W22-CSC430/530
Lecture 15

Feb 2nd, 2022

§ Equivalence of Sets of Functional Dependencies:

Cover: A set of FD's $\mathcal{F}$ is said to cover another set of FD's $\mathcal{E}$ if every FD in $\mathcal{E}$ is also in $\mathcal{F}^+$; i.e. if every dependency in $\mathcal{E}$ can be inferred from $\mathcal{F}$

$\supseteq$

↳ we can say "$\mathcal{E}$" is covered by $\mathcal{F}$" $\mathcal{F} \supseteq \mathcal{E}$

Equivalence: Two sets of FD's $\mathcal{E}$ and $\mathcal{F}$ are equivalent if $\mathcal{E}^+ = \mathcal{F}^+$.

↳ Equivalence means that every FD in $\mathcal{E}$ can be inferred from $\mathcal{F}$; i.e $\mathcal{E}$ is equivalent to $\mathcal{F}$ if both the conditions — $\mathcal{E}$ covers $\mathcal{F}$ and $\mathcal{F}$ covers $\mathcal{E}$ — hold.

$\mathcal{E} \supseteq \mathcal{F}$ $\quad$ $\mathcal{F} \supseteq \mathcal{E}$

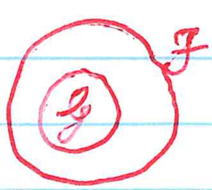
Example: Let $\mathcal{F}$ and $\mathcal{G}$ be two sets of FD's on a relation $R(A, C, D, E, H)$

fd's $\mathcal{F}$: $\{A \rightarrow C$
$AC \rightarrow D$
$E \rightarrow AD$
$E \rightarrow H\}$

fd's $\mathcal{G}$: $\{A \rightarrow CD$
$E \rightarrow AH\}$

show that $\mathcal{F}$ and $\mathcal{G}$ are equivalent.

we need to show that $\mathcal{F} \supseteq \mathcal{G}$ & $\mathcal{G} \supseteq \mathcal{F}$

$\mathcal{F} \supseteq \mathcal{G}$ 

$\mathcal{G} \supseteq \mathcal{F}$ 

$R(A, C, D, E, H)$

Step 1:

$\mathcal{F}$: $A \to C$ $AC \to D$ $E \to AD$ $E \to H$	$\mathcal{G}$: $A \to CD$ $E \to AH$
A^+: AC^+: E^+:	A^+: E^+:

Step 2:

$\mathcal{F}$: $A \to C$ $AC \to D$ $E \to AD$ $E \to H$	$\mathcal{G}$: $A \to CD$ $E \to AH$.
compute closure of using $\mathcal{G}$	compute closure of using $\mathcal{F}$
A^+: ACD AC^+: ACD E^+: $EADCH$	A^+: ACD E^+: $EADCH$
This means that $\mathcal{F} \geq \mathcal{G}$	This means that $\mathcal{G} \geq \mathcal{F}$

Step 3

Step 4: Since $\mathcal{F} \geq \mathcal{G}$ and $\mathcal{G} \geq \mathcal{F}$ both $\mathcal{F}^+ = \mathcal{G}^+$

Try this yourself:

Example: Given $R(PQRS)$ and Fd's
$X = \{P \to Q,\ Q \to R,\ R \to S\}$
$Y = \{P \to QR,\ R \to S\}$
which of the following holds true:
a) $X \supseteq Y$
b) $Y \supseteq X$
c) $X^+ = Y^+$
d) $X^+ \neq Y^+$

Example: State (True or False) given $R(ABC)$ ~~and~~, the Fd's $F = \{A \to B,\ B \to C,\ C \to A\}$ and
$G = \{A \to BC,\ B \to A,\ C \to A\}$ are equivalent.

Example: Given $R(VWXYZ)$ and function dependencies
$F = \{W \to X,\ WX \to Y,\ Z \to WY,\ Z \to V\}$ &
$G = \{W \to XY,\ Z \to WX\}$ prove/disprove
$G \supseteq F$.

∴ ℰ **Cannonical form of functional dependencies:**
↳ aka. minimal sets of FD's / Irreducible sets of FD's.

Objective: to reduce a set of FD's "$\mathcal{F}$" to its minimal set "$\mathcal{E}$".

note: we define a set of functional dependancies as minimal if it satisfies, the following conditions:

a) Every dependency in "$\mathcal{F}$" has a single attribute for its right hand side (RHS)

b) We cannot replace any dependency $X \rightarrow A$ in $\mathcal{F}$ with a dependency $Y \rightarrow A$, where Y is a proper ~~set~~ subset of X, and still have a set of dependencies that is equivalent to $\mathcal{F}$

c) We cannot remove any dependency from $\mathcal{F}$ and still have a set of dependencies that is equivalent to $\mathcal{F}$.

