The Distributional Welfare Costs of Inflation Revisited in a Heterogeneous-agent Monetary-search Model

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Abstract

This paper develops a micro-founded general equilibrium model of payments to examine the distributional welfare costs of inflation. In equilibrium, low-income households use cash, while high-income households use deposits. I calibrate the model to the U.S. economy using 2017–2021 data. Counterfactual analysis shows that reducing inflation from 10% to 0% yields welfare gains for the bottom 20% of consumers—ranked by their consumption expenditure—that are 100 times larger than the average gain for the top 80%. Consumers who switch from cash to deposits as inflation falls experience the largest welfare gain, amounting to 2.7%.

1 Introduction

This paper examines the long-run real distributional effects of inflation in a heterogeneous agent New Monetarist model. The question of distributional welfare costs of inflation has been studied in the literature with heterogeneous households. This strand of literature is motivated by the following two stylized facts: low-income people more frequently use cash for transactions, and a non-negligible fraction of households lack access to checking accounts, debit cards, or credit cards for payment. However, in these work, either there is no micro-founded role for money or there is only one medium of exchange and households are heterogeneous only in terms of money holdings but not means of payment used. In this paper, I build a model of endogenous variation in means of payment used across transactions based on the interaction between the heterogeneity in the cost structures of payment methods and the heterogeneity of consumer preferences, and I use the framework to examine the role of monetary policy, as well as quantitatively calculate distributional welfare costs of inflation for consumers.

In practice, when a customer makes a purchase offline, let it be a meal at a food court or daily necessities from a grocery store, she often has different payment methods at her disposal. As to which means of payment are accepted in a specific transaction, it can depend on both the size of the merchant and how big the purchase is. Daily life experiences suggest that cash is the most commonly used means of payment for transactions at small merchants, such as food stall owners. In Singapore's hawker centers[1], approximately 90% of transactions were conducted using cash according to a 2016 KPMG survey. Larger merchants also accept bank deposits (i.e., credit card, debit card or checks), although the acceptance can be conditioned on the transaction value. Sometimes, card payment or its equivalents is accepted when the transaction value is above some thresholds. In 2010, Congress passed the Dodd-Frank Wall Street Reform and Consumer Protection Act, which stipulates that merchants are legally allowed to set a minimum purchase amount of up to \$10 for credit card transactions. Merchants' decision on which payment instrument(s) to accept is made partly based on their different cost structures and the size of customers' portfolios. For example, cash is costly to accept due to risk of theft, and resources spent on counting, sorting, and reconciling works. When merchants accept deposit, they often have to wait for some time before the funds are transferred to their accounts.

Motivated by these features of the data, I adopt the search-theoretical model in New Monetarist where media of exchange are essential for trade in the decentralised market where buyers and sellers trade in bilateral meetings, extended with *ex ante* heterogeneous consumers. The model features a two-sided market, where merchants make decisions on which mean(s) of payment to accept across transactions with heterogeneous size, and consumers with heterogeneous consumption needs decide on their portfolio choices. Deposits are issued by banks with heterogeneous investment returns who have to borrow from households whenever feasible.

The first main result of this paper is that there exists a cutoff of the amount of money held by buyers, above which card payment will be accepted by sellers and as a result buyers will bring only deposits in their portfolios. For portfolios with size below the cutoff, deposits will not be accepted by sellers and hence buyers only accumulate cash. I refer to the first type of meetings as deposit meetings and the latter as cash meetings. The sorting into deposits and cash meetings is a result of how cost is modeled in the framework. Motivated by the distinguishing features of cash and deposits discussed in the next section, I assume that sellers incur a proportional handling cost when cash is used in a transaction and a fixed user cost when deposits are used instead. Therefore, card payment is desirable from merchants' point of view only when the amount of

[1] A hawker center, commonly found in Singapore and Malaysia, is a food court that offers a variety of affordable local dishes.

money carried by consumers is large enough. Moreover, buyers will have the freedom to choose between cash and deposits when deposits are accepted by sellers, and they choose to bring only deposits because deposits are less costly to carry. A direct consequence of the existence of the cutoff is the differential inflation costs for consumers in cash and deposit meetings.

