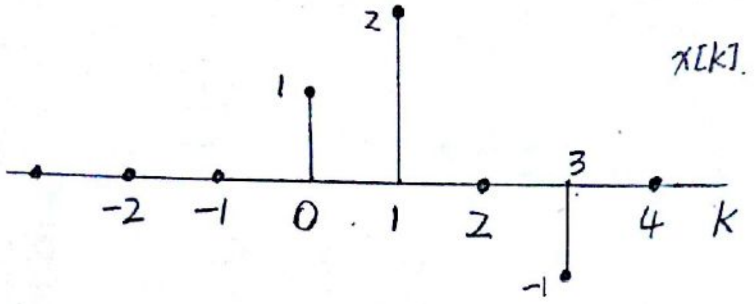
P86/2.1 (C)

$x[k]=\delta[k]+2\delta[k-1]-\delta[k-3]$，如下图：



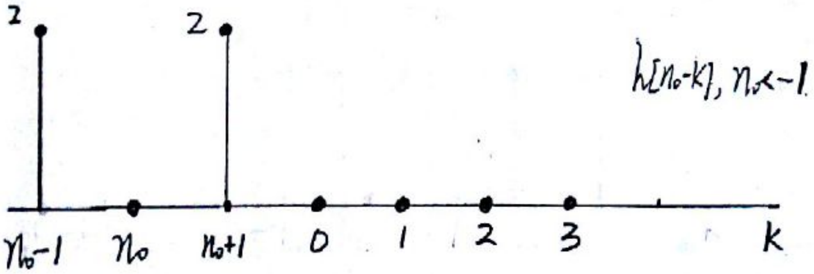
$x[k]$.

$h[n]=2\delta[n+1]+2\delta[n-1]$

$h[n+2]=2\delta[n+3]+2\delta[n+1]$.

记 $n_0=n+2$.

则 $h[n_0]=2\delta[n_0+1]+2\delta[n_0-1]$



$h[n_0-k],\ n_0<-1$

① 当 $n_0<-1$，即 $n<-3$ 时，

$x[k]$ 与 $h[n_0-k]$ 非零部分不重合.

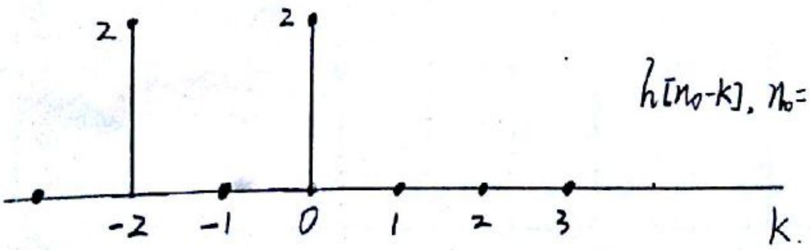
故 $y[n]=0$.

② 当 $-1\le n_0\le 4$，即 $-3\le n\le 2$ 时.

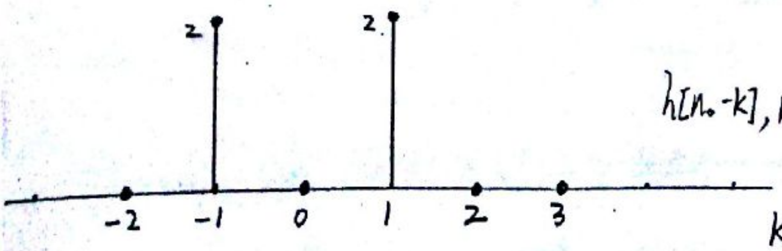
$$y[n]=\sum_{k=-\infty}^{+\infty}x[k]h[n_0-k]=\sum_{n_0=-\infty}^{+\infty}x[k]h[n_0-k]$$

$$=2+4+2+2+0-2$$

$$=8$$



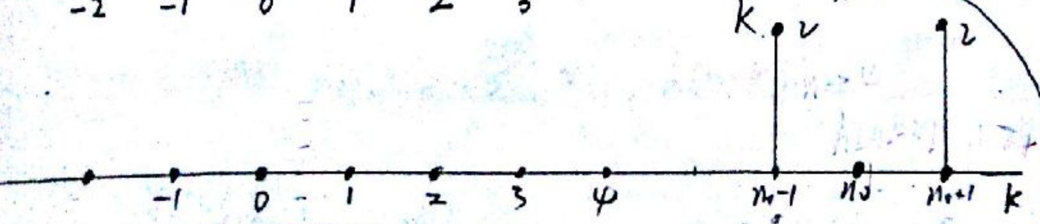
$h[n_0-k],\ n_0=-1$.



