Questoin 3:

Using only the five axioms and two theorems stated above as assumptions, prove that there are at least four students in the club.
Justify why each line of your proof follows from the assumptions or previous lines.
[Note: your proof may not require the use of all of the axioms and theorems; do not worry if one or more of the axioms/theorems seem irrelevant to your answer to this question.]

Prove:

Since proving “there are at least 4 student in the club” is the same as proving “there cannot less than 4 student in the club” ∩ "there can be 4 student in the club";

\

- **Part 1: Proving there can be 4 member in a club**
  Let K representing the club, assumethere are 4 members in the K,the members are x, y, z, w such that $\exists! x, y, z, w \in K$.
  Given by axiom ©, every pair of student have one committee in common $\therefore \exists$ committee A, B, C, D, E, F such that $\{\exists! x, y \in A \cap \exists! x, z \in B \cap \exists! x, w \in C \cap \exists! y, z \in D \cap \exists! y, w \in E \cap \exists! z, w \in F\}$
  The relation can be shown in the following graph

Club	Committee	student
K	A	x, y
K	B	x, z
K	C	x, w
K	D	y, z
K	E	y, w
K	F	z, w

  Checking for each axiom:
  (a) Since there are 4 student in the club, the statement follows the axiom a

(b) In this case every comittee is made of 2 student , the statment follows the axiom b

© In this case for each pair of student $\{x, y; x, z; x, w; y, z; y, w; z, w\}$ there is one comittee every pair both serve which is $\{A, B, C, D, E, F\}$, the statment follows the axiom c

(d)Here no committee is no such committee G such that $\{x, y, z, w \in G\}$, the statment follows the axiom d

(e) For committee A, z, w are members that is not in it, both z, w have one and only committee F such that $\{x, y \in A \cap x, y \notin F\}$; similarly, for $\{x, z \in B\}$ $\{y, z \in C\}$ $\{x, z \in D\}$ $\{y, w \in E\}$ $\{z, w \in F\}$, there are respectively $\{x, z \notin E\}$ $\{x, w \notin D\}$ $\{y, z \notin C\}$ $\{y, w \notin B\}$ $\{z, w \notin A\}$, the statment follows the axiom e/

$\therefore$ the previous distribution of 4 student into 6 committees in 1 club fllows all the axioms.

$\therefore$ the statement is true.

\

- **Part 2: Proving there cannot be only 3 members in a club**

  Let K representing the club, assumethere are 3 members in the K,the members are x, y, z such that $\exists! x, y, z \in K$.

  Given by axiom ©, every pair of student have one committee in common: $\therefore \exists$ committees A, B, C, such that $\{A : x, y \in A\} \cap \{B : x, z \in B\} \cap \{C : y, z \in C\}$;

  Given by axiom (d), there is no single committe containning all the members of the club $\therefore \nexists$ committee containning members x, y, z at the same time

  Given by axiom (e), $\forall$ members not in a committee, $\exists$ a committee that none of the members in first committee are contained: $\exists$ committe $\therefore \exists$ committees D, E, F such that $\{z \in D, x, y \notin D\} \cap \{y \in E, x, z \notin E\} \cap \{x \in F, y, z \notin F\}$

  Also given by the axiom (e), there is only one committee containning the members not in the committee $\therefore$ here $\{x \notin C \cap x \notin D \cap x \notin E\}$ violate the axiom given

  $\therefore \nexists$ a club K made of exactlly three members

\

- **Part 3: Proving there cannot be only 2 members in a club**
  Let K representing the club, assumethere are 2 members in the K,the members are x, y such that $\exists! x, y \in K$.
  Given by the axiom © for each pair of student, there is one committee they both serve $\therefore \exists$ such committee A such that $\{x, y \in A\}$
  Given by the axiom (d) no committee can have all the student in club $\therefore$ the existance of $\{\exists! x, y \in A \cap \exists! x, y \in K\}$ voilate the axioms
  $\therefore \nexists$ a club K made of exactlly two members

\

- **Part 4: Proving there cannot be only 1 members in a club**
  Let K representing the club, assumethere are 1 members in the K,the members are x such that $\exists! x \in K$.
  Given by axiom (a), there are at least two student in one club $\therefore$ the assumption that $\{\exists! x \in\}$ violate the axiom
  $\therefore \nexists$ a club K made of exactlly one member

Here, in Part 1, there is such configuration that follows all the configurations, therefore proves the first statment “there can be four student in the club”;

Since number of student is discrete, there cannot be non-interger number of student such as a half student or a quater of student, and cannot be less than zero, “less than 4 student” is the same as “one or two or three student”. Part 1,2,3 combines and proves “there cannot be less than foru student in a club”.

Therefore “there are at least four student in the club” is a true statment $\square$