The second set of results concerns how the equilibrium changes when policy variables change. Firstly, when deposits become less costly to accept, the threshold of buyers' portfolio size for deposit meetings drops. More buyers accumulate deposits in their portfolios, and this increases buyers' aggregate demand for deposits. As a result, both the equilibrium deposit rate and the cut-off productivity for the marginal bank who earns zero profit decline, and more bank projects are getting financed. Secondly, when inflation is higher, bankers are making less profit through their investment in reserves. Consequently, the banking sector can only afford a lower deposit rate. With a lower deposit rate, trade surplus from deposit meetings decreases. Therefore, the threshold increases and less bankers can get financed by issuing deposits. Based on these two results, one potential policy recommendation for central banks aiming to enhance financial inclusion is to provide merchants accepting digital payment with subsidies or to implement a stricter monetary policy. This could increase the proportion of deposit meetings, thereby improving access to financial services for a larger segment of the population. The first part of the second set of results is consistent with what happens at Singapore hawker centers after the Hawkers Go Digital initiative was launched in 2018. In a registered stall, consumers can scan the stall's QR code and pay through apps like GrabPay, Paynow, and Paylah. The programme offers incentives like cash payouts and the waiver of transaction fees for registered stallowners. Additionally, e-payment transfers are processed in real-time, making transactions quicker and more efficient for both vendors and customers. As of late 2023, consumers in Singapore can use QR code payments at 11,000 of the 18,000 local market stalls. This is a significant increase from the end of 2018, by when only about 20% of stallholders had adopted card payments or other forms of e-payment solutions.

I then calibrate the model to the US economy using data from 2017 to 2021 in order to quantify the distributional welfare costs of inflation. I use consumption equivalents as the measure of inflation costs. The counterfactual analysis shows that for the bottom 20% of consumers ranked by their consumption expenditure, reducing inflation from 10% to 0% results in a welfare gain slightly above 1% for the bottom 10% of consumers and 0.8% for the remaining 10% ; while the number is much smaller for the rest of the distribution, with an average of 0.008%. In the calibrated model, all consumers in the first decile and some in the second decile participate in cash meetings. Hence their welfare gains from reduced inflation is much larger than those engaged in deposit meetings. Particularly, for the group of consumers who switch from carrying cash to deposits when inflation falls from 10% to 0%, they experience a welfare gain of 2.7%. This is precisely because they are deposits users when there is no inflation and become cash users when there is 10% net inflation.

My paper is related to the search-theoretical literature that integrates the Lagos and Wright framework with competition among multiple means of payment. For example, Zhang (2014) and Camera et al. (2004) study competition between two fiat monies; Lagos and Rocheteau (2008) and Geromichalos et al. (2007) model competition between fiat money and real asset or capital as media of exchanges. Among this strand of literature, the most relevant works are those considering outside and inside money such as He et al. (2008), Li (2011), and Yu (2023). In Li (2011), agents' means-of-payment decisions are also modelled explicitly, and there exists an equilibrium where checks are only used in large transactions. However, the distribution of asset holdings is degenerate since agents are ex ante identical in her model. Moreover, unlike their models where a representative agent holds both outside and inside money in some equilibria, although cash and bank deposit are both used by consumers in the decentralized market in my model, a particular consumer holds either cash or bank deposits and not both except for the cutoff consumer who is indifferent between carrying cash or deposits. In other words, the two media of exchange will not coexist within a specific meeting. The heterogeneity in buyers' portfolios results from decisions made by both sides of the market, which makes it feasible for me to examine the distributional welfare costs of inflation. The question of distributional impacts of policy shocks is often considered in the context of heterogeneous-agent New Keynesian models, exemplified by Nakamura (2024) and Kaplan et al. (2018). There have also been some attempts to incorporate heterogeneity in monetary search models so as to study the welfare impacts of monetary policies such as Chiu and Molico (2010), Jin and Zhu (2022) and Bustamante (2023). In their work, households hold heterogeneous portfolios with only one means of payment available. Overall, my paper closely follows the spirit of Erosa and Ventura (2002). Erosa and Ventura (2002) build a monetary growth model with a cash-in-advance constraint where low-income and high-income households choose between cash and costly credit to purchase goods. In their paper, inflation acts as a non-linear regressive tax since low income households are more exposed to it by holding more cash. Cao et al. (2021) extends their work with an overlapping generations model with age and cohort effects. My analysis thus revisits the concept of regressive inflation tax, incorporating a micro-founded role of money.

