# April 25, 2021 (Due: 08:00 May 11, 2021)

**1.** Try to find constants $c_1$, $c_2$, and $c_3$ such that the degree of exactness of the following quadrature rule is maximized:

$$\int_{-2a}^{2a} f(x)\,\mathrm{d}x \approx c_1 f(-a) + c_2 f(0) + c_3 f(a).$$

**2.** (a) Determine the degree of exactness of the quadrature rule

$$\int_a^b f(x)\,\mathrm{d}x \approx \frac{b-a}{2}\left(f\Big(\frac{2a+b}{3}\Big) + f\Big(\frac{a+2b}{3}\Big)\right).$$

(b) By partitioning $[a,b]$ into $n$ subintervals of equal length and applying the quadrature rule from part (a) on each subinterval, we obtain a composite quadrature rule. Implement such a composite quadrature rule and determine the asymptotic behavior of the error ($O(h)$, $O(h^2)$, $O(h^3)$, or $\ldots$) when integrating

$$\int_0^{\pi} \sin x\,\mathrm{d}x = 2,$$

where $h = (b-a)/n$.

**3.** Suppose that $f\colon [a,b] \to \mathbb{R}$ is twice continuously differentiable.

(a) Show that the remainder (i.e., truncation error) of the composite midpoint rule is given by

$$\frac{(b-a)^3}{24n^2} f''(\xi)$$

for some $\xi \in (a,b)$.

(b) If we use the composite midpoint rule to approximate $\int_0^1 \mathrm{e}^x\,\mathrm{d}x$, how many sampling points do we need in order to obtain six correct digits after the decimal point?

**4.** Use Romberg's method to compute $\int_0^1 \mathrm{e}^x\,\mathrm{d}x$ and $\int_0^1 x^{3/2}\,\mathrm{d}x$ with five correct digits after the decimal point. Keep a record for intermediate results. What can you say about the convergence?

**5.** (optional) Let us ignore all planets in the solar system and assume that the orbit of Halley's comet is a perfect ellipse. Try to estimate the total orbit length of Halley's comet.

Note that the circumference of an ellipse cannot be represented by an elementary function in terms of its major and minor axes. You will need to do the calculation using numerical integration.

Here are some useful data:

- The eccentricity of the orbit is 0.967.

- The shortest distance between the comet and the sun is 88 million kilometers.

You can also search online to find some other information about the comet.