$h[n_0-k],\ n_0=0$

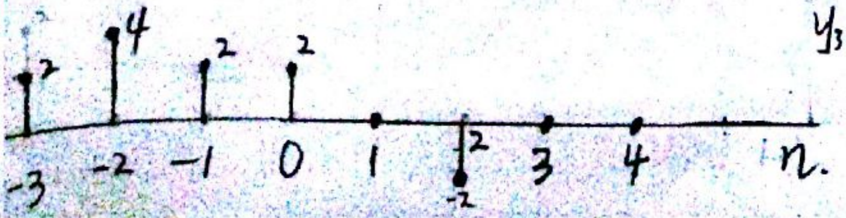
③ 当 $n_0>4$，即 $n>2$ 时

$y[n]=0$.



$n>4$

可得 $y_3[n]$ 图像



$y_3[n]$

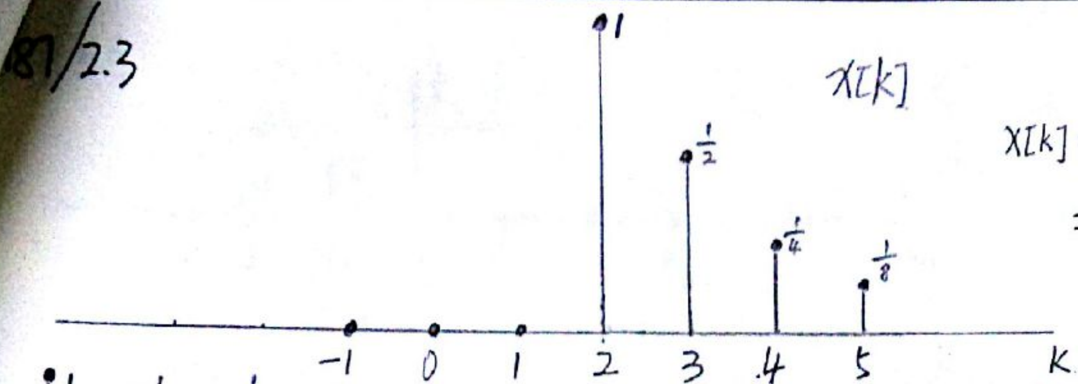
$y_3[n]=x[n]*h[n+2]$

$=2\delta[n+3]+4\delta[n+2]+2\delta[n+1]$

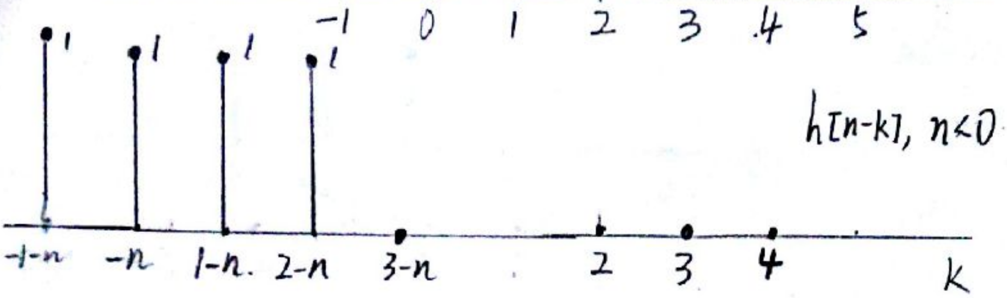
$+2\delta[n]-2\delta[n-2]$

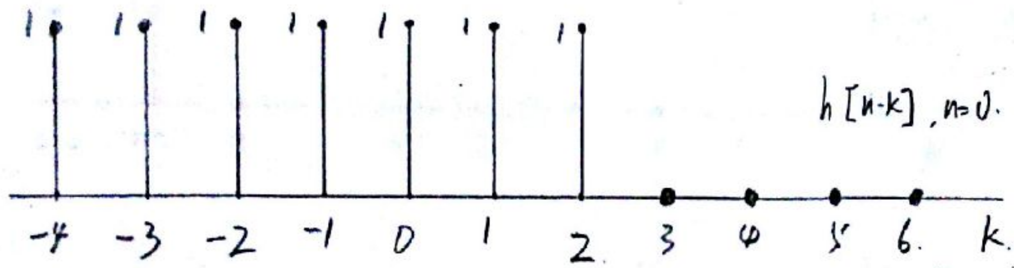
Scanned by CamScanner

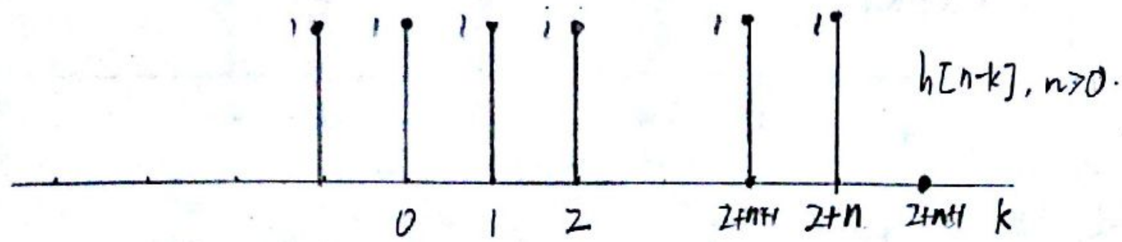
187/2.3



$$x[k] = \left(\tfrac{1}{2}\right)^{k-2} u[k-2] = \begin{cases} \left(\tfrac{1}{2}\right)^{k-2}, & k \geq 2 \\ 0, & k < 2 \end{cases}$$







