Homework XII

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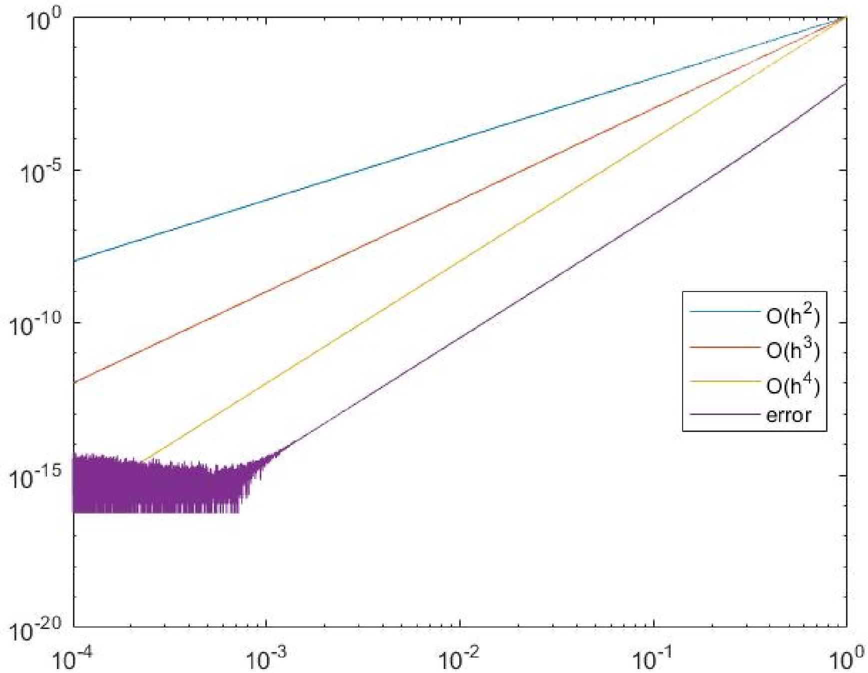
Abstract

This is Daniel's homework of "Numerical Algorithms with Case Studies II".

1 Problems

Q1 Implement the code and examine the truncation error of RK4 on loglog() plots.



(a) truncation error

Most of the truncation error satisfy $O(h^4)$ level, while further accuracy cannot be reached because of rounding error.

Q2 In this implicit method, we will have the following loop invariant in each iteration.

$$u_{k+1} = u_k + h(\theta f(t_k + h, u_{k+1}) + (1-\theta)f(t_k, u_k)) \tag{1}$$
$$= u_k - h\theta\lambda u_{k+1} - h(1-\theta)\lambda u_k \tag{2}$$

So after each iteration, we will have,

$$u_{k+1} = \frac{1 - h(1-\theta)\lambda}{1 + h\theta\lambda} u_k \tag{3}$$

And to make $u_k \to 0$ when $k \to \infty$, we have to let,

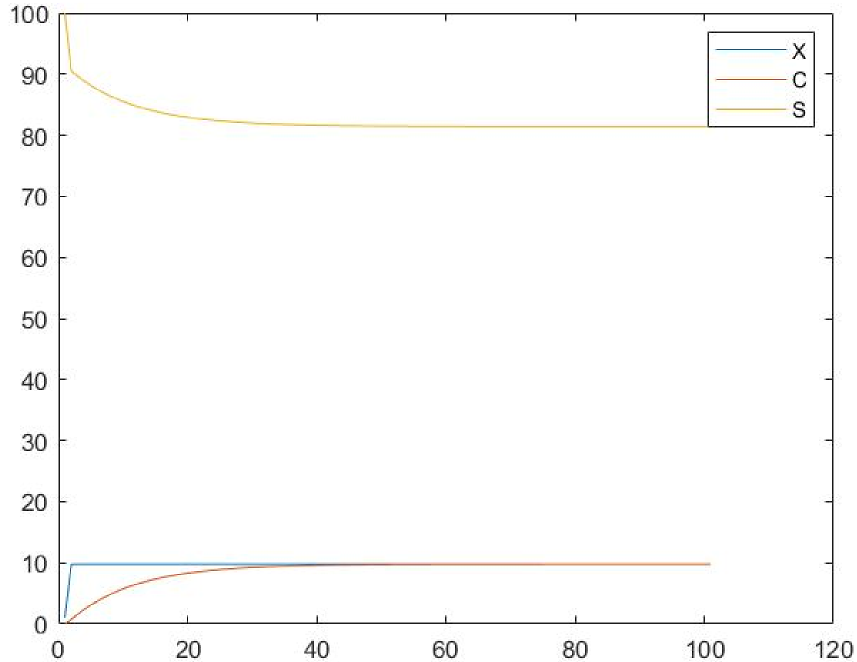
$$\frac{1 - h(1-\theta)\lambda}{1 + h\theta\lambda} < 1 \tag{4}$$

Which gives $h > 0$.
However, this is not sufficient for convergence, because we must take a good step h to make sure every implicit equation converges within each iteration. That gives the condition that $x = (1 - h(1-\theta)\lambda)y - h\theta\lambda x$ must converge.

$$\|x_k + 1 - x_k\| = h\theta\lambda \|x_k - x_{k-1}\| \tag{5}$$
$$h < \frac{1}{\theta\lambda} \tag{6}$$

Therefore, $h \in (0, \frac{1}{\theta\lambda})$. However, numerical studies show that there could be an even tighter upper bound for stepsize.

Q3 The stationary concentrations are approximately $X = 0.097754, C = 0.097749, S = 0.814495$



(b) solution

Q4