting 5, 8, 2, 1, 4, 9, 6, 7 into binary search tree:**



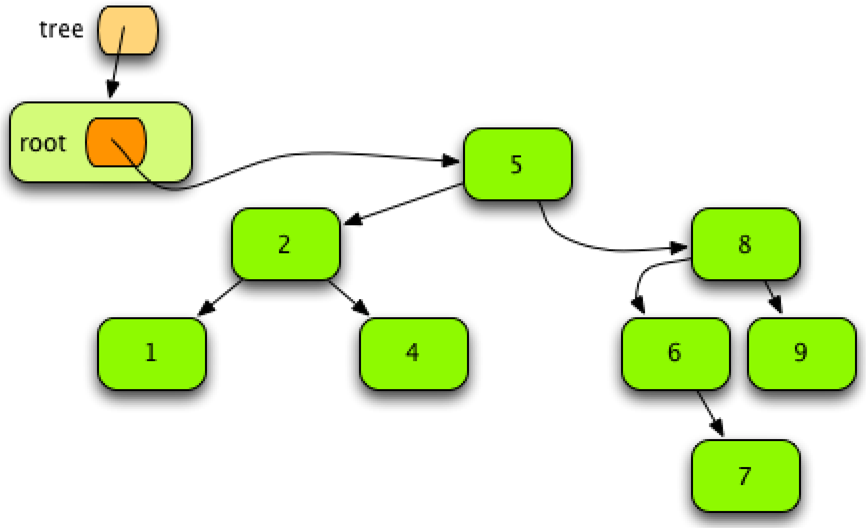
BINARY SEARCH TREES: INSERTION (2)

- **Inserting 5, 8, 2, 1, 4, 9, 6, 7 into binary search tree (cont.):**



BINARY SEARCH TREES: INSERTION (3)

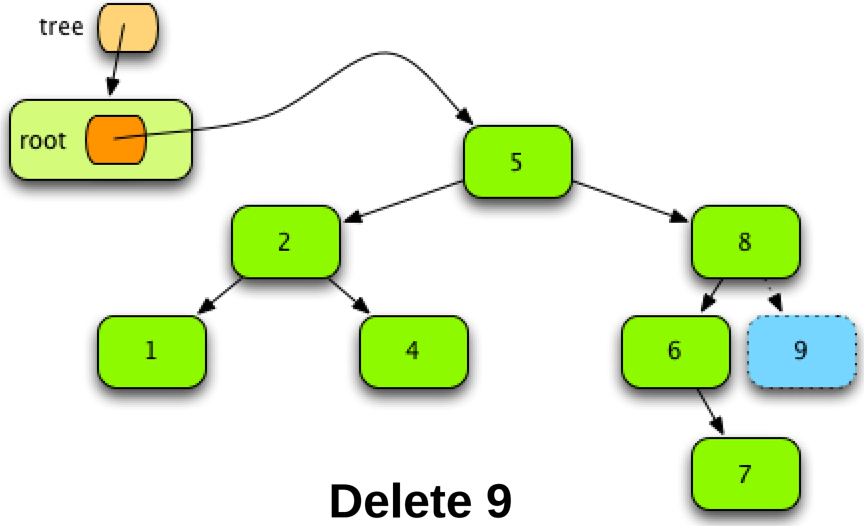
- **Inserting 5, 8, 2, 1, 4, 9, 6, 7 into binary search tree (cont.):**