当 $n<0$ 时，$x[k]$ 与 $h[n-k]$ 非零部分不重合，故 $y[n]=0$.

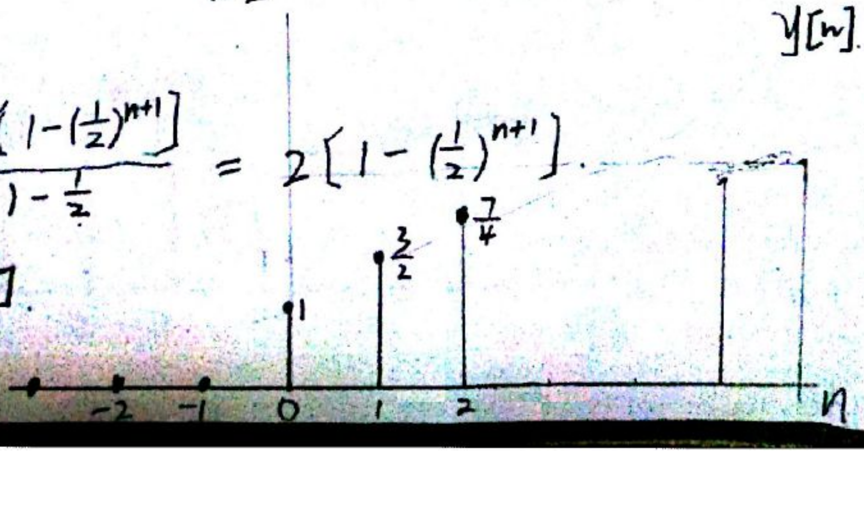
当 $n \geq 0$ 时，$y[n] = \sum_{k=2}^{2+n} x[k]\,h[n-k] = \sum_{k=2}^{n+2} \left(\frac{1}{2}\right)^{k-2}$

令 $r=k-2$，$k=r+2$.

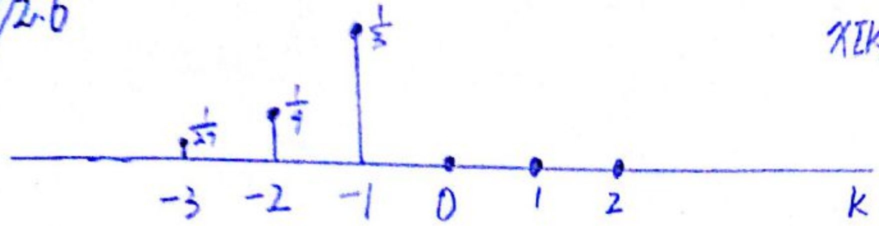
$$y[n] = \sum_{r=0}^{n} \left(\frac{1}{2}\right)^{r} = \frac{1\left[1-\left(\frac{1}{2}\right)^{n+1}\right]}{1-\frac{1}{2}} = 2\left[1-\left(\frac{1}{2}\right)^{n+1}\right]$$

综上，$y[n] = 2\left[1-\left(\frac{1}{2}\right)^{n+1}\right] u[n]$.

