# Lesson 6: Relational Algebra

CSC430/530 – DATABASE MANAGEMENT SYSTEMS

# OUTLINE

- Introduction.

- Unary relational operations.
  - SELECT, PROJECT, RENAME.

- Set theory relational operations.
  - UNION, INTERSECTION, SET DIFFERENCE, CARTESIAN PRODUCT.

- Binary relational operations.
  - JOIN, DIVISION.

- Additional relational operations.
  - Aggregation and grouping.

# INTRODUCTION

- Why relational algebra?
  - Formal foundation for **relational** model **operations**.
  - Basis of **query implementation** and **optimization**.
  - Many concepts are incorporated in **SQL**.

- Relational algebra defines the basic **set of operations** for the relational data model.
  - Operations allow user to specify basic **retrieval** request.
    - Result of such a retrieval query is a **new relation**.
    - New relation can be further manipulated using same operations, forming **relational algebra expression**.

# UNARY RELATIONAL OPERATIONS: SELECT (1)

- Selects a **subset** of the tuples from a relation based on a **selection condition**.
  - Denoted by σ (*sigma*).
  - Acts as a **filter** – keeps only those tuples that **satisfy** the selection condition.

- σ<sub>\<selection condition\></sub>(R)
  - Selection condition – **Boolean expression** on attributes of relation R.
    - *\<attribute name\>\<comparison op\>\<constant value\> or \<attribute name\>\<comparison op\>\<attribute name\>*

- **Examples**:
  - *Retrieve the EMPLOYEE tuples whose department number is 4:*

    σ <sub>Dno = 4</sub> (EMPLOYEE)
  - *Retrieve the EMPLOYEE tuples whose salary is greater than $30,000:*

    σ <sub>Salary > 30000</sub> (EMPLOYEE)

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|-------|-------|-------|-----|-------|---------|-----|--------|-----------|-----|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

- **Properties**:
  - Operation produces new relation that has **same attributes** as R.
  - The number of **tuples** in resulting relation is always **less than** (or equal to) the number of tuples in R.
  - Operation is **commutative**.
    - $\sigma_{<cond1>}(\sigma_{<cond2>}(R)) = \sigma_{<cond2>}(\sigma_{<cond1>}(R))$
  - Can be **cascaded** into single operation.
    - $\sigma_{<cond1>}(\sigma_{<cond2>}(\sigma_{<cond3>}(R))) = \sigma_{<cond1> \text{ AND } <cond2> \text{ AND } <cond3>}(R)$

# UNARY RELATIONAL OPERATIONS: SELECT (3)

- **Example**: *Retrieve employees who either work in department 4 and make over $25,000 per year or work in department 5 and make over $30,000.*

  - $\sigma_{\text{(Dno = 4 AND Salary > 25000) OR (Dno = 5 AND Salary > 30000)}}$ (EMPLOYEE)

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |

# UNARY RELATIONAL OPERATIONS: PROJECT (1)

- Keeps certain **columns** (*attributes*) from a relation and **discards** the other columns.
  - Only the **list of specified** columns (*attributes*) is kept in each tuple.
  - Denoted by $\pi$ (pi).

- $\pi_{\text{<attribute list>}}(R)$
  - Attribute list – **desired list** of attributes from relation R.

- **Example**:
  - *List each employee's last name, first name, and salary.*

    $\pi_{\text{Lname, Fname, Salary}}(EMPLOYEE)$

- Removes any **duplicate** tuples.
  - Because the result of operation is a new **relation** (*set of tuples*).

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|-------|-------|-------|-----|-------|---------|-----|--------|-----------|-----|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

- **Properties**:
  - The number of **tuples** in the resulting relation is **less than** or equal to the number of tuples in R.
    - **Equals** only when the list of attributes includes a **key** of R.
  - Operation is not (*generally*) **commutative**.
    - $\pi_{<list1>} (\pi_{<list2>} (R)) = \pi_{<list2>} (\pi_{<list1>} (R))$, only if <list2> contains the **same attributes** as in <list1>.

- **Examples**:
  - $\pi_{\text{Lname, Fname, Salary}}$ (EMPLOYEE)

| Lname | Fname | Salary |
|---|---|---|
| Smith | John | 30000 |
| Wong | Franklin | 40000 |
| Zelaya | Alicia | 25000 |
| Wallace | Jennifer | 43000 |
| Narayan | Ramesh | 38000 |
| English | Joyce | 25000 |
| Jabbar | Ahmad | 25000 |
| Borg | James | 55000 |

- $\pi_{\text{Sex, Salary}}$ (EMPLOYEE)

| Sex | Salary |
|---|---|
| M | 30000 |
| M | 40000 |
| F | 25000 |
| F | 43000 |
| M | 38000 |
| M | 25000 |
| M | 55000 |

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

# UNARY RELATIONAL OPERATIONS: RENAME (1)

- Relational operations can be **nested** to an arbitrary depth.
  - *Retrieve first name, last name and salary of employees who work for department 5.*
  - $\pi_{\text{Fname, Lname, Salary}} (\sigma_{\text{Dno} = 5} (\text{EMPLOYEE}))$

- Alternatively, same operation can be **rewritten** by the **sequence** of operation, giving **names** to intermediate relations.
  - DEP5_EMPS $\leftarrow \sigma_{\text{Dno} = 5} (\text{EMPLOYEE})$
  
    RESULT $\leftarrow \pi_{\text{Fname, Lname, Salary}} (\text{DEP5\_EMPS})$

- **Attributes names** can also be **renamed** if needed.
  - TEMP $\leftarrow \sigma_{\text{Dno} = 5} (\text{EMPLOYEE})$
  
    R(First_name, Last_name, Salary) $\leftarrow \pi_{\text{Fname, Lname, Salary}} (\text{TEMP})$

# UNARY RELATIONAL OPERATIONS: RENAME (2)

- General **RENAME** operation ρ (*rho*) can be expressed by any of the following forms:
  - ρ $_{S (B1, B2, ..., Bn)}$ (R) changes both:
    - the relation name to S, and
    - the column (attribute) names to $B_1$, $B_2$, ..., $B_n$.
  - ρ $_S$ (R) changes:
    - the relation name only, to S.
  - ρ $_{(B1, B2, ..., Bn )}$ (R) changes:
    - the column (attribute) names only, to $B_1$, $B_2$, ..., $B_n$.

- **Example**: *Retrieve first name, last name and salary of employees who work for department 5.*
  - $\pi_{Fname, Lname, Salary} (\sigma_{Dno=5} (EMPLOYEE))$

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

| Fname | Lname | Salary |
|---|---|---|
| John | Smith | 30000 |
| Franklin | Wong | 40000 |
| Ramesh | Narayan | 38000 |
| Joyce | English | 25000 |

- $TEMP \leftarrow \sigma_{Dno = 5} (EMPLOYEE)$

  $R(First\_name, Last\_name, Salary) \leftarrow \pi_{Fname, Lname, Salary} (TEMP)$

**TEMP**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston,TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston,TX | M | 40000 | 888665555 | 5 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble,TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |

**R**

| First_name | Last_name | Salary |
|---|---|---|
| John | Smith | 30000 |
| Franklin | Wong | 40000 |
| Ramesh | Narayan | 38000 |
| Joyce | English | 25000 |

# UNION, INTERSECTION & SET DIFFERENCE (1)

- **Union** operation:  A $\cup$ B.
  - The result is a **relation** that includes all tuples that are **either** in **A or** in **B or** in **both**.
  - **Duplicate** tuples are **eliminated**.

