

Value at Risk and Expected Shortfall

CQF

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The Word Risk

risk, *noun*, situation involving exposure to danger.

Origin: Mid 17th century: from French *risque* (noun), *risquer* (verb), from Italian *risco* 'danger' and *rischiare* 'run into danger'.

Oxford English Dictionary

risk management is the identification, assessment, and prioritization of risks followed by coordinated and economical application of resources to minimize, monitor, and control the probability and/or impact of unfortunate events or to maximize the realization of opportunities.

Hubbard, Douglas (2009). *The Failure of Risk Management: Why It's Broken and How to Fix It*. John Wiley & Sons.

Earthquakes around the world

VALDIVIA EARTHQUAKE

MAY
22
1960

Most powerful earthquake on record, comparable to **1,000 atomic bombs** detonating at once

\$1 billion
in damage

9.5 magnitude

Valdivia, Chile

Triggered tsunamis in Hawaii and Japan



6,000
Deaths



165,000
Injured



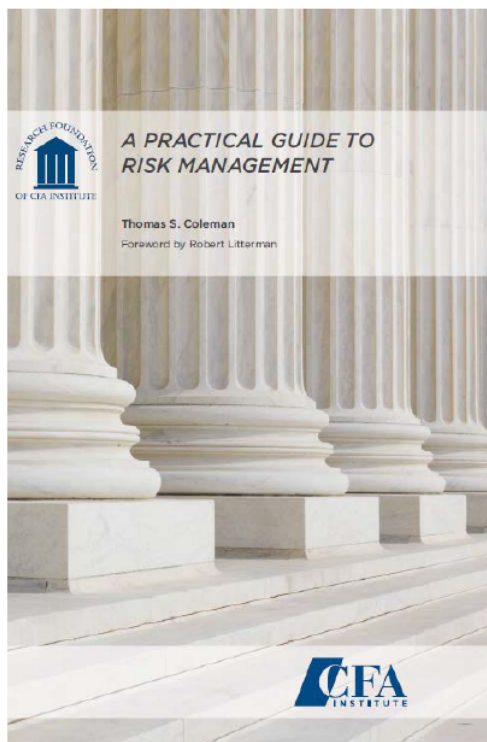
20,000
Homeless

© Mapsoft/World 2014

It occurred in the afternoon (19:11 GMT, 15:11 local time), and lasted approximately 10 minutes. **The resulting tsunami** affected southern Chile, Hawaii, Japan, the Philippines, eastern New Zealand, southeast Australia, and the Aleutian Islands.

The 1960 Valdivia earthquake or Great Chilean earthquake (Gran terremoto de Chile) of Sunday, 22 May 1960 was **the most powerful earthquake ever recorded**, rating a 9.5 on the moment magnitude scale.





Coleman, Tom, *A Practical Guide to Risk Management* (July 27, 2011). CFA Institute Research Foundation M2011-2.

Available at SSRN:

<http://ssrn.com/abstract=2586032>

Risk measurement has three goals:

- **Uncovering “known” risks** faced by the portfolio or the firm. By “known” risks, I mean risks that can be identified and understood with study and analysis because these or similar risks have been experienced in the past by this particular firm or others. Such risks often are not obvious or immediately apparent, possibly because of the size or diversity of a portfolio, but these risks can be uncovered with diligence.
- **Making the known risks easy to see**, understand, and compare—in other words, the effective, simple, and transparent display and reporting of risk. Value at risk, or VaR, is a popular tool in this arena, but there are other, complementary, techniques and tools.
- **Trying to understand and uncover the “unknown”** or unanticipated risks—those that may not be easy to understand or anticipate, for example, because the organization or industry has not experienced them before.

Risk representation*

Overall P&L Distribution

Table 1.3. Portfolio Sensitivity to One Standard Deviation Moves in Specific Market Risk Factors

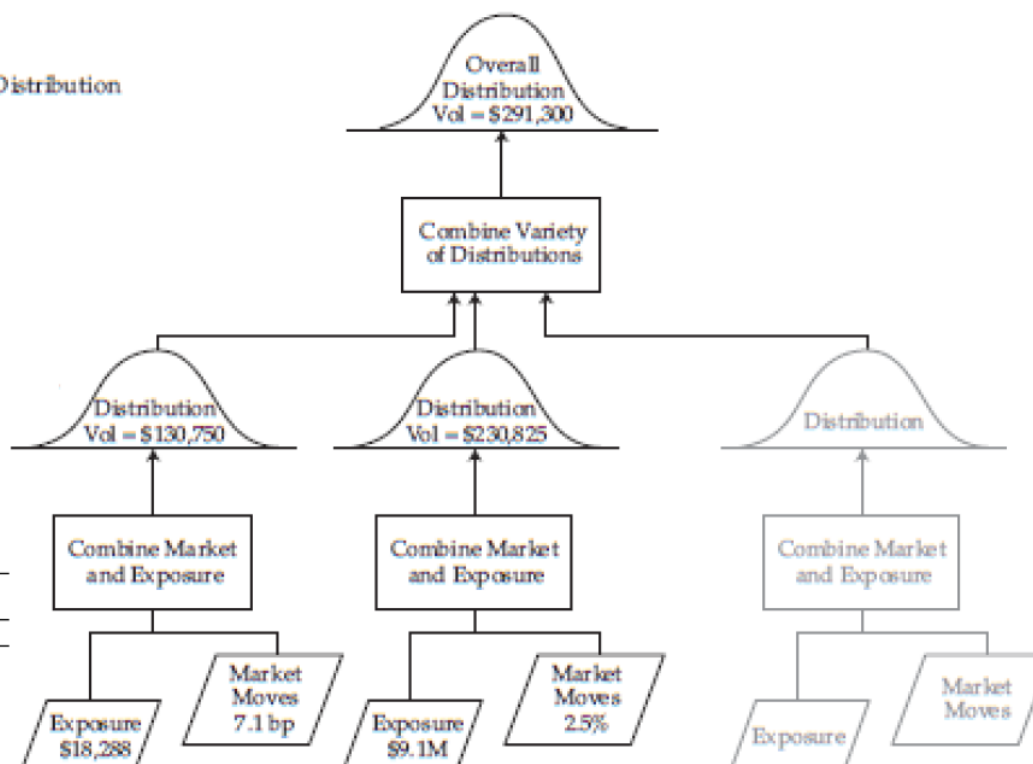
Yield Curve (yield down)		Equity (index up)	
10-year par yield	\$130,750	CAC	\$230,825

Table 1.2. Volatility or Standard Deviation of Individual Market Yield Moves

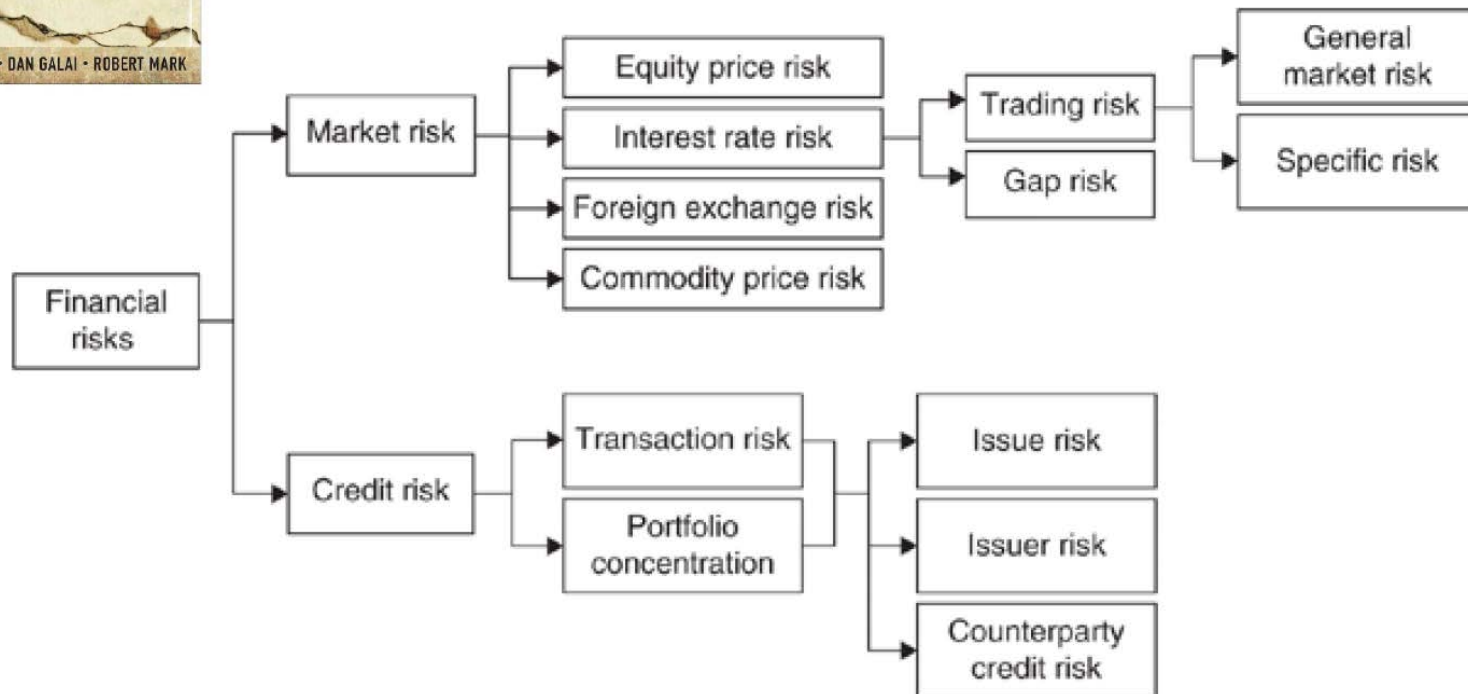
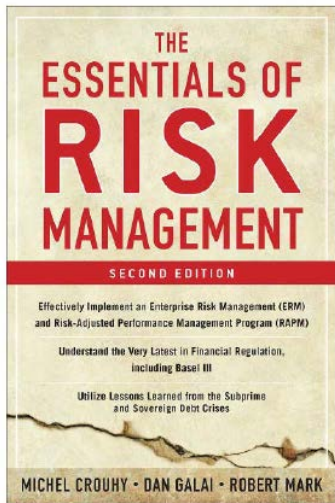
Yield Curve (bps per day)		Equity (% per day)	
10-year par yield	7.15	CAC	2.54

Table 1.1. Sample Exposure Report

Yield Curve (per 1 bp down)		Equity (beta-equivalent notional)	
10-year par yield	\$18,288	CAC	\$9,100,000

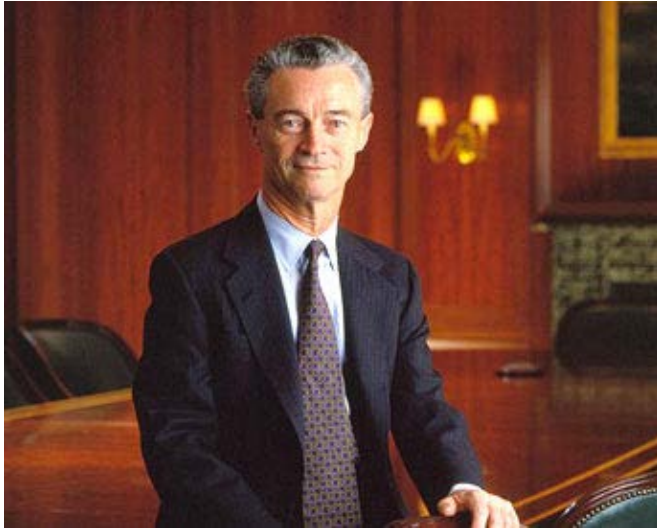


*after Coleman (2011)



Value at Risk (VaR)

JP Morgan and “inventing VaR”