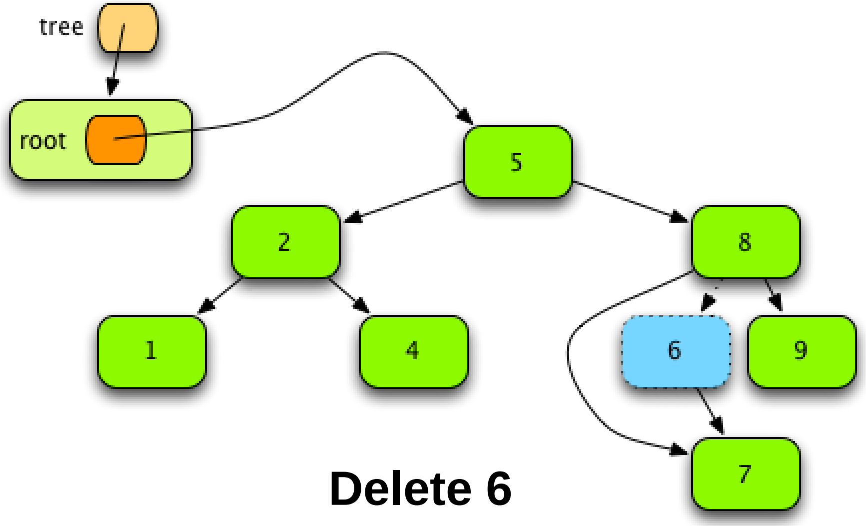
Final tree

BINARY SEARCH TREES: DELETION

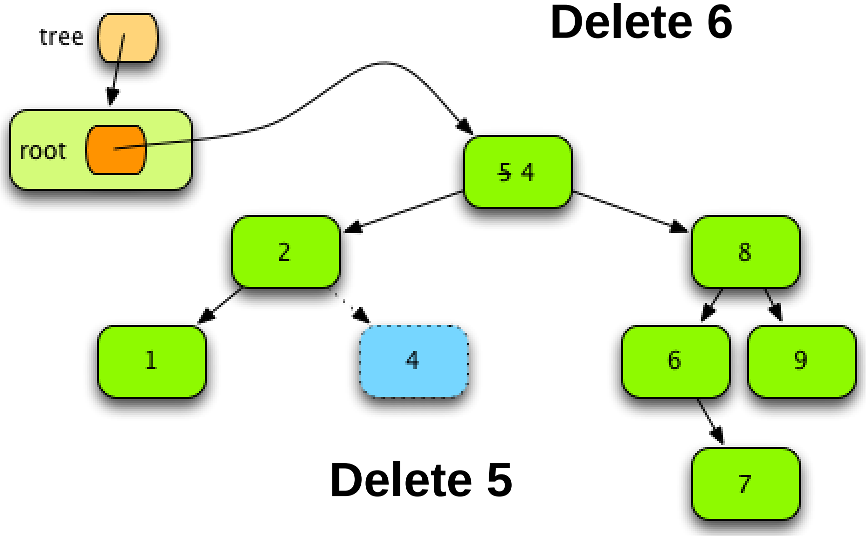
- **Deleting 9, 6, 5 from binary search tree:**



Delete 9



Delete 6



Delete 5

SEARCH SPACES

- Normally, **problems** consist of many different **states**.
  - **Goal** (*end state*) = finding a **solution** to the problem.
  - **Collection** of all problem **states** = **search space**.
  - A **solution** can be found by applying a **depth first search** on the problem search space.
- **Depth first search** of a problem space:
  - Check for a **solution** by making a **guess**.
  - If **guess** does **not lead** to a solution, then **backtrack**.
    - **Backtracking** – undoing unsuccessful guess and trying next guess.
  - Keep **trying** guesses and **backtracking** until solution is **found**.

BALANCED BINARY SEARCH TREES

- **Unbalanced binary search trees.**
  - Problem: BST operations computational complexity grows once the tree becomes unbalanced.
  - Solution: **Balanced** BST structure with guaranteed *logn* height, *Θ(nlogn)* to build a tree, *Θ(n)* for in-order traversal.
    - **AVL trees & splay trees.**
- **Balanced binary search trees** are used when many **insert/delete/lookup** operations & many **iterations** over items in **ascending/descending** order are required.

AVL TREES

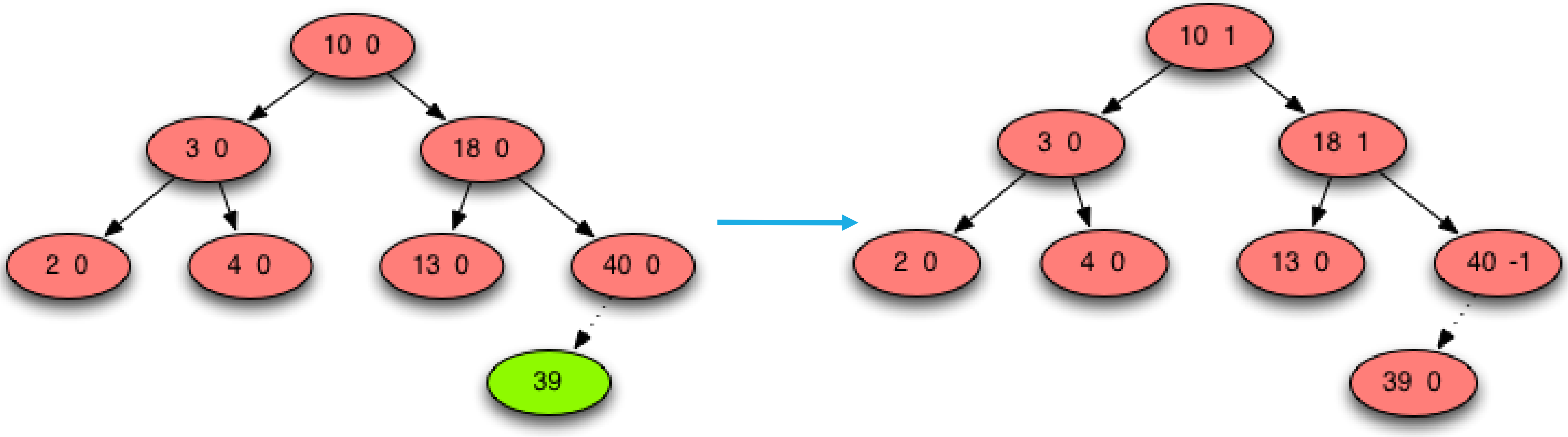
- **AVL trees.**
  - **Binary search trees** with an additional information to maintain the **balance**.
  - **Height** is *logn*.
  - **Lookup/insert/delete** operations are *Θ(logn).*
  - Tree is **built** in *Θ(nlogn)*.
  - **In-order traversing** (yields sorted items) is *Θ(n).*
- **AVL tree definitions:**
  - <u>Height</u> = 1 + the maximum height of subtrees.
    - Height of leaf node = 1.
  - <u>Balance</u> = (height of right subtree) – (height of left subtree).
  - <u>AVL tree</u> = balance of every node must be in [−1, 0, 1].
- **Balance** is maintained using **height** or **balance** of **each node** in the tree.

AVL TREES: INSERTION (1)

- **Inserting into AVL trees:**
  - Initially, **same process** as for BST – **start** from the **root**, **find** correct **position**.
    - In addition, **keep track** of the nodes along the path.
    - Traversed nodes are pushed onto the stack (**path stack**).
  - Once node is inserted, nodes **balances** are **adjusted**.
    - **Pop** nodes off the **path stack** and **adjust balances**.
    - If found node with **balance != 0** before adjusting – **pivot node**.
  - **Based on the pivot node and location of inserted node, three cases must be considered:**
    - **Case 1:** No pivot found.
    - **Case 2:** Pivot found; no rotation needed.
    - **Case 3:** Pivot found; rotation needed.
      - Single rotation or double rotation.