- **Intersection** operation:  A $\cap$ B.
  - The result is a **relation** that includes all tuples that are in **both A** and **B**.
  - The **attribute names** in the result will be the **same** as the attribute names in A.

- **Set difference** operation:  A – B.
  - The result is a **relation** that includes all tuples that are in **A** but **not** in **B**.
  - The **attribute names** in the result will be the **same** as the attribute names in A.

- The two operand relations A and B must be **type compatible**.
  - **Same number** of attributes.
  - **Pair** of **attributes** must be from **same** (*compatible*) **domain**.
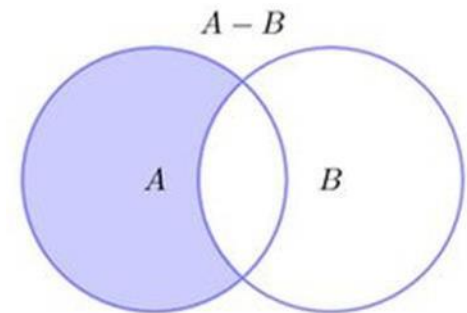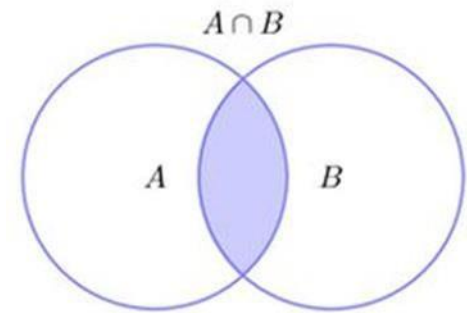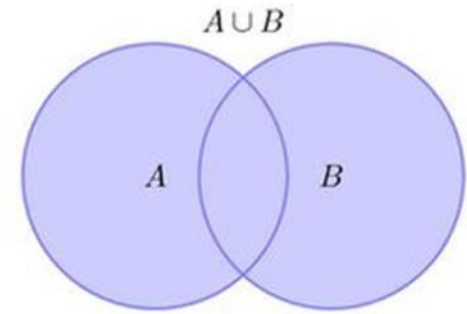
# UNION, INTERSECTION & SET DIFFERENCE (2)

- **Examples**:
  - (a) Two compatible relations.
  - (b) STUDENT ∪ INSTRUCTOR.
  - (c) STUDENT ∩ INSTRUCTOR.
  - (d) STUDENT – INSTRUCTOR.
  - (e) INSTRUCTOR – STUDENT.

**(a) STUDENT**

| Fn | Ln |
|---|---|
| Susan | Yao |
| Ramesh | Shah |
| Johnny | Kohler |
| Barbara | Jones |
| Amy | Ford |
| Jimmy | Wang |
| Ernest | Gilbert |

**INSTRUCTOR**

| Fname | Lname |
|---|---|
| John | Smith |
| Ricardo | Browne |
| Susan | Yao |
| Francis | Johnson |
| Ramesh | Shah |

**(b)**

| Fn | Ln |
|---|---|
| Susan | Yao |
| Ramesh | Shah |
| Johnny | Kohler |
| Barbara | Jones |
| Amy | Ford |
| Jimmy | Wang |
| Ernest | Gilbert |
| John | Smith |
| Ricardo | Browne |
| Francis | Johnson |

**(c)**

| Fn | Ln |
|---|---|
| Susan | Yao |
| Ramesh | Shah |

**(d)**

| Fn | Ln |
|---|---|
| Johnny | Kohler |
| Barbara | Jones |
| Amy | Ford |
| Jimmy | Wang |
| Ernest | Gilbert |

**(e)**

| Fname | Lname |
|---|---|
| John | Smith |
| Ricardo | Browne |
| Francis | Johnson |

- **Properties**:
  - **Union** & **intersection** are **commutative**.
    - $A \cup B = B \cup A$, and $A \cap B = B \cap A$
  - **Union** & **intersection** are **associative** and can be treated as **n-ary** operations.
    - $A \cup (B \cup C) = (A \cup B) \cup C$
    - $(A \cap B) \cap C = A \cap (B \cap C)$
  - **Set difference** is NOT **commutative**.
    - $A - B \neq B - A$

# CARTESIAN PRODUCT (1)

- Used to **combine tuples** from two relations in a **combinatorial** fashion.
  - $R(A_1, A_2, ..., A_n)$ x $S(B_1, B_2, ..., B_m)$

- Results in a **relation Q** with *n + m* **attributes**.
  - $Q(A_1, A_2, ..., A_n, B_1, B_2, ..., B_m)$, in that order.

- **State** of **relation Q** has one tuple for each **combination** of tuples – one from **R** and one from **S**.

# CARTESIAN PRODUCT (2)

- Generally, the result of **Cartesian product** is not **meaningful**.
  - It only becomes meaningful when **followed** by other **operations**.

- **Example**: *List names of female employees and their dependents.*
  - FEMALE_EMPS ← $\sigma_{Sex = 'F'}$ (EMPLOYEE)

    EMPNAMES ← $\pi_{Fname, Lname, Ssn}$ (FEMALE_EMPS)

    EMP_DEPENDENTS ← EMPNAMES x DEPENDENT

**DEPENDENT**

| Essn | Dependent_name | Sex | Bdate | Relationship |
|---|---|---|---|---|
| 333445555 | Alice | F | 1986-04-05 | Daughter |
| 333445555 | Theodore | M | 1983-10-25 | Son |
| 333445555 | Joy | F | 1958-05-03 | Spouse |
| 987654321 | Abner | M | 1942-02-28 | Spouse |
| 123456789 | Michael | M | 1988-01-04 | Son |
| 123456789 | Alice | F | 1988-12-30 | Daughter |
| 123456789 | Elizabeth | F | 1967-05-05 | Spouse |

**FEMALE_EMPS**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| Alicia | J | Zelaya | 999887777 | 1968-07-19 | 3321Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |

**EMPNAMES**

| Fname | Lname | Ssn |
|---|---|---|
| Alicia | Zelaya | 999887777 |
| Jennifer | Wallace | 987654321 |
| Joyce | English | 453453453 |

- Generally, the result of **Cartesian product** is not **meaningful**.
  - It only becomes meaningful when **followed** by other **operations**.