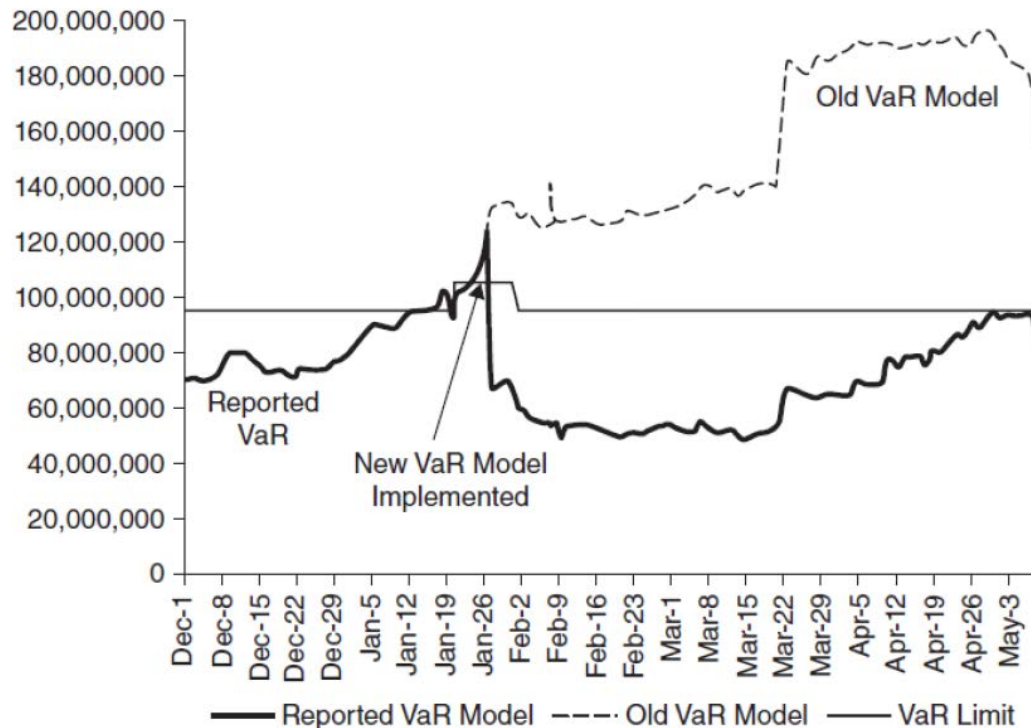


JPMorgan is credited with helping to make VaR a widely used measure. The Chairman, Sir Dennis Weatherstone was dissatisfied with the long-risk reports he received every day. These contained a huge amount of detail on the Greek letters for different exposures, but very little that was really useful to top management. He asked for something simpler that focused on the bank's total exposure over the next 24 hours measured across the bank's entire trading portfolio.

At first his subordinates said this was impossible, but eventually they adapted the Markowitz portfolio theory to develop a VaR report. This became known as the 4:15 report because it was placed on the chairman's desk at 4:15 pm every day after the close of trading.

John C. Hull, Risk Management and Financial Institutions, Wiley, 2012

JP Morgan and the “London Whale”



United States Senate Permanent Subcommittee on Investigations, *JP Morgan Chase Whale Trades: A Case History of Derivatives Risks and Abuses*, Hearing, March 15, 2013, Exhibits.

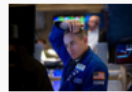
During the first half of 2012, JPMorgan Chase lost billions of dollars from exposure to a massive credit derivative portfolio.

The losses were the result of the so-called "London Whale" trades executed by traders in its London office. Initially dismissed by the bank's chief executive as a "tempest in a teapot," the trading losses quickly doubled and then tripled.

In contrast to JPMorgan Chase's reputation for best-in-class risk management, the whale trades exposed a bank culture in which risk limit breaches were routinely disregarded, risk metrics were frequently criticized or downplayed, and risk evaluation models were targeted by bank personnel seeking to produce artificially lower capital requirements."



Argentina Debt Deal
Promises Holdout
Creditors a Big
Payday



Dow Industrials
Notch Gains for
February



The Worst Market of
All: One Without a
Story



ANALYSIS
Stuck: The Problem
with China's New
Stimulus

MARKETS

‘London Whale’ Breaks Silence

Bruno Iksil says J.P. Morgan Chase made him a scapegoat

Updated Feb. 22, 2016 9:01 p.m. ET

The trader at the center of the “London whale” trading debacle broke nearly four years of silence by taking aim at former employer [J.P. Morgan Chase & Co.](#), saying he was made a scapegoat for trades that were “initiated, approved, mandated and monitored” by senior management.

Bruno Iksil also said that he resents the London whale nickname, which was devised by rival traders to dramatize the size of J.P. Morgan’s bets in corporate-debt markets.

In a single-spaced letter exceeding three pages and sent to publications including Financial News, Mr. Iksil contends the bank and the news media [misrepresented his role in the 2012 episode](#), which led to more than \$6 billion in losses for the nation’s largest bank and a handful of personnel changes.

“For no good reason, I was singled out by the media,” Mr. Iksil writes.

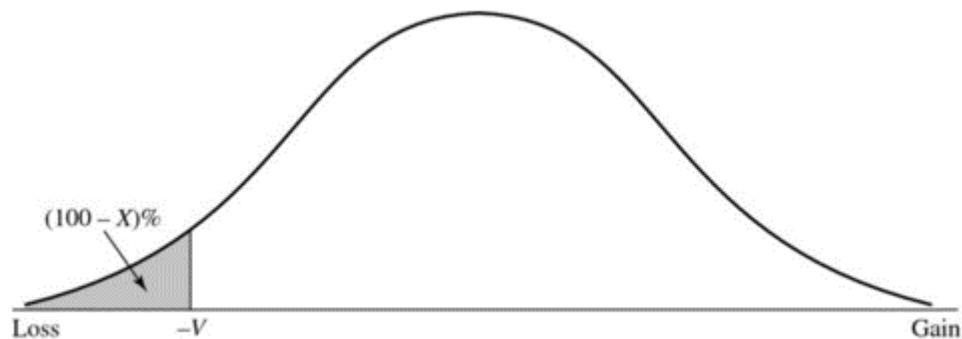
The losses represented a stain on the reputation of Chief Executive [James Dimon](#), who originally dismissed the idea that the bank was at risk of big losses.

Definition: Value at Risk (VaR)

Value-at-risk (VaR) can be defined as the worst loss that might be expected from holding a security or portfolio over a given period of time (say one day, or 10 days for the purpose of regulatory capital reporting), given a specified level of probability (confidence level).

Example:

let's say that a position has a daily VaR of \$10 million at the 99 percent confidence level, we mean that the realized daily losses from the position will on average be higher than \$10 million on only one day in every 100 trading days (i.e., two to three days each year).



Interpretation of VaR

VaR is **not** the answer to the question: *How much can I lose on my portfolio over a given period of time?* The answer to this question is "everything," or any fraction value of the portfolio.

Instead, VaR offers a probability statement about the **potential change** in the value of a portfolio resulting from a **change in market factors** over a specified period of time.

Crucially, the VaR measure also does not state by how much actual losses are likely to exceed the VaR figure; it simply states how likely (or unlikely) it is that the VaR measure will be exceeded.

Calculating VaR

First, derive the forward distribution of the returns on the portfolio, at the chosen horizon (in this case, one day).

This distribution can be derived using three different approaches: historical price distributions (nonparametric VaR); assumptions about normal distributions (parametric VaR); and Monte Carlo simulation.

This distribution is then plotted to indicate how likely it is (vertical axis) that losses of a particular dollar value (horizontal axis) will occur.

Second, identify the required percentile of this distribution so that a particular loss number can be identified.

By choosing the first percentile of the (empirical) distribution then the VaR is measured at the 99 percent confidence level. If we assume that the distribution is a normal, rather than an arbitrary distribution that could be skewed towards particularly light/heavy losses) then at a confidence level of 99 percent corresponds to a VaR of 2.33 standard deviations.

From 1-Day VaR to 10-Day VaR

$$T\text{-day VaR} = 1\text{-day VaR} \times \sqrt{T}$$

VaR is often used to manage market risk over a 1-day time horizon. For this purpose, it's necessary to derive VaR from the daily distribution of the portfolio values.

However, regulators have set a time horizon of 10 days for the purpose of VaR calculations that are used to report regulatory capital requirements. Ideally, this "10-day VaR" would be derived from a corresponding distribution of results over a 10-day horizon. This is problematic, however, as it implies that the time series of data used for the analysis must be much longer-indeed, 10 times longer-than that employed in any one-day VaR analysis.

As a result, many banks employ a work-around that allows them to derive an approximation of 10-day VaR from daily VaR data by multiplying the daily VaR by the square root of time (here, 10 days). The "square root of time" rule is endorsed by the regulators.

Methods for Estimating VaR

The Three VaR Methods:

First Method: analytic variance-covariance approach (aka parametric VaR)

Under the analytic variance-covariance approach or "delta normal" approach, we assume that the risk factors and the portfolio values are log-normally distributed or, equivalently, that their log returns (the log of the returns) are normally distributed.

This makes the calculation much simpler, since the normal distribution is completely characterized by its first two moments, and the analyst can derive the mean and the variance of the portfolio return distribution from

- (a) The multivariate distribution of the risk factors
- (b) The composition of the portfolio

$$\text{VaR} = \mu + \sigma N^{-1}(X)$$

The Three VaR Methods:

Second Method: historical simulation (aka non-parametric VaR)

The historical simulation approach to VaR calculation is conceptually simple and does not oblige the user to make any assumptions about the distribution. However, at least two or three years of historical data are necessary to produce meaningful results. Three steps are involved:

- (1) Select a sample of actual daily risk factor changes over a given period of time, say 500 days (i.e., two years' worth of trading days), using the same period of time for all the factors.
- (2) Apply those daily changes to the current value of the risk factors, revaluing the current portfolio as many times as the number of days in the historical sample. Sum these changes across all positions, keeping the days synchronized.
- (3) Construct the histogram of portfolio values and identify the VaR that isolates the first percentile of the distribution in the left-hand tail (assuming VaR is derived at the 99 percent confidence level).

Advantages

The major attraction of historical simulation is that the method is completely **nonparametric** (i.e., we don't need to worry about setting parameters) and does not depend on any assumptions about the distribution of the risk factors.

The nonparametric nature of historical simulation also obviates the need to estimate **volatilities and correlations**. Historical volatilities and correlations are already reflected in the data set, so all we need to calculate are the synchronous risk-factor returns over a given historical period.

Historical simulation has also no problem accommodating **fat tails in distributions**, since the historical returns already reflect actual synchronous moves in the market across all risk factors.

Disadvantages

The main drawback of historical simulation is its **complete dependence** on a particular set of historical data.

The underlying assumption is that the past, as captured in this historical data set, is a **reliable representation of the future**.

The Three VaR Methods:

Third Method: Monte Carlo Simulation

Consists of repeatedly simulating the random processes that govern market prices and rates. Each simulation (scenario) generates a possible value for the portfolio at the target horizon (e.g., 10 days). It involves three steps:

- (a) Specify all the relevant riskfactors, their stochastic processes and parameter estimates
- (b) Construct price paths. Price paths are constructed using random numbers produced and advancing one step at a time (daily) the numerical solution to the stochastic processes
- (c) Value the portfolio for each path (scenario).

The process is repeated a large number of times, say 10,000 times, to generate the distribution, at the risk horizon, of the portfolio return. VaR at the 99 percent confidence level is then simply derived as the distance to the mean of the first percentile of the distribution, as for our other calculation methods.

Advantages

It can **accommodate** any distribution of risk factors to allow for fat-tailed distributions, where extreme events are expected to occur more commonly than in normal distributions, and "jumps" or discontinuities in price processes.

Monte Carlo simulation, like historical simulation, allows the analyst to calculate the **confidence interval** of VaR

Monte Carlo simulation permits to carry out **sensitivity analyses** by changing the market parameters used in the analysis, such as the term structure of interest rates.

Disadvantages

Good estimates of the **parameters** of the distributions, such as the means, the variances, and the covariances are required.

A major limitation is the amount of **computer** resources it requires, especially for large complex portfolios.

Expected Shortfall

Expected Shortfall (ES)

Expected Shortfall also sometimes referred to as conditional value at risk, conditional tail expectation, or expected tail loss.

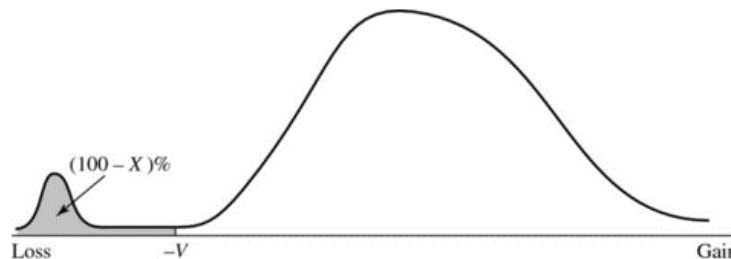
Whereas VaR asks the question: "How bad can things get?" expected shortfall asks: **"If things do get bad, what is the expected loss?"**

Expected shortfall, like VaR, is a function of two parameters: T (the time horizon) and X (the confidence level).