AVL TREES: INSERTION (2)

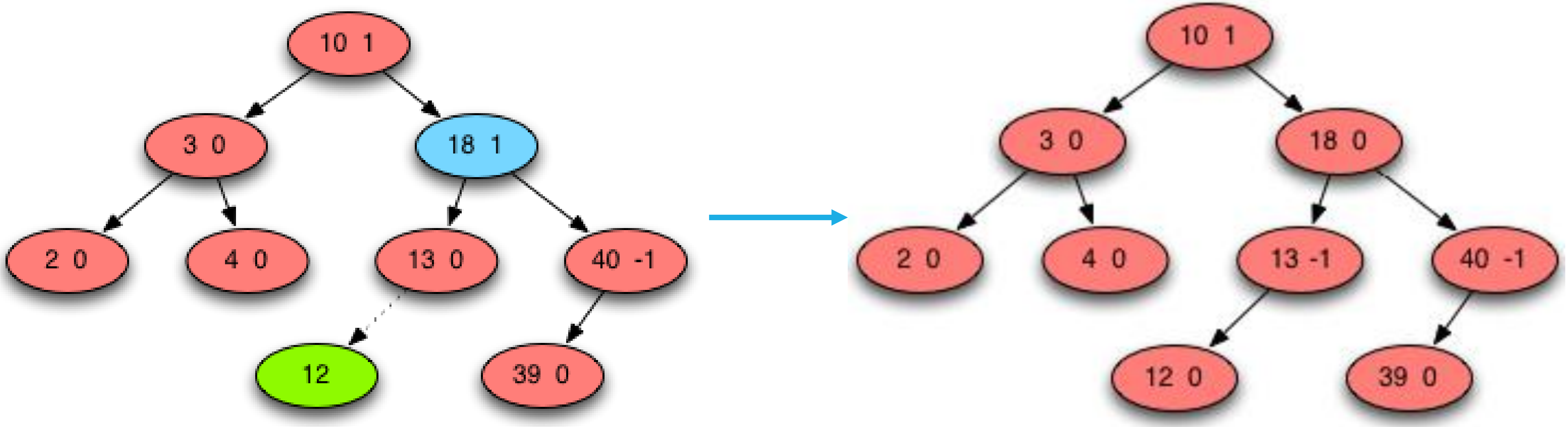
- **Case 1: No pivot found.**
  - Balance of each node along the path == 0.
  - Adjust balance of each node in path stack.



Inserting 39, no pivot

AVL TREES: INSERTION (3)

- **Case 2: Pivot found, no rotation.**
  - Subtree of the pivot node (where new node inserted) has smaller height.
  - Adjust balance from new nodes up to pivot node.
  - Balances of nodes above the pivot are not affected.
    - Height of the subtree rooted at the pivot node is not changed by the insertion of the new node.



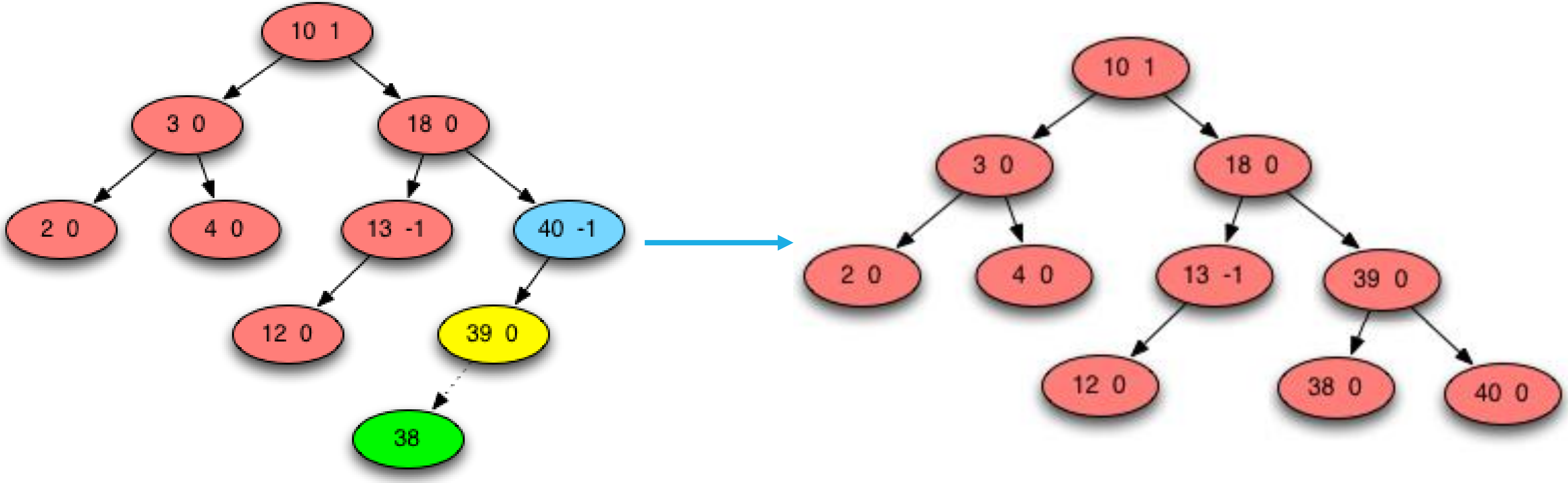
Inserting 12, pivot = 18

AVL TREES: INSERTION (4)

- **Case 3: Pivot found, rotation needed.**
  - Node is added to subtree of larger height.
  - Pivot balance becomes -2 or 2 -> tree is no longer AVL.
    - Single or double rotation must be performed.
  - **Bad child** – pivot child in the direction of the imbalance.
  - **Bad grandchild** – child of a bad child in the direction of the imbalance.
- **Two subcases must be considered:**
  - **Subcase A: Single rotation.**
    - Node is added to the subtree of a bad child in the **direction** of the imbalance.
    - **Rotate** at the **pivot** in the **direction opposite** to the imbalance.
  - **Subcase B: Double rotation.**
    - Node is added to the subtree of a bad child in the **direction opposite** to the imbalance.
    - **Rotate** at the **bad child** in the **direction** of the imbalance.
    - **Rotate** at the **pivot** in the **direction opposite** to the imbalance.

