April 6, 2021 (Due: 08:00 April 13, 2021)

1. In the context of data fitting, a few nonlinear models are called intrinsically linear because they can be converted to linear models by certain transformations. Show that the following nonlinear models are intrinsically linear (under mild assumptions) by finding appropriate transformations.
(1) $y = a\exp(bx + cx^2)$.
(2) $y = 1/\big(1 + \exp(a + bx)\big)$.
(3) $y = ax/(b + \sqrt{x}\,)^2$.
2. In the homework on March 9, you have been asked to find the root of $x^{64} - 0.1 = 0$ in $[0, 1]$ using bisection and *regula falsi*. Try to fit the history of residuals using a simple model.
3. Suppose that there is a bimodal function of the form

$$y = \alpha_1 g\big(\beta_1(x - \gamma_1)\big) + \alpha_2 g\big(\beta_2(x - \gamma_2)\big),$$

where $g(x) = \exp(-x^2)$. Can you find out the parameters from the following *noisy* data set sampled from this model?

x_i	y_i
-4.00000	0.00001
-3.50000	0.00726
-3.00000	0.25811
-2.50000	1.87629
-2.00000	1.55654
-1.50000	0.17209
-1.00000	0.00899
-0.50000	0.05511
0.00000	0.24564
0.50000	0.60455
1.00000	0.89370
1.50000	1.03315
2.00000	0.51633
2.50000	0.18032
3.00000	0.04287
3.50000	0.00360
4.00000	0.00045

4. Chebyshev polynomials have explicit expressions $T_n(x) = \cos(n \arccos x)$ for $x \in [-1, 1]$. Can you find an explicit expression of $T_n(x)$ for $x \in (1, +\infty)$?
5. (optional) In the homework on March 30, you have been asked to interpolate temperature in human body using cubic spline. In fact, interpolation is not a good

idea because the data are noisy. Try to fit the data using a periodic cubic spline with equispaced nodes $\{1, 4, 7, \dots, 19, 22, 25\}$. Plot your solution for a two-day-period, and compare it with the interpolating spline.

6. (optional) Suppose that you are given a tool that can draw a cubic Bézier curve (i.e., a Bézier curve with *exactly* four control points). Make use of this tool to draw a segment of the parabola $y = ax^2 + bx + c$ for $x_1 \leq x \leq x_2$. You may use pseudo code to answer this question.