The rest of the paper is organized as follows. Section 2 builds the model. Section 3 characterizes the equilibrium by solving sellers' means-of-payment decisions and buyers' portfolio decisions. Section 4 conducts welfare analysis for different agents. Section 5 calibrates the model for counterfactual analysis. Section 6 concludes.

2 The model

The model is based on the framework of Lagos and Wright (2005). Time is discrete and with infinite horizon. There are four types of agents: a unit measure of buyers, a finite number of N merchants, a continuum of bankers, and the central bank. Each period is divided into two subperiods: a frictional decentralized market (DM) and a Walrasian centralized market (CM). Buyers and merchants first meet in the DM, then they enter the CM. There are two perishable goods which are produced and consumed in the two subperiods respectively: DM good and CM good. Buyers and merchants live forever and the discount factor across periods is $\beta \in (0,1)$.

In the DM, buyers and merchants meet bilaterally, with each merchant trading with a measure $\frac{1}{N}$ of customers. A major departure of my model from standard monetary-search models is that buyers are ex ante heterogeneous in terms of their DM preference. By consuming q units of the DM good, an α-type buyer's utility is $\alpha u(q_\alpha)$ with $u'(0) = \infty$, $u' > 0$, and $u'' < 0$. The distribution of α is summarized by the cumulative distribution function $F(\cdot)$ with support $[0,1]$. The distribution over α can be interpreted as capturing in a reduced-form way the heterogeneity in willingness to consume for households with heterogeneous income.

Merchants incur a linear cost in producing q units of DM good. Hence, the efficient amount of DM trade with an α-type buyer, denoted as q_α^{**}, solves $\alpha u'(q_\alpha^{**}) = 1$. There is no commitment and no record-keeping technology among buyers and merchants so DM trade must be quid pro quo. Buyers will have to use a means of payment in order to consume in the DM. I will elaborate on the available means of payment later. In the CM, both buyers and merchants can work and consume the CM good, x. Agents can produce one unit of CM good with unitary labor input, which generates one unit of disutility. Their preference for CM consumption is $U(x)$ with $U'(0) = \infty$, $U' > 0$, and $U'' < 0$. In summary, buyer's and merchant's period utilities in an α-type meeting are

$$U^b(q,x,h) = \alpha u(q_\alpha) + U(x) - h,$$
$$U^s(q,x,h) = -q_\alpha + U(x) - h.$$

Bankers of a given generation are born in the CM, and they become old the next period and die in the next CM. They only consume when they are old. A new cohort of bankers is born each CM. Young bankers are endowed with an investment project that they have no internal funds to finance. Therefore, they choose to borrow from households by issuing deposits with an interest rate. The deposit can be used as a means of payment in the DM. Investment projects take one unit of current CM good as input and have heterogeneous returns in the next CM as in Keister and Sanches (2022). There is a total measure η of bankers whose project returns are uniformly distributed in the support $[0, \bar{\gamma}]$. Banks' project returns are not fully pledgeable. Young bankers can only pledge up to a fraction ε of their project returns to depositors. Moreover, banks are subject to a reserve requirement that they can only invest a fraction $1-\mu$ of issued deposits into projects. The rest goes into reserves at the central bank.

The central bank issues two types of liabilities: cash and reserves. Both of them earn zero interest rate. Cash can be used to facilitate trade in the DM while reserve is illiquid. Thus there are two available means of payment in the DM: cash and bank deposits.

The flow of payments and goods is as follows. In the CM, merchants decide on their rules for accepting deposits. Note that sellers' acceptance rule is contingent on the type of buyer that they trade with. If the seller accepts deposits in the following DM, he has to bear a fixed cost f in the DM. Given merchants' means-of-payment decision, buyers with different values of α will accumulate different portfolios of cash and deposits in the CM. Deposit are issued by young bankers to finance their projects. In the following DM, each seller trades with a measure $\frac{1}{N}$ of buyers. Since the production function in the DM is linear, I can normalize the number of sellers to be 1. With this normalization, I have a representative merchant, who trade with a continuum of heterogeneous buyers in the DM. In the second subperiod, merchants redeem deposit received with old bankers and consume. Old bankers consume their project returns after paying back deposit and interest. Buyers receive lump-sum transfers and adjust their balances of cash and deposits. Figure 1 summarizes activities carried out by private agents in a timeline.

