1)

Consider a **symmetric** random walk which starts with a marker placed at a point x at time s; written (x, s). Suppose at a later time $t > s$ the marker is at y; the future state denoted (y, t). The marker can move in step sizes of δy in a time step of δt. At the previous step the marker must have been at one of $(y - \delta y, t - \delta t)$ or $(y + \delta y, t - \delta t)$. The transition probability density function of the position y of the diffusion at a later time t, is written $p(x, s; y, t)$. Derive the Forward Equation

$$\frac{\partial p}{\partial t} = \frac{1}{2}\frac{\partial^2 p}{\partial y^2}. \tag{6.1}$$

You may omit the dependence on (x, s) in your working as they will not change.
Assume a solution of (6.1) exists and takes the following form

$$p(y, t) = t^{-1/2} f(\eta)\,; \ \eta = \frac{y}{t^{1/2}}.$$

Solve (6.1) to show that a particular solution of this is

$$p(x, s; y, t) = \frac{1}{\sqrt{2\pi(t - s)}} \exp\left(-\frac{(y - x)^2}{2(t - s)}\right).$$

You may use the result $\int_{\mathbb{R}} e^{-x^2} dx = \sqrt{\pi}$, in your working.

2)

Portfolio Risk warm up. Consider a position of £5 million in a single asset X with daily volatility of 1%. What are the annualised and 10-day standard deviations? Using the Normal factor calculate 99%/10day VaR in money terms.

3)

Now, consider a portfolio of two assets X and Y, £100,000 investment each. The daily volatilities of both assets are 1% and correlation between their returns is $\rho_{XY} = 0.3$. Calculate 99%/5day Analytical VaR (in money terms) for this portfolio.

4) In the class, you computed VAR by using experimental data directly. Use the same data to find VAR and expected shortfall using an analytical model (normal distribution).