ELEN 472: Introduction to Digital Control Systems

HW3

Q1. Sketch the root locus diagram for the following analog system with the loop gain:

$$L(s) = \frac{1}{s(s+2)(s+5)}$$

Q2.1 Given damping ratio $\zeta = 0.5$ and natural frequency $\omega_n = 1$ rad/s. Find the following time-domain specifications:

1) Percentage Overshoot
2) Peak Time
3) Settling time

Q2.2 Fill the blank space:

For continuous time systems, the real part of the pole determines the rate of _____, and the imaginary part determines the _____.

Q2.3 For a position control system with the continuous time transfer function:

$$G(s) = \frac{1}{s(s+5)}$$

Design a proportional controller to make the damping ratio $\zeta = 0.1$

Q3. Find the critical gain of the following systems:

1) $G(z) = \frac{K}{z-0.4}$
2) $G(z) = \frac{K}{(z+0.9)(z-0.9)}$

Q4.1 Fill the blank space:

If the Laplace transform $F(s)$ of a continuous-time function $f(t)$ has a **pole** $\boldsymbol{p_s}$, then the z-transform $F(z)$ of its sampled counterpart $f(kT)$ has a **pole** at _____.

The Frequency of Oscillations (ω_d) can be calculated with ζ and ω_n by using the equation: _____.

Q4.2 Design proportional controllers for the digital system

$$G(z) = \frac{K}{(z + 0.9)(z - 0.9)}$$

to meet the following specifications separately (you need to find two values of K to meet condition 1 and 2 separately):

1) A damping ratio of 0.7.
2) A steady-state error of 10% due to a unit step input (sampling period $T = 0.02$ s).

Q5.1 Apply the **forward difference** approximation of the following second-order analog controller:

$$C_a(s) = \frac{4}{s^2 + 2s + 4}$$

to find the digital controller ($T = 1$ s).

Q5.2 Repeat the previous question with a **backward difference**.

Q6.1 Design a digital filter by applying the bilinear transformation to the analog filter

$$C_a(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

With $T = 0.1$ s.

Q6.2 Repeat the previous question and apply the prewarping at $\omega_0 = 1$ rad/s to find the new digital filter.

Q7. Answer the following questions to design a digital controller for the analog plant:

$$G(s) = \frac{1}{(s+1)(s+5)}$$

to

1) In order to obtain zero steady-state error, what kind of controller (i.e., PI, PD, or PID) should we choose?
2) To obtain a settling time less than $T_s < 2$ s and the natural frequency of $\omega_n = 5$ rad/s, what is the value of damping ratio ζ?
3) What is the range of suitable sampling frequency?
4) Find the bilinear transform of the analog controller that you selected in question 1 ($T = 1$ s).