A buyer with a higher α can be interpreted as having a higher income than her peers with lower values of α. Higher-income buyers will spend more and, consequently, consume more in the DM. One possible motivation for the higher consumption of high-income households is that they have more consumption commitments, which are goods that incur transaction costs and are adjusted infrequently. For example, consumption on housing, vehicles and insurance are considered to be less adjustable and more committed. When high-income people have more of these goods, they will have a larger utility or gas bill than low-income ones.

In reality, there are many features that distinguish bank deposits from physical currency in terms of their role as media of exchange. In the model, I consider two prominent characteristics that differentiate bank deposits with cash when a merchant decides on which means of payment to accept. When a merchant accepts cash, he has to bear costs associated with counting and sorting cash. He also has to pay for services which can break large denomination bills and provide change. For businesses operating vending machines that sell drinks, rail tickets, and take meal orders, which accept cash, they face similar costs. There is significant human effort involved in ensuring these machines are properly stocked and maintained. Moreover, there are risks of receiving counterfeit currency as well as theft and loss. For example, cashiers may steal some money from the cash register during their shifts. These costs are summarized as the handling cost and are proportionate to the amount of cash received. When a merchant accepts deposits, it takes time for the network to process transactions. Moreover, if the initial payment attempt fails for any reason, the seller must reenter the transaction amount and prompt customers to tap their card again, resulting in a loss of time. He also has to incur some fixed infrastructure cost, including card readers, internet connectivity for payment authorization and clearing, and software to record and analyze payments. Meanwhile, he has to train staff to be technology-savvy. These costs are invariant to the transaction value, and are considered the fixed user cost. This fixed user cost associated with accepting card payment is well motivated in Li et al. (2020). There is also a proportional interchange fee imposed by card companies on merchants when accepting card payment. However, this interchange fee of using debit or credit cards can be smaller than the handling cost of cash. For example, In the EU, the Interchange Fee Regulation (IFR) caps debit card interchange fees at 0.2% and credit card fees at 0.3% of the transaction amount. Large merchants, such as supermarkets, usually have the bargaining power to negotiate a lower interchange fee with the card company. Therefore, I can normalize the proportional cost of accepting deposits to be zero in the model and the proportional cost of accepting cash would be the excess amount beyond deposits. Given the proportional handling cost of accepting cash and the fixed cost of accepting deposits per transaction, merchants have to decide whether they want to accept deposits in a transaction with an α-type buyer. Merchants must accept cash as dictated by law in most of the world[2], but they have the discretion to refuse bank deposit in their own interests. To illustrate this setup in a real-world context, consider transactions taking place in a supermarket. Every day, there are lots of patrons visiting the supermarket, and for each transaction, the supermarket gets to decide whether the card reader shall be turned on so that customers can pay with cards.

3 Equilibrium characterization

I solve the equilibrium in three steps. First, I solve buyers' portfolio decisions under two payment scenarios: one where only cash is accepted by the merchant, and another where both cash and deposits are accepted. Contingent on the seller's acceptance decision over deposits and buyers' DM preference, different portfolios will be accumulated in the CM and carried into the DM. Second, I solve the merchant's means-of-payment problem given a general portfolio consisting of both cash and deposits, and terms of trade determined by proportional bargaining protocol. Specifically, the

[2]However, in some countries such as Sweden, Canada and Singapore, a merchant can legally refuse cash payments.

