May 11, 2021 (Due: 08:00 May 25, 2021)

1. Show that the DFT matrix F_n diagonalizes

$$
J_n = \begin{bmatrix}
0 & 1 & & & \\
& 0 & 1 & & \\
& & \ddots & \ddots & \\
& & & 0 & 1 \\
1 & & & & 0
\end{bmatrix},
$$

i.e., $F_n^{-1} J_n F_n$ is diagonal.

2. Plot the discrete Fourier transform of 1024 equispaced sampling points of the function $f(x) = \sin(3x)$ over $[0, 2\pi)$. Where are the peaks? Can you explain what you have observed?

Repeat the experiment for $f(x) = \sin x + \sin(\sqrt[12]{128}\, x)$ over $[0, 12\pi)$ and explain your observation.

3. Derive the circular convolution theorem based on the following convention of DFT:

$$
\widehat{u}_k = \sum_{k=0}^{n-1} \exp\left(-\frac{2jk\pi \mathrm{i}}{n}\right) u_j, \qquad (u \in \mathbb{C}^n).
$$

4. Write a program to compute the product of two complex polynomials using fast convolution algorithms. You can make use of the MATLAB/Octave function `conv()` to check the correctness of your implementation. Make a plot to demonstrate that the complexity of your implementation is $\Theta(n \log n)$. What is the complexity of `conv()` in MATLAB/Octave?

5. Create a discontinuous function and smoothen it by convolving with Gaussian functions. Make plots to visualize the results.

6. You are given an audio file `DTMF_dialing.ogg`, which contains 80 touch tones from a DTMF keyboard. Try to determine the keys corresponding to the tones according to the following table.

	1209 Hz	1336 Hz	1477 Hz	1633 Hz
697 Hz	1	2	3	A
770 Hz	4	5	6	B
852 Hz	7	8	9	C
941 Hz	*	0	#	D

7. (optional) Prove that

$$
\delta(x) = \frac{1}{\pi} \lim_{\eta \to 0+} \frac{\eta}{x^2 + \eta^2}.
$$

It suffices to show
$$\lim_{\eta \to 0+} \int_{-1}^{1} \frac{\eta f(x)}{x^2 + \eta^2}\, \mathrm{d}x = \pi f(0)$$
for any continuous function $f(x)$.

FYI. Other frequently used approximations to $\delta(x)$ include
$$\delta(x) = \lim_{\sigma \to 0+} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) = \lim_{\eta \to 0+} \frac{1}{\pi x} \sin\frac{x}{\eta}.$$

8. (optional) Implement Radix-3 FFT and Radix-5 FFT. Make sure your implementations have complexity $\Theta(n \log n)$.