- **Example**: *List names of female employees and their dependents.*
  - FEMALE_EMPS ← σ $_{Sex = 'F'}$ (EMPLOYEE)
    EMPNAMES ← π $_{Fname, Lname, Ssn}$ (FEMALE_EMPS)
    EMP_DEPENDENTS ← EMPNAMES x DEPENDENT

**EMP_DEPENDENTS**

| Fname | Lname | Ssn | Essn | Dependent_name | Sex | Bdate | . . . |
|-------|-------|-----|------|----------------|-----|-------|-------|
| Alicia | Zelaya | 999887777 | 333445555 | Alice | F | 1986-04-05 | . . . |
| Alicia | Zelaya | 999887777 | 333445555 | Theodore | M | 1983-10-25 | . . . |
| Alicia | Zelaya | 999887777 | 333445555 | Joy | F | 1958-05-03 | . . . |
| Alicia | Zelaya | 999887777 | 987654321 | Abner | M | 1942-02-28 | . . . |
| Alicia | Zelaya | 999887777 | 123456789 | Michael | M | 1988-01-04 | . . . |
| Alicia | Zelaya | 999887777 | 123456789 | Alice | F | 1988-12-30 | . . . |
| Alicia | Zelaya | 999887777 | 123456789 | Elizabeth | F | 1967-05-05 | . . . |
| Jennifer | Wallace | 987654321 | 333445555 | Alice | F | 1986-04-05 | . . . |
| Jennifer | Wallace | 987654321 | 333445555 | Theodore | M | 1983-10-25 | . . . |
| Jennifer | Wallace | 987654321 | 333445555 | Joy | F | 1958-05-03 | . . . |
| Jennifer | Wallace | 987654321 | 987654321 | Abner | M | 1942-02-28 | . . . |
| Jennifer | Wallace | 987654321 | 123456789 | Michael | M | 1988-01-04 | . . . |
| Jennifer | Wallace | 987654321 | 123456789 | Alice | F | 1988-12-30 | . . . |
| Jennifer | Wallace | 987654321 | 123456789 | Elizabeth | F | 1967-05-05 | . . . |
| Joyce | English | 453453453 | 333445555 | Alice | F | 1986-04-05 | . . . |
| Joyce | English | 453453453 | 333445555 | Theodore | M | 1983-10-25 | . . . |
| Joyce | English | 453453453 | 333445555 | Joy | F | 1958-05-03 | . . . |
| Joyce | English | 453453453 | 987654321 | Abner | M | 1942-02-28 | . . . |
| Joyce | English | 453453453 | 123456789 | Michael | M | 1988-01-04 | . . . |
| Joyce | English | 453453453 | 123456789 | Alice | F | 1988-12-30 | . . . |
| Joyce | English | 453453453 | 123456789 | Elizabeth | F | 1967-05-05 | . . . |

- Generally, the result of **Cartesian product** is not **meaningful**.
  - It only becomes meaningful when **followed** by other **operations**.

- **Example**: *List names of female employees and their dependents.*

  - FEMALE_EMPS ← σ $_{Sex = 'F'}$ (EMPLOYEE)
    EMPNAMES ← π $_{Fname, Lname, Ssn}$ (FEMALE_EMPS)
    EMP_DEPENDENTS ← EMPNAMES x DEPENDENT

  - To make it meaningful, we can add **Select** and **Project** operations.

    ACTUAL_DEPS ← σ $_{Ssn = Essn}$ (EMP_DEPENDENTS)

    RESULT ← π $_{Fname, Lname, Dependent\_name}$ (ACTUAL_DEPS)

**EMP_DEPENDENTS**

| Fname | Lname | Ssn | Essn | Dependent_name | Sex | Bdate | . . . |
|---|---|---|---|---|---|---|---|
| Alicia | Zelaya | 999887777 | 333445555 | Alice | F | 1986-04-05 | . . . |
| Alicia | Zelaya | 999887777 | 333445555 | Theodore | M | 1983-10-25 | . . . |
| Alicia | Zelaya | 999887777 | 333445555 | Joy | F | 1958-05-03 | . . . |
| Alicia | Zelaya | 999887777 | 987654321 | Abner | M | 1942-02-28 | . . . |
| Alicia | Zelaya | 999887777 | 123456789 | Michael | M | 1988-01-04 | . . . |
| Alicia | Zelaya | 999887777 | 123456789 | Alice | F | 1988-12-30 | . . . |
| Alicia | Zelaya | 999887777 | 123456789 | Elizabeth | F | 1967-05-05 | . . . |
| Jennifer | Wallace | 987654321 | 333445555 | Alice | F | 1986-04-05 | . . . |
| Jennifer | Wallace | 987654321 | 333445555 | Theodore | M | 1983-10-25 | . . . |
| Jennifer | Wallace | 987654321 | 333445555 | Joy | F | 1958-05-03 | . . . |
| Jennifer | Wallace | 987654321 | 987654321 | Abner | M | 1942-02-28 | . . . |
| Jennifer | Wallace | 987654321 | 123456789 | Michael | M | 1988-01-04 | . . . |
| Jennifer | Wallace | 987654321 | 123456789 | Alice | F | 1988-12-30 | . . . |
| Jennifer | Wallace | 987654321 | 123456789 | Elizabeth | F | 1967-05-05 | . . . |
| Joyce | English | 453453453 | 333445555 | Alice | F | 1986-04-05 | . . . |
| Joyce | English | 453453453 | 333445555 | Theodore | M | 1983-10-25 | . . . |
| Joyce | English | 453453453 | 333445555 | Joy | F | 1958-05-03 | . . . |
| Joyce | English | 453453453 | 987654321 | Abner | M | 1942-02-28 | . . . |
| Joyce | English | 453453453 | 123456789 | Michael | M | 1988-01-04 | . . . |
| Joyce | English | 453453453 | 123456789 | Alice | F | 1988-12-30 | . . . |
| Joyce | English | 453453453 | 123456789 | Elizabeth | F | 1967-05-05 | . . . |

**ACTUAL_DEPENDENTS**

| Fname | Lname | Ssn | Essn | Dependent_name | Sex | Bdate | . . . |
|---|---|---|---|---|---|---|---|
| Jennifer | Wallace | 987654321 | 987654321 | Abner | M | 1942-02-28 | . . . |

**RESULT**

| Fname | Lname | Dependent_name |
|---|---|---|
| Jennifer | Wallace | Abner |

# BINARY RELATIONAL OPERATIONS: JOIN (1)

- Combines **related** tuples from two relations into **single** "longer" **tuple**.
  - Allows processing **relationships** among relations.
  - Denoted by ⋈

- R⋈<sub><join condition></sub> S
  - Join condition is specified on **attributes** from two relations and is **evaluated** for **each combination** of tuples.
  - If condition is True then the combination is included in the resulting relation as a **single combined tuple**.

- Each **join condition** has a general form $A_i \; \Theta \; B_j$, where
  - $A_i$ is an attribute of R.
  - $B_j$ is an attribute of S.
  - $\Theta$ (theta) is one of the comparisons operators ($=, >, \geq, <, \leq, \neq$).