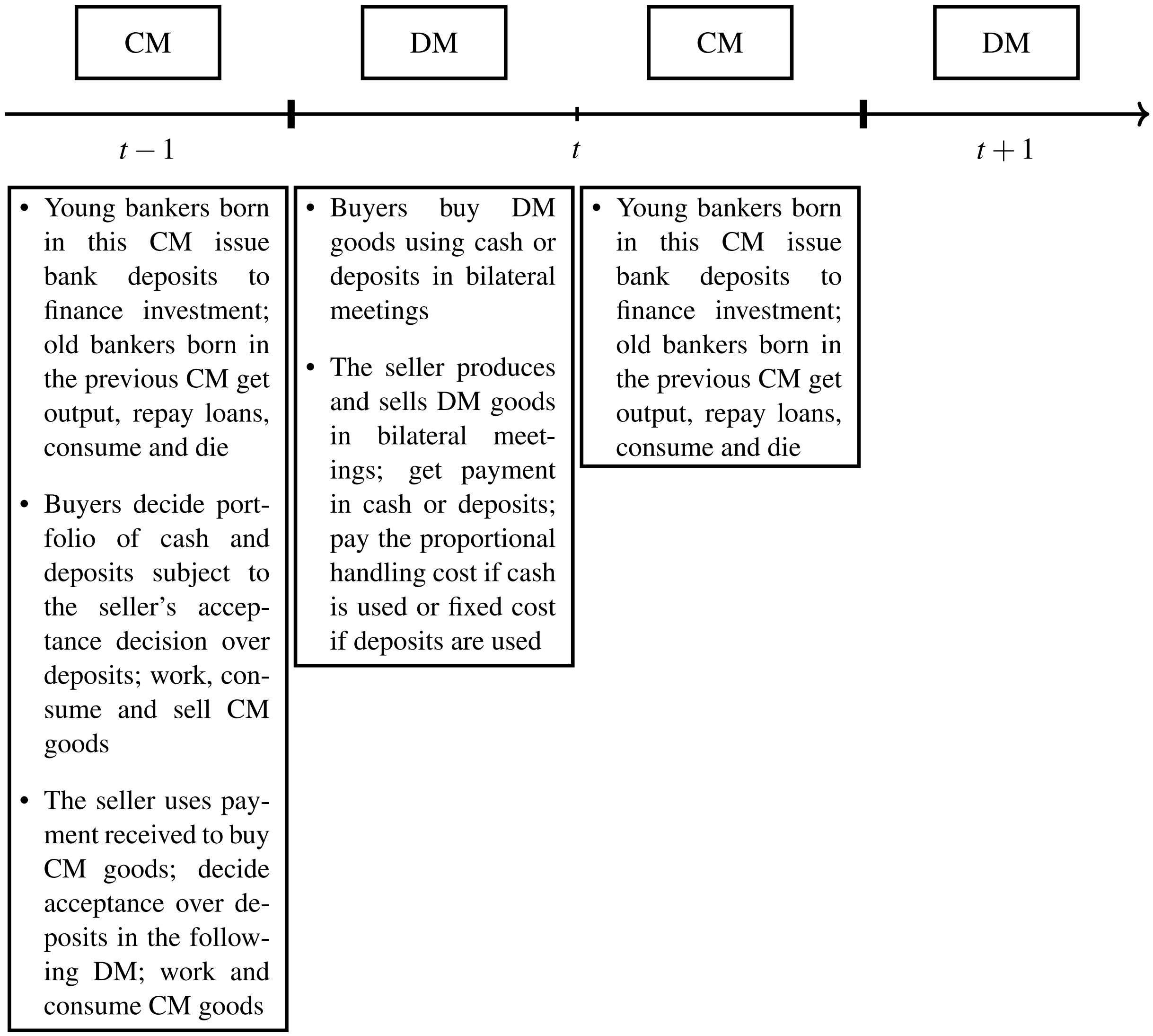

Figure 1: Timeline

merchant has to decide whether to accept deposits when trading with an α-type buyer carrying a general portfolio. Using the derived merchant's acceptance rule, I can deduce buyers' portfolios based on the value of α. Finally, I equate the demand and supply of bank deposits to obtain the equilibrium deposit rate.

