W23 CSC430/530

LECTURE 15

JAN 30, 2023

Recap: Hidden Functional dependencies:

Functional dependencies are constraints between sets of attributes in a relation

FD: $A \to B$

means: the values of B are determined by A

or A is said to functionally determine B

Some times FD's are riddled by redundancies.

ACC-NO, TITLE → YR-PUB.

ACC-NO → YR-PUB

ACC-NO → YR-PUB, TITLE ← redundant.

ACC-NO → S-NAME

[ACC-NO → S-ADDR] ← redundant {Hidden Dependancy}

S-NAME → S-ADDR

Observation: FD's can be exhaustive, and therefore require a "technique" to determine closure.

The closure of FD refers to the complete set of all possible attributes that can be functionally derived from a given set of FD's.

represented as (F^+) / $\{F^+\}$ / F^+

Eg: Given $R = (A, B, C, D, E, F)$

fd's:
1) $A \rightarrow B$
2) $A \rightarrow C$
3) $CD \rightarrow E$
4) $CD \rightarrow F$
5) $B \rightarrow E$

which of the following are the hidden functional dependencies.

a) $A \rightarrow E$
b) $A \rightarrow BC$
c) $AD \rightarrow E$
d) $AD \rightarrow F$

To compute the closure of an attribute given a set of FD's, we use the Armstrong Axioms

E Armstrong Axioms:

Input: set of FD's and the relational schema

Method 3 rules/axioms:
(i) Reflexivity rule
(ii) Augmentation rule
(iii) Transitive rule

Output: The closure of the attribute

§ The **Rules**:

(i) Reflexive rule: if $B \subseteq A$ then $A \to B$ holds.
(ii) Augmentation rule: if $A \to B$ then $AC \to BC$ holds
(iii) Transitivity rule: if $A \to B$ & $B \to C$ then $A \to C$ holds.

Objective: Apply any of these rules to FD's until no new FD's are obtained.

checks: (i) soundness: Any new FD's/FD derived by the axioms are/is indeed members of the closure.

(ii) Completeness: All elements of the closure can be determined by repeated application of the axioms

§ Other Rules:

(iv) Union Rule: if $A \to B$ & $A \to X$ then $A \to BX$ holds
(v) Decomposition: if $A \to BX$ holds then $A \to B$ & $A \to X$ also holds.
(vi) Pseudo-transitivity: if $A \to B$ & $CB \to D$ holds then $CA \to D$ also holds.

Examples: Given $R(A,B,C)$ & fd: $\{A \to B, B \to C\}$ determine the closure of A, B, & C.

To find the closure of $\{A\}^+$ or A^+

$A^+ = A$
$= AB$
$= ABC$ $\therefore A^+ = [ABC]$

$B^+ = B$
$= BC$
$\therefore B^+ = [BC]$

<u>Example 2:</u> given $R(A\,B\,C\,D\,E\,F)$

FD's: $A \to B$

$C \to DE$

$AC \to F$

$D \to AF$

$E \to CF$

State True/False:

$D^+ = A\,B\,D\,F$

soln: $D^+ = D$

$= DAF$ {using $D \to AF$}

$= ABDF$ {using $A \to B$}

$\therefore$ True.

$DE^+ = ABCDEF$

<u>Example 3:</u> $R(A, B, C, D, E, F, G)$

fd: $\{A \to B,\ BC \to DE,\ AEG \to G\}$

* determine AC^+

<u>Example 4:</u> $R(A\,B\,C\,D\,E\,F\,G\,H)$

fd: $\{A \to BC$

$CD \to E$

$E \to C$

$D \to AEH$

$ABH \to BD$

$DH \to BC\}$

* Determine if $BCD \to H$ is a valid FD.

Example 5: Given $R(ABCDE)$ and fd's: $\{A \to BC$

$CD \to E$

$B \to D$

$E \to A\}$

determine if AB is a candidate key of R

Example 6: State (True/False) Given $R(ABCDEF)$ and fds: $\{AB \to C, BC \to AD, D \to E, CF \to B\}$

AB is "not" the primary key.