Homework VI

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Abstract

This is Daniel's homework of "Numerical Algorithms with Case Studies II".

1 Problems

Q1

(a) We have,

$$\frac{y}{a} = e^{bx+cx^2} \tag{1}$$

$$log(y) - log(a) = bx + cx^2 \tag{2}$$

If we reset $log(y) = \tilde{y}$, we will have a linear model,

$$\tilde{y} = \tilde{a} + bx + cx^2 \tag{3}$$

(b) We have,

$$\frac{1}{y} = 1 + e^{a+bx} \tag{4}$$

$$log(\frac{1}{y} - 1) = a + bx \tag{5}$$

If we reset $log(\frac{1}{y} - 1) = \tilde{y}$, we will have a linear model,

$$\tilde{y} = a + bx \tag{6}$$

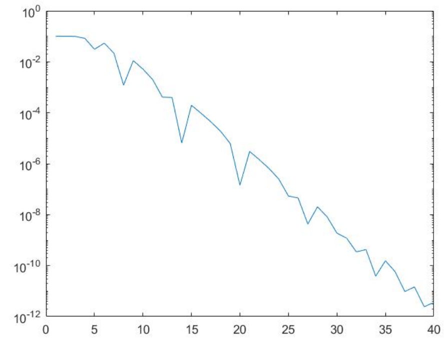
(c) We have,

$$y = \frac{a}{(\frac{b}{\sqrt{x}} + 1)^2} \tag{7}$$

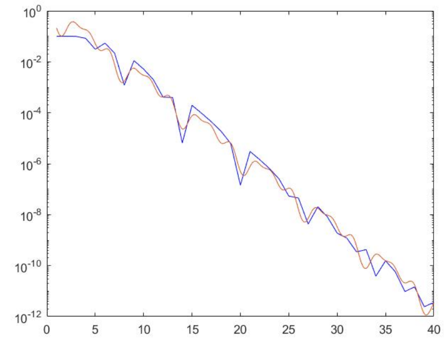
If we reset $\frac{1}{\sqrt{y}} = \tilde{y}$ and $\frac{1}{\sqrt{x}} = \tilde{x}$ and $\sqrt{a} = \tilde{a}$, we will have a linear model,

$$\tilde{a}\tilde{y} = b\tilde{x} + 1 \tag{8}$$

Q2 First check the semilogy() plot on bisection residuals, and we immediately discover that it should be fitted with $log(y)$ instead of y. Choose several basis $\{x, sin(x), cos(x), sin(2x), cos(2x), sin(3x), cos(3x)\}$

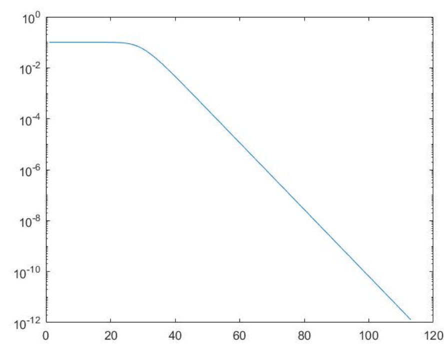


(a) residual

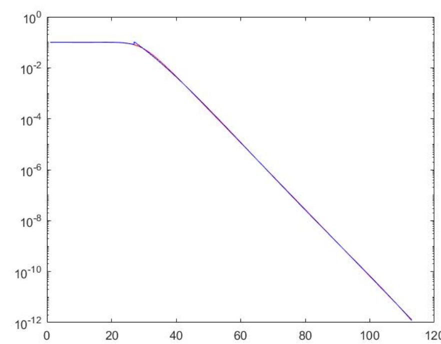


(b) fit

Next the residual of $regular\ falsi$ is segmented into two parts.

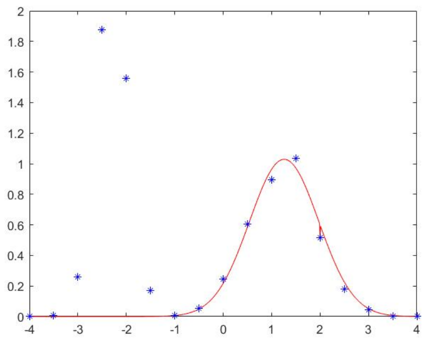


(c) residual

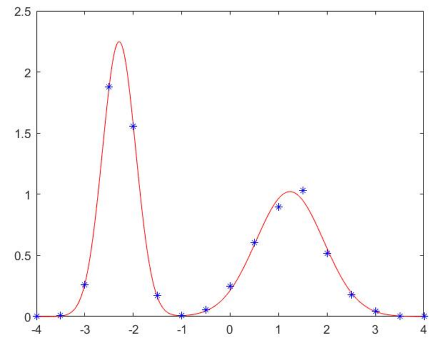


(d) fit

Q3 The most important thing of this model is that we must start our searching from two different basis with $\gamma_1 = -\gamma_2$, because otherwise we are just trying to fit with only one basis, and can never converge to a good result (see the poor result).



(e) poor



(f) good

Through experiment, I discover that α and β are not so sensitive to initial values.

Q4 We discover with joy that hyperbolic triangular functions have similar properties as basic triangular functions, i.e.

$$cosh(a+b) = \frac{e^{a+b}+e^{-a-b}}{2} \tag{9}$$
$$= \frac{e^a+e^{-a}}{2} \cdot \frac{e^b+e^{-b}}{2} - \frac{e^a-e^{-a}}{2} \cdot \frac{e^b-e^{-b}}{2} \tag{10}$$
$$= cosh(a) \cdot cosh(b) - sinh(a) \cdot sinh(b) \tag{11}$$

We define Chebyshev polynomial on $(1, +\infty)$ as $T_n(x) = cosh(n \cdot arccosh(x))$. Check the assumption.

$$2xT_n(x) = 2cosh(arccosh(x)) \cdot cosh(n \cdot arccosh(x)) \tag{12}$$
$$= cosh((n+1)arccosh(x)) + cosh((n-1)arccosh(x)) \tag{13}$$
$$= T_{n-1}(x) + T_{n+1}(x) \tag{14}$$