Homework XIII

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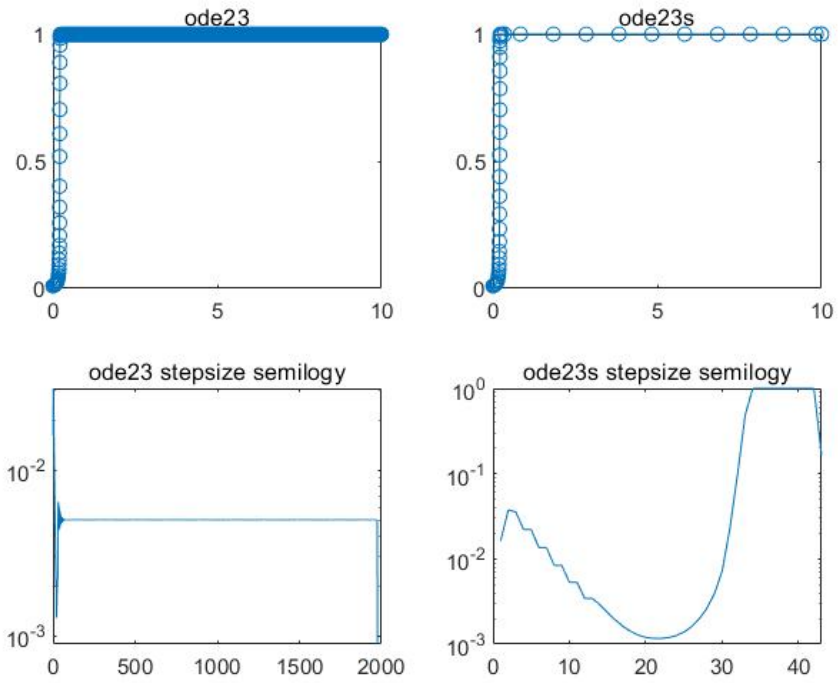
Abstract

This is Daniel's homework of "Numerical Algorithms with Case Studies II".

1 Problems

Q1 The comparison between ode23 and ode23s is plotted below with the solution and stepsize(in semilogy). The running time of ode23 is 0.871743s and ode23s is 0.197925s.



(a) Comparison

ode23s have better stepsize choices and termination condition than ode23.

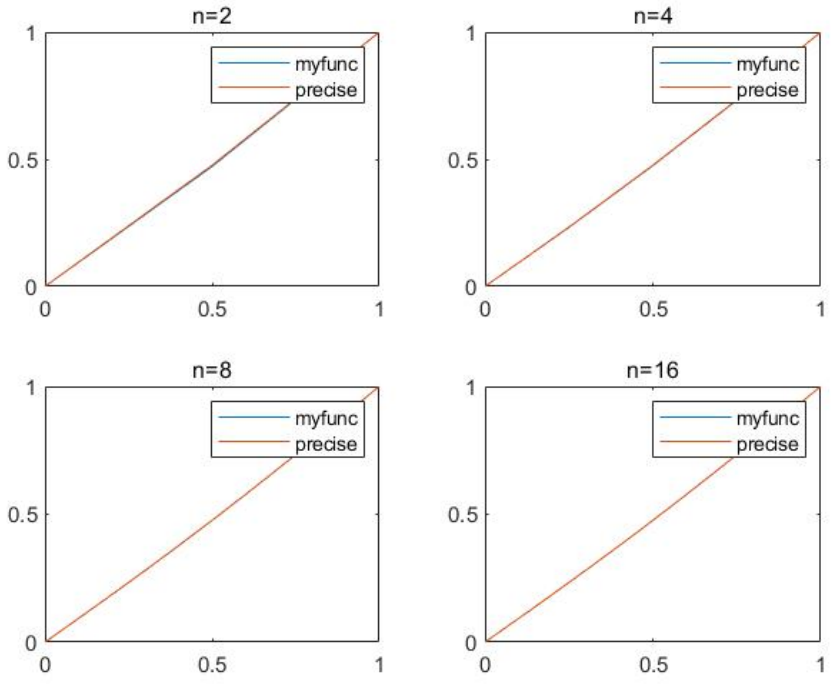
Q2 The second order differential equation could be solved using differential operator.

$$-(D^2-1)(u) = -(D-1)(D+1)(u) \tag{1}$$
$$= -(D+1)(z) = -e^{-x}D(e^x z) \tag{2}$$

$$\therefore -e^x z = x^2 e^x - 2xe^x + 2e^x + C_1 \tag{3}$$
$$(D-1)u = -x^2 + 2x - 2 + C_1 e^{-x} \tag{4}$$

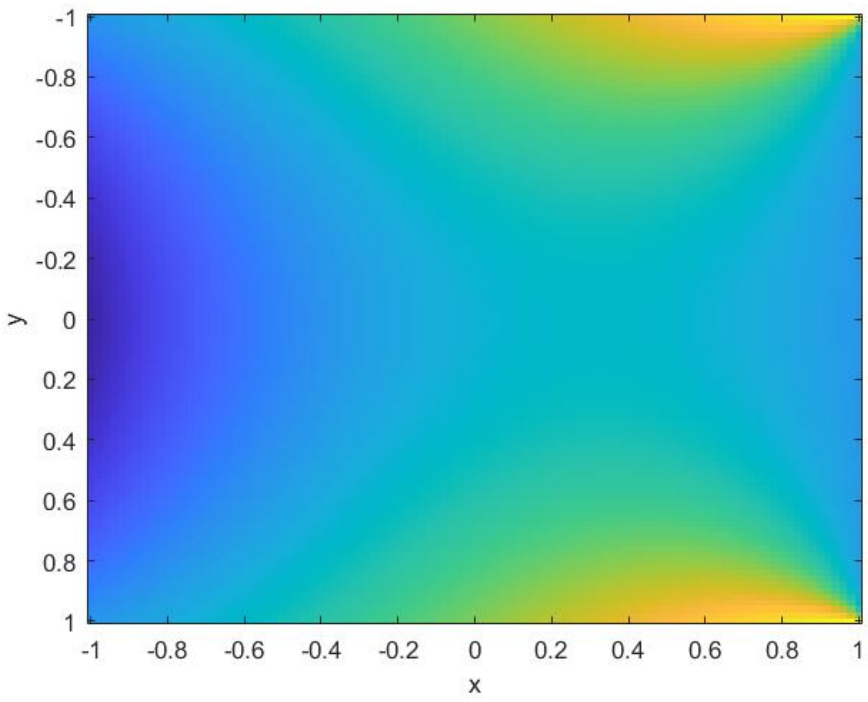
$$\therefore D(e^{-x}u) = -x^2 e^{-x} + 2xe^{-x} - 2e^{-x} + C_1 e^{-2x} \tag{5}$$
$$\therefore u = x^2 + 2 + C_1 e^{-x} + C_2 e^x \tag{6}$$

Since $u(0) = 0$, $u(1) = 1$, we have $C_1 = -\dfrac{2}{e^{-1}+1}$, $C_2 = -\dfrac{2}{e+1}$. Here is the comparison with my numerical experiments.



(b) My result

Q3 The solution is visualized as follows,



(c) Solution