AVL TREES: INSERTION (5)

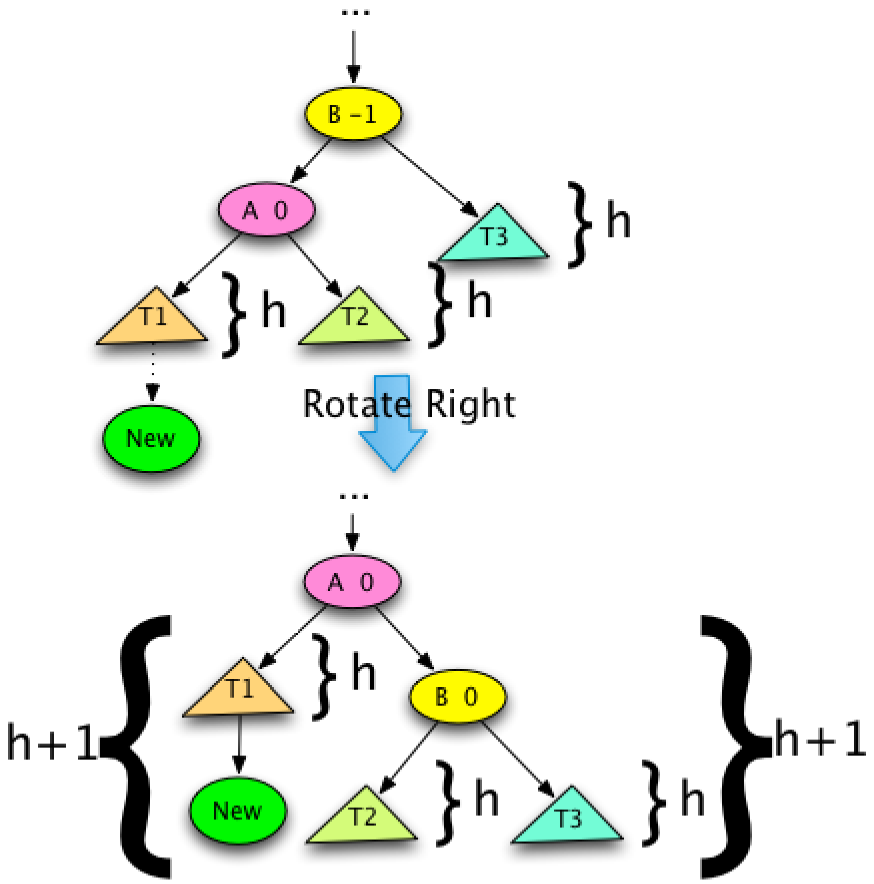
- **Case 3: Pivot found, rotation needed (cont.)**
  - **Subcase A: Single rotation.**
    - Node is added to the subtree of a bad child in the direction of the imbalance.
    - Rotate at the pivot in the direction opposite to the imbalance.



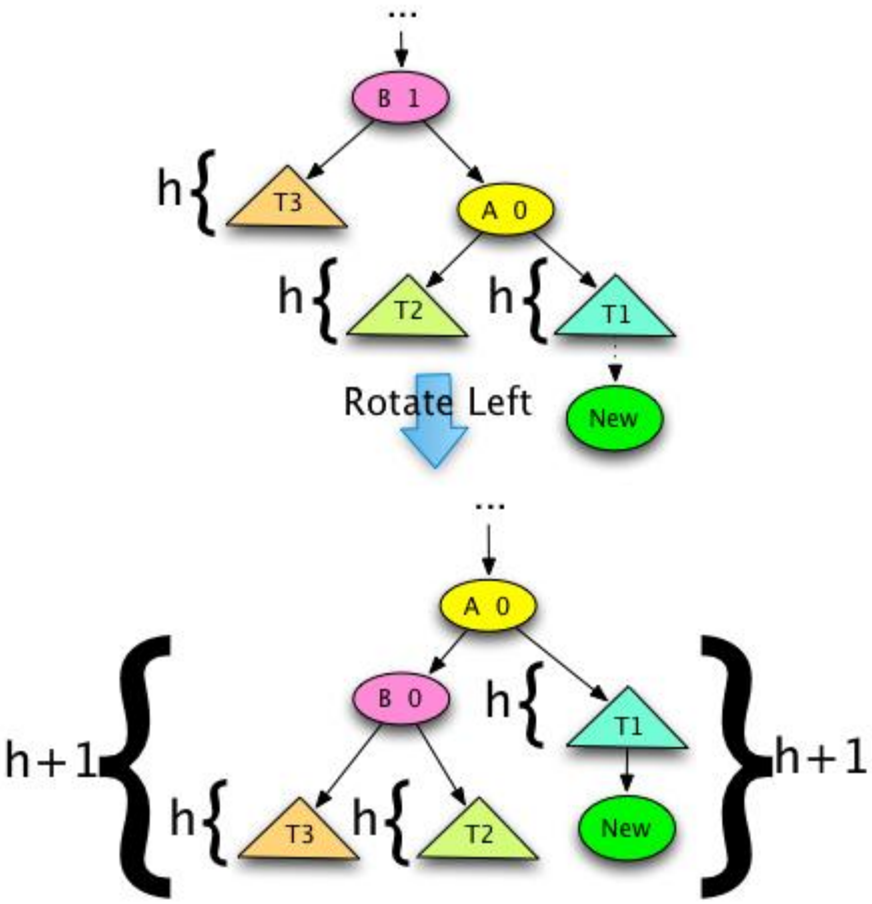
Inserting 38, pivot = 40, bad child = 39