Example: Given $R(W X Y Z)$ with fd's $\mathcal{F} = \{X \rightarrow W,\ WZ \rightarrow XY,\ Y \rightarrow WXZ\}$, can we derive the minimal set of fd's.

Algorithm: Finding a Minimal set (cover) F for a set of FD's E.

Input: A set of FD's E.
Output: A minimal set of FD's F.

Step 1: Set $F := E$

Step 2: Replace each FD $X \rightarrow \{A_1, A_2, \ldots, A_n\}$ in F by the form $X \rightarrow A_1$, $X \rightarrow A_2$, … $X \rightarrow A_n$ (using decomposition rule)

Step 3: a) For each FD $X \rightarrow A$ in F determine X^+

b) Determine closure without the FD $X \rightarrow A$.

↳ if there is a change then the FD $X \rightarrow A$ is essential, else discard $X \rightarrow A$ from F.

Soln: R(WXYZ)

fd E: $\{X \rightarrow W$
$WZ \rightarrow XY$
$Y \rightarrow WXZ\}$

Step 1: $F := E$

Step 2: Apply decomposition rule to every FD in F

F: {
1) $X \rightarrow W$
2) $WZ \rightarrow X$
3) $WZ \rightarrow Y$
4) $Y \rightarrow W$
5) $Y \rightarrow X$
6) $Y \rightarrow Z$ }

$F = \{$
1) $X \rightarrow W$
2) $WZ \rightarrow X$
3) $WZ \rightarrow Y$
4) $Y \rightarrow W$
5) $Y \rightarrow X$
6) $Y \rightarrow Z$
$\}$

Step 3a: determine closure of each FD

1) $X \rightarrow W$

$X^+ = XW$ —(a)

Step 3b: determine closure without FD.

$X^+ = X$ —(b)

since (a) $\neq$ (b) the FD is essential.

2) $WZ \rightarrow X$

$WZ^+ = WZXY$ —(a)

$WZ^+ = WZYX$ —(b)

since (a) = (b) this FD is discarded from F

3) $WZ \rightarrow Y$

$WZ^+ = WZYX$ —(a)

$WZ^+ = WZ$ —(b)

since (a) $\neq$ (b) this FD is essential.

4) $Y \rightarrow W$

$Y^+ = YWXZ$ —(a)

$Y^+ = YXZW$ —(b)

since (a) = (b) this FD is discarded.

5) $Y \rightarrow X$

$Y^+ = YXZW$ —(a)

$Y^+ = YZ$ —(b) essential

6) $Y \rightarrow Z$

$Y^+ = YZX$ —(a)

$Y^+ = YX$ —(b) essential

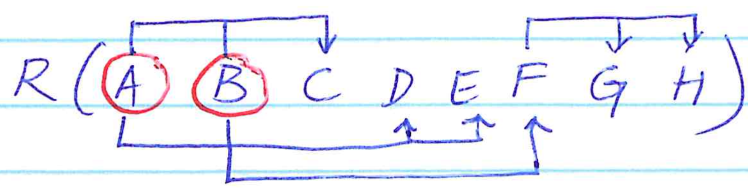
Output: $F = \{X \to W,\ WZ \to Y,\ Y \to XZ\}$
as the minimal cover of $R(WXYZ)$

Example: $R(ABCD)$ & fd's $F = \{A \to B,\ C \to B,\ D \to ABC,\ AC \to D\}$
determine the minimal cover.

§ Finding Candidate keys using edge diagrams:

Given $R(A\ B\ C\ D\ E\ F\ G\ H)$ &
fd: $\{AB \to C$
$A \to DE$
$B \to F$
$F \to GH\}$

convert the FD's into the edge diagram:



Find those attributes that do not have incomming edges.
∴ AB are ~~potential~~ attributes of potential candidate keys.

$AB^+ = ABCDEFGH$ ∴ AB is a candidate key.

Example: $R(A\ B\ C\ D\ E\ F\ G\ H)$

$$fd's = \{ AB \to C,\quad BD \to EF,\quad AD \to G,\quad A \to H \}$$

is ABD a candidate key.

Soln: $R(A\ B\ C\ D\ E\ F\ G\ H)$

$\therefore$ A B D are attributes of a candidate key.

$ABD^+ = ABCDEFGH$ $\therefore$ is a candidate key

Example: $R(A\ B\ C\ D\ E)$

$$fd's = \{ BC \to ADE,\ D \to B \}$$

Find the super keys.

Example: $R(W\ X\ Y\ Z)$

$$fd's = \{ Z \to W,\ Y \to XZ,\ WX \to Y \}$$

state True/False that Y is the primary key.