图像如右图所示：

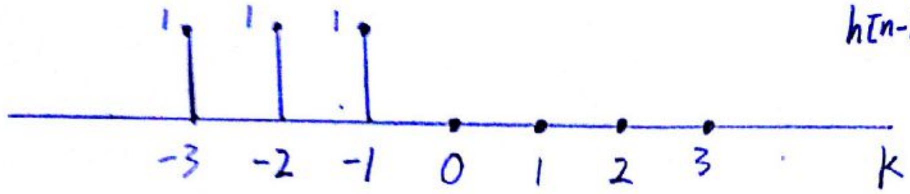


87/2.6

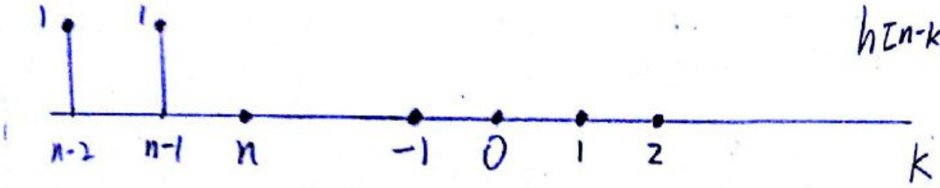


x[k]

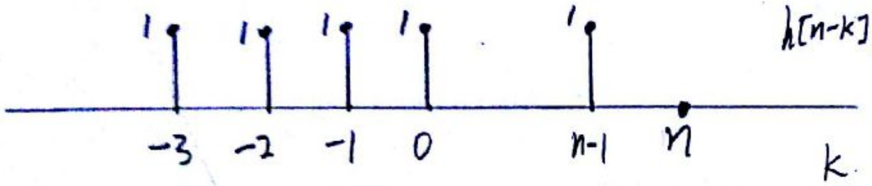
$$x[k]=\begin{cases}(\frac{1}{3})^{-k}, & k\le -1\\ 0, & k>-1\end{cases}$$



$h[n-k],\ n=0.$



$h[n-k],\ n<0$

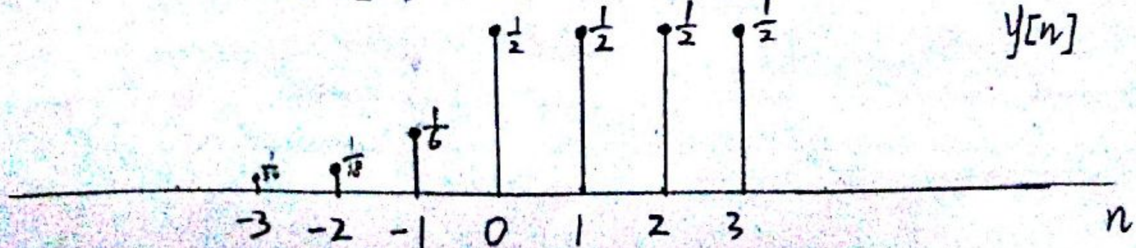


$h[n-k],\ n>0.$

$n\ge 0$时, $y[n]=\sum_{k=-\infty}^{-1}(\frac{1}{3})^{-k}=\sum_{-k=1}^{+\infty}(\frac{1}{3})^{-k}=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{1}{2}.$

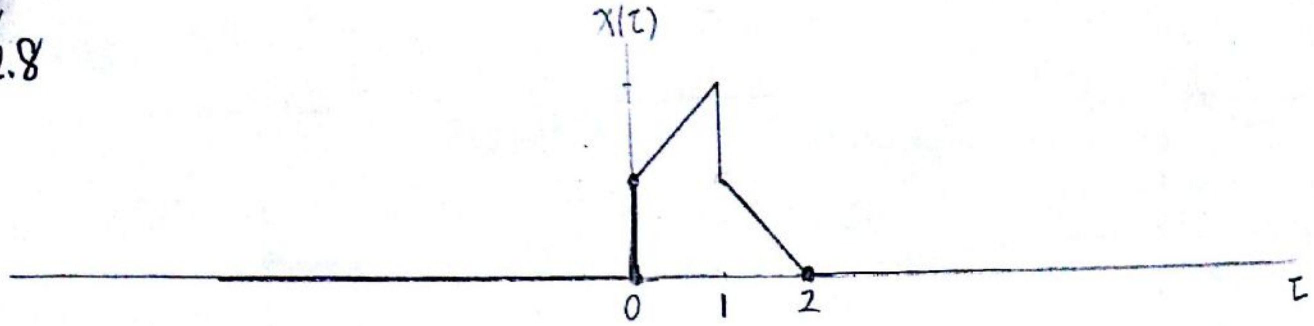
$n<0$时, $y[n]=\sum_{k=-\infty}^{n-1}(\frac{1}{3})^{-k}=\sum_{-k=1-n}^{+\infty}(\frac{1}{3})^{-k}=\frac{(\frac{1}{3})^{1-n}}{1-\frac{1}{3}}=\frac{3^n}{2}$

