

April 13, 2021 (Due: 08:00 April 20, 2021)

1. Find

$$\min_{a,b,c} \int_0^{\pi/2} |\sin x - ax^2 - bx - c|^2 dx$$

without programming.

2. Find

$$\min_{a,b} \max_{-1 \leq x \leq 2} |x^3 + ax + b|$$

without programming.

3. Find

$$\min_{a,b,c} \max_{0 \leq x \leq 1} |\ln(1+x) + ax^2 + bx + c|$$

using Remez algorithm. Visualize the approximation error and the (nonuniform) alternating set by taking a few snapshots. (E.g., you may plot the error curve for the initial guess, two intermediate solutions, as well as the final solution.)

4. Compute a few low degree Padé approximants of $f(x) = (1+x)^{-1/2}$ and visualize the approximation errors over $[0, 2]$.

5. (optional) Show that in each step of Remez algorithm, the linear system has a unique solution.