

May 25, 2021 (Due: 08:00 June 8, 2021)

1. Solve the IVP

$$\begin{cases} u'(t) = 500u^2(1-u), & t > 0, \\ u(0) = 0.01. \end{cases}$$

Try MATLAB functions `ode23` and `ode23s` and compare the solutions, step sizes, execution time, etc.

(GNU Octave may not have `ode23s`. If you are using GNU Octave, you can play with other stiff ODE solvers.)

2. Use finite difference method (with $n+1$ equispaced nodes) to solve the boundary value problem

$$\begin{cases} -u''(x) + u(x) = x^2, & (0 < x < 1) \\ u(0) = 0, \quad u(1) = 1. \end{cases}$$

Try a few different values of n and compare your solutions with the exact one.

3. Solve the partial differential equation

$$\begin{cases} \frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2} = 0, & (-1 < x < 1, -1 < y < 1) \\ u(x, -1) = u(x, 1) = x + 1, & (-1 < x < 1) \\ u(-1, y) = y^2 - 1, \quad u(1, y) = y^2 + 1, & (-1 < y < 1) \end{cases}$$

using the finite difference method. Visualize your solution.

4. (optional) Visualize the stability region of RK4.