3.1 The value functions and terms of trade

I first lay out buyers' CM problem. Define $\vec{a_\alpha} = (c_\alpha, d_\alpha)$ as the portfolio vector of real cash and deposits balances accumulated by an α-type buyer. Let $\vec{R} = (R_c, R_d)$ be the vector of real gross returns offered by cash and deposits. Let W and V denote buyers' CM and DM value functions. In the CM, buyers with different values of α choose their consumption of the CM good x^b, labor h, and portfolios $\vec{a'_\alpha}$ carried into the next DM.

$$W(\vec{a_\alpha}) = \max_{x^b,h,\vec{a'_\alpha}} \quad U(x^b) - h + \beta V(\vec{a'_\alpha})$$
$$\text{subject to} \quad x^b + \vec{1}\cdot\vec{a'_\alpha} = h + \vec{R}\cdot\vec{a_\alpha} + T,$$

where $\vec{1} = (1,1)$, "$\cdot$" is the inner product, and T is lump-sum transfers received by buyers in real term.

Assuming an interior solution for h and substituting h from the budget constraint, the CM value function can be rewritten as:

$$W(\vec{a_\alpha}) = R\cdot\vec{a_\alpha} + T + \max_{x^b} \quad \{U(x^b) - x^b\} + \max_{\vec{a'_\alpha}} \quad \{-\vec{1}\cdot\vec{a'_\alpha} + \beta V(\vec{a'_\alpha})\}.$$

The first-order condition with respect to $\vec{a'_\alpha}$ is

$$-1 + \beta\frac{\partial V(\vec{a'_\alpha})}{\partial a} \leq 0, \text{holding with equality if} \quad a'_\alpha > 0 \quad \text{for} \quad a = (c,d). \tag{1}$$

Buyers' DM value function is

$$V(\vec{a_\alpha}) = \alpha u(q_\alpha) + W(\vec{a_\alpha} - \vec{p_\alpha}),$$

where $(\vec{p_\alpha} \equiv (p_\alpha(c), p_\alpha(d)), q_\alpha)$ are the terms of trade, representing payment in cash and deposits, and the amount of DM goods traded. Note that $\vec{p_\alpha}$ is dependent on the merchant's acceptance decision over deposits. Thus, I have that

$$\frac{\partial V(\vec{a_\alpha})}{\partial c} = \alpha u'(q_\alpha)\frac{\partial q_\alpha}{c} + R^c(1-\frac{\partial p_\alpha(c)}{\partial c}) - R^d\frac{\partial p_\alpha(d)}{\partial c},$$
$$\frac{\partial V(\vec{a_\alpha})}{\partial d} = \alpha u'(q_\alpha)\frac{\partial q_\alpha}{d} + R^d(1-\frac{\partial p_\alpha(d)}{\partial d}) - R^c\frac{\partial p_\alpha(c)}{\partial d}.$$

A marginal increase in the amount of cash or deposits held changes buyers' DM value function due to changes in the terms of trade. It is worth emphasizing that when a buyer decides to carry more cash (or deposits) in the CM, the payment in deposits (or cash) may also change since cash and deposits are not perfect substitutes from the perspective of buyers. Buyers have a pecking order of usage of payment: deposits are preferred to cash. This pecking order stems from the marginal cost of using cash. Although buyers are not the direct bearer of the proportional handling cost associated with using cash, they will still take it into account since their surplus from DM trade is related to it.

I use proportional bargaining as the trading protocol in the DM, where buyers' bargaining power is θ. Given that an α-type buyer brings $\vec{a_\alpha} = (c_\alpha, d_\alpha)$ into the DM, the terms of trade solves the following problem:

$$\max_{q_\alpha, \vec{p_\alpha}} \quad S^b_\alpha$$
$$\text{subject to:} \quad S^b_\alpha = \frac{\theta}{1-\theta} S^s_\alpha,$$
$$0 \le \vec{p_\alpha} \le f(\vec{a_\alpha}), \vec{a_\alpha} = (c,d),$$

