STAT 5213/6213/MATH 4903 Exam 1. Provide full solutions to the problems on blank sheets of paper. You must show your work to receive credit. Use LaTeX or write clearly, boxing in final numerical answers. Use R for calculating sample statistics and computing probabilities and critical values from distributions. You are not permitted to receive assistance from a classmate or anyone else. Nor may you use artificial intelligence. Each numbered problem is worth 10 points.

Name: ____ **By signing your name you are pledging to have followed the instructions given above. Receiving prohibited assistance or using AI will result in penalties.**

1. Suppose $M(t) = (1-2t)^{-10}$, $t < \frac{1}{2}$, is the moment generating function of a random variable X.

   (a) Compute the mean μ of X.

   (b) Compute the standard devaition σ of X.

   (c) Use R to compute $P(X > 9)$.

   (d) Use R to compute $P(|X - \mu| < \sigma)$

2. Suppose the lifetime in months of an engine, working under hazardous conditions, has a gamma distribution with a mean of eight months and a variance of 16 months^2.

   (a) Compute the median lifetime of the engine.

   (b) Suppose each engine is termed successful if its lifetime exceeds 12 months. In a sample of 10 engines, determine the probability of at least three successful engines.

3. Suppose a beta distribution has a pdf of the form

$$f(x) = \begin{cases} cx^2(1-x)^8 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   where c is a constant. Compute the value of c.

4. Suppose a population is governed by a random variable X that is Bernoulli with parameter p. That is $P(X = 1) = p$ and the $P(X = 0) = 1 - p$. The probability mass function for X can be written

$$p(x) = \left\{ \begin{array}{cc} p^x(1-p)^{1-x} & \text{if } \ x = 0 \text{ or } 1 \\ 0 & \text{otherwise} \end{array} \right\}.$$

   Show that for the random sample $X_1, ..., X_n$, from the population the maximum likelihood estimator of p is $\hat{p} = \frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}$.

5. The sample variance S^2 that a random sample $X_1, ..., X_n$, of a population gives is defined by $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$, where $\bar{X}$ is the sample mean. Assuming that the population has mean μ and standard deviation σ, show that S^2 is an unbiased estimator of σ^2.

6. Calculate a 99% confidence for the mean of a normal population based on the random sample

$$12,\ 15,\ 8,\ 20,\ 15,\ 16,\ 17,\ 11,\ 14,\ 10,$$

   of ten values from the population. Round the endpoints of the interval off accurately to two decimal places.

7. In a random sample of 500 registered voters a pollster discovers that 212 support a certain candidate. Calculate a 95% confidence interval for the proportion of registered voters who support the candidate. How many voters would the pollster need to survey to make the margin of error of the 95% confidence interval at most four percent (making the width of the confidence interval at most 0.08).

8. Calculate a 90% confidence for the standard deviation of a normal population based on the random sample

$$12,\ 15,\ 8,\ 20,\ 15,\ 16,\ 17,\ 11,\ 14,\ 10,$$

   of ten values from the population. Round the endpoints of the interval off accurately to two decimal places.

9. Suppose Population 1 and Population 2 have the same variance and both are normal. Calculate a 95% confidence interval for the difference in means of the two populations, $\mu_1 - \mu_2$, based on independent random samples of the two populations with the following summary statistics: $n_1 = 15$, $\bar{x}_1 = 8.2$, $s_1 = 1.25$, $n_2 = 12$, $\bar{x}_2 = 5.7$, $s_2 = 1.10$.

10. Obtain 20-50 values from a real life data set. For example, the number of points the New Orleans Pelicans scored in each of their first 30 regular season games of the 2025-2026 season. Do not use this particular example. Choose one that is so odd, that it's highly unlikely a class mate will select the same data set. Then, acting as if your list of values is a random sample from a population, do the following (using R as needed):

    (a) State what data you're using.

    (b) Compute the sample mean and sample standard deviation

    (b) Construct a box plot

    (c) List all "potential outliers" according to the technical definition on p. 259 of the textbook.

    (d) Construct a histogram

    (e) Compute the proportion of your values that are within one sample standard deviation of the sample mean.