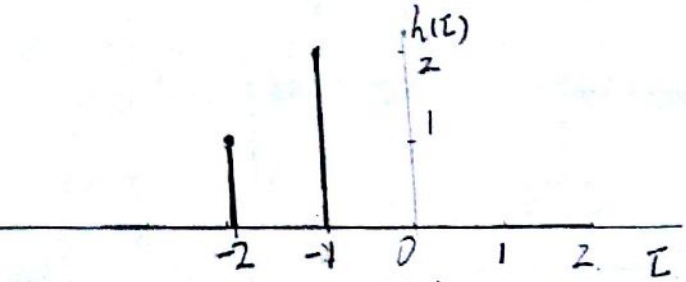
综上, 有 $y[n]=\begin{cases}\frac{3^n}{2}, & n<0\\ \frac{1}{2}, & n\ge 0\end{cases}$

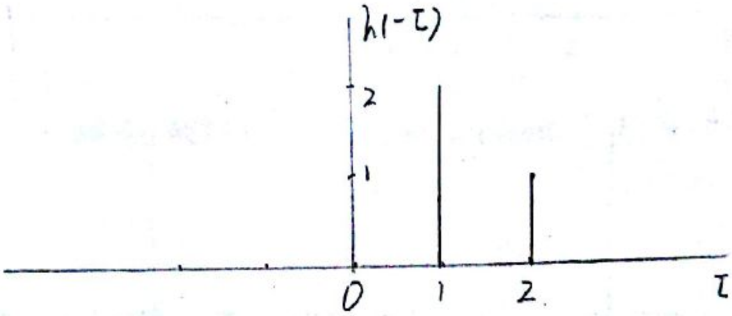


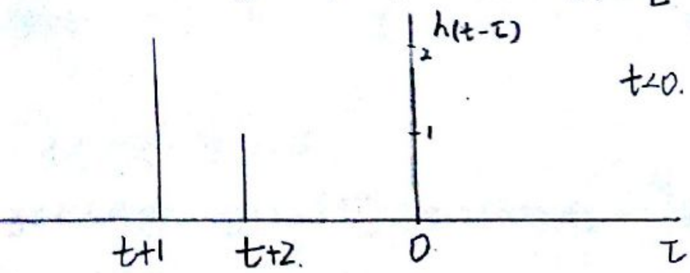
y[n]

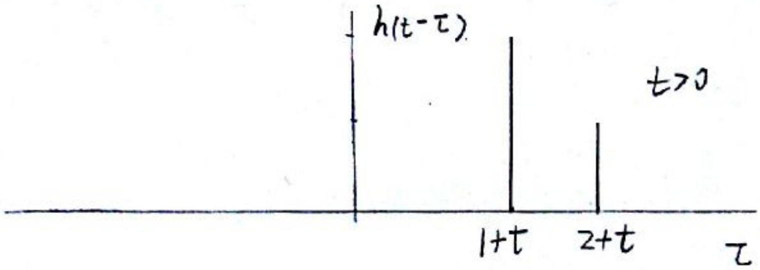
2.8











$t \geq 1$ 时, $x(\tau)\cdot h(t-\tau)=0$, $y(t)=0$.

$0<t<1$ 时, $y(t)=\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = 2\cdot[2-(1+t)] = 2-2t$

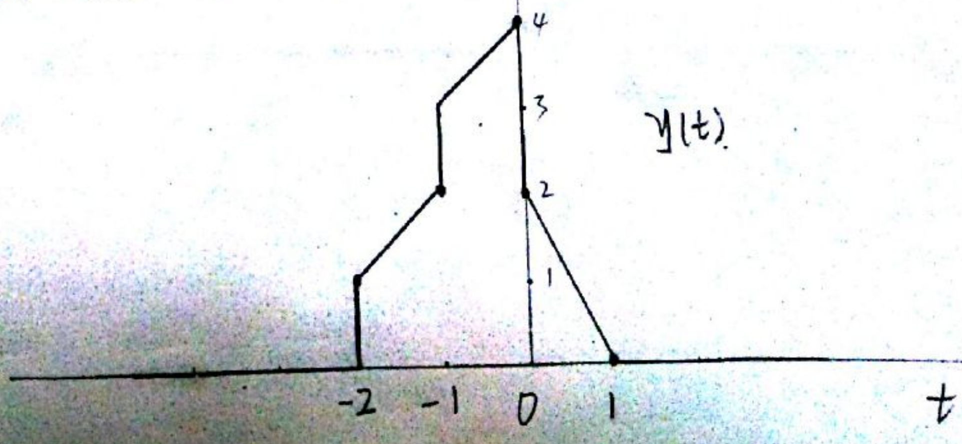
$-1<t\leq 0$ 时, $y(t)=\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = 2[(1+t)+1]+[2-(2+t)] = t+4$

