Homework VII

Name: Shao Yanjun, Number: 19307110036

April 16, 2021

Abstract

This is Daniel's homework of "Numerical Algorithms with Case Studies II".

1 Problems

Q1 We calculate the integral first (eliminate the constant because it's useless)

$$r(a,b,c) = \int_0^{\frac{\pi}{2}} |sin(x) - ax^2 - bx - c|^2 dx = \int_0^{\frac{\pi}{2}} sin^2(x) - 2(ax^2 - bx - c)sin(x) + (ax^2 - bx - c)^2 dx \tag{1}$$

$$= -(4a - 2c)cos(x) - 2bsin(x) + 2bxcosx - 4axsin(x) + 2ax^2cosx \tag{2}$$

$$+ c^2x + x^3(b^2/3 + (2ac)/3) + (a^2 * x^5)/5 + (abx^4)/2 + bcx^2 \Big|_0^{\frac{\pi}{2}} \tag{3}$$

$$= 4a - 2b - 2c - 2\pi a + \frac{\pi}{2}c^2 + \frac{\pi^3}{24}(b^2 + 2ac) + \frac{\pi^5}{160}a^2 + \frac{\pi^4}{32}ab + \frac{\pi^2}{4}bc \tag{4}$$

Find derivative of $r(a,b,c)$,

$$\begin{cases} r'_a(a,b,c) = \dfrac{\pi^4}{32}a + \dfrac{\pi^3}{12}b + \dfrac{\pi^2}{4}c - 2 = 0 \\ r'_b(a,b,c) = \dfrac{\pi^5}{80}a + \dfrac{\pi^4}{32}b + \dfrac{\pi^3}{12}c + 4 - 2\pi = 0 \\ r'_c(a,b,c) = \dfrac{\pi^3}{12}a + \dfrac{\pi^2}{4}b + \pi c - 2 = 0 \end{cases} \tag{5}$$

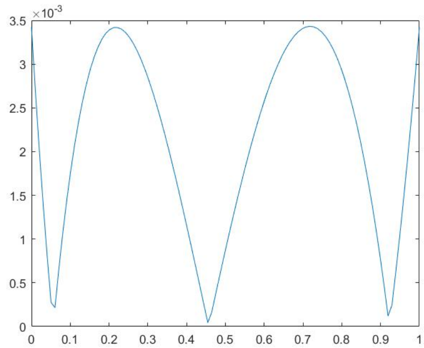
And the solution is $a = 240\pi^{-3} + 2880\pi^{-4} - 11520\pi^{-5}$, $b = -144\pi^{-2} - 1344\pi^{-3} + 5760\pi^{-4}$ and $c = 18\pi^{-1} + 96\pi^{-2} - 480\pi^{-3}$. The minimum value is 7.105584603731387.

Q2 We will approximate x^3 with a line, which means there are 3 alternation points. $r(x) = x^3 + ax + b$

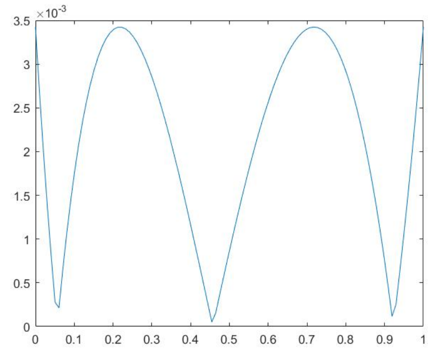
$$\begin{cases} r(-1) = -1 - a + b = 8 + 2a + b = r(2) \\ r'(x_1) = 3x_1^2 + a = 0 \\ r(x_1) = x_1^3 + ax_1 + b = 1 + a - b = -r(-1) \end{cases} \tag{6}$$

The solution is $a = -3$ and $b = 0$. The optimal value is 14.

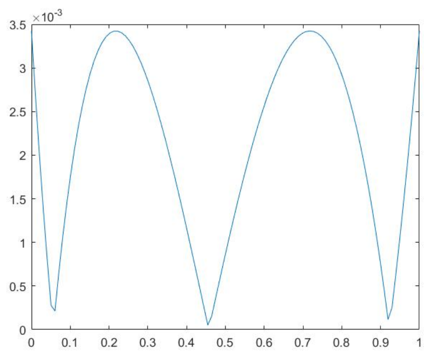
Q3 Use Remez's method, examine $|ln(1 + x) + ax^2 + bx + c|$ for several iterations. It converges very quickly, thanks to my good choice of the initial points. (All kinds of initial conditions produce fast convergence though...)