- **Example**: *List department managers.*
  - DEPT_MGR ← DEPARTMENT ⋈ $_{Mgr\_ssn = Ssn}$ EMPLOYEE

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

**DEPARTMENT**

| Dname | Dnumber | Mgr_ssn | Mgr_start_date |
|---|---|---|---|
| Research | 5 | 333445555 | 1988-05-22 |
| Administration | 4 | 987654321 | 1995-01-01 |
| Headquarters | 1 | 888665555 | 1981-06-19 |

**DEPT_MGR**

| Dname | Dnumber | Mgr_ssn | · · · | Fname | Minit | Lname | Ssn | · · · |
|---|---|---|---|---|---|---|---|---|
| Research | 5 | 333445555 | · · · | Franklin | T | Wong | 333445555 | · · · |
| Administration | 4 | 987654321 | · · · | Jennifer | S | Wallace | 987654321 | · · · |
| Headquarters | 1 | 888665555 | · · · | James | E | Borg | 888665555 | · · · |

- **Special cases** of Join operation:
  - **EQUIJOIN**.
    - Only **"="** comparison operator is used.
    - Most **common** use of JOIN operation.
    - Results in one or more pairs of attributes that have **identical values** in every tuple.
  - **NATURAL JOIN**.
    - **Removes** second (*superfluous*) attribute in EQUIJOIN.
    - Two **join attributes** must have the same **name** in both relations.
      - A **renaming** operation might be applied first.
    - Denoted by **\*.**

# BINARY RELATIONAL OPERATIONS: JOIN (4)

- **Examples**: *List departments' locations.*
  - DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS

**DEPARTMENT**

| Dname | Dnumber | Mgr_ssn | Mgr_start_date |
|-------|---------|---------|----------------|
| Research | 5 | 333445555 | 1988-05-22 |
| Administration | 4 | 987654321 | 1995-01-01 |
| Headquarters | 1 | 888665555 | 1981-06-19 |

**DEPT_LOCATIONS**

| Dnumber | Dlocation |
|---------|-----------|
| 1 | Houston |
| 4 | Stafford |
| 5 | Bellaire |
| 5 | Sugarland |
| 5 | Houston |

**DEPT_LOCS**

| Dname | Dnumber | Mgr_ssn | Mgr_start_date | Location |
|-------|---------|---------|----------------|----------|
| Headquarters | 1 | 888665555 | 1981-06-19 | Houston |
| Administration | 4 | 987654321 | 1995-01-01 | Stafford |
| Research | 5 | 333445555 | 1988-05-22 | Bellaire |
| Research | 5 | 333445555 | 1988-05-22 | Sugarland |
| Research | 5 | 333445555 | 1988-05-22 | Houston |

# BINARY RELATIONAL OPERATIONS: JOIN (4)

- **Examples**: *List projects' departments.*
  - PROJ_DEPT ← PROJECT * ρ$_{(Dname, Dnum, Mgr\_ssn, Mgr\_start\_date)}$ (DEPARTMENT)

**PROJECT**

| Pname | Pnumber | Plocation | Dnum |
|---|---|---|---|
| ProductX | 1 | Bellaire | 5 |
| ProductY | 2 | Sugarland | 5 |
| ProductZ | 3 | Houston | 5 |
| Computerization | 10 | Stafford | 4 |
| Reorganization | 20 | Houston | 1 |
| Newbenefits | 30 | Stafford | 4 |

**DEPARTMENT**

| Dname | Dnumber | Mgr_ssn | Mgr_start_date |
|---|---|---|---|
| Research | 5 | 333445555 | 1988-05-22 |
| Administration | 4 | 987654321 | 1995-01-01 |
| Headquarters | 1 | 888665555 | 1981-06-19 |

**PROJ_DEPT**

| Pname | Pnumber | Plocation | Dnum | Dname | Mgr_ssn | Mgr_start_date |
|---|---|---|---|---|---|---|
| ProductX | 1 | Bellaire | 5 | Research | 333445555 | 1988-05-22 |
| ProductY | 2 | Sugarland | 5 | Research | 333445555 | 1988-05-22 |
| ProductZ | 3 | Houston | 5 | Research | 333445555 | 1988-05-22 |
| Computerization | 10 | Stafford | 4 | Administration | 987654321 | 1995-01-01 |
| Reorganization | 20 | Houston | 1 | Headquarters | 888665555 | 1981-06-19 |
| Newbenefits | 30 | Stafford | 4 | Administration | 987654321 | 1995-01-01 |

# COMPLETE SET OF RELATIONAL OPERATIONS

- The set of operations $\{\sigma, \pi, \cup, -, \rho, x\}$ is called a **complete set**.
  - Any other relational algebra expression can be **expressed** by a **combination** of these operations.

- **Examples:**
  - $R \cap S = (R \cup S) - ((R - S) \cup (S - R))$
  - $R \bowtie _{<join\ condition>} S = \sigma _{<join\ condition>} (R\ x\ S)$

# BINARY RELATIONAL OPERATIONS: DIVISION

- Denoted by ÷ and applied to two relations $R(Z) \div S(X) = T(Y)$.
  - **Attributes** of *S* are a **subset** of the **attributes** of *R*.
  - Tuples in *S* **restrict** *R* by selecting those tuples that **match** all values present in S.

- **Example**:
  - *Retrieve the names of employees who work on all the projects that John Smith works on.*
    - SMITH ← σ <sub>Fname = 'John" AND Lname = 'Smith'</sub> (EMPLOYEE)

      SMITH_PNOS ← π <sub>Pno</sub> (WORKS_ON ⋈ <sub>Essn = Ssn</sub> SMITH)

      SSN_PNOS ← π <sub>Essn, Pno</sub> (WORKS_ON)

      SSNS(Ssn) ← SSN_PNOS ÷ SMITH_PNOS

      RESULT ← π <sub>Fname, Lname</sub> (SSNS * EMPLOYEE)

**WORKS_ON**

| Essn | Pno | Hours |
|---|---|---|
| 123456789 | 1 | 32.5 |
| 123456789 | 2 | 7.5 |
| 666884444 | 3 | 40.0 |
| 453453453 | 1 | 20.0 |
| 453453453 | 2 | 20.0 |
| 333445555 | 2 | 10.0 |
| 333445555 | 3 | 10.0 |
| 333445555 | 10 | 10.0 |
| 333445555 | 20 | 10.0 |
| 999887777 | 30 | 30.0 |
| 999887777 | 10 | 10.0 |
| 987987987 | 10 | 35.0 |
| 987987987 | 30 | 5.0 |
| 987654321 | 30 | 20.0 |
| 987654321 | 20 | 15.0 |
| 888665555 | 20 | NULL |

**SMITH_PNOS**

| Pno |
|---|
| 1 |
| 2 |

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

# BINARY RELATIONAL OPERATIONS: DIVISION

- Denoted by ÷ and applied to two relations $R(Z) \div S(X) = T(Y)$.
  - **Attributes** of $S$ are a **subset** of the **attributes** of $R$.
  - Tuples in $S$ **restrict** $R$ by selecting those tuples that **match** all values present in S.