It is the expected loss during time T conditional on the loss being greater than the Xth percentile of the loss distribution.

Example: suppose that X=99, T is 10 days, and the VaR is \$64 million. The expected shortfall is the average amount lost over a 10-day period assuming that the loss is greater than \$64 million.

$$ES = \mu + \sigma \frac{e^{-Y^2/2}}{\sqrt{2\pi}(1-X)}$$



Extreme Value Theory (EVT)

RISK MANAGEMENT

Extreme value theory has hidden risks, research finds

Method for calculating capital based on sparse data can lead to additional model risk



Alexander Campbell

14 January 2016



Risks of extreme value theory have been hidden until now

Extreme value theory (EVT) has been hailed as a solution to the problem of calculating capital requirements based on sparse data about tail risks. But academics in Regensburg and Sydney have discovered that the use of EVT may in fact involve more model risk than traditional methods.

Extreme value theory (EVT)

EVT can be viewed as an extension of the central limit theorem, which states that the average of independent random variables tends to the normal distribution, irrespective of the original distribution. This deals with the mean, or center, of the distribution.

For risk management purposes, the tails of the distribution are of interest. The EVT theorem says that the limit distribution for values x beyond a cutoff point u belongs to the family below where $y=(x-u)/P$. To simplify, we defined the loss x as a positive number so that y is also positive.

The distribution is characterized (a) a scale parameter, and (b) a shape parameter that determines the speed at which the tail disappears.

$$F(y) = 1 - (1 + \xi y)^{-1/\xi}, \quad \xi \neq 0$$
$$F(y) = 1 - \exp(-y), \quad \xi = 0$$

Extreme value theory (EVT)

VaR, as well as CVaR, can be derived in closed-form solution from the analytical distribution of EVT.

This requires estimation of the tail parameter and of the dispersion parameter.

This can be performed using a variety of statistical approaches. One method is maximum likelihood. First, we define a cutoff point u . This needs to be chosen so that there are a sufficient number of observations in the tail. However, the theory is most valid far into the tail. A good, ad hoc, choice is to choose u so as to include 5% of the data in the tail. For example, if we have $T = 1,000$ observations, we would consider only the 50 in the left tail. Second, we consider only losses beyond u and then maximize the likelihood of the observations over the two parameters.

VaR Laboratory: General Electric



GENERAL ELECTRIC COMPANY (NYSE:GE)


[QUOTE](#) [COMPANY INFORMATION](#) [NEWS](#) [SEC FILINGS](#) [OPTIONS](#)

QUOTE

General Electric Co

LAST PRICE	DAY CHANGE	BID	BID SIZE	ASK	ASK SIZE
29.14	-0.26 -0.88%	29.13	20	29.20	4
MON FEB 29, 2016 04:00 PM USD DELAYED Closed					
OPEN	52WK LOW	52W L (DATE)	52WK HIGH	52W H (DATE)	MKT CAP
29.44	19.37	08-24-2015	31.49	12-31-2015	297.21 bil
				VOLUME	PREV CLOSE
				33.56mil	29.40

PRICE CHART

[COMPARE](#) [INDICATORS](#) [DISPLAY](#)

Mar 01, 2006

Feb 26, 2016

Daily

[1 D](#) [5 D](#) [1 Mth](#) [3 Mth](#) [YTD](#) [1 Yr](#) [3 Yr](#) [5 Yr](#) [10 Yr](#) [Max](#)

Mar 01, 2006 - Feb 26, 2016 • GE


[Share this chart](#) | [Export data to file](#)

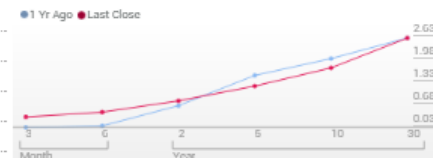
INDICES

INDEX	VALUE	CHANGE \$ (%)
NYSE Composite	\$9,559.53	-\$60.25 (-0.62%)
NYSE U.S. 100 Index	\$7,849.17	-\$75.04 (-0.94%)
Dow Jones	\$16,516.50	-\$123.47 (-0.74%)
S&P 500	\$1,932.23	-\$15.81 (-0.81%)

As of 9:31 PM EST, February 29, 2016

TREASURY YIELDS

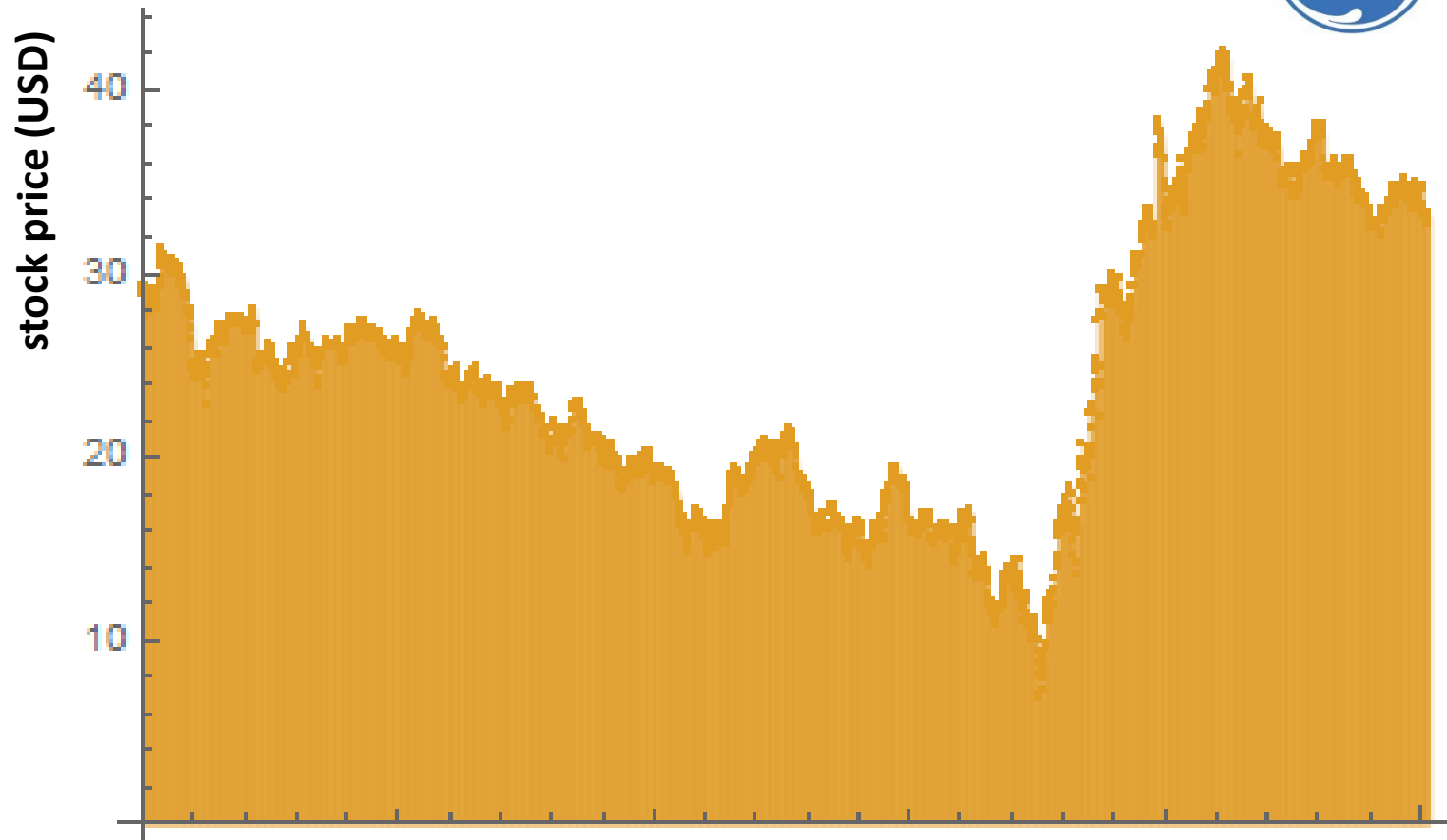
MATURITY	%YIELD	1 Yr Ago	Last Close
6 Month	0.47		
2 Years	0.80		
5 Years	1.23		
10 Years	1.76		
30 Years	2.63		