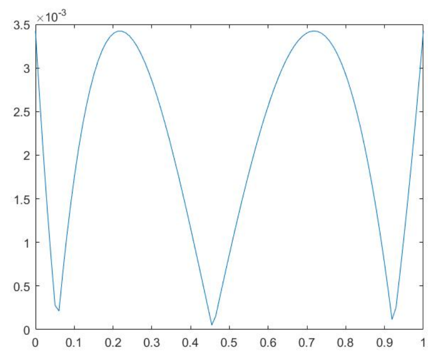
(a) iter=1



(b) iter=3

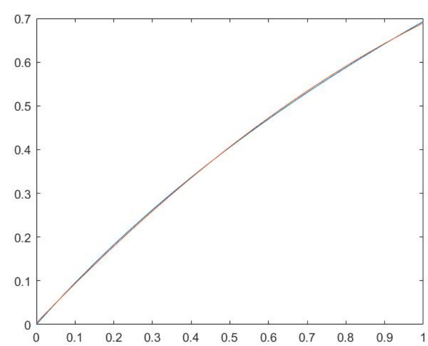


(c) iter=5

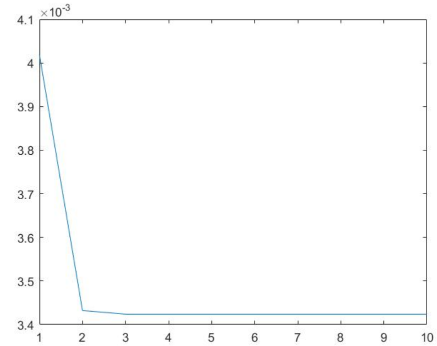


(d) iter=7

And here is the comparison between the polynomial and the $f(x)$, and the best polynomial is $-0.239030719054479x^2 + 0.925329938214002x + 0.003423980700211$. And the points should be chosen as $\{1, 0.718069281999180, 0.217518624597682, 0\}$



(e) approximation



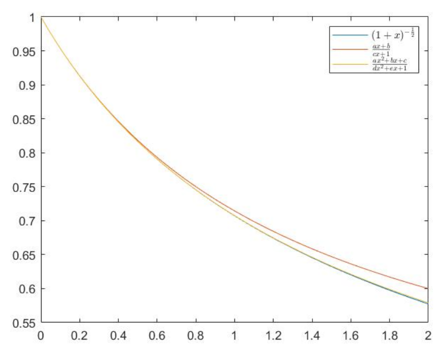
(f) residual

Q4 Do Taylor expansion of $f(x) = (1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{16}{5}x^3 + \frac{35}{128}x^4$. Solve the following linear systems and get an idea of Pade approximation,

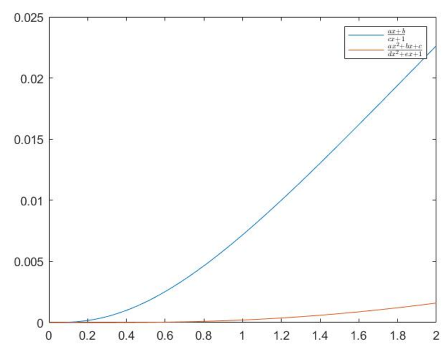
$$\frac{ax+b}{cx+1} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 \tag{7}$$

$$\frac{ax^2+bx+c}{dx^2+ex+1} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{16}{5}x^3 + \frac{35}{128}x^4 \tag{8}$$

$p_1(x) = \frac{\frac{1}{4}x+1}{\frac{3}{4}x+1}$ and $p_2(x) = \frac{\frac{1}{16}x^2+\frac{3}{4}x+1}{\frac{5}{16}x^2+\frac{5}{4}+1}$. Check the polynomial vs $f(x)$



(g) pade approximation



(h) pade error