- **Example**:
  - *Retrieve the names of employees who work on all the projects that John Smith works on.*
    - SMITH ← σ $_{Fname = 'John'' AND Lname = 'Smith'}$ (EMPLOYEE)

      SMITH_PNOS ← π $_{Pno}$ (WORKS_ON ⋈ $_{Essn = Ssn}$ SMITH)

      SSN_PNOS ← π $_{Essn, Pno}$ (WORKS_ON)

      SSNS(Ssn) ← SSN_PNOS ÷ SMITH_PNOS

      RESULT ← π $_{Fname, Lname}$ (SSNS * EMPLOYEE)

**SSN_PNOS**

| Essn | Pno |
|---|---|
| 123456789 | 1 |
| 123456789 | 2 |
| 666884444 | 3 |
| 453453453 | 1 |
| 453453453 | 2 |
| 333445555 | 2 |
| 333445555 | 3 |
| 333445555 | 10 |
| 333445555 | 20 |
| 999887777 | 30 |
| 999887777 | 10 |
| 987987987 | 10 |
| 987987987 | 30 |
| 987654321 | 30 |
| 987654321 | 20 |
| 888665555 | 20 |

**SMITH_PNOS**

| Pno |
|---|
| 1 |
| 2 |

**SSNS**

| Ssn |
|---|
| 123456789 |
| 453453453 |

**RESULT**

| Fname | Lname |
|---|---|
| John | Smith |
| Joyce | English |

# SUMMARY OF RELATIONAL OPERATIONS (1)

| OPERATION | PURPOSE | NOTATION |
|---|---|---|
| SELECT | Selects all tuples that satisfy the selection condition from a relation $R$. | $\sigma_{<\text{selection condition}>}(R)$ |
| PROJECT | Produces a new relation with only some of the attributes of $R$, and removes duplicate tuples. | $\pi_{<\text{attribute list}>}(R)$ |
| THETA JOIN | Produces all combinations of tuples from $R_1$ and $R_2$ that satisfy the join condition. | $R_1 \bowtie_{<\text{join condition}>} R_2$ |
| EQUIJOIN | Produces all the combinations of tuples from $R_1$ and $R_2$ that satisfy a join condition with only equality comparisons. | $R_1 \bowtie_{<\text{join condition}>} R_2$, OR $R_1 \bowtie_{(<\text{join attributes 1}>), (<\text{join attributes 2}>)} R_2$ |
| NATURAL JOIN | Same as EQUIJOIN except that the join attributes of $R_2$ are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all. | $R_1 *_{<\text{join condition}>} R_2$, OR $R_1 *_{(<\text{join attributes 1}>), (<\text{join attributes 2}>)} R_2$ OR $R_1 * R_2$ |

# SUMMARY OF RELATIONAL OPERATIONS (2)

| OPERATION | PURPOSE | NOTATION |
|---|---|---|
| UNION | Produces a relation that includes all the tuples in $R_1$ or $R_2$ or both $R_1$ and $R_2$; $R_1$ and $R_2$ must be union compatible. | $R_1 \cup R_2$ |
| INTERSECTION | Produces a relation that includes all the tuples in both $R_1$ and $R_2$; $R_1$ and $R_2$ must be union compatible. | $R_1 \cap R_2$ |
| DIFFERENCE | Produces a relation that includes all the tuples in $R_1$ that are not in $R_2$; $R_1$ and $R_2$ must be union compatible. | $R_1 - R_2$ |
| CARTESIAN PRODUCT | Produces a relation that has the attributes of $R_1$ and $R_2$ and includes as tuples all possible combinations of tuples from $R_1$ and $R_2$. | $R_1 \times R_2$ |
| DIVISION | Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in $R_1$ in combination with every tuple from $R_2(Y)$, where $Z = X \cup Y$. | $R_1(Z) \div R_2(Y)$ |

# AGGREGATION & GROUPING (1)

- Specifying simple **mathematical aggregate** functions on collections of values from the database cannot be **directly expressed** in the **basic** relational algebra.
  - For example - get **average** or **total** salary of all employees or the **total number** of employee tuples.
    - **SUM**, **AVERAGE**, **MAXIMUM**, **MINIMUM**, **COUNT**.

- **Grouping operation and aggregate functions** are used for this purpose.

  - <grouping attributes> $\Im$ <function list> (R)
    - Grouping attributes - list of **attributes** of the relation R.
    - Function list - list of (*<function> <attribute>*) pairs.

# AGGREGATION & GROUPING (2)

- **Example**: *Retrieve each department number, the number of employees in the department, and their average salary.*

  - $\rho_{R(Dno, No\_of\_employees, Average\_sal)}(_{Dno}\mathfrak{I}_{COUNT\ Ssn,\ AVERAGE\ Salary}(EMPLOYEE))$

**R**

| Dno | No_of_employees | Average_sal |
|-----|-----------------|-------------|
| 5   | 4               | 33250       |
| 4   | 3               | 31000       |
| 1   | 1               | 55000       |

  - $_{Dno}\mathfrak{I}_{COUNT\ Ssn,\ AVERAGE\ Salary}(EMPLOYEE)$

| Dno | Count_ssn | Average_salary |
|-----|-----------|----------------|
| 5   | 4         | 33250          |
| 4   | 3         | 31000          |
| 1   | 1         | 55000          |

  - $\mathfrak{I}_{COUNT\ Ssn,\ AVERAGE\ Salary}(EMPLOYEE)$

| Count_ssn | Average_salary |
|-----------|----------------|
| 8         | 35125          |

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|-------|-------|-------|-----|-------|---------|-----|--------|-----------|-----|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

# SUMMARY

- Unary operations.
  - Select, project, rename.

- Set theory operations.
  - Union, intersection, set difference, Cartesian product.

- Binary operations.
  - Joins, division.

- Additional relational operations.
  - Aggregation, grouping.

- **Query 1.**
  - Retrieve the name and address of all employees who work for the 'Research' department.

