Homework V

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Abstract

This is Daniel's homework of "Numerical Algorithms with Case Studies II".

1 Problems

Q1 First, calculate the third-order derivative of $s(x)$.

$$s_0'''(x) = 12\frac{y_0}{h_0^3} - 12\frac{y_1}{h_0^3} + 6\frac{k_0}{h_0^2} + 6\frac{k_1}{h_0^2} \tag{1}$$

$$s_1'''(x) = 12\frac{y_1}{h_1^3} - 12\frac{y_2}{h_1^3} + 6\frac{k_1}{h_1^2} + 6\frac{k_2}{h_1^2} \tag{2}$$

$$s_{n-1}'''(x) = 12\frac{y_{n-1}}{h_{n-1}^3} - 12\frac{y_n}{h_{n-1}^3} + 6\frac{k_{n-1}}{h_{n-1}^2} + 6\frac{k_n}{h_{n-1}^2} \tag{3}$$

$$s_n'''(x) = 12\frac{y_n}{h_n^3} - 12\frac{y_{n+1}}{h_n^3} + 6\frac{k_n}{h_n^2} + 6\frac{k_{n+1}}{h_n^2} \tag{4}$$

We apply the not-a-knot condition to $s_0'''(x) = s_1'''(x)$ and $s_(n-1)'''(x) = s_n'''(x)$, therefore,

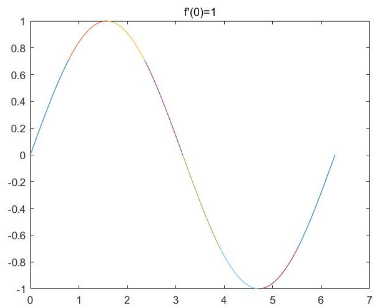
$$2(\frac{y_0 - y_1}{h_0^3} - \frac{y_1 - y_2}{h_1^3}) = -k_0\frac{1}{h_0^2} + k_1(\frac{1}{h_1^2} - \frac{1}{h_0^2}) + k_2\frac{1}{h_1^2} \tag{5}$$

$$2(\frac{y_{n-1} - y_n}{h_{n-1}^3} - \frac{y_n - y_{n+1}}{h_n^3}) = -k_{n-1}\frac{1}{h_{n-1}^2} + k_n(\frac{1}{h_n^2} - \frac{1}{h_{n-1}^2}) + k_{n+1}\frac{1}{h_n^2} \tag{6}$$

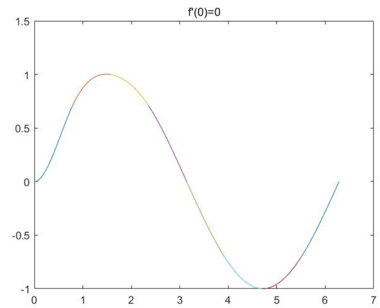
Maintain the rest of the equations, we have,

$$\begin{pmatrix}
-\frac{1}{h_0^2} & \frac{1}{h_1^2} - \frac{1}{h_0^2} & \frac{1}{h_1^2} & \cdots & 0 & 0 & 0 \\
2\frac{1}{h_0} & 4\frac{1}{h_0} + 4\frac{1}{h_1} & 2\frac{1}{h_1} & \cdots & 0 & 0 & 0 \\
0 & 2\frac{1}{h_1} & 4\frac{1}{h_1} + 4\frac{1}{h_2} & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 4\frac{1}{h_{n-2}} + 4\frac{1}{h_{n-1}} & 2\frac{1}{h_{n-1}} & 0 \\
0 & 0 & 0 & \cdots & 2\frac{1}{h_{n-1}} & 4\frac{1}{h_{n-1}} + 4\frac{1}{h_n} & 2\frac{1}{h_n} \\
0 & 0 & 0 & \cdots & -\frac{1}{h_{n-1}^2} & \frac{1}{h_n^2} - \frac{1}{h_{n-1}^2} & \frac{1}{h_n^2}
\end{pmatrix}
\begin{pmatrix} k_0 \\ k_1 \\ k_2 \\ \vdots \\ k_{n-1} \\ kn \\ k_{n+1} \end{pmatrix}
=
\begin{pmatrix}
2(\frac{y_0 - y_1}{h_0^3} - \frac{y_1 - y_2}{h_1^3}) \\
6(\frac{y_2 - y_1}{h_1^2} + \frac{y_1 - y_0}{h_0^2}) \\
\vdots \\
6(\frac{y_{n+1} - y_n}{h_n^2} + \frac{y_n - y_{n-1}}{h_{n-1}^2}) \\
2(\frac{y_{n-1} - y_n}{h_{n-1}^3} - \frac{y_n - y_{n+1}}{h_n^3})
\end{pmatrix} \tag{7}$$

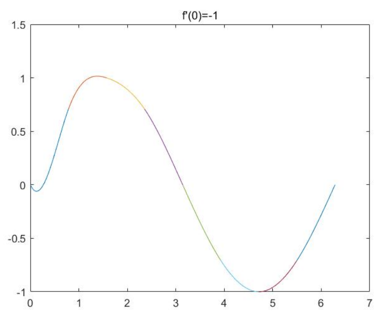
Q2 The error is small for $f'(0) = 0$, but the plot start to become strange and recognizable when $f'(0)$ was too far away.