$-2\leq t\leq -1$ 时, $y(t)=\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = (2+t)+1 = t+3$

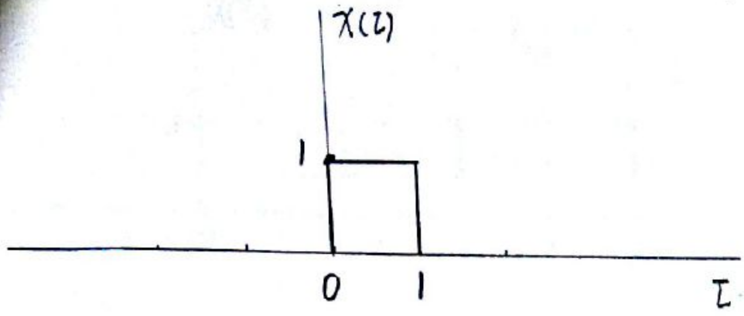
$t<-2$ 时, $y(t)=0$

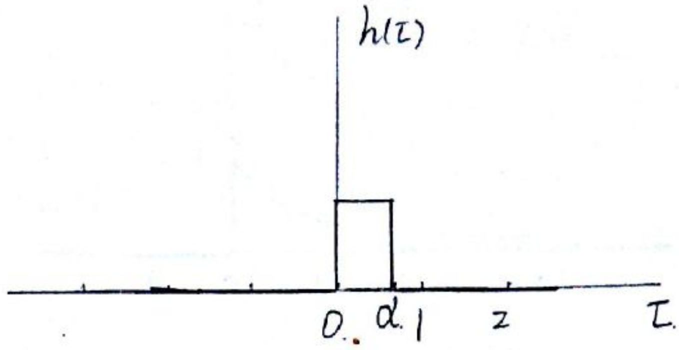
综上, 有

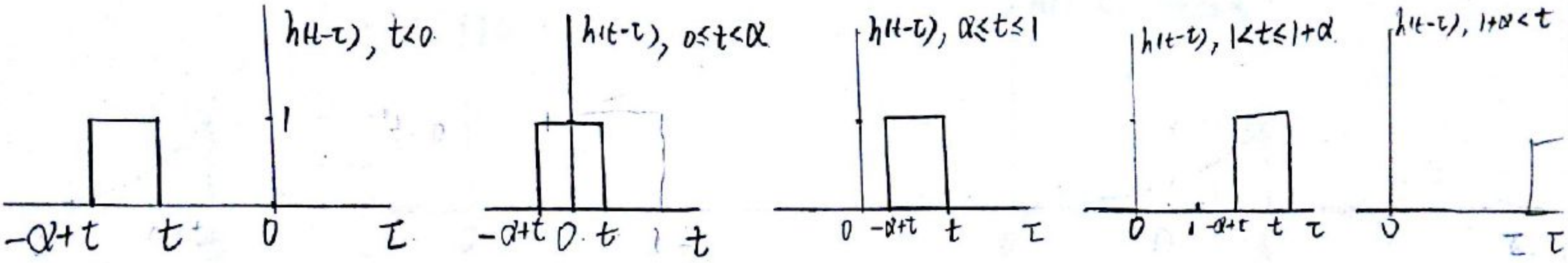
$$y(t)=\begin{cases} t+3, & -2\leq t\leq -1 \\ t+4, & -1<t\leq 0 \\ 2-2t, & 0<t<1 \\ 0, & \text{其他} \end{cases}$$



P87/2.10 (a)







$t<0$ 时, $y(t)=0$.

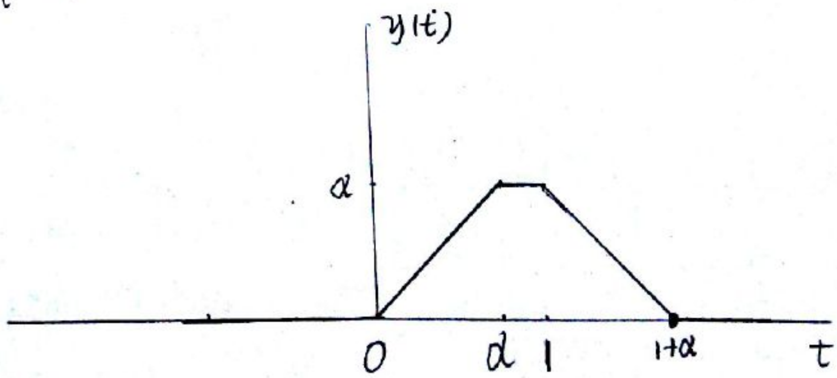
$0\le t<\alpha$ 时, $y(t)=\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)\,d\tau=\int_0^t 1\cdot d\tau=t$.