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|-------|-------|-------|-----|-------|---------|-----|--------|-----------|-----|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

**DEPARTMENT**

| Dname | Dnumber | Mgr_ssn | Mgr_start_date |
|-------|---------|---------|----------------|
| Research | 5 | 333445555 | 1988-05-22 |
| Administration | 4 | 987654321 | 1995-01-01 |
| Headquarters | 1 | 888665555 | 1981-06-19 |

# EXAMPLES OF QUERIES

- **Query 1.**
  - Retrieve the name and address of all employees who work for the 'Research' department.

$$\text{RESEARCH\_DEPT} \leftarrow \sigma_{Dname='Research'}(\text{DEPARTMENT})$$
$$\text{RESEARCH\_EMPS} \leftarrow (\text{RESEARCH\_DEPT} \bowtie_{Dnumber=Dno} \text{EMPLOYEE})$$
$$\text{RESULT} \leftarrow \pi_{Fname, Lname, Address}(\text{RESEARCH\_EMPS})$$

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

**DEPARTMENT**

| Dname | Dnumber | Mgr_ssn | Mgr_start_date |
|---|---|---|---|
| Research | 5 | 333445555 | 1988-05-22 |
| Administration | 4 | 987654321 | 1995-01-01 |
| Headquarters | 1 | 888665555 | 1981-06-19 |

# EXAMPLES OF QUERIES

- **Query 1.**
  - Retrieve the name and address of all employees who work for the 'Research' department.

$$\text{RESEARCH\_DEPT} \leftarrow \sigma_{\text{Dname='Research'}}(\text{DEPARTMENT})$$
$$\text{RESEARCH\_EMPS} \leftarrow (\text{RESEARCH\_DEPT} \bowtie_{\text{Dnumber=Dno}} \text{EMPLOYEE})$$
$$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname, Address}}(\text{RESEARCH\_EMPS})$$

  - Single in-line version:

$$\pi_{\text{Fname, Lname, Address}} (\sigma_{\text{Dname='Research'}}(\text{DEPARTMENT} \bowtie_{\text{Dnumber=Dno}}(\text{EMPLOYEE})))$$

  - **Note:** This query could be specified in other ways. For example, the order of the JOIN and SELECT operations could be reversed, or the JOIN could be replaced by a NATURAL JOIN after renaming one of the join attributes to match the other join attribute name.

# EXAMPLES OF QUERIES

- **Query 2.**
  - For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

**PROJECT**

| Pname | Pnumber | Plocation | Dnum |
|---|---|---|---|
| ProductX | 1 | Bellaire | 5 |
| ProductY | 2 | Sugarland | 5 |
| ProductZ | 3 | Houston | 5 |
| Computerization | 10 | Stafford | 4 |
| Reorganization | 20 | Houston | 1 |
| Newbenefits | 30 | Stafford | 4 |

**DEPARTMENT**

| Dname | Dnumber | Mgr_ssn | Mgr_start_date |
|---|---|---|---|
| Research | 5 | 333445555 | 1988-05-22 |
| Administration | 4 | 987654321 | 1995-01-01 |
| Headquarters | 1 | 888665555 | 1981-06-19 |

- **Query 2.**
  - For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.

$$\text{STAFFORD\_PROJS} \leftarrow \sigma_{Plocation='Stafford'}(\text{PROJECT})$$
$$\text{CONTR\_DEPTS} \leftarrow (\text{STAFFORD\_PROJS} \bowtie_{Dnum=Dnumber} \text{DEPARTMENT})$$
$$\text{PROJ\_DEPT\_MGRS} \leftarrow (\text{CONTR\_DEPTS} \bowtie_{Mgr\_ssn=Ssn} \text{EMPLOYEE})$$
$$\text{RESULT} \leftarrow \pi_{Pnumber, Dnum, Lname, Address, Bdate}(\text{PROJ\_DEPT\_MGRS})$$

**PROJECT**

| Pname | Pnumber | Plocation | Dnum |
|---|---|---|---|
| ProductX | 1 | Bellaire | 5 |
| ProductY | 2 | Sugarland | 5 |
| ProductZ | 3 | Houston | 5 |
| Computerization | 10 | Stafford | 4 |
| Reorganization | 20 | Houston | 1 |
| Newbenefits | 30 | Stafford | 4 |

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

**DEPARTMENT**

| Dname | Dnumber | Mgr_ssn | Mgr_start_date |
|---|---|---|---|
| Research | 5 | 333445555 | 1988-05-22 |
| Administration | 4 | 987654321 | 1995-01-01 |
| Headquarters | 1 | 888665555 | 1981-06-19 |

- **Query 2.**
  - For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birth date.

$$STAFFORD\_PROJS \leftarrow \sigma_{Plocation='Stafford'}(PROJECT)$$
$$CONTR\_DEPTS \leftarrow (STAFFORD\_PROJS \bowtie_{Dnum=Dnumber} DEPARTMENT)$$
$$PROJ\_DEPT\_MGRS \leftarrow (CONTR\_DEPTS \bowtie_{Mgr\_ssn=Ssn} EMPLOYEE)$$
$$RESULT \leftarrow \pi_{Pnumber, Dnum, Lname, Address, Bdate}(PROJ\_DEPT\_MGRS)$$

  - **Note:** In this example, we first select the projects located in Stafford, then join them with their controlling departments, and then join the result with the department managers. Finally, we apply a project operation on the desired attributes.

- **Query 3.**
  - Find the names of employees who work on all the projects controlled by department number 5.

**WORKS_ON**

| Essn | Pno | Hours |
|---|---|---|
| 123456789 | 1 | 32.5 |
| 123456789 | 2 | 7.5 |
| 666884444 | 3 | 40.0 |
| 453453453 | 1 | 20.0 |
| 453453453 | 2 | 20.0 |
| 333445555 | 2 | 10.0 |
| 333445555 | 3 | 10.0 |
| 333445555 | 10 | 10.0 |
| 333445555 | 20 | 10.0 |
| 999887777 | 30 | 30.0 |
| 999887777 | 10 | 10.0 |
| 987987987 | 10 | 35.0 |
| 987987987 | 30 | 5.0 |
| 987654321 | 30 | 20.0 |
| 987654321 | 20 | 15.0 |
| 888665555 | 20 | NULL |

**PROJECT**

| Pname | Pnumber | Plocation | Dnum |
|---|---|---|---|
| ProductX | 1 | Bellaire | 5 |
| ProductY | 2 | Sugarland | 5 |
| ProductZ | 3 | Houston | 5 |
| Computerization | 10 | Stafford | 4 |
| Reorganization | 20 | Houston | 1 |
| Newbenefits | 30 | Stafford | 4 |

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

- **Query 3.**
  - Find the names of employees who work on all the projects controlled by department number 5.

$$\text{DEPT5\_PROJS} \leftarrow \rho_{(Pno)}(\pi_{Pnumber}(\sigma_{Dnum=5}(PROJECT)))$$
$$\text{EMP\_PROJ} \leftarrow \rho_{(Ssn, Pno)}(\pi_{Essn, Pno}(WORKS\_ON))$$
$$\text{RESULT\_EMP\_SSNS} \leftarrow \text{EMP\_PROJ} \div \text{DEPT5\_PROJS}$$
$$\text{RESULT} \leftarrow \pi_{Lname, Fname}(\text{RESULT\_EMP\_SSNS} * \text{EMPLOYEE})$$

**WORKS_ON**

| Essn | Pno | Hours |
|------|-----|-------|
| 123456789 | 1 | 32.5 |
| 123456789 | 2 | 7.5 |
| 666884444 | 3 | 40.0 |
| 453453453 | 1 | 20.0 |
| 453453453 | 2 | 20.0 |
| 333445555 | 2 | 10.0 |
| 333445555 | 3 | 10.0 |
| 333445555 | 10 | 10.0 |
| 333445555 | 20 | 10.0 |
| 999887777 | 30 | 30.0 |
| 999887777 | 10 | 10.0 |
| 987987987 | 10 | 35.0 |
| 987987987 | 30 | 5.0 |
| 987654321 | 30 | 20.0 |
| 987654321 | 20 | 15.0 |
| 888665555 | 20 | NULL |