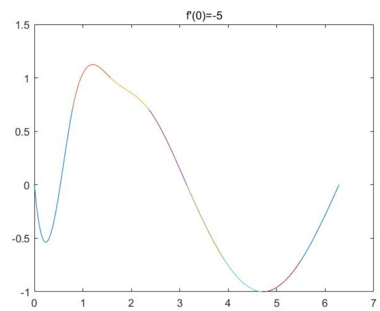
(a) $f'(0) = 1$



(b) $f'(0) = 0$

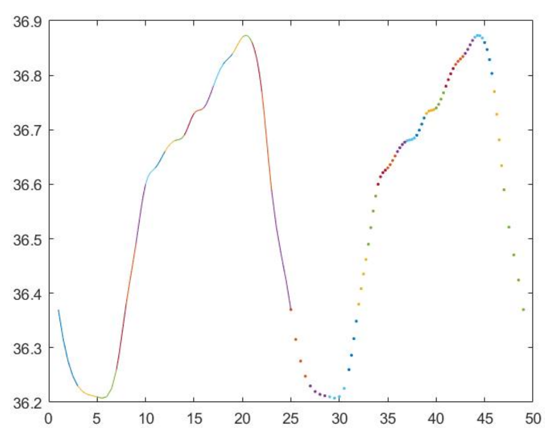


(c) $f'(0) = -1$



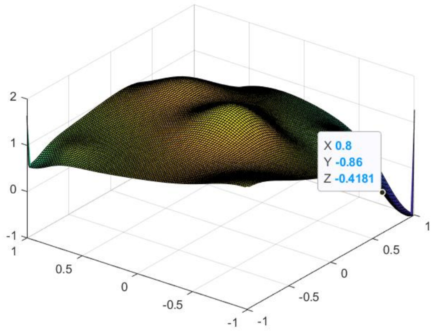
(d) $f'(0) = -5$

Q3 We assume that human body temperature remain parellel between two consecutive days. Also we connect $f(23)$ with $f(1 + 24)$ to maintain the periodicity. The dotted line represent estimated data, while the solid line represent the interpolated data.

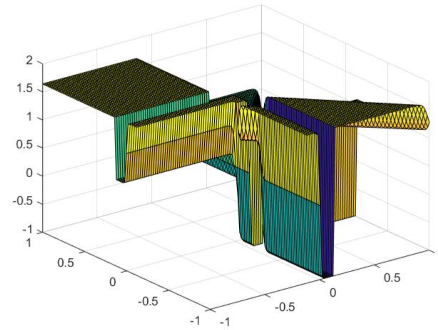


(e) human body temperature in 2 days

Q4 The result was rather funny if I set $p = 100$ for $\|\cdot\|_p$, but the interpolation under $\|\cdot\|_2$ remain plausible.



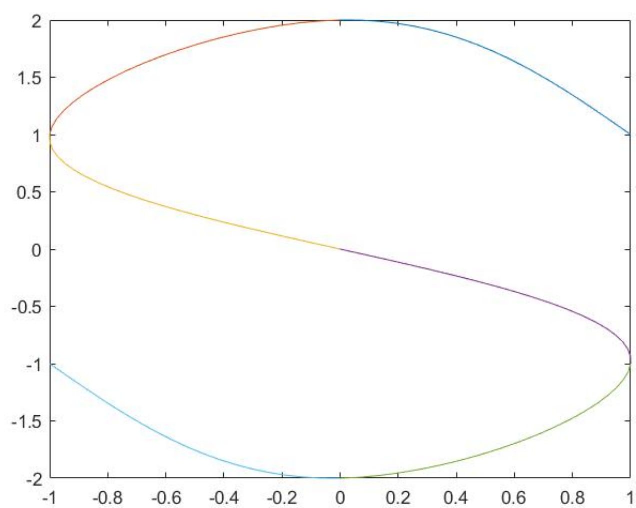
(f) $\|\cdot\|_2$



(g) $\|\cdot\|_p$

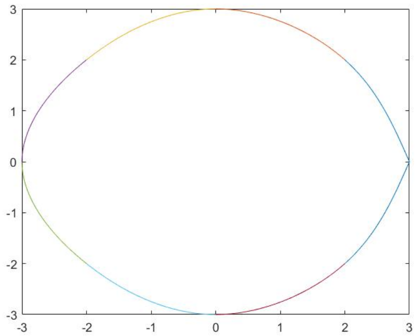
Q5

(a) It turns out to be an "S" curve. Use the natural condition.

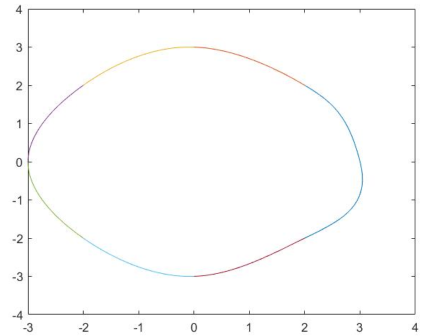


(h) curve 1

(b) The bad news is that using natural condition makes the closed curve look not smooth enough. We applied periodic condition to make the head and tail meet smoothly.



(i) poor curve



(j) smooth curve