ELEN 472 – Introduction to Digital Control Systems

HW 4

Q1. For the following Continuous-time State Space Models, classify their **linearity** and **time variance**:

(a) $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \sin(t) & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(b) $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 1 & 5 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(c) $$\dot{x} = -2x^2 + 7x + xu$$

$$y = 3x$$

Q2. Linearize the nonlinear state-space model:

$$\dot{x}_1 = x_1^2 + \sin x_2 - 1$$
$$\dot{x}_2 = -x_2^3 + u$$

Around the equilibrium point $x_{10} = 1, x_{20} = 0, u_0 = 0.$

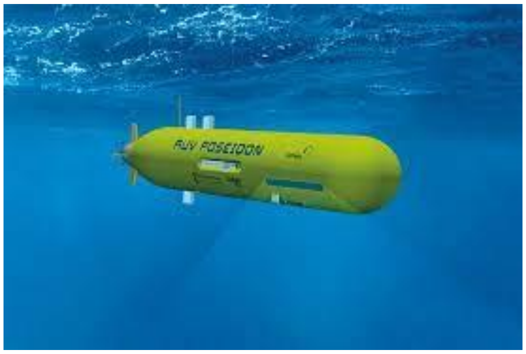
Q3. Obtain the state transition matrix, i.e., e^{At}, of the following state matrix A:

(a) $A = \mathrm{diag}\{-3,\ -5,\ -7\}$

(b) $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -6 & 0 & 0 \end{bmatrix}$

You can use MATLAB for this question (reference code is at Lecture 12 – Page 16).

Q4. Autonomous Underwater Vehicles (AUV) are robotic submarines that can be used for a variety of studies of the underwater environment. The following state-space model is a linearized model for the horizontal plane motion of the AUV.



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -0.14 & -0.69 & 0.0 \\ -0.19 & -0.048 & 0.0 \\ 0.0 & 1.0 & 0.0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0.056 \\ -0.23 \\ 0.0 \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Obtain the discrete state-space model for the system with a sampling period of 50 ms (i.e., 0.05 s). You can use MATLAB for this question (reference code is at Lecture 12 – Page 24).

Q5. Find the Equilibrium State of the following system:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.5 & -0.1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Q6. Determine the Asymptotic Stability of the following systems:

(a) $$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.1 & 0 \\ 1 & 0.2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \mathbf{u}(k)$$

$$y(k) = [1 \quad 1] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

(b) $$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} -0.2 & 0.2 & 0 \\ 0 & 1 & 0.1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \mathbf{u}(k)$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

Q7. Determine the Controllability and Observability of the following systems (You can use MATLAB for matrix calculation):

(a)
$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.1 & 0 \\ 1 & 0.2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \mathbf{u}(k)$$

$$y(k) = [1 \quad 1] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

(b)
$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} -0.2 & 0.2 & 0 \\ 0 & 1 & 0.1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \mathbf{u}(k)$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

Q8. For the following system pair, determine the state feedback control action K to assign the eigenvalues to the set $\{-2 \pm 2j\}$

$$A = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(Hint: You need to show manual calculation procedures)

Q9: Determine the state estimator gain L for the following system pair to assign the observer eigenvalues set as $\{-8, -10\}$:

$$A = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

(Hint: You need to show manual calculation procedures)