**PROJECT**

| Pname | Pnumber | Plocation | Dnum |
|-------|---------|-----------|------|
| ProductX | 1 | Bellaire | 5 |
| ProductY | 2 | Sugarland | 5 |
| ProductZ | 3 | Houston | 5 |
| Computerization | 10 | Stafford | 4 |
| Reorganization | 20 | Houston | 1 |
| Newbenefits | 30 | Stafford | 4 |

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|-------|-------|-------|-----|-------|---------|-----|--------|-----------|-----|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

# EXAMPLES OF QUERIES

- **Query 3.**
  - Find the names of employees who work on all the projects controlled by department number 5.

$$\text{DEPT5\_PROJS} \leftarrow \rho_{(Pno)}(\pi_{Pnumber}(\sigma_{Dnum=5}(\text{PROJECT})))$$
$$\text{EMP\_PROJ} \leftarrow \rho_{(Ssn, Pno)}(\pi_{Essn, Pno}(\text{WORKS\_ON}))$$
$$\text{RESULT\_EMP\_SSNS} \leftarrow \text{EMP\_PROJ} \div \text{DEPT5\_PROJS}$$
$$\text{RESULT} \leftarrow \pi_{Lname, Fname}(\text{RESULT\_EMP\_SSNS} * \text{EMPLOYEE})$$

- **Note:** In this query, we first create a table DEPT5_PROJS that contains the project numbers of all projects controlled by department 5. Then we create a table EMP_PROJ that holds (Ssn, Pno) tuples, and apply the division operation. Notice that we renamed the attributes so that they will be correctly used in the division operation. Finally, we join the result of the division, which holds only Ssn values, with the EMPLOYEE table to retrieve the Fname, Lname attributes from EMPLOYEE.

# EXAMPLES OF QUERIES

- **Query 4.**
  - Make a list of project numbers for projects that involve an employee whose last name is 'Smith', either as a worker or as a manager of the department that controls the project.

$$SMITHS(Essn) \leftarrow \pi_{Ssn} (\sigma_{Lname='Smith'}(EMPLOYEE))$$
$$SMITH\_WORKER\_PROJS \leftarrow \pi_{Pno}(WORKS\_ON * SMITHS)$$
$$MGRS \leftarrow \pi_{Lname, Dnumber}(EMPLOYEE \bowtie_{Ssn=Mgr\_ssn} DEPARTMENT)$$
$$SMITH\_MANAGED\_DEPTS(Dnum) \leftarrow \pi_{Dnumber} (\sigma_{Lname='Smith'}(MGRS))$$
$$SMITH\_MGR\_PROJS(Pno) \leftarrow \pi_{Pnumber}(SMITH\_MANAGED\_DEPTS * PROJECT)$$
$$RESULT \leftarrow (SMITH\_WORKER\_PROJS \cup SMITH\_MGR\_PROJS)$$

- **Note:** In this query, we retrieved the project numbers for projects that involve an employee named Smith as a worker in SMITH_WORKER_PROJS. Then we retrieved the project numbers for projects that involve an employee named Smith as manager of the department that controls the project in SMITH_MGR_PROJS. Finally, we applied the UNION operation on SMITH_WORKER_PROJS and SMITH_MGR_PROJS.

# EXAMPLES OF QUERIES

- **Query 5.**
  - List the names of all employees with more than two dependents.

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|-------|-------|-------|-----|-------|---------|-----|--------|-----------|-----|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

**DEPENDENT**

| Essn | Dependent_name | Sex | Bdate | Relationship |
|------|----------------|-----|-------|--------------|
| 333445555 | Alice | F | 1986-04-05 | Daughter |
| 333445555 | Theodore | M | 1983-10-25 | Son |
| 333445555 | Joy | F | 1958-05-03 | Spouse |
| 987654321 | Abner | M | 1942-02-28 | Spouse |
| 123456789 | Michael | M | 1988-01-04 | Son |
| 123456789 | Alice | F | 1988-12-30 | Daughter |
| 123456789 | Elizabeth | F | 1967-05-05 | Spouse |

- **Query 5.**
  - List the names of all employees with more than two dependents.

$$T1(Ssn, No\_of\_dependents) \leftarrow {}_{Essn}\mathfrak{J}_{COUNT\ Dependent\_name}(DEPENDENT)$$

$$T2 \leftarrow \sigma_{No\_of\_dependents>2}(T1)$$

$$RESULT \leftarrow \pi_{Lname,\ Fname}(T2 * EMPLOYEE)$$

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

**DEPENDENT**

| Essn | Dependent_name | Sex | Bdate | Relationship |
|---|---|---|---|---|
| 333445555 | Alice | F | 1986-04-05 | Daughter |
| 333445555 | Theodore | M | 1983-10-25 | Son |
| 333445555 | Joy | F | 1958-05-03 | Spouse |
| 987654321 | Abner | M | 1942-02-28 | Spouse |
| 123456789 | Michael | M | 1988-01-04 | Son |
| 123456789 | Alice | F | 1988-12-30 | Daughter |
| 123456789 | Elizabeth | F | 1967-05-05 | Spouse |

# EXAMPLES OF QUERIES

- **Query 5.**
  - List the names of all employees with more than two dependents.

$$T1(\text{Ssn, No\_of\_dependents}) \leftarrow {}_{\text{Essn}} \Im_{\text{COUNT Dependent\_name}}(\text{DEPENDENT})$$
$$T2 \leftarrow \sigma_{\text{No\_of\_dependents}>2}(T1)$$
$$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(T2 * \text{EMPLOYEE})$$

  - **Note:** For this query, we have to use the AGGREGATE FUNCTION operation with the COUNT aggregate function. We assume that dependents of the same employee have distinct Dependent_name values.

- **Query 6.**
  - Retrieve the names of employees who have no dependents.