$\alpha\le t\le 1$ 时, $y(t)=\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)\,d\tau=\int_{-\alpha+t}^{t} 1\,d\tau=\alpha$

$1<t\le 1+\alpha$ 时, $y(t)=\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)\,d\tau=\int_{-\alpha+t}^{1} 1\cdot d\tau=1+\alpha-t$

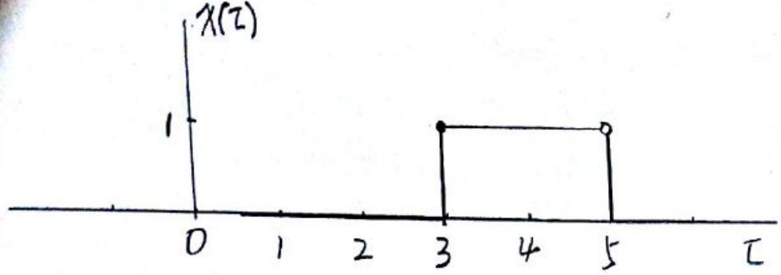
$t>1+\alpha$ 时, $y(t)=0$

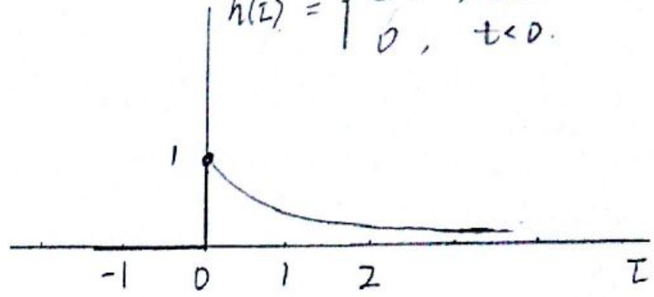
综上, $y(t)=\begin{cases} t, & 0\le t<\alpha \\ \alpha, & \alpha\le t\le 1 \\ 1+\alpha-t, & 1<t\le 1+\alpha \\ 0, & \text{其他} \end{cases}$

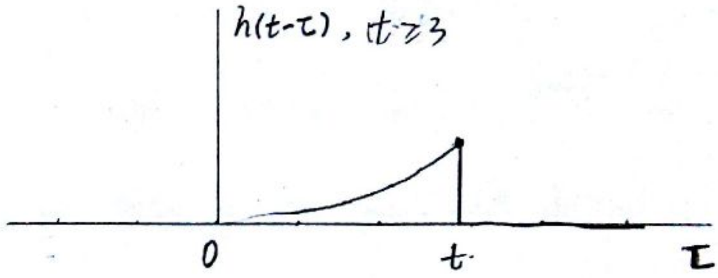


P87/2.11 (a)

$$h(\tau)=\begin{cases} e^{-3t}, & t\geq 0 \\ 0, & t<0 \end{cases}$$







$t<3$ 时, $y(t)=0$

$3<t\leq 5$ 时, $y(t)=\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_3^t e^{-3(t-\tau)}d\tau = \frac{1}{3}e^{-3(t-\tau)}\Big|_3^t = \frac{1-e^{3(3-t)}}{3}$

$t>5$ 时, $y(t)=\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_3^5 e^{-3(t-\tau)}d\tau = \frac{1}{3}e^{-3(t-\tau)}\Big|_3^5 = \frac{(1-e^{-6})e^{-3(t-5)}}{3}$

综上.

$$y(t)=\begin{cases} 0, & t<3 \\ \frac{1-e^{-3(t-3)}}{3}, & 3<t\leq 5 \\ \frac{(1-e^{-6})e^{-3(t-5)}}{3}, & t>5 \end{cases}$$

2.13

(a) $h[n] = (\frac{1}{5})^n u[n]$

$h[n-1] = (\frac{1}{5})^{n-1} u[n-1]$

令 $h[n] - A h[n-1] = \delta[n]$

则 $(\frac{1}{5})^n u[n] - A(\frac{1}{5})^{n-1} u[n-1] = \delta[n]$

$\because \delta[n] = u[n] - u[n-1]$

$\therefore [(\frac{1}{5})^n - 1] \cdot u[n] = [A(\frac{1}{5})^{n-1} - 1] \cdot u[n-1]$

$$\therefore \begin{cases} (\frac{1}{5})^n - 1 = A \cdot (\frac{1}{5})^{n-1} - 1 \\ (\frac{1}{5})^n - 1 = 0 \ , \ n=0 \end{cases}$$

得 $A = \frac{1}{5}$.