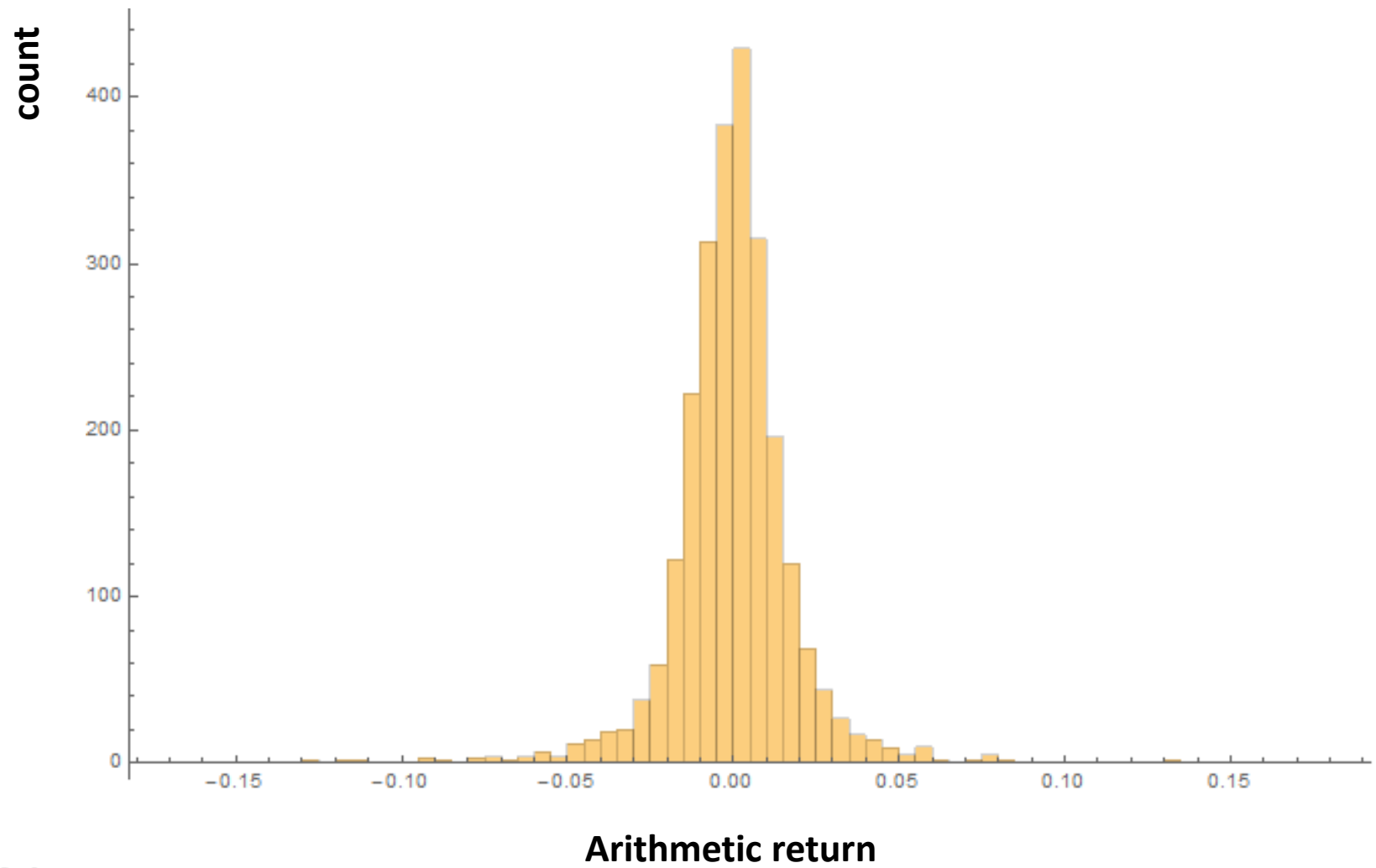
Data as of last close

COMMODITIES

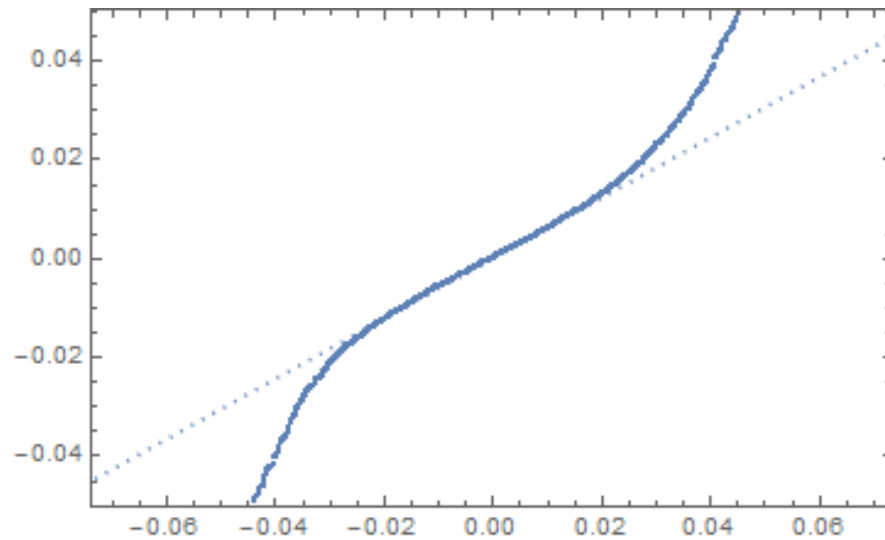
COMMODITY	EXPIRY	VALUE	CHANGE \$ (%)
NYSE Liffe Gold	Apr 16	\$1,234.00	-\$0.40 -0.03%
ICE Brent Crude	Mar 16	\$36.69	+\$0.11 +0.32%
UK Natural Gas	Mar 16	\$29.31	-\$0.03 -0.10%



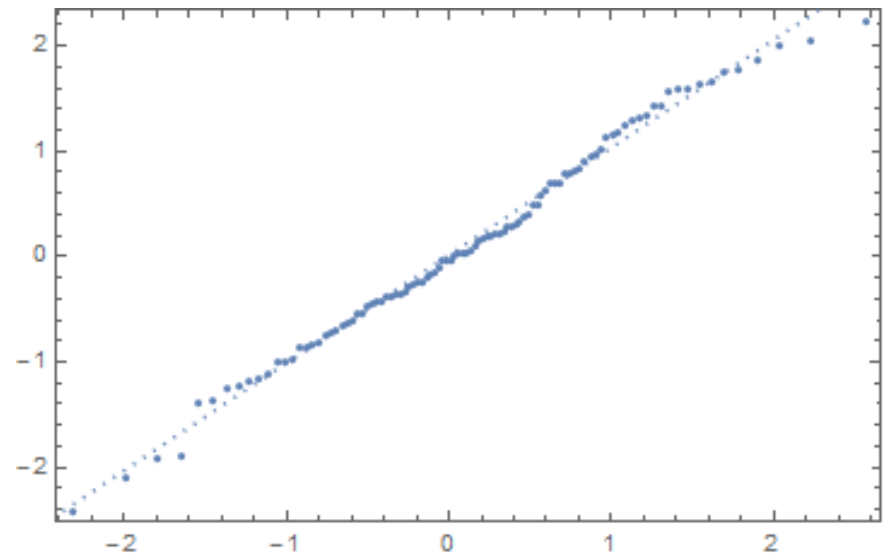
2006-2016 (ten years)



CQF



QQ plot (GE)



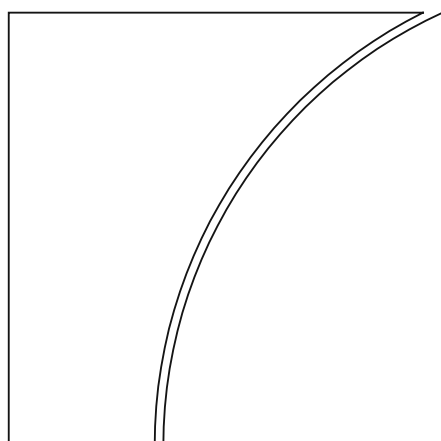
QQ plot (normal deviates)

EXCERPT

Full report publicly available at: <http://www.bis.org/bcbs/publ/d352.htm>

Basel Committee on Banking Supervision

STANDARDS



Minimum capital requirements for market risk

January 2016



BANK FOR INTERNATIONAL SETTLEMENTS

2. Structure of the standardised approach

(i) Overview of the structure of the standardised approach

47. The standardised approach capital requirement is the simple sum of three components: the risk charges under the sensitivities based method, the default risk charge, and the residual risk add-on.

- (a) The risk charge under the Sensitivities-based Method must be calculated by aggregating the following risk measures:
 - (i) *Delta*: A risk measure based on sensitivities of a bank's trading book to regulatory delta risk factors. Delta sensitivities are to be used as inputs into the aggregation formula which delivers the capital requirement for the Sensitivities-based method.
 - (ii) *Vega*: A risk measure that is also based on sensitivities to regulatory vega risk factors to be used as inputs to a similar aggregation formula as for Delta risks.
 - (iii) *Curvature*: A risk measure which captures the incremental risk not captured by the delta risk of price changes in the value of an option. Curvature risk is based on two stress scenarios involving an upward shock and a downward shock to a given risk factor. The worst loss of the two scenarios is the risk position (defined in paragraph 48) to be used as an input into the aggregation formula which delivers the capital charge.
- (b) In order to address the risk that correlations may increase or decrease in periods of financial stress, three risk charge figures must be calculated for each risk class defined under the sensitivities based method (see paragraphs 54 to 55 for details), based on three different scenarios on the specified values for the correlation parameter ρ_{kl} (ie correlation between risk factors within a bucket) and γ_{bc} (ie correlation across buckets within a risk class). There must be no diversification benefit recognised between individual risk classes.
- (c) The bank must determine each delta and vega sensitivity and curvature scenario based on instrument prices or pricing models that an independent risk control unit within a bank uses to report market risks or actual profits and losses to senior management.
- (d) The default risk charge captures the jump-to-default risk in three independent capital charge computations for default risk of non-securitisations, securitisations (non-correlation trading portfolio) and securitisation correlation trading portfolio. It is calibrated based on the credit risk treatment in the banking book in order to reduce the potential discrepancy in capital requirements for similar risk exposures across the bank. Some hedging recognition is allowed within a risk weight bucket. There must be no diversification benefit recognised between different buckets.
- (e) Additionally, the Committee acknowledges that not all market risks can be captured in the standardised approach, as this might necessitate an unduly complex regime. A residual risk add-on is thus introduced to ensure sufficient coverage of market risks.

(ii) Sensitivities-based Method: main definitions

48. The following definitions cover the main concepts of the standardised approach:

- (a) Risk class: The seven risk classes defined for the Sensitivities-based Method are general interest rate risk, credit spread risk: non-securitisation, credit spread risk: securitisations (non-correlation trading portfolio), credit spread risk: securitisations (correlation trading portfolio), equity risk, commodity risk and foreign exchange risk (defined in Section 4).
- (b) Risk factor: variables (eg a given vertex of a given interest rate curve or an equity price) within a pricing function decomposed from trading book instruments and which fall within scope of the risk factor definitions in Section 3. Risk factors are mapped to a risk class.

- (c) Risk position: the main input that enters the risk charge computation. For delta and vega risks, it is a sensitivity to a risk factor. For curvature risk, it is the worst loss of two stress scenarios.
- (d) Risk charge: the amount of capital that a bank should hold as a consequence of the risks it takes; it is computed as an aggregation of risk positions first at the bucket level, and then across buckets within a risk class defined for the Sensitivities-based Method.
- (e) Bucket: a set of risk positions which are grouped together by common characteristics, as defined within Section 4, paragraphs 74 to 121.

(iii) Sensitivities-based Method: instruments subject to delta, vega and curvature

49. A key assumption of the standardised approach for market risk is that a bank's pricing model used in actual profit and loss reporting provide an appropriate basis for the determination of regulatory capital requirements for all market risks. To ensure such adequacy, banks must at a minimum establish a framework for prudent valuation practices that include the requirements of paragraph 718(c) to 718(cxii) of Basel II. Additionally:

- (a) Each instrument with optionality¹⁰ is subject to vega risk and curvature risk. Instruments without optionality are not subject to vega risk and curvature risk.
- (b) An instrument with an embedded prepayment option¹¹ is an instrument with optionality according to paragraph 49(a). Accordingly, the embedded option is subject to vega and curvature risk with respect to the interest rate risk and credit spread risk (non-securitisation and securitisation) risk classes. When the prepayment option is a behavioural option the instrument may also be subject to the residual risk add-on as per paragraph 58. The pricing model of the bank must reflect such behavioural patterns where relevant. For securitisation tranches, instruments in the securitised portfolio may have embedded prepayment options as well. In this case the securitisation tranche may be subject to the residual risk add-on.
- (c) Instruments whose cash flows can be written as a linear function of underlying notional are instruments without optionality (eg cash flows generated by a coupon bearing bond can be written as a linear function) are not subject to vega risk nor curvature risk charges. Similarly, the cash flows generated by a plain-vanilla option cannot be written as a linear function (as they are the maximum of the spot and the strike). Therefore all options are subject to vega risk and curvature risk.
- (d) A non-exhaustive list of example instruments with optionality includes: calls, puts, caps, floors, swaptions, barrier options and exotic options.

(iv) Sensitivities-based Method: delta and vega

50. Delta and vega risks consist of a set of prescribed risk factors and sensitivities which are defined in Section 3. The *net sensitivities* for each risk factor within a risk class is multiplied by a respective risk weight provided in Section 4 and 5. These weighted sensitivities are then aggregated by prescribed formulae using correlations provided in Sections 4 and 5. This sub-section provides the aggregation

¹⁰ For example, each instrument that is an option or that includes an option (eg an embedded option such as convertibility or rate dependent prepayment and that is subject to the capital requirements for market risk).

¹¹ An instrument with a prepayment option is a debt instrument which grants the debtor the right to repay part or the entire principal amount before the contractual maturity without having to compensate for any foregone interest. The debtor can exercise this option with a financial gain to can obtain funding over the remaining maturity of the instrument at a lower rate in other ways in the market.

formula for calculating the capital requirement within each bucket, as well as the formula for calculating the capital requirement across buckets, for each risk class that is covered under the delta and vega risk framework.

51. Delta and vega risks are computed using the same aggregation formulae on all relevant risk factors in the Sensitivities-based Method. However, delta and vega risks must be calculated separately, with no diversification benefit recognised between delta and vega risk factors. Delta and vega risks are captured using the same aggregation formulae through the following step-by-step approach:

- (a) Find a net sensitivity s_k across instruments to each risk factor k (defined in Section 3). For instance, all sensitivities to the vertex 1 year of the swap curve Euribor 3 months should offset, irrespective of the instrument from which they derive.¹²
- (b) The weighted sensitivity WS_k is the product of the net sensitivity s_k and the corresponding risk weight RW_k as defined in Sections 4 and 5.

$$WS_k = RW_k s_k$$

- (c) The risk position for Delta (respectively Vega) bucket b , K_b , must be determined by aggregating the weighted sensitivities to risk factors within the same bucket using the corresponding prescribed correlation ρ_{kl} set out in the following formula:

$$K_b = \sqrt{\sum_k WS_k^2 + \sum_{k \neq l} \rho_{kl} WS_k WS_l}$$

where the quantity within the square root function is floored at zero.

- (d) The Delta (respectively Vega) risk charge is determined from risk positions aggregated between the Delta (respectively Vega) buckets within each risk class, using the corresponding prescribed correlations γ_{bc} as set out in the following formula:

$$\text{Delta (respectively Vega)} = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c}$$

where $S_b = \sum_k WS_k$ for all risk factors in bucket b and $S_c = \sum_k WS_k$ in bucket c .

If these values for S_b and S_c produce a negative number for the overall sum of $\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c$:

- The bank is to calculate the Delta (respectively Vega) risk charge using an alternative specification whereby $S_b = \max[\min(\sum_k WS_k, K_b), -K_b]$ for all risk factors in bucket b and $S_c = \max[\min(\sum_k WS_k, K_c), -K_c]$ for all risk factors in bucket c .

