March 30, 2021 (Due: 08:00 April 6, 2021)

1. Similar to complete, natural, and periodic cubic splines, when the "not-a-knot" condition is used in cubic spline interpolation, the computational kernel is also to solve a sparse linear system. Try to derive the corresponding linear system.

2. When performing interpolation with a complete cubic spline, the choice of derivatives on the boundary is important. Suppose that Bob wants to interpolate the sine function $f(x) = \sin x$ at nine equispaced nodes over $[0, 2\pi]$, with $f'(0) = f'(2\pi) = 1$. Unfortunately, he made a typo on $f'(0)$ in his program and observed some strange results. Try to reproduce Bob's result with a few different values of $f'(0)$. For instance, $f'(0) = 0$, $f'(0) = -1$, etc.

3. The temperature in the human body is not a constant, but rather follows a daily rhythm driven by an internal biological clock. The following table lists 20 averaged values of temperature measurements taken from 70 English sailors in an experiment done in 1971.

Temperature in human body

Time (hour) t	Temperature (°C) $T(t)$
1	36.37
3	36.23
5	36.21
7	36.26
8	36.38
9	36.49
10	36.60
11	36.63
12	36.66
13	36.68
14	36.69
15	36.73
16	36.74
17	36.78
18	36.82
19	36.84
20	36.87
21	36.86
22	36.77
23	36.59

Interpolate the data with a periodic cubic spline and plot your solution for a two-day-period.

4. Interpolate the following data set and visualize your solution on $[-1,1]\times[-1,1]$.

x_i	y_i	z_i
-1.0000	-1.0000	1.6389
-1.0000	1.0000	0.5403
1.0000	-1.0000	-0.9900
1.0000	1.0000	0.1086
-0.7313	0.6949	0.9573
0.5275	-0.4899	0.8270
-0.0091	-0.1010	1.6936
0.3031	0.5774	1.3670

You are free to choose any 2D interpolation strategy learned from the lecture. (If Delaunay triangularization is used, you can make use of the MATLAB/Octave function `delaunay`.) Note that different interpolation strategies will lead to different results.

5. (optional) Sometimes we are interested in finding a curve to (approximately) pass through the given data points (x_1,y_1), (x_2,y_2), ..., (x_n,y_n). The curve is not necessarily of the form $y=f(x)$ because a straight line parallel to the y-axis may intersect with the curve at multiple points. One strategy is to perform two cubic spline interpolations (or cubic spline fittings) $x=x(t)$ and $y=y(t)$ by choosing an appropriate sequences of t_i's. According to differential geometry, the best parameterization is to choose t as the arc length. In our case, we can replace arc length by straight line distance since we only have a discrete data set. Use this strategy to interpolate the following data sets and visualize your results.

(1) A smooth curve that connects (in turn)

$$(1,1),\ (0,2),\ (-1,1),\ (0,0),\ (1,-1),\ (0,-2),\ (-1,-1).$$

(2) A smooth closed curve that connects (in turn)

$$(3,0),\ (2,2),\ (0,3),\ (-2,2),\ (-3,0),\ (-2,-2),\ (0,-3),\ (2,-2).$$