Homework XIV

Name: Shao Yanjun, Number: 19307110036

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Abstract

This is Daniel's homework of "Numerical Algorithms with Case Studies II".

1 Problems

Q1 If we have $x_{k+1} - x_k = h$, we will have our basis,

$$v_k(x) = \frac{1}{h}(H(x - x_k)(x_{k+1} - x) + H(x_k - x)(x - x_{k-1}) + H(x_{k-1} - x)(x_{k-1} - x) + H(x - x_{k+1})(x - x_{k+1})) \tag{1}$$

Take the derivative,

$$v_k'(x) = \frac{1}{h}(\delta(x - x_k)(x_{k+1} - x) - \delta(x_k - x)(x - x_{k-1}) - \delta(x_{k-1} - x)(x_{k-1} - x) \tag{2}$$

$$+ \delta(x - x_{k+1})(x - x_{k+1}) - H(x - x_k) + H(x_k - x) - H(x_{k-1} - x) + H(x - x_{k+1})) \tag{3}$$

Q2 The second order differential equation could be solved using differential operator.

$$-(D^2 - 1)(u) = -(D - 1)(D + 1)(u) \tag{4}$$

$$= -(D + 1)(z) = -e^{-x} D(e^x z) \tag{5}$$

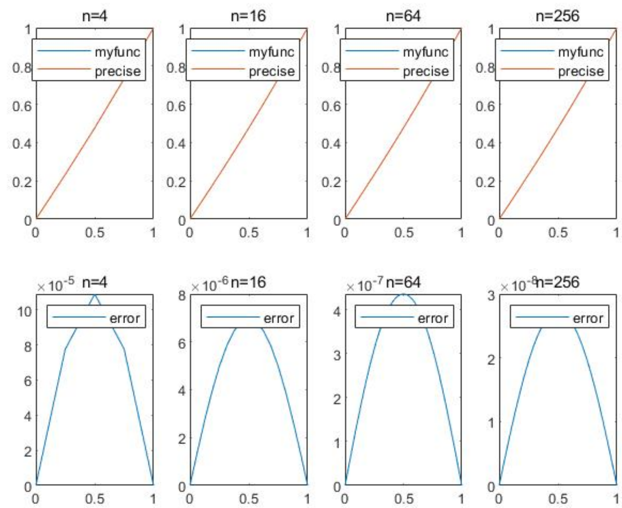
$$\therefore -e^x z = x^2 e^x - 2xe^x + 2e^x + C_1 \tag{6}$$

$$(D - 1)u = -x^2 + 2x - 2 + C_1 e^{-x} \tag{7}$$

$$\therefore D(e^{-x} u) = -x^2 e^{-x} + 2xe^{-x} - 2e^{-x} + C_1 e^{-2x} \tag{8}$$

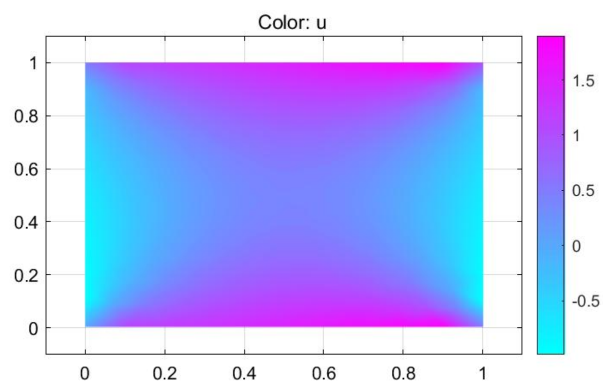
$$\therefore u = x^2 + 2 + C_1 e^{-x} + C_2 e^x \tag{9}$$

Since $u(0) = 0$, $u(1) = 1$, we have $C_1 = -\dfrac{2}{e^{-1} + 1}$, $C_2 = -\dfrac{2}{e + 1}$. Here is the comparison with my numerical experiments.



(a) My result

Q3 The solution is visualized as follows,



(b) My result