(v) Sensitivities-based Method: curvature

52. The curvature risk charge consists of a set of stress scenario on given risk factors which are defined in Section 3. Two stress scenarios are to be computed per risk factor (an upward shock and a downward shock) with the delta effect, already captured by the delta risk charge, being removed. The two scenarios are shocked by risk weights and the worst loss is aggregated by correlations provided in

¹² This example can be generalised as follows: if a bank's portfolio is made of two interest rate swaps on Euribor 3 months with the same fixed rate and same notional but of opposite direction, the general interest rate risk on that portfolio would be zero.

Section 6. The purpose of this subsection is to provide the aggregation formulae within buckets, and across buckets within a risk class.

53. The following step-by-step approach to capture curvature risk must be separately applied to each risk class (apart from default risk):

- (a) Find a net curvature risk charge CVR_k across instruments to each curvature risk factor k . For instance, all vertices of all the curves within a given currency (eg Euribor 3 months, Euribor 6 months, Euribor 1 year, etc for Euro) must be shifted upward. The potential loss, after deduction of the delta risk positions, is the outcome of the first scenario. The same approach must be followed on a downward scenario. The worst loss (expressed as a positive quantity), after deduction of the delta risk position, is the curvature risk position for the considered risk factor. If the price of an option depends on several risk factors, the curvature risk is determined separately for each risk factor.
- (b) The curvature risk charge for curvature risk factor k can be formally written as follows:

$$CVR_k = -\min \left[\begin{array}{l} \sum_i \{V_i(x_k^{(RW^{(curvature)+})}) - V_i(x_k) - RW_k^{(curvature)} \cdot s_{ik}\} \\ \sum_i \{V_i(x_k^{(RW^{(curvature)-})}) - V_i(x_k) + RW_k^{(curvature)} \cdot s_{ik}\} \end{array} \right]$$

where:

- i is an instrument subject to curvature risks associated with risk factor k ;
 - x_k is the current level of risk factor k ;
 - $V_i(x_k)$ is the price of instrument i depending on the current level of risk factor k ;
 - $V_i(x_k^{(RW^{(curvature)+})})$ and $V_i(x_k^{(RW^{(curvature)-})})$ both denote the price of instrument i after x_k is shifted (ie "shocked") upward and downward.
 - under the FX and Equity risk classes:
 - $RW_k^{(curvature)}$ is the risk weight for curvature risk factor k for instrument i determined in accordance with paragraph 131.
 - s_{ik} is the delta sensitivity of instrument i with respect to the delta risk factor that corresponds to curvature risk factor k .
 - under the GIRR, CSR and Commodity risk classes:
 - $RW_k^{(curvature)}$ is the risk weight for curvature risk factor k for instrument i determined in accordance with paragraph 132
 - s_{ik} is the sum of delta sensitivities to all tenors of the relevant curve of instrument i with respect to curvature risk factor k .
- (c) The aggregation formula for curvature risk distinguishes between positive curvature and negative curvature risk exposures. The negative curvature risk exposures are ignored, unless they hedge a positive curvature risk exposure. If there is a negative net curvature risk exposure from an option exposure, the curvature risk charge is zero.
 - (d) The curvature risk exposure must be aggregated within each bucket using the corresponding prescribed correlation ρ_{kl} as set out in the following formula:

$$K_b = \sqrt{\max\left(0, \sum_k \max(CVR_k, 0)^2 + \sum_k \sum_{k \neq l} \rho_{kl} CVR_k CVR_l \psi(CVR_k, CVR_l)\right)}$$

where $\psi(CVR_k, CVR_l)$ is a function that takes the value 0 if CVR_k and CVR_l both have negative signs. In all other cases, $\psi(CVR_k, CVR_l)$ takes the value of 1.

- (e) Curvature risk positions must then be aggregated across buckets within each risk class, using the corresponding prescribed correlations γ_{bc} .

$$\text{Curvature risk} = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c \psi(S_b, S_c)}$$

where:

- $S_b = \sum_k CVR_k$ for all risk factors in bucket b , and $S_c = \sum_k CVR_k$ in bucket c ; and
- $\psi(S_b, S_c)$ is a function that takes the value 0 if S_b and S_c both have negative signs. In all other cases, $\psi(S_b, S_c)$ takes the value of 1.

If these values for S_b and S_c produce a negative number for the overall sum of $\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{bc} S_b S_c \psi(S_b, S_c)$

- the bank is to calculate the curvature risk charge using an alternative specification whereby $S_b = \max[\min(\sum_k CVR_k, K_b), -K_b]$ for all risk factors in bucket b and $S_c = \max[\min(\sum_k CVR_k, K_c), -K_c]$ for all risk factors in bucket c .

(vi) **Sensitivities-based Method: correlation scenarios and aggregation of risk charges**

54. In order to address the risk that correlations increase or decrease in periods of financial stress, three risk charge figures are to be calculated for each risk class, corresponding to three different scenarios on the specified values for the correlation parameter ρ_{kl} (correlation between risk factors within a bucket) and γ_{bc} (correlation across buckets within a risk class).

- Under the first scenario, "high correlations", the correlation parameters ρ_{kl} and γ_{bc} that are specified in Sections 4, 5 and 6 are uniformly multiplied by 1.25, with ρ_{kl} and γ_{bc} subject to a cap at 100%.
- Under the second scenario, "medium correlations", the correlation parameters ρ_{kl} and γ_{bc} remain unchanged from those specified in Sections 4, 5 and 6.
- Under the third scenario, "low correlations", the corresponding prescribed correlations are the correlations given in Section 4, 5 and 6 uniformly multiplied by 0.75.

55. For each scenario, the bank must determine a scenario-related risk charge at the portfolio level as the simple sum of the risk charges at risk class level for that scenario. The ultimate portfolio level risk capital charge is the largest of the three scenario-related portfolio level risk capital charges.

(vii) **The Default Risk Charge**

56. The default risk charge is intended to capture jump-to-default-risk. It is described in detail in Section 7. The purpose of this subsection is to provide the offsetting rules as well as the hedging formula which can be applied within the default risk buckets.

57. The following step-by-step approach to capture jump-to-default risk must be followed:

- Compute the jump-to-default risk of each instrument separately. The jump-to-default risk is a function of notional amount (or face value) and market value of the instruments and prescribed LGD.

- (b) Offsetting rules are specified in Section 7, which enables the derivation of “net jump-to-default” (net JTD) risk positions.
- (c) Net JTD risk positions are then allocated to buckets and weighted by prescribed risk weights. For securitisation (both those in correlation trading portfolios and others), the risk weights are to be computed applying the banking book regime. Within a given default risk bucket, the weighted short risk positions can be deducted from the weighted long risk positions in a proportion equal to the ratio of the long divided by the sum of the long and short non-weighted risk positions. For non-securitisation and securitisation non-correlation trading portfolio, the default risk charge is then the simple sum of bucket level default risks. For the correlation trading portfolio, in order to constrain hedging benefit recognition, the default risk charge is the simple sum of the bucket level default risks when they are positive, and half the bucket level default risks when they are negative.

(viii) **The Residual Risk Add-On**

58. The residual risk add-on is to be calculated for all instruments bearing residual risk separately and in addition to other components of the capital requirement under the standardised approach for market risk.

- (a) The residual risk add-on must be calculated in addition to any other capital requirements within the standardised approach.
- (b) The scope of instruments that are subject to the residual risk add-on must not have an impact in terms of increasing or decreasing the scope of risk factors subject to the delta, vega, curvature or default risk capital treatments in the standardised approach.
- (c) The residual risk add-on is the simple sum of gross notional amounts of the instruments bearing residual risks, multiplied by a risk weight of 1.0% for instruments with an exotic underlying and a risk weight of 0.1% for instruments bearing other residual risks.¹³
- (d) Instruments with an exotic underlying are trading book instruments with an underlying exposure that is not within the scope of delta, vega or curvature risk treatment in any risk class under the Sensitivities-based Method or default risk charges in the standardised approach.¹⁴
- (e) Instruments bearing other residual risks are those that meet criteria (i) and (ii) below:
 - (i) instruments subject to vega or curvature risk capital charges in the trading book and with pay-offs that cannot be written or perfectly replicated as a finite linear combination of vanilla options with a single underlying equity price, commodity price, exchange rate, bond price, CDS price or interest rate swap; or
 - (ii) instruments which fall under the definition of the Correlation Trading Portfolio (CTP) in paragraph 61, except for those instruments which are recognised in the Market Risk Framework as eligible hedges of risks within the CTP.
- (f) In cases where a transaction exactly matches with a third-party transaction (ie a back-to-back transaction), the instruments used in both transactions must be excluded from the residual risk

¹³ Where the bank cannot satisfy the supervisor that the residual risk add-on provides a sufficiently prudent capital charge, the supervisor will address any potentially under-capitalised risks by imposing a conservative additional capital charge under Pillar 2.

¹⁴ Examples of exotic underlying exposures include: longevity risk, weather, natural disasters, future realised volatility (as an underlying exposure for a swap).

add-on charge. Any instrument that is listed and/or eligible for central clearing must be excluded from the residual risk add-on.

- (g) A non-exhaustive list of other residual risks types and instruments that may fall within the criteria set out in paragraphs 58(e) include:
- *Gap risk*: risk of a significant change in vega parameters in options due to small movements in the underlying, which results in hedge slippage. Relevant instruments subject to gap risk include all path dependent options, such as barrier options, and Asian options, as well as all digital options.
 - *Correlation risk*: risk of a change in a correlation parameter necessary for determination of the value of an instrument with multiple underlyings. Relevant instruments subject to correlation risk include all basket options, best-of-options, spread options, basis options, Bermudan options and quanto options.
 - *Behavioural risk*: risk of a change in exercise/prepayment outcomes such as those that arise in fixed rate mortgage products where retail clients may make decisions motivated by factors other than pure financial gain (such as demographical features and/or and other social factors). A callable bond may only be seen as possibly having behavioural risk if the right to call lies with a retail client.
- (h) When an instrument is subject to one or more of the following risk types, this by itself will not cause the instrument to be subject to the residual risk add-on:
- (i) Risk from a cheapest-to-deliver option;
 - (ii) Smile risk – the risk of a change in an implied volatility parameter necessary for determination of the value of an instrument with optionality relative to the implied volatility of other instruments optionality with the same underlying and maturity, but different moneyiness.
 - (iii) Correlation risk arising from multi-underlying European or American plain vanilla options where all underlyings have sensitivities for delta risk of the same sign, and from any options that can be written as a linear combination of such options. This exemption applies in particular to the relevant index options.
 - (iv) Dividend risk arising from a derivative instrument whose underlying does not consist solely of dividend payments.

3. Sensitivities-based Method: risk factor and sensitivity definitions

(i) Risk factor definitions

59. General Interest Rate Risk (GIRR) risk factors

- (a) Delta GIRR: The GIRR delta risk factors are defined along two dimensions: a risk-free yield curve for each currency in which interest rate-sensitive instruments are denominated and the following vertices: 0.25 years, 0.5 years, 1 year, 2 years, 3 years, 5 years, 10 years, 15 years, 20 years, 30 years, to which delta risk factors are assigned.¹⁵

¹⁵ Assignment of risk factors to the specified vertices should be performed by linear interpolation or a method that is most consistent with the pricing functions used by the independent risk control function of a bank to report market risks or profits and losses to senior management.

68. **Vega risk sensitivities:**

- (a) The option-level vega risk sensitivity to a given risk factor is the product (ie multiplication) of the vega and implied volatility of the option.²¹ To determine this product, the bank must use the instrument's vega and implied volatility contained within the pricing models used by the independent risk control unit of a bank.
- (b) The portfolio-level vega risk sensitivity to a given vega risk factor is equal to the simple sum of option-level vega risk sensitivities to that risk factor, across all options in the portfolio.
- (c) The following sets out how vega risk sensitivities are to be derived in specific cases:
 - (i) With regard to options that do not have a maturity, assign those options to the longest prescribed maturity vertex, and assign these options also to the residual risks add-on;
 - (ii) With regard to options that do not have a strike or barrier and options that have multiple strikes or barriers, apply the mapping to strikes and maturity used internally to price the option, and assign those instruments also to the residual risks add-on;
 - (iii) With regard to CTP securitisation tranches which do not have an implied volatility, do not compute vega risk for such an instrument. Such instruments may not, however, be exempt from delta and curvature risk charges.

