Question 1:

The state vector is defined as:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix}$$

$$u = \delta$$

$$\dot{x_1} = \dot{\alpha} = -0.313\alpha + 56.7q + 0.232\delta$$

$$\dot{x_2} = \dot{q} = -0.0139\alpha - 0.426q + 0.0203\delta$$

$$\dot{x_3} = \dot{\theta} = q$$

The continuous-time state-space matrices are

$$A = \begin{bmatrix} -0.313 & 56.7 & 0 \\ -0.0139 & -0.426 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0.232 \\ 0.0232 \\ 0 \end{bmatrix} u$$

$$y = [0 \quad 0 \quad 1]x + 0 \times u$$

```
A=  [-0.313 56.7 0
    -0.0139 -0.426 0
    0 1 0];
B=  [0.232
    0.0203
    0];
C = [0 0 1];
D = 0;

% Build a continuous-time state-space model
sys_ss = ss(A, B, C, D);
% Convert to a transfer function
G = tf(sys_ss)
```

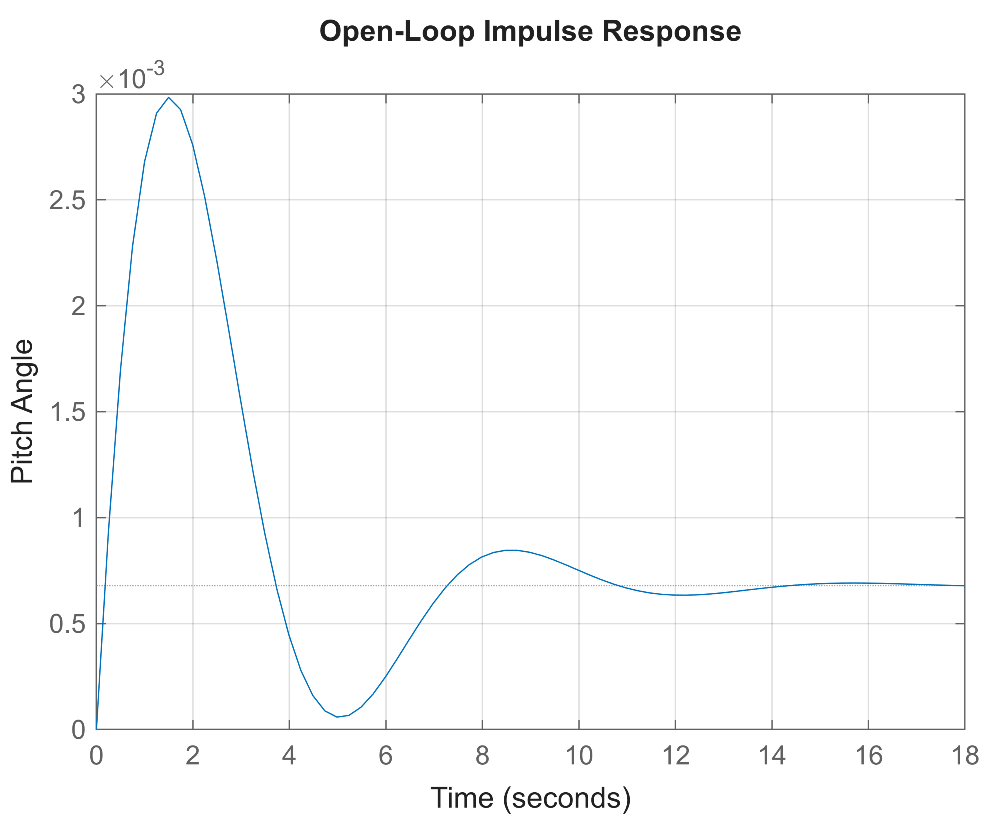
```
G =

     0.0203 s + 0.003129
  --------------------------
  s^3 + 0.739 s^2 + 0.9215 s

Continuous-time transfer function.
Model Properties
```

```

```

Question 2:

```
% Define state-space matrices
A = [ -0.313   56.7     0
      -0.0139  -0.426   0
       0        1       0 ];
```