AVL TREES: INSERTION (6)

- **Case 3: Pivot found, rotation needed (cont.)**
  - **Subcase A: Single rotation.**



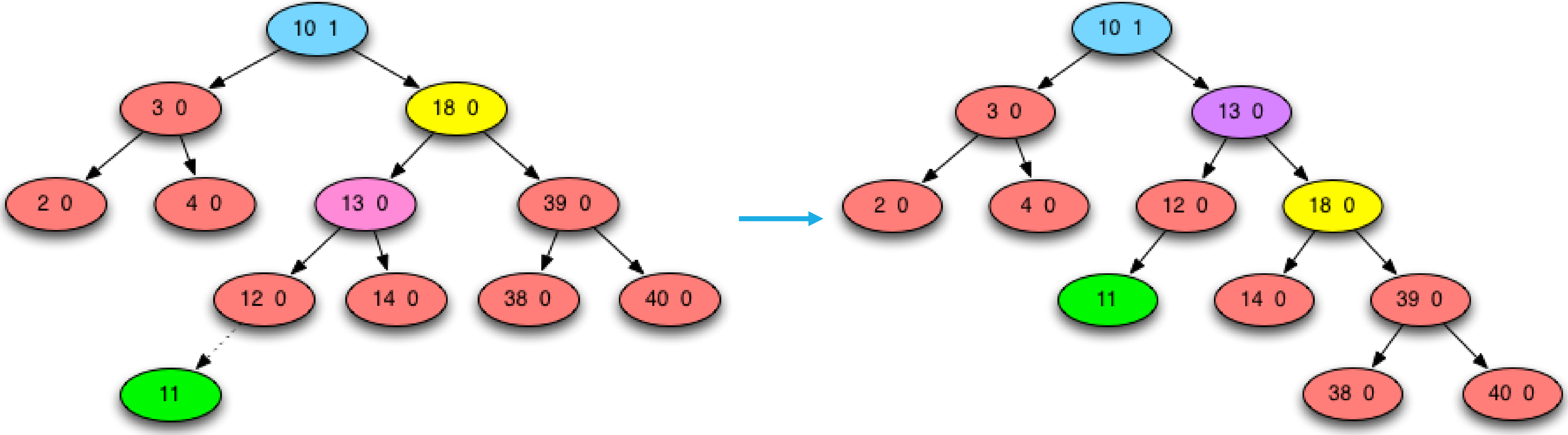
Case 3A right rotation



Case 3A left rotation

AVL TREES: INSERTION (7)

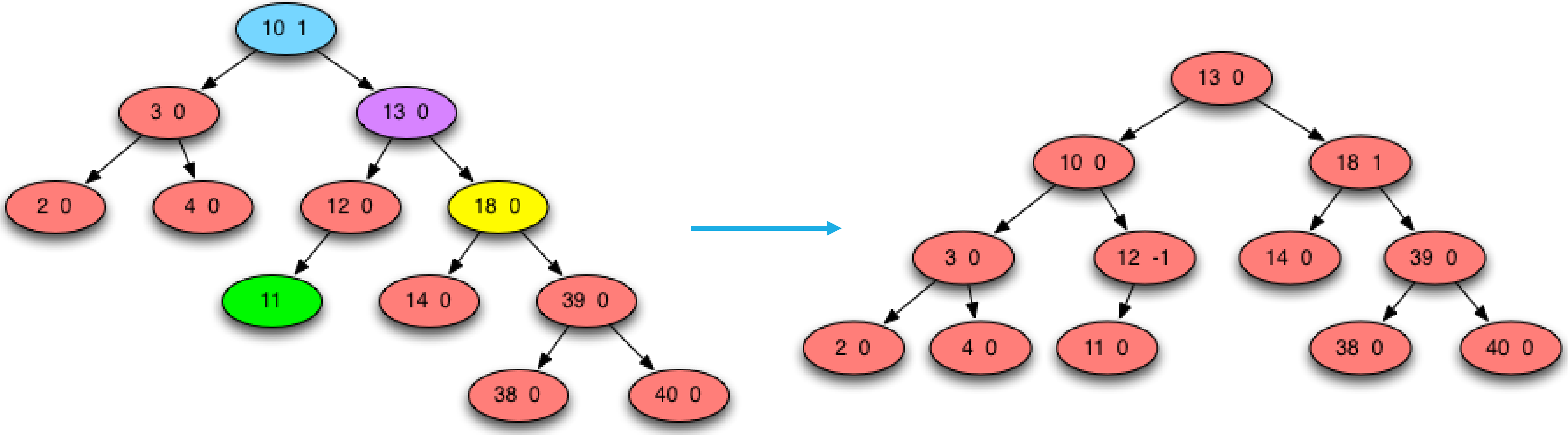
- **Case 3: Pivot found, rotation needed (cont.)**
  - **Subcase B: Double rotation.**
    - Node is added to the subtree of a bad child in the direction opposite to the imbalance.
    - **Rotate at the bad child in the direction of the imbalance.**
    - Rotate at the pivot in the direction opposite to the imbalance.