(iii) **Treatment of index instruments and multi-underlying options**

69. In the delta risk context:

- (a) For index instruments and multi-underlying options where all index constituents/option underlyings have delta risk sensitivities of the same sign, a look-through approach must be used. The sensitivities to constituent risk factors from index instruments and multi-underlying options are allowed to net with sensitivities to single name instruments without restrictions, although this does not apply to the correlation trading portfolio.
- (b) As per the requirement in paragraph 15, an equity investment in a fund in which the bank cannot look through the fund daily must be assigned to the banking book.

70. In the delta and vega risk context:

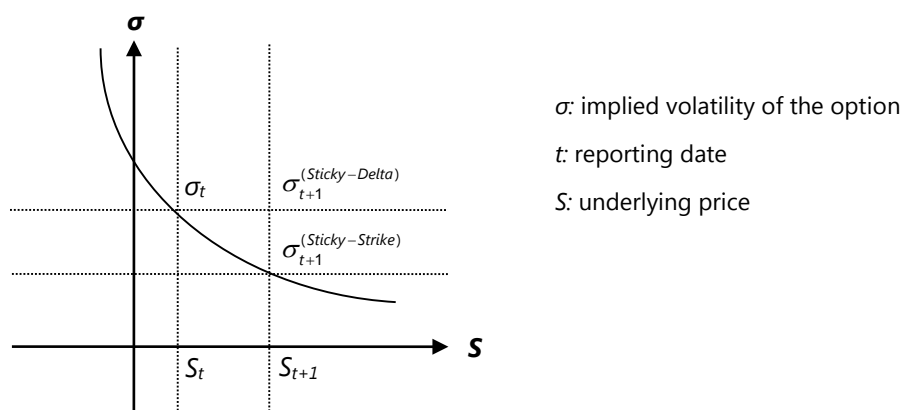
- (a) Multi-underlying options with delta risk sensitivities of different signs are exempted from delta and vega risk but may be subject to the residual risk add-on if they fall within the definitions set out in paragraph 58.
- (b) Multi-underlying options (including index options) are usually priced based on the implied volatility of the option, rather than the implied volatility of its underlying constituents.²²

²¹ As specified in the vega risk factor definitions in Section 3, the implied volatility of the option must be mapped to one or more maturity vertices.

²² As specified in the vega risk factor definitions in Section 3, the implied volatility of an option must be mapped to one or more maturity vertices.

(iv) Requirements on sensitivity computations

71. When computing a first-order sensitivity for instruments subject to optionality, banks should assume that the implied volatility remains constant, consistent with a "sticky delta" approach. This concept is illustrated in the following graph:



72. When computing a vega GIRR or CSR sensitivity, banks may use either the lognormal or normal assumptions. When computing a vega Equity, Commodity or FX sensitivity, banks must use the lognormal assumption.²³

- (a) If, for internal risk management, a bank computes sensitivities using definitions differing from the definitions provided in the present standards, this bank may use linear transformations to deduce from the sensitivities it computes the one to be used for the vega risk measure, knowing that the difference between these transformations and the exact price movements shall be captured through the curvature risk measure.
- (b) All sensitivities must be computed ignoring the impact of Credit Valuation Adjustments (CVA).

4. Sensitivities-based Method: delta risk weights and correlations

73. The prescribed risk weights and correlations in this section have been calibrated to the liquidity adjusted time horizon related to each risk class.

(i) Delta GIRR

Buckets

74. Each bucket represents an individual currency exposure to GIRR.

²³ Since the vega ($\frac{\partial V}{\partial \sigma_i}$) on an instrument is multiplied by its implied volatility (σ_i), the vega risk sensitivity for that instrument will be the same under the lognormal assumption and the normal assumption. As a consequence, banks may use a lognormal or normal assumption for GIRR and CSR (in recognition of the trade-offs between constrained specification and computational burden for a standardised approach). For the other risk classes, banks must only use a lognormal assumption (in recognition that this is aligned with common practices across jurisdictions).

Risk weights

75. The risk weights are set as follows:

Vertex	0.25 year	0.5 year	1 year	2 year	3 year
Risk weight (percentage points)	2.4%	2.4%	2.25%	1.88%	1.73%

Vertex	5 year	10 year	15 year	20 year	30 year
Risk weight (percentage points)	1.5%	1.5%	1.5%	1.5%	1.5%

- (a) A risk weight of 2.25% is set for the inflation risk factor and the cross currency basis risk factors, respectively.
- (b) For selected currencies by the Basel Committee,²⁴ the above risk weights may at the discretion of the bank be divided by the square root of 2.

Correlations

76. The delta risk correlation ρ_{kl} is set at 99.90% between sensitivities WS_k and WS_l within the same bucket (ie same currency), same assigned vertex, but different curves.

77. The delta risk correlation ρ_{kl} between sensitivities WS_k and WS_l within the same bucket (ie same currency) with different vertex and same curve is set at $\max \left[e^{\left(-\theta \cdot \frac{|T_k - T_l|}{\min\{T_k, T_l\}} \right)}; 40\% \right]$,²⁵ where:

- (a) T_k (respectively T_l) is the vertex that relates to WS_k (respectively WS_l)
- (b) θ set at 3%.

78. Between two sensitivities WS_k and WS_l within the same bucket (ie same currency), different vertex and different curves, the correlation ρ_{kl} is equal to the correlation parameter specified in paragraph 77 multiplied by 99.90%.²⁶

79. The delta risk correlation ρ_{kl} between a sensitivity WS_k to the inflation curve and a sensitivity WS_l to a given vertex of the relevant yield curve is 40%.

80. The delta risk correlation ρ_{kl} between a sensitivity WS_k to a cross currency basis curve and a sensitivity WS_l to either a given vertex of the relevant yield curve, the inflation curve or another cross currency basis curve (if relevant) is 0%.

81. The parameter $\gamma_{bc} = 50\%$ must be used for aggregating between different currencies.

²⁴ Selected currencies by the Basel Committee are: EUR, USD, GBP, AUD, JPY, SEK, CAD as well as the domestic reporting currency of a bank.

²⁵ For example, the correlation between a sensitivity to the vertex 1 year of the Eonia swap curve and the a sensitivity to the vertex 5 year of the Eonia swap curve in the same currency is $\max \left[e^{\left(-3\% \cdot \frac{|1-5|}{\min\{1;5\}} \right)}; 40\% \right] = 88.69\%$.

²⁶ For example, the correlation between a sensitivity to the vertex 1 year of the Eonia swap curve and a sensitivity to the vertex 5 year of the Euribor 3M swap curve in the same currency is $(88.69\%) \cdot (0.999) = 88.60\%$.

(ii) Delta CSR non-securitisations

Buckets

82. Sensitivities or risk exposures should first be assigned to a bucket according to the following table:

Bucket number	Credit quality	Sector
1	Investment grade (IG)	Sovereigns including central banks, multilateral development banks
2		Local government, government-backed non-financials, education, public administration
3		Financials including government-backed financials
4		Basic materials, energy, industrials, agriculture, manufacturing, mining and quarrying
5		Consumer goods and services, transportation and storage, administrative and support service activities
6		Technology, telecommunications
7		Health care, utilities, professional and technical activities
8		Covered bonds ²⁷
9	High yield (HY) & non-rated (NR)	Sovereigns including central banks, multilateral development banks
10		Local government, government-backed non-financials, education, public administration
11		Financials including government-backed financials
12		Basic materials, energy, industrials, agriculture, manufacturing, mining and quarrying
13		Consumer goods and services, transportation and storage, administrative and support service activities
14		Technology, telecommunications
15		Health care, utilities, professional and technical activities
16	Other sector ²⁸	

83. To assign a risk exposure to a sector, banks must rely on a classification that is commonly used in the market for grouping issuers by industry sector. The bank must assign each issuer to one and only one of the sector buckets in the table under paragraph 82. Risk positions from any issuer that a bank cannot assign to a sector in this fashion must be assigned to the "other sector" (ie bucket 16).

²⁷ Covered bonds must meet the definition provided in paragraphs 68, 70 and 71 in the following publication:

Basel Committee on Banking Supervision. *Standards Supervisory framework for measuring and controlling large exposures*. April 2014, www.bis.org/publ/bcbs283.pdf

²⁸ Credit quality is not a differentiating consideration for this bucket.

Risk weights

84. The risk weights for the buckets 1 to 16 are set out in the following table. Risk weights are the same for all vertices (ie 0.5 year, 1 year, 3 year, 5 year, 10 year) within each bucket:

Bucket number	Risk weight (percentage points)
1	0.5%
2	1.0%
3	5.0%
4	3.0%
5	3.0%
6	2.0%
7	1.5%
8	4.0%
9	3.0%
10	4.0%
11	12.0%
12	7.0%
13	8.5%
14	5.5%
15	5.0%
16	12.0%

Correlations

85. Between two sensitivities WS_k and WS_l within the same bucket, the correlation parameter ρ_{kl} is set as follows:

$$\rho_{kl} = \rho_{kl}^{(name)} \cdot \rho_{kl}^{(tenor)} \cdot \rho_{kl}^{(basis)}$$

where:

- $\rho_{kl}^{(name)}$ is equal to 1 where the two names of sensitivities k and l are identical, and 35% otherwise;
- $\rho_{kl}^{(tenor)}$ is equal to 1 if the two vertices of the sensitivities k and l are identical, and to 65% otherwise;
- $\rho_{kl}^{(basis)}$ is equal to 1 if the two sensitivities are related to same curves, and 99.90% otherwise.

For example, a sensitivity to the 5Y Apple bond curve and a sensitivity to the 10Y Google CDS curve would be $35\% \cdot 65\% \cdot 99.90\% = 22.73\%$.

86. Consistently with paragraph 60(c), the correlation parameter ρ_{kl} as defined in paragraphs 85 is not applicable in the curvature risk context.

87. The correlations above do not apply to the other sector bucket. The within "other sector" bucket capital requirement for the delta and vega risk aggregation formula would be equal to the simple sum of the absolute values of the net weighted sensitivities allocated to this bucket:

$$K_{b(\text{other bucket})} = \sum_k |WS_k|$$

This "other sector" bucket level capital will be added to the overall risk class level capital, with no diversification or hedging effects recognised with any bucket.

88. The correlation parameter γ_{bc} is set as follows:

$$\gamma_{bc} = \gamma_{bc}^{(rating)} \cdot \gamma_{bc}^{(sector)}$$

where:

- $\gamma_{bc}^{(rating)}$ is equal to 1 where the two buckets b and c have the same rating category (either IG or HY/NR), and 50% otherwise;
- $\gamma_{bc}^{(sector)}$ is equal to 1 if the two buckets have the same sector, and to the following numbers otherwise:

Bucket	1 / 9	2 / 10	3 / 11	4 / 12	5 / 13	6 / 14	7 / 15	8
1 / 9		75%	10%	20%	25%	20%	15%	10%
2 / 10			5%	15%	20%	15%	10%	10%
3 / 11				5%	15%	20%	5%	20%
4 / 12					20%	25%	5%	5%
5 / 13						25%	5%	15%
6 / 14							5%	20%
7 / 15								5%
8								

(iii) Delta CSR Securitisations (correlation trading portfolio)

Buckets

89. Sensitivities to CSR arising from the correlation trading portfolio and its hedges are treated as a separate risk class, for which the same bucket structure and correlation structure apply as those for the CSR non-securitisation framework, but for which the risk weights and correlations of the Delta CSR non-securitisations are modified to reflect longer liquidity horizons and larger basis risk.

Risk weights

90. Risk weights are the same for all vertices (ie 0.5 yr, 1 yr, 3 yr, 5 yr, 10 yr) within each bucket:

Bucket number	Risk weight (in percentage points)
1	4.0%
2	4.0%
3	8.0%
4	5.0%
5	4.0%
6	3.0%
7	2.0%
8	6.0%
9	13.0%
10	13.0%
11	16.0%
12	10.0%
13	12.0%
14	12.0%
15	12.0%
16	13.0%

Correlations

116. For the purpose of correlation recognition, any two commodities are considered distinct commodities if there exists in the market two contracts differentiated only by the underlying commodity to be delivered against each contract. For example, in bucket 2 ("Energy – Liquid Combustibles") WTI and Brent would typically be treated as distinct commodities.