(b) $\because S_2$是线性时不变的

$\therefore h[n] * h_1[n] = \delta[n]$

$\because h_1[n] = g[n]$

$\delta[n] = h[n] - \frac{1}{5} h[n-1]$

$\therefore h[n] * g[n] = h[n] - \frac{1}{5} h[n-1]$

$\therefore g[n] = \delta[n] - \frac{1}{5}\delta[n-1]$.

2.14 (b)

$h_2(t) = e^{-t}\cos(2t)u(t)$

$\int_{-\infty}^{+\infty} |h_2(t)| dt = \int_0^{+\infty} |e^{-t}\cos 2t| dt$

$= \int_0^{+\infty} e^{-t} |\cos 2t| dt$

$e^{-t}|\cos 2t|$ 数值呈指数形式下降.

$h_2(t)$ 对应稳定的线性时不变系统

2.15 (b)

$h_2[n] = 3^n u[-n+10]$

$\sum_{k=-\infty}^{+\infty} |h_2[k]| = \sum_{k=-\infty}^{10} |3^k| \xlongequal{令r=-k} \sum_{r=-10}^{+\infty} (\frac{1}{3})^r$

$= \frac{(\frac{1}{3})^{-10}}{1-\frac{1}{3}} = \frac{3^{11}}{2} < \infty$

$\therefore h_2[n]$ 对应稳定的线性时不变系统.

2.17 (a)

记 $y(t)=y_p(t)+y_h(t)$.　　$t>0$ 时，$x(t)=e^{(-1+3j)t}$

设 $t>0$ 时，一个解的形式为 $y_p(t)=Ye^{(-1+3j)t}$　　$t>0$.

代入 $\frac{d}{dt}y(t)+4y(t)=x(t)$，有 $Y(-1+3j)e^{(-1+3j)t}+4Ye^{(-1+3j)t}=e^{(-1+3j)t}$

化简有 $(-1+3j)Y+4Y=(3+3j)Y=1$　　$\therefore Y=\frac{1}{3+3j}=\frac{1}{6}-\frac{1}{6}j$

$\therefore y_p(t)=(\frac{1}{6}-\frac{1}{6}j)e^{(-1+3j)t}$，$t>0$

假定 $y_h(t)=Ae^{st}$，代入 $\frac{d}{dt}y(t)+4y(t)=0$ 中有：

$Ase^{st}+4Ae^{st}=\cancel{As+4A}\ Ae^{st}(s+4)=0$.　　则 $s=-4$.

$\therefore y(t)=Ae^{-4t}+(\frac{1}{6}-\frac{1}{6}j)e^{(-1+3j)t}$.

将 $t=0$，$y(0)=0$ 代入上式，则 $0=A+\frac{1}{6}-\frac{1}{6}j$　　$\therefore A=-\frac{1}{6}+\frac{1}{6}j$

对 $t>0$，有 $y(t)=(\frac{1}{6}-\frac{1}{6}j)[e^{(-1+3j)t}-e^{-4t}]$

而对 $t<0$ 有 $y(t)=0$（由于初始松弛）. 结合后可得完全解为

$$y(t)=(\frac{1}{6}-\frac{1}{6}j)[e^{(-1+3j)t}-e^{-4t}]u(t)$$

2.18

当 $n \leqslant 0$ 时，$x[n]=0$，由初始松弛条件 $y[n]=0$.

初始条件为 $y[0]=0$.

对于 $n>0$.

$$y[1]=\frac{1}{4}y[0]+\delta[0]=1.$$

$$y[2]=\frac{1}{4}y[1]+\delta[1]=\frac{1}{4}$$

$$y[3]=\frac{1}{4}y[2]+\delta[2]=\left(\frac{1}{4}\right)^2$$

$$\vdots$$

$$y[n]=\frac{1}{4}y[n-1]+\delta[n-1]=\left(\frac{1}{4}\right)^{n-1}.$$

当 $n\leq 0$ 时，$y[n]=0$

当 $n>0$ 时，$y[n]=\left(\frac{1}{4}\right)^{n-1}$

综上，有 $y[n]=\left(\frac{1}{4}\right)^{n-1}\cdot u[n-1]$.