Inserting 11, pivot = 10, bad child = 18, bad grandchild = 13

AVL TREES: INSERTION (8)

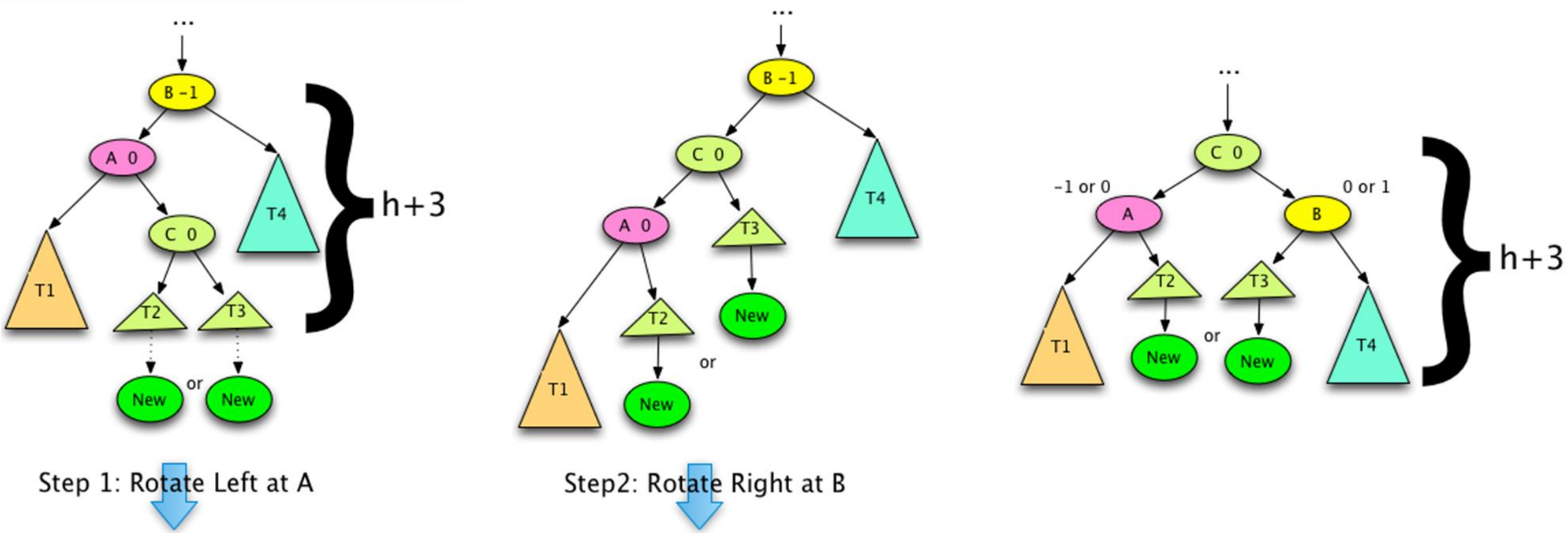
- **Case 3: Pivot found, rotation needed (cont.)**
  - **Subcase B: Double rotation.**
    - Node is added to the subtree of a bad child in the direction opposite to the imbalance.
    - Rotate at the bad child in the direction of the imbalance.
    - **Rotate at the pivot in the direction opposite to the imbalance.**



Inserting 11, pivot = 10, bad child = 18, bad grandchild = 13

AVL TREES: INSERTION (9)

- **Case 3: Pivot found, rotation needed (cont.)**
  - **Subcase B: Double rotation.**



SPLAY TREES

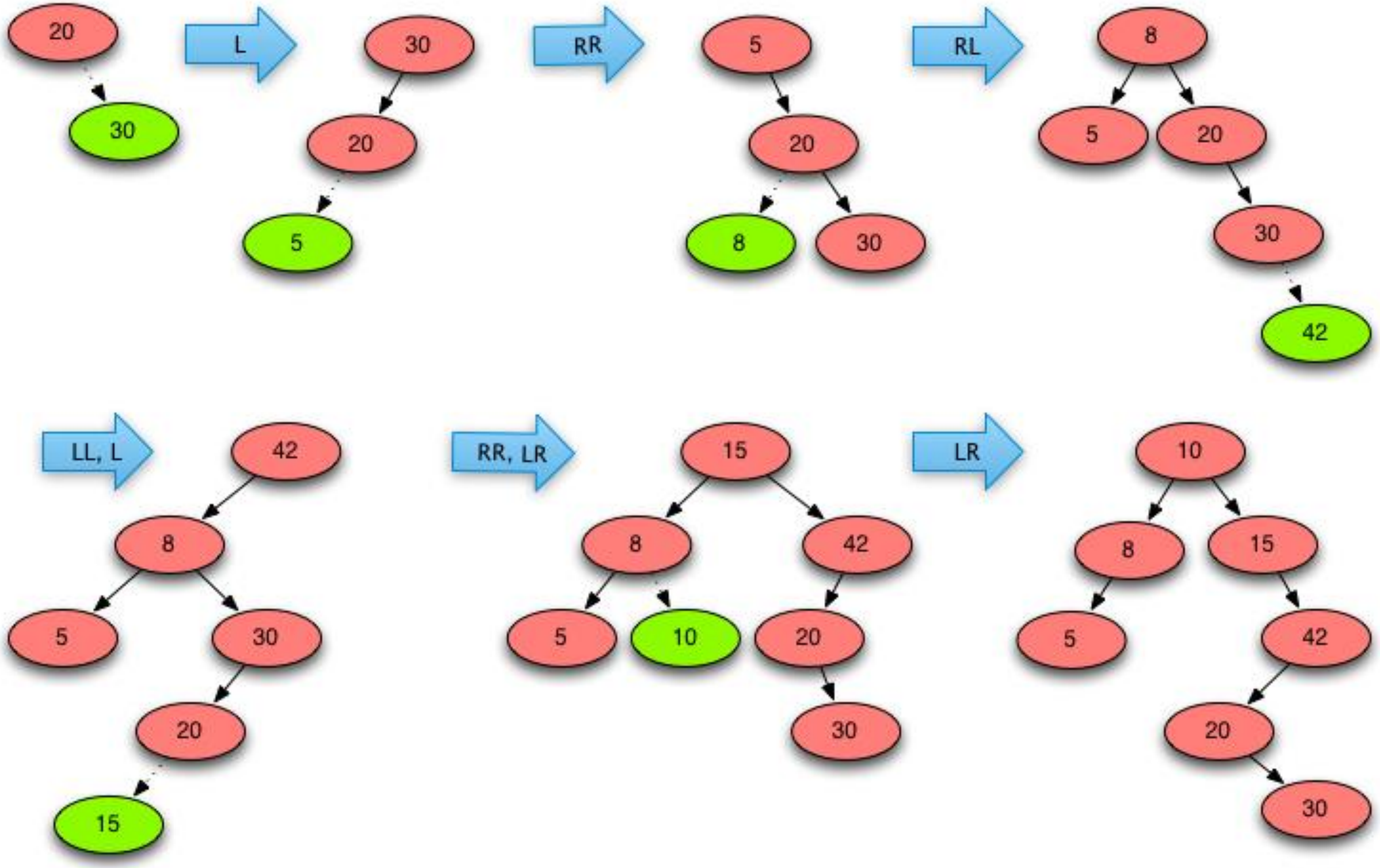
- AVL tree **drawback** – node must maintain the **balance**.
  - **Extra work** to calculate + **extra space** to store.
  - **Improvement** – exploiting **spatial locality**.
- **Spatial locality.**
  - In large data sets a relatively **small subset** of data is **accessed** over a **short period** of time.
- **Splay tree** – *relatively* **balanced** binary search tree with no stored **balance**.
  - **Insert/lookup** moves the node to the root of the tree (**splaying** through **rotations**).
  - **Deletion** of item splays the parent node to the root of the tree.
- **Splay trees** performance is **same/better** than AVL trees on **random** data sets.
  - **Balanced enough** to get **amortized complexity** of *$\Theta(\log n)$* for insert/lookup/delete operations.