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|-------|-------|-------|-----|-------|---------|-----|--------|-----------|-----|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

**DEPENDENT**

| Essn | Dependent_name | Sex | Bdate | Relationship |
|------|----------------|-----|-------|--------------|
| 333445555 | Alice | F | 1986-04-05 | Daughter |
| 333445555 | Theodore | M | 1983-10-25 | Son |
| 333445555 | Joy | F | 1958-05-03 | Spouse |
| 987654321 | Abner | M | 1942-02-28 | Spouse |
| 123456789 | Michael | M | 1988-01-04 | Son |
| 123456789 | Alice | F | 1988-12-30 | Daughter |
| 123456789 | Elizabeth | F | 1967-05-05 | Spouse |

# EXAMPLES OF QUERIES

- **Query 6.**
  - Retrieve the names of employees who have no dependents.

$$\text{ALL\_EMPS} \leftarrow \pi_{Ssn}(\text{EMPLOYEE})$$
$$\text{EMPS\_WITH\_DEPS}(Ssn) \leftarrow \pi_{Essn}(\text{DEPENDENT})$$
$$\text{EMPS\_WITHOUT\_DEPS} \leftarrow (\text{ALL\_EMPS} - \text{EMPS\_WITH\_DEPS})$$
$$\text{RESULT} \leftarrow \pi_{Lname, Fname}(\text{EMPS\_WITHOUT\_DEPS} * \text{EMPLOYEE})$$

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

**DEPENDENT**

| Essn | Dependent_name | Sex | Bdate | Relationship |
|---|---|---|---|---|
| 333445555 | Alice | F | 1986-04-05 | Daughter |
| 333445555 | Theodore | M | 1983-10-25 | Son |
| 333445555 | Joy | F | 1958-05-03 | Spouse |
| 987654321 | Abner | M | 1942-02-28 | Spouse |
| 123456789 | Michael | M | 1988-01-04 | Son |
| 123456789 | Alice | F | 1988-12-30 | Daughter |
| 123456789 | Elizabeth | F | 1967-05-05 | Spouse |

# EXAMPLES OF QUERIES

- **Query 6.**
  - Retrieve the names of employees who have no dependents.

$$ALL\_EMPS \leftarrow \pi_{Ssn}(EMPLOYEE)$$
$$EMPS\_WITH\_DEPS(Ssn) \leftarrow \pi_{Essn}(DEPENDENT)$$
$$EMPS\_WITHOUT\_DEPS \leftarrow (ALL\_EMPS - EMPS\_WITH\_DEPS)$$
$$RESULT \leftarrow \pi_{Lname, Fname}(EMPS\_WITHOUT\_DEPS * EMPLOYEE)$$

  - **Note:** We first retrieve a relation with all employee Ssns in ALL_EMPS. Then we create a table with the Ssns of employees who have at least one dependent in EMPS_WITH_DEPS. Then we apply the SET DIFFERENCE operation to retrieve employees Ssns with no dependents in EMPS_WITHOUT_DEPS, and finally join this with EMPLOYEE to retrieve the desired attributes.

- **Query 7.**
  - List the names of managers who have at least one dependent.

**DEPENDENT**

| Essn | Dependent_name | Sex | Bdate | Relationship |
|------|----------------|-----|-------|--------------|
| 333445555 | Alice | F | 1986-04-05 | Daughter |
| 333445555 | Theodore | M | 1983-10-25 | Son |
| 333445555 | Joy | F | 1958-05-03 | Spouse |
| 987654321 | Abner | M | 1942-02-28 | Spouse |
| 123456789 | Michael | M | 1988-01-04 | Son |
| 123456789 | Alice | F | 1988-12-30 | Daughter |
| 123456789 | Elizabeth | F | 1967-05-05 | Spouse |

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|-------|-------|-------|-----|-------|---------|-----|--------|-----------|-----|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

**DEPARTMENT**

| Dname | Dnumber | Mgr_ssn | Mgr_start_date |
|-------|---------|---------|----------------|
| Research | 5 | 333445555 | 1988-05-22 |
| Administration | 4 | 987654321 | 1995-01-01 |
| Headquarters | 1 | 888665555 | 1981-06-19 |

- **Query 7.**
  - List the names of managers who have at least one dependent.

$$MGRS(Ssn) \leftarrow \pi_{Mgr\_ssn}(DEPARTMENT)$$
$$EMPS\_WITH\_DEPS(Ssn) \leftarrow \pi_{Essn}(DEPENDENT)$$
$$MGRS\_WITH\_DEPS \leftarrow (MGRS \cap EMPS\_WITH\_DEPS)$$
$$RESULT \leftarrow \pi_{Lname,\ Fname}(MGRS\_WITH\_DEPS * EMPLOYEE)$$

**DEPENDENT**

| Essn | Dependent_name | Sex | Bdate | Relationship |
|---|---|---|---|---|
| 333445555 | Alice | F | 1986-04-05 | Daughter |
| 333445555 | Theodore | M | 1983-10-25 | Son |
| 333445555 | Joy | F | 1958-05-03 | Spouse |
| 987654321 | Abner | M | 1942-02-28 | Spouse |
| 123456789 | Michael | M | 1988-01-04 | Son |
| 123456789 | Alice | F | 1988-12-30 | Daughter |
| 123456789 | Elizabeth | F | 1967-05-05 | Spouse |

**EMPLOYEE**

| Fname | Minit | Lname | Ssn | Bdate | Address | Sex | Salary | Super_ssn | Dno |
|---|---|---|---|---|---|---|---|---|---|
| John | B | Smith | 123456789 | 1965-01-09 | 731 Fondren, Houston, TX | M | 30000 | 333445555 | 5 |
| Franklin | T | Wong | 333445555 | 1955-12-08 | 638 Voss, Houston, TX | M | 40000 | 888665555 | 5 |
| Alicia | J | Zelaya | 999887777 | 1968-01-19 | 3321 Castle, Spring, TX | F | 25000 | 987654321 | 4 |
| Jennifer | S | Wallace | 987654321 | 1941-06-20 | 291 Berry, Bellaire, TX | F | 43000 | 888665555 | 4 |
| Ramesh | K | Narayan | 666884444 | 1962-09-15 | 975 Fire Oak, Humble, TX | M | 38000 | 333445555 | 5 |
| Joyce | A | English | 453453453 | 1972-07-31 | 5631 Rice, Houston, TX | F | 25000 | 333445555 | 5 |
| Ahmad | V | Jabbar | 987987987 | 1969-03-29 | 980 Dallas, Houston, TX | M | 25000 | 987654321 | 4 |
| James | E | Borg | 888665555 | 1937-11-10 | 450 Stone, Houston, TX | M | 55000 | NULL | 1 |

**DEPARTMENT**

| Dname | Dnumber | Mgr_ssn | Mgr_start_date |
|---|---|---|---|
| Research | 5 | 333445555 | 1988-05-22 |
| Administration | 4 | 987654321 | 1995-01-01 |
| Headquarters | 1 | 888665555 | 1981-06-19 |

# EXAMPLES OF QUERIES

- **Query 7.**
  - List the names of managers who have at least one dependent.

$$\text{MGRS(Ssn)} \leftarrow \pi_{\text{Mgr\_ssn}}(\text{DEPARTMENT})$$
$$\text{EMPS\_WITH\_DEPS(Ssn)} \leftarrow \pi_{\text{Essn}}(\text{DEPENDENT})$$
$$\text{MGRS\_WITH\_DEPS} \leftarrow (\text{MGRS} \cap \text{EMPS\_WITH\_DEPS})$$
$$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(\text{MGRS\_WITH\_DEPS} * \text{EMPLOYEE})$$

- **Note:** In this query, we retrieve the Ssns of managers in MGRS, and the Ssns of employees with at least one dependent in EMPS_WITH_DEPS, then we apply the SET INTERSECTION operation to get the Ssns of managers who have at least one dependent.