117. Formally, between two sensitivities WS_k and WS_l within the same bucket, the correlation parameter ρ_{kl} is set as follows:

$$\rho_{kl} = \rho_{kl}^{(cty)} \cdot \rho_{kl}^{(tenor)} \cdot \rho_{kl}^{(basis)}$$

Where:

- $\rho_{kl}^{(cty)}$ is equal to 1 where the two commodities of sensitivities k and l are identical, and to the intra-bucket correlations in the table below otherwise;
- $\rho_{kl}^{(tenor)}$ is equal to 1 if the two vertices of the sensitivities k and l are identical, and to 99.00% otherwise;
- $\rho_{kl}^{(basis)}$ is equal to 1 if the two sensitivities are identical in both (i) contract grade of the commodity, and (ii) delivery location of a commodity, and 99.90% otherwise.

For example, the correlation between the sensitivity to Brent, tenor 1Y, for delivery in Le Havre and the sensitivity to WTI, tenor 5Y, for delivery in Oklahoma is $95\% \cdot 99.00\% \cdot 99.90\% = 93.96\%$.

Bucket	Commodity bucket	Correlation (ρ_{kl})
1	Energy - Solid combustibles	55%
2	Energy - Liquid combustibles	95%
3	Energy - Electricity and carbon trading	40%
4	Freight	80%
5	Metals – non-precious	60%
6	Gaseous combustibles	65%
7	Precious metals (including gold)	55%
8	Grains & oilseed	45%
9	Livestock & dairy	15%
10	Softs and other agriculturals	40%
11	Other commodity	15%

118. The correlation parameters γ_{bc} applying to sensitivity or risk exposure pairs between different buckets is set at:

- 20% if bucket b and bucket c fall within bucket numbers 1 to 10.
- 0% if either bucket b or bucket c is bucket number 11.

119. Further definitions related to delivery time are as follows:

- For bucket 3, each time interval at which the electricity can be delivered and that is subject to a contract that is made on a financial market is considered a distinct electricity commodity (just as silver and gold are considered distinct precious metals). Electricity produced in various areas such as Electricity NE, Electricity SE, Electricity North should also be considered distinct electricity commodities and therefore the correlation parameters in the preceding paragraphs should apply between sensitivities to each of those electricity types. In addition, the electricity

risk factor can either be the spot or the forward price, as transactions on the forward price are more frequent than transactions on spot price.

- For bucket 4 ("Freight"), each combination of freight route and each week at which a good has to be delivered is a distinct commodity.

(vii) Foreign exchange risk

Risk weights

120. A unique relative risk weight equal to 30% applies to all the FX sensitivities or risk exposures.
- (a) For the specified currency pairs by the Basel Committee,³¹ the above risk weight may at the discretion of the bank be divided by the square root of 2.

Correlations

121. A uniform correlation parameter γ_{bc} equal to 60% applies to FX sensitivity or risk exposure pairs.

5. Sensitivities-based Method: vega risk weights and correlations

(i) The vega buckets

122. The delta buckets are replicated in the vega context, unless specified otherwise in the preceding paragraphs within Section 3 and Section 4.
123. The bucket remains the first level of aggregation between vega risk positions within a risk class, ie the steps in paragraph 51 are to be performed.

(ii) The vega risk weights

124. The risk of market illiquidity is incorporated into the determination of vega risk factors, through the assignment of different liquidity horizons for each risk class. The risk weight for a given vega risk factor k (RW_k) is determined by the following function:

$$RW_k = \min \left[RW_\sigma \cdot \frac{\sqrt{LH_{risk\ class}}}{\sqrt{10}}; 100\% \right]$$

where:

- RW_σ is set at 55%;
- $LH_{risk\ class}$ is the regulatory liquidity horizon to be prescribed in the determination of each vega risk factor k . $LH_{risk\ class}$ is specified as follows:

³¹ Selected currency pairs by the Basel Committee are: USD/EUR, USD/JPY, USD/GBP, USD/AUD, USD/CAD, USD/CHF, USD/MXN, USD/CNY, USD/NZD, USD/RUB, USD/HKD, USD/SGD, USD/TRY, USD/KRW, USD/SEK, USD/ZAR, USD/INR, USD/NOK, USD/BRL, EUR/JPY, EUR/GBP, EUR/CHF and JPY/AUD.

Risk class	$LH_{risk\ class}$
GIRR	60
CSR non-securitisations	120
CSR securitisations (CTP)	120
CSR securitisations (non-CTP)	120
Equity (large cap)	20
Equity (small cap)	60
Commodity	120
FX	40

(iii) The vega correlations

125. Between vega risk sensitivities within the same bucket of the GIRR risk class, the correlation parameter ρ_{kl} is set as follows:

$$\rho_{kl} = \min [\rho_{kl}^{(option\ maturity)} \cdot \rho_{kl}^{(underlying\ maturity)}, 1]$$

where:

- $\rho_{kl}^{(option\ maturity)}$ is equal to $e^{-\alpha \frac{|T_k - T_l|}{\min\{T_k; T_l\}}}$ where α is set at 1%, T_k (respectively T_l) is the maturity of the option from which the vega sensitivity VR_k (VR_l) is derived, expressed as a number of years;
- $\rho_{kl}^{(underlying\ maturity)}$ is equal to $e^{-\alpha \frac{|T_k^U - T_l^U|}{\min\{T_k^U; T_l^U\}}}$, where α is set at 1%, T_k^U (respectively T_l^U) is the maturity of the underlying of the option from which the sensitivity VR_k (VR_l) is derived, expressed as a number of years after the maturity of the option.

126. Between vega risk sensitivities within a bucket of the other risk classes (ie not GIRR), the correlation parameter ρ_{kl} is set as follows:

$$\rho_{kl} = \min [\rho_{kl}^{(DELTA)} \cdot \rho_{kl}^{(option\ maturity)}, 1]$$

where:

- $\rho_{kl}^{(DELTA)}$ is equal to the correlation that applies between the delta risk factors that correspond to vega risk factors k and l . For instance, if k is the vega risk factor from equity option X and l is the vega risk factor from equity option Y then $\rho_{kl}^{(DELTA)}$ is the delta correlation applicable between X and Y; and
- $\rho_{kl}^{(option\ maturity)}$ is defined as in paragraph 125.

127. With regard to vega risk sensitivities between buckets within a risk class (GIRR and non-GIRR), the same correlation parameters for γ_{bc} , as specified for delta correlations for each risk class in Section 4, are to be used in the vega risk context (eg $\gamma_{bc} = 50\%$ is to be used for aggregation of vega risk sensitivities across different GIRR buckets).

128. There is no diversification or hedging benefit recognised in the standardised approach between vega and delta risk factors. Vega and delta risk charges are aggregated by simple summation.

6. Sensitivities-based Method: curvature risk weights and correlations

(i) The curvature buckets

129. The delta buckets are replicated in the curvature context, unless specified otherwise in the preceding paragraphs within Section 3 and Section 4.

130. The bucket remains the first level of aggregation between curvature risk positions within each risk class.

(ii) The curvature risk weights

131. For FX and Equity curvature risk factors, the curvature risk weights are relative shifts ("shocks") equal to the delta risk weights.

132. For GIRR, CSR and Commodity curvature risk factors, the curvature risk weight is the parallel shift of all the vertices for each curve based on the highest prescribed delta risk weight for each risk class. For example, in the case of GIRR the risk weight assigned to the 0.25 year vertex (ie most punitive vertex risk weight) is applied to all the vertices simultaneously for each risk-free yield curve (consistent with a "translation", or "parallel shift" risk calculation).

(iii) The curvature correlations

133. Between curvature exposures, each delta correlation parameters ρ_{kl} and γ_{bc} should be squared. For instance, between CVR_{EUR} and CVR_{USD} in the GIRR context, the correlation should be $50\%^2 = 25\%$.

7. The Default Risk Charge

134. The approach for the standardised default risk capital charge comprises a multi-step procedure. In the first step, JTD loss amounts for each instrument subject to default risk are determined; second, offsetting of the JTD amounts of long and short exposures with respect to the same obligor (where permissible) produces net long and net short amounts in distinct obligors; third, the net short exposures are discounted by a hedge benefit ratio; and finally, default risk weights are applied to arrive at the capital charge. The procedure is specified in the material below. In the procedure, offsetting refers to the netting of exposures to the same obligor (where a short exposure may be subtracted in full from a long exposure), while hedging refers to the application of a partial hedge benefit from the short exposures (where the risk of long and short exposures in distinct obligors do not fully offset due to basis or correlation risks).

135. The default risk charge for non-securitisations and securitisations is independent from the other capital charges in the Standardised Approach for Market Risk; in particular it is independent from the CSR capital charge.

136. For the correlation trading portfolio (CTP), the capital charge includes the default risk for securitisation exposures and for non-securitisation hedges. These hedges are to be removed from the default risk non-securitisation calculations. There must be no diversification benefit between the default risk charge for non-securitisations, default risk charge for securitisations (non-correlation trading portfolio) and default risk charge for the securitisation correlation trading portfolio.

137. In line with criteria set out in other parts of the Capital Accord, at national discretion claims on sovereigns, public sector entities and multilateral development banks may be subject to a zero default risk weight. National authorities may apply a non-zero risk weight to securities issued by certain foreign governments, including to securities denominated in a currency other than that of the issuing government.

C. Market risk – The Internal Models Approach

1. General criteria

176. The use of an internal model for the purposes of regulatory capital determination will be conditional upon the explicit approval of the bank's supervisory authority. Home and host country supervisory authorities of banks that carry out material trading activities in multiple jurisdictions intend to work cooperatively to ensure an efficient approval process.

177. The supervisory authority will only give its approval if at a minimum:

- (a) It is satisfied that the bank's risk management system is conceptually sound and is implemented with integrity;
- (b) The bank has, in the supervisory authority's view, sufficient numbers of staff skilled in the use of sophisticated models not only in the trading area but also in the risk control, audit and, if necessary, back office areas;
- (c) The bank's models have, in the supervisory authority's judgement, a proven track record of reasonable accuracy in measuring risk;
- (d) The bank regularly conducts stress tests along the lines discussed in paragraphs 195 to 202 below; and
- (e) The positions included in the internal model for regulatory capital determination are held in approved trading desks that have passed the required tests described in paragraph 182.

178. Supervisory authorities will be able to insist on a period of initial monitoring and live testing of a bank's internal model before it is used for supervisory capital purposes.

179. In addition to these general criteria, banks using internal models for capital purposes will be subject to the additional requirements detailed below.

2. Qualitative standards

180. Supervisory authorities must be able to assure themselves that banks using internal models have market risk management systems that are conceptually sound and implemented with integrity. Accordingly, the bank must meet the following *qualitative criteria* on an ongoing basis. Supervisors must assess that banks have met the criteria before they are permitted to use a models-based approach. These qualitative criteria include:

- (a) The bank must have an independent risk control unit that is responsible for the design and implementation of the bank's risk management system. The unit should produce and analyse daily reports on the output of the bank's risk measurement model, including an evaluation of the relationship between measures of risk exposure and trading limits. This unit must be independent from business trading units and should report directly to senior management of the bank.
- (b) The unit must conduct regular backtesting and profit and loss (P&L) attribution programmes, ie an ex-post comparison of the risk measure and P&L values generated by the model against actual daily changes in portfolio values over longer periods of time, as well as hypothetical changes based on static positions. Both of these exercises must be conducted at a trading desk level, while regular backtesting must also be conducted on the firm-wide internal model for regulatory capital determination level.
- (c) A distinct unit must conduct the initial and ongoing validation of all internal models. Internal models must be validated on at least an annual basis.