SPLAY TREES: OPERATIONS & SPLAYING

- **Operations:**
  - **Insertion.**
    - Value splayed to the root.
  - **Lookup.**
    - Value found - splayed to the root.
    - Value not found - would-be parent splayed to the root.
  - **Deletion.**
    - Parent is splayed to the root.
- Value is splayed to the top of the tree through **rotations**:
  - <u>Single rotations:</u> *zig* or *zag*.
  - <u>Double rotations:</u> *zig-zig* or *zig-zag*.
- Series of **double rotations** take node to the **root** or to the **root child**.
  - If at root child – additional **single rotation** to get to the root.

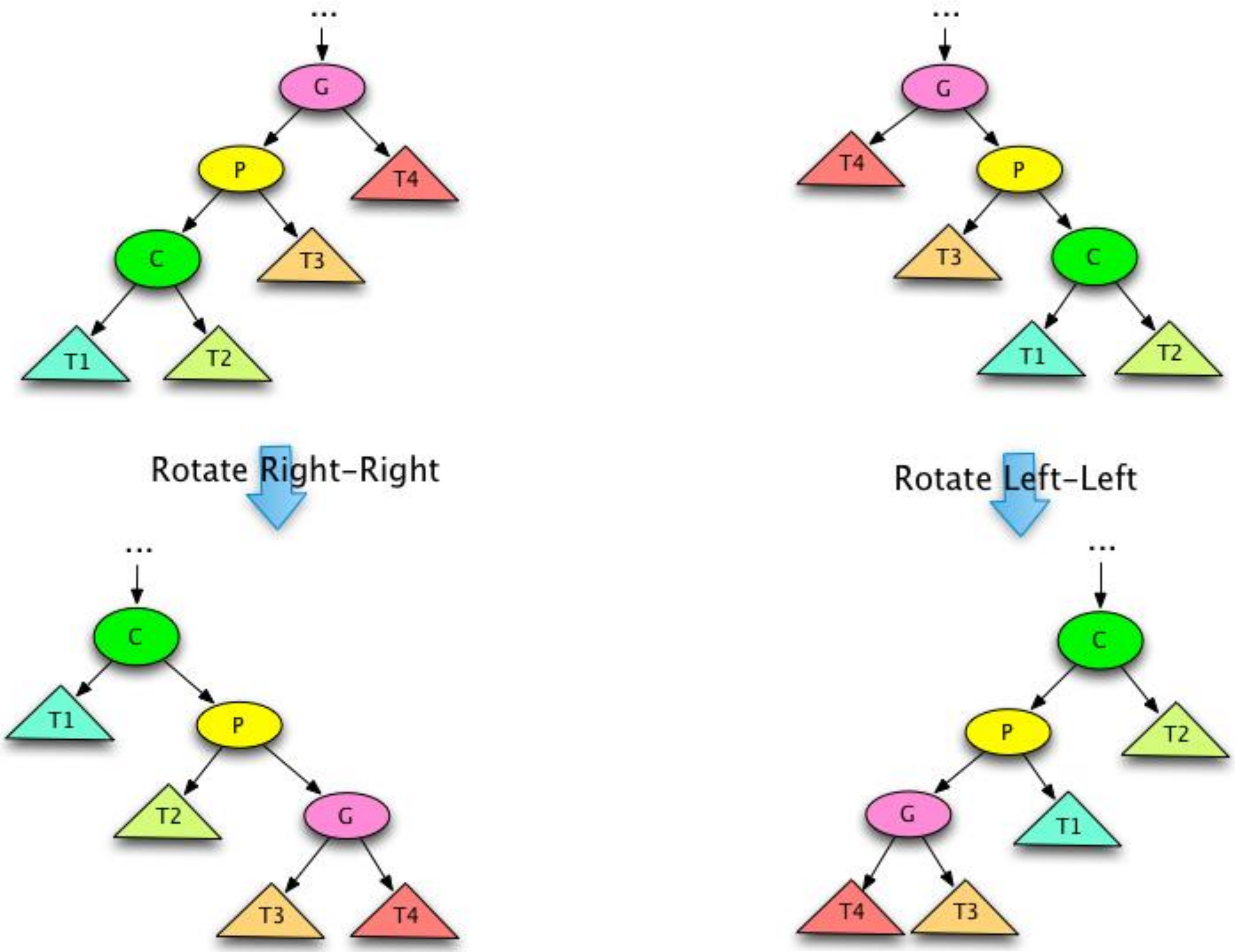
SPLAY TREES: EXAMPLE

- Inserting values 30, 5, 8, 42, 15, 10 into Splay tree.



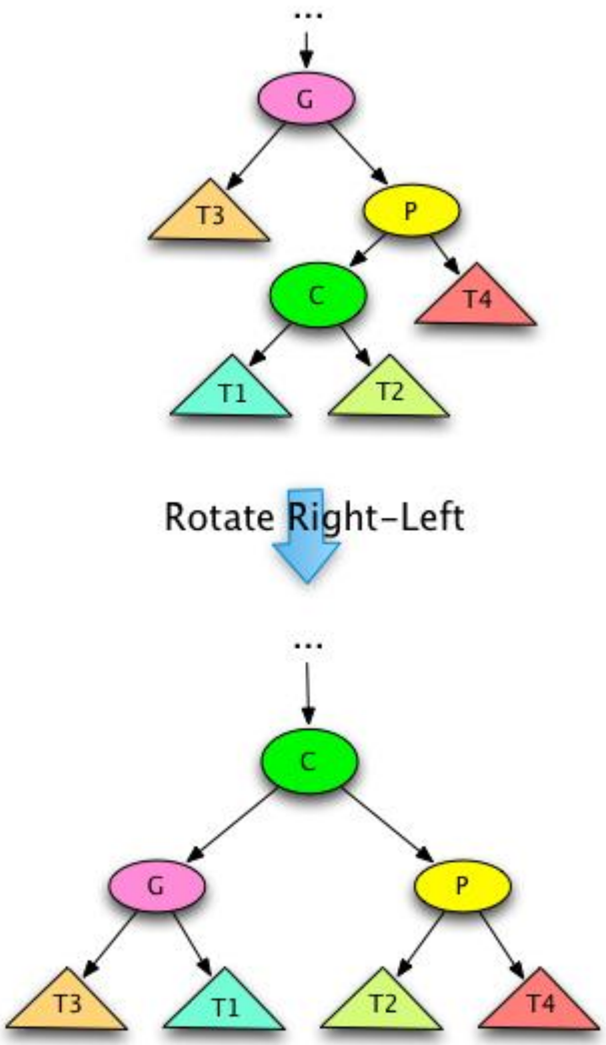
SPLAY TREES: ROTATIONS (1)

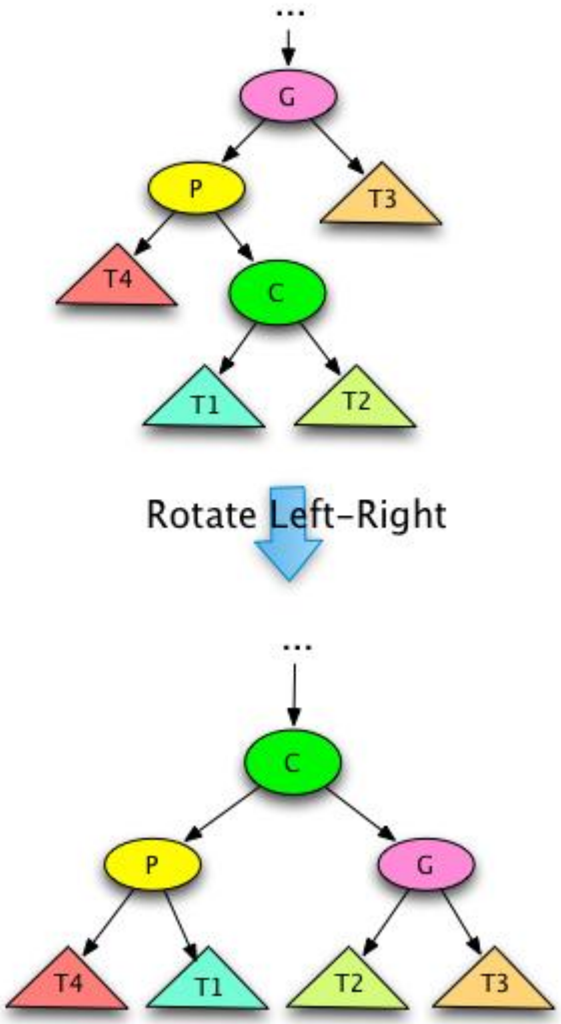
- **Zig-zig rotations.**



SPLAY TREES: ROTATIONS (2)

- **Zig-zag rotations.**





Zig-zag / right-left / left-right rotations

SUMMARY

- Trees.
- Binary search trees.
  - Properties, Insertion, deletion, traversal.
- Search spaces.
  - DFS + backtracking.
- Unbalanced vs. balanced trees.
- AVL trees.
  - Properties, insertions.
- Splay trees.
  - Spatial locality, properties, rotations, operations.