Hough Transform Line Parametrization

**Task:** Answer the Following Questions. You can just use this word template and insert your solution.

Q1. Show that if you use the line equation  , each image point (x, y) results in a sinusoid in () Hough space. Relate the amplitude and phase of the sinusoid to the point (x,y).

In the Hough Transform, the parametric equation of a line in polar coordinates is:

This means that when a point (x, y) in the image is given, as \theta changes, \rho also changes, forming a sinusoidal curve with respect to \theta. Once the point (x, y) is fixed, the slope and position of the line will vary with \theta, but all these lines will pass through the point (x, y).

For each different \theta, the equation computes a different value of \rho, which represents the perpendicular distance from the line to the origin.

Since both cos(theta) and sin(theta) are periodic functions, their linear combination also results in a periodic sinusoidal function. Therefore, for any given image point (x, y), the equation essentially describes a sinusoidal relationship between \rho and \theta.

In Hough space, the sinusoidal curve corresponding to each image point (x, y) represents all possible lines passing through that point. When multiple points lie on the same line, their respective sinusoidal curves intersect at a particular (p，theta) point in Hough space. This intersection point corresponds to the line in the image space. Thus, the formation of sinusoidal curves in Hough space helps us identify lines in the image by finding where these curves intersect.

Q2. Why do we parametrize the line in terms () instead of the slope and intercept (m, c)? Express the slope and intercept in terms of ().

1. The slope-intercept form (m, c) cannot represent vertical lines because the slope m is infinite, whereas theta = 90 form can represent vertical lines.

2. In the slope-intercept form, the slope m can take values from -infinity to +infinity which complicates the handling of slope in the Hough Transform. However, in the (p，theta) form, theta is limited to the range of 0 to 2pi (or 0 to 180 degrees).

3. In image processing tasks (such as edge detection), using polar coordinates to represent lines makes it easier to analyze lines in different directions and positions. The parameters directly correspond to the distance of the line from the origin and its orientation, which is more intuitive for detecting and analyzing lines.

4. In image processing, lines are often composed of a series of discrete pixel points, especially in the presence of noise or discontinuities along the edges. The (p，theta) form makes it easier to handle these discrete points.

Q3. Assuming that the image points (x, y) are in an image of width W and height H, that is, , what is the maximum absolute value of , and what is the range for ?

The parameter \rho represents the perpendicular distance from the origin (0, 0) to a line.The max p thus is：

sqrt(W^2 + H^2)

The parameter theta represents the angle of the line’s normal vector with respect to the x-axis. For lines in an image, \theta typically ranges from 0 to 180

The maximum absolute value of p is sqrt(W^2 + H^2) .The range of theta is (0, 180)