- (d) Board of directors and senior management must be actively involved in the risk control process and need to regard risk control as an essential aspect of the business to which significant resources are devoted. In this regard, the daily reports prepared by the independent risk control unit must be reviewed by a level of management with sufficient seniority and authority to enforce both reductions of positions taken by individual traders and reductions in the bank's overall risk exposure.
- (e) Internal models used to calculate market risk charges are likely to differ from those used by banks in their day-to-day internal management functions. Nevertheless, the starting point for the design of both the regulatory and the internal risk models should be the same. In particular, the valuation models that are embedded in both should be similar. These valuation models must be an integral part of the internal identification, measurement, management and internal reporting of price risks within the firm. As well, internal risk models should, at a minimum, cover the positions covered by the regulatory models, although they may cover more. In the construction of their regulatory capital models, banks must start from the methodologies used in their internal models with regard to risk factor identification, parameter estimation and proxy concept and deviate only if this is appropriate due to regulatory constraints. It is expected that the same risk factors are covered in the regulatory models as in the internal models.
- (f) A routine and rigorous programme of stress testing is required as a supplement to the risk analysis based on the output of the bank's risk measurement model. The results of stress testing must be reviewed at least monthly by senior management, used in the internal assessment of capital adequacy, and reflected in the policies and limits set by management and the board of directors. Where stress tests reveal particular vulnerability to a given set of circumstances, prompt steps must be taken to mitigate those risks appropriately (eg by hedging against that outcome or reducing the size of the bank's exposures, or increasing capital).
- (g) Banks need to have a routine in place for ensuring compliance with a documented set of internal policies, controls and procedures concerning the operation of the risk measurement system. The bank's risk measurement system must be well documented, for example, through a comprehensive risk management manual that describes the basic principles of the risk management system and that provides a detailed explanation of the empirical techniques used to measure market risk.
- (h) Any significant changes to a regulatory-approved model must be approved by the supervisor prior to being implemented.
- (i) Risk measures must be calculated on the full set of positions which are in the scope of application of the model. The risk measures must be based on a sound theoretical basis, calculated correctly, and reported accurately.
- (j) An independent review of the risk measurement system must be carried out regularly by either the bank's own internal auditing process or an external auditor. This review must include both the activities of the business trading units and of the independent risk control unit. The review must be sufficiently detailed to determine for any failings which desks are impacted. A review of the overall risk management process should take place at regular intervals (not less than once a year) and must specifically address, at a minimum:
- The organisation of the risk control unit;
 - The adequacy of the documentation of the risk management system and process;
 - The accuracy and appropriateness of the risk measurement system (including any significant changes);
 - The verification of the consistency, timeliness and reliability of data sources used to run internal models, including the independence of such data sources;

- The approval process for risk pricing models and valuation systems used by front and back-office personnel;
- The scope of market risks captured by the risk measurement model;
- The integrity of the management information system;
- The accuracy and completeness of position data;
- The accuracy and appropriateness of volatility and correlation assumptions;
- The accuracy of valuation and risk transformation calculations; and
- The verification of the model's accuracy through frequent backtesting and P&L attribution as described in Appendix B: *Supervisory framework for the use of backtesting in conjunction with the internal models approach to market risk capital requirements*.

3. Quantitative standards

181. Banks will have flexibility in devising the precise nature of their models, but the following minimum standards will apply for the purpose of calculating their capital charge. Individual banks or their supervisory authorities will have discretion to apply stricter standards.

- "Expected shortfall" must be computed on a daily basis for the bank-wide internal model for regulatory capital purposes. Expected shortfall must also be computed on a daily basis for each trading desk that a bank wishes to include within the scope for the internal model for regulatory capital purposes.
- In calculating the expected shortfall, a 97.5th percentile, one-tailed confidence level is to be used.
- In calculating the expected shortfall, the liquidity horizons described in paragraph 181(k) must be reflected by scaling an expected shortfall calculated on a base horizon. The expected shortfall for a liquidity horizon must be calculated from an expected shortfall at a base liquidity horizon of 10 days with scaling applied to this base horizon result as follows:

$$ES = \sqrt{\left(ES_T(P)\right)^2 + \sum_{j \geq 2} \left(ES_T(P, j) \sqrt{\frac{(LH_j - LH_{j-1})}{T}}\right)^2}$$

where:

- ES is the regulatory liquidity-adjusted expected shortfall;
- T is the length of the base horizon, ie 10 days;
- $ES_T(P)$ is the expected shortfall at horizon T of a portfolio with positions $P = (p_i)$ with respect to shocks to all risk factors that the positions P are exposed to;
- $ES_T(P, j)$ is the expected shortfall at horizon T of a portfolio with positions $P = (p_i)$ with respect to shocks for each position p_i in the subset of risk factors $Q(p_i, j)$, with all other risk factors held constant;
- the ES at horizon T , $ES_T(P)$ must be calculated for changes in the risk factors, and $ES_T(P, j)$ must be calculated for changes in the relevant subset $Q(p_i, j)$ of risk factors, over the time interval T without scaling from a shorter horizon;
- $Q(p_i, j)$ is the subset of risk factors whose liquidity horizons, as specified in paragraph 181(k), for the desk where p_i is booked are at least as long as LH_j according to the table

below. For example, $Q(p_i, 4)$ is the set of risk factors with a 60-day horizon and a 120-day liquidity horizon. Note that $Q(p_i, j)$ is a subset of $Q(p_i, j-1)$;

- the time series of changes in risk factors over the base time interval T may be determined by overlapping observations; and
- LH_j is the liquidity horizon j , with lengths in the following table:

j	LH_j
1	10
2	20
3	40
4	60
5	120

- (d) The expected shortfall measure must be calibrated to a period of stress. Specifically, the measure must replicate an expected shortfall charge that would be generated on the bank's current portfolio if the relevant risk factors were experiencing a period of stress. This is a joint assessment across all relevant risk factors, which will capture stressed correlation measures. This calibration is to be based on an "indirect" approach using a reduced set of risk factors. Banks are to specify a reduced set of risk factors that are relevant for their portfolio and for which there is a sufficiently long history of observations. This reduced set of risk factors is subject to supervisory approval and must meet the data quality requirements for a modellable risk factor as outlined in paragraph 183(c). The identified reduced set of risk factors must be able to explain a minimum of 75% of the variation of the full ES model (ie the ES of the reduced set of risk factors should be at least equal to 75% of the fully specified ES model on average measured over the preceding 12 week period).

The expected shortfall for the portfolio using this set of risk factors, calibrated to the most severe 12-month period of stress available over the observation horizon, is calculated. That value is then scaled up by the ratio of the current expected shortfall using the full set of risk factors to the current expected shortfall measure using the reduced set of factors. The expected shortfall for risk capital purposes is therefore:

$$ES = ES_{R,S} \cdot \frac{ES_{F,C}}{ES_{R,C}}$$

where the expected shortfall for capital purposes (ES) is equal to the expected shortfall based on a stressed observation period using a reduced set of risk factors ($ES_{R,S}$) multiplied by the ratio of the expected shortfall measure based on the current (most recent) 12-month observation period with a full set of risk factors ($ES_{F,C}$) and the expected shortfall measure based on the current period with a reduced set of risk factors ($ES_{R,C}$). For the purpose of this calculation, the ratio is floored at 1.

- (e) For measures based on current observations ($ES_{F,C}$), banks must update their *data sets* no less frequently than once every month and must also reassess them whenever market prices are subject to material changes. This updating process must be flexible enough to allow for more frequent updates. The supervisory authority may also require a bank to calculate its Expected Shortfall using a shorter observation period if, in the supervisor's judgement; this is justified by a significant upsurge in price volatility. In this case, however, the period should be no shorter than 6 months.
- (f) For measures based on stressed observations ($ES_{R,S}$), banks must identify the 12-month period of stress over the observation horizon in which the portfolio experiences the largest loss. The observation horizon for determining the most stressful 12 months must, at a minimum, span

back to and including 2007. Observations within this period must be equally weighted. Banks must update their 12-month stressed periods no less than monthly, or whenever there are material changes in the risk factors in the portfolio.

- (g) No particular type of expected shortfall model is prescribed. So long as each model used captures all the material risks run by the bank, as confirmed through P&L attribution and backtesting, and conforms to each of the requirements set out above and below, supervisors may permit banks to use models based on either historical simulation, Monte Carlo simulation, or other appropriate analytical methods.
- (h) Banks will have discretion to recognise empirical *correlations* within broad regulatory risk factor classes (interest rate risk, equity risk, foreign exchange risk, commodity risk and credit risk, including related options volatilities in each risk factor category). Empirical correlations across broad risk factor categories will be constrained by the supervisory aggregation scheme, as described in paragraph 189, and must be calculated and used in a manner consistent with the applicable liquidity horizons, clearly documented and able to be explained to supervisors on request.
- (i) Banks' models must accurately capture the unique risks associated with *options* within each of the broad risk categories. The following criteria apply to the measurement of options risk:
- Banks' models must capture the *non-linear price characteristics* of options positions;
 - Each bank's risk measurement system must have a set of risk factors that captures the *volatilities of the rates and prices* underlying option positions, ie vega risk. Banks with relatively large and/or complex options portfolios must have detailed specifications of the relevant volatilities. This means that banks must model the volatility surface across both strike price and vertex (ie tenor).
- (j) Each bank must meet, on a daily basis, a *capital requirement* C_A expressed as the higher of (1) its previous day's aggregate capital charge for market risk; and (2) an average of the daily capital measures in the preceding 60 business days according to the parameters specified in paragraphs 187 to 194 for the following formula:
- $$C_A = \max\{IMCC_{t-1} + SES_{t-1}; m_c \cdot IMCC_{avg} + SES_{avg}\}$$
- (k) As set out in paragraph 181(c), a scaled expected shortfall **must** be calculated based on the liquidity horizon n defined below. n is calculated using the following conditions:
- banks must map each risk factor on to one of the risk factor categories shown below using consistent and clearly documented procedures;
 - the mapping must be (i) set out in writing; (ii) validated by the bank's risk management; (iii) made available to supervisors; and (iv) subject to internal audit; and
 - n is determined for each broad category of risk factor as set out in the following table. However, on a desk-by-desk basis n can be increased relative to the values in the table below (ie the liquidity horizon specified below can be treated as a floor). Where n is increased, the increased horizon must be 20, 40, 60 or 120 days and the rationale must be documented and be subject to supervisory approval. Furthermore, liquidity horizons should be capped at the maturity of the related instrument:

Risk factor category	<i>n</i>	Risk factor category	<i>n</i>
Interest rate: specified currencies - EUR, USD, GBP, AUD, JPY, SEK, CAD and domestic currency of a bank	10	Equity price (small cap): volatility	60
Interest rate: – unspecified currencies	20	Equity: other types	60
Interest rate: volatility	60	FX rate: specified currency pairs ³⁷	10
Interest rate: other types	60	FX rate: currency pairs	20
Credit spread: sovereign (IG)	20	FX: volatility	40
Credit spread: sovereign (HY)	40	FX: other types	40
Credit spread: corporate (IG)	40	Energy and carbon emissions trading price	20
Credit spread: corporate (HY)	60	Precious metals and non-ferrous metals price	20
Credit spread: volatility	120	Other commodities price	60
Credit spread: other types	120	Energy and carbon emissions trading price: volatility	60
		Precious metals and non-ferrous metals price: volatility	60
Equity price (large cap)	10	Other commodities price: volatility	120
Equity price (small cap)	20	Commodity: other types	120
Equity price (large cap): volatility	20		

4. Model validation standards

182. Banks must have processes in place to ensure that their internal models have been adequately validated by suitably qualified parties independent of the development process to ensure that they are conceptually sound and adequately capture all material risks. This validation must be conducted when the model is initially developed and when any significant changes are made to the model. Models must be periodically revalidated, particularly when there have been significant structural changes in the market or changes to the composition of the portfolio which might lead to the model no longer being adequate. Model validation must not be limited to P&L attribution and backtesting, but must, at a minimum, also include the following:

- (a) Tests to demonstrate that any assumptions made within the internal model are appropriate and do not underestimate risk. This may include the assumption of the normal distribution and any pricing models.
- (b) Further to the regulatory backtesting programmes, testing for model validation must use hypothetical changes in portfolio value that would occur were end-of-day positions to remain unchanged. It therefore excludes fees, commissions, bid-ask spreads, and intraday trading. Moreover, additional tests are required which may include, for instance:

³⁷ USD/EUR, USD/JPY, USD/GBP, USD/AUD, USD/CAD, USD/CHF, USD/MXN, USD/CNY, USD/NZD, USD/RUB, USD/HKD, USD/SGD, USD/TRY, USD/KRW, USD/SEK, USD/ZAR, USD/INR, USD/NOK, USD/BRL, EUR/JPY, EUR/GBP, EUR/CHF and JPY/AUD.