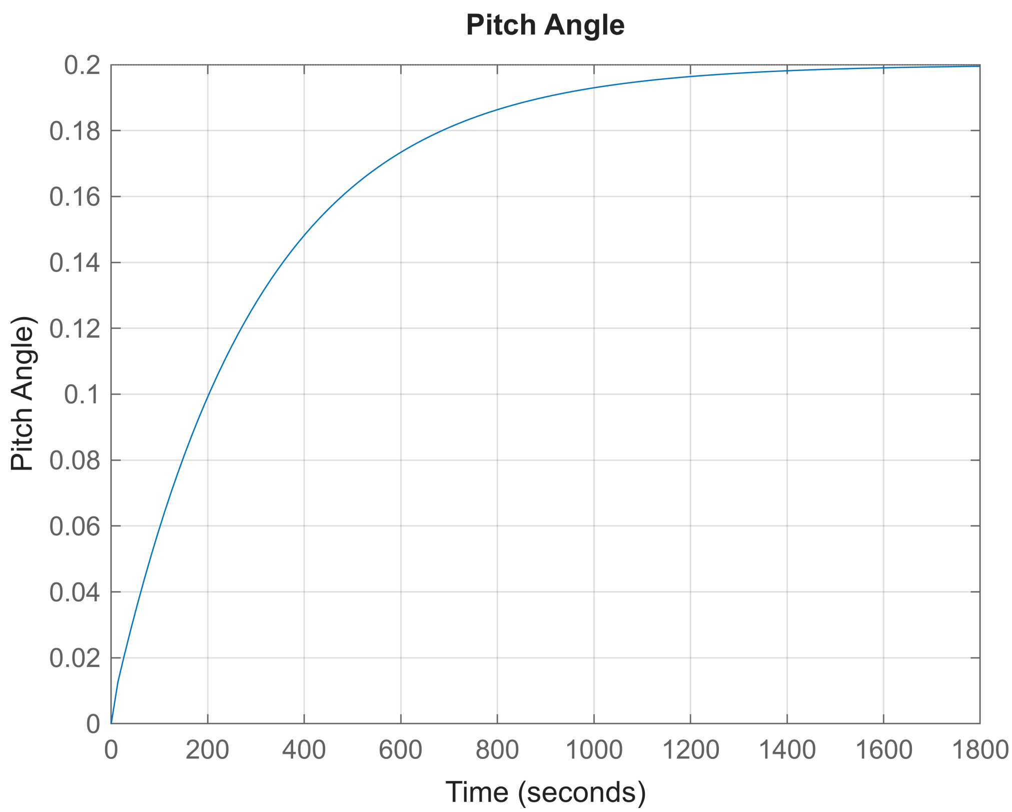
```
B = [ 0.232
      0.0203
      0     ];

C = [ 0  0  1 ];
D = 0;

% Create state-space system
sys_ss = ss(A, B, C, D);

% Convert state-space to transfer function
G = tf(sys_ss);

% Impulse response with amplitude scaling (0.2)
impulse(0.2 * G);
title('Open-Loop Impulse Response');
xlabel('Time');
ylabel('Pitch Angle');
grid on;
```



Question 3:

```
% System Matrices
A = [ -0.313   56.7    0
```

```
      -0.0139  -0.426  0
       0        1      0 ];
B = [ 0.232
      0.0203
      0      ];
C = [ 0  0  1 ];
D = 0;

% Create continuous-time state-space system
sys_ss = ss(A, B, C, D);

% Open-Loop Transfer Function
G = tf(sys_ss);

% Controller: C(s) = K = 1 (Proportional Gain)
K = 1;

% Closed-Loop System with Unity Feedback
G_cl = feedback(K * G, 1);

% Step Response with a 0.2 rad Reference (Pitch Angle)
step(0.2 * G_cl);
title('Pitch Angle');
xlabel('Time');
ylabel('Pitch Angle)');
grid on;
```



```

```

Question 4:

```
% Continuous-time model (A, B, C, D)
A = [ -0.313   56.7     0
      -0.0139  -0.426   0
       0        1       0 ];
B = [ 0.232
      0.0203
      0      ];
C = [ 0  0  1 ];
D = 0;

% Build the continuous-time state-space model
sys_ss = ss(A, B, C, D);

% Convert to transfer function (for reference, if needed)
G = tf(sys_ss);

% Discretize using zero-order hold
Ts = 0.02;  % Sampling period
sys_d = c2d(sys_ss, Ts, 'zoh');
Gd = c2d(G, Ts, 'zoh');

% Extract the discrete-time state-space matrices
[Ad, Bd, Cd, Dd] = ssdata(sys_d);

% Construct and display the controllability matrix
Co = ctrb(Ad, Bd);
r = rank(Co);
n = size(Ad, 1);

fprintf('Rank of controllability matrix: %d\n', r);
```

```
Rank of controllability matrix: 3
```

```
fprintf('Number of states: %d\n', n);
```

```
Number of states: 3
```

```

if r == n
    disp('Discrete-time system is fully controllable.');
else
    disp('Discrete-time system is NOT fully controllable.');
end
```

```
Discrete-time system is fully controllable.
```