where $S^b_\alpha = \alpha u(q_\alpha) + W(\vec{a_\alpha} - \vec{p_\alpha}) - W(\vec{a_\alpha}) = \alpha u(q_\alpha) - R \cdot \vec{p_\alpha}$, and $S^s_\alpha = -q_\alpha + X(\vec{a_s}, \vec{p_\alpha}) - X(\vec{a_s}, 0) = -q_\alpha + R \cdot \vec{p_\alpha} - R^c \nu p_\alpha(c)$. S^b_α and S^s_α are the α-type buyer's and the seller's surplus from DM trade. Specifically, the α-type buyer's trade surplus is simply her gain from DM consumption minus the value of payment transferred. $X(\vec{a_s}, \vec{p_\alpha})$ is the seller's CM value function when he accumulates portfolio $\vec{a_s}$ and receives payment $\vec{p_\alpha}$. ν is the proportional handling cost associated with accepting cash, which decreases seller's trade surplus. $f(\vec{a_\alpha})$ is the buyer's usable liquidity subject to the seller's acceptance decision over deposits. If deposits are not accepted by the seller, $f(\vec{a_\alpha}) = (c_\alpha, 0)$; $f(\vec{a_\alpha}) = (c_\alpha, d_\alpha)$ if deposits are accepted by the seller.

Next, I will characterize solutions for the terms of trade problem above for the two cases where deposits are accepted and are not accepted. The solutions depend not only on the merchant's acceptance decision over deposits, but also on how much cash and deposits are carried by buyers.

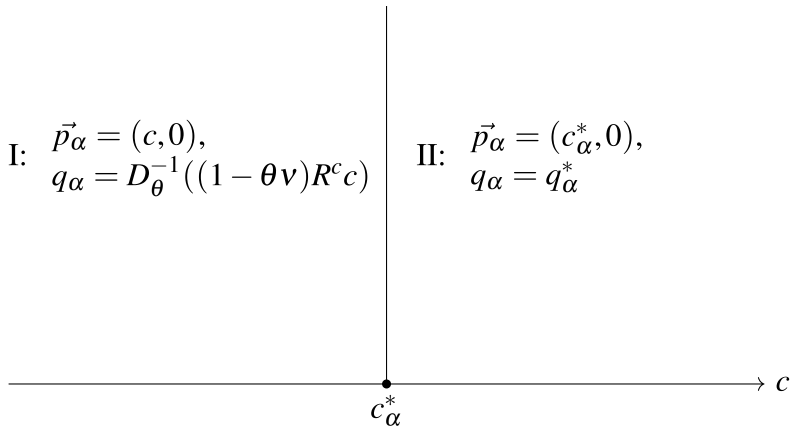

Figure 2: Terms of Trade in Cash Meetings

When cash is the only means of payment accepted,

$$q_\alpha = \begin{cases} D_\theta^{-1}((1-\theta\nu)R^c c_\alpha)) & \text{for} \quad c_\alpha < c^*_\alpha, \\ q^*_\alpha & \text{for} \quad c_\alpha \geq c^*_\alpha, \end{cases} \tag{2}$$

$$\vec{p}_\alpha = \begin{cases} (c_\alpha, 0) & \text{for} \quad c_\alpha < c^*_\alpha, \\ (c^*_\alpha, 0) & \text{for} \quad c_\alpha \geq c^*_\alpha, \end{cases} \tag{3}$$

where $D_\theta(q_\alpha) = (1-\theta)\alpha u(q_\alpha) + \theta q_\alpha$ is the amount of payment (after accounting for rates of return) needed to trade q_α in the DM. q^*_α solves $\alpha u'(q^*_\alpha) = \frac{1}{1-\nu}$. It is the second-best quantity that can be traded in the DM when cash is used in the transaction, and it is smaller than q^{**}_α, the first-best. Accordingly, $c^*_\alpha \equiv \frac{D_\theta(q^*_\alpha)}{(1-\theta\nu)R^c}$ is the amount of cash needed to achieve q^*_α. In other words, when the buyer brings sufficient cash into the DM, the second-best can be achieved and she will pay c^*_α. Otherwise, the buyer will exhaust all her cash holdings. Figure 2 is a graphical representation of equations (2) and (3). The horizontal axis represents the amount of cash brought into the DM by an α-type buyer.

When deposits are accepted by the merchant,

$$q_\alpha = \begin{cases} D_\theta^{-1}((1-\theta\nu)R^c c_\alpha + R^d d_\alpha)) & \text{for} \quad d_\alpha < d_\alpha^*, c_\alpha < c_\alpha^*(d), \\ q_\alpha^* & \text{for} \quad d_\alpha < d_\alpha^*, c_\alpha \geq c_\alpha^*(d), \\ D_\theta^{-1}(R^d d_\alpha) & \text{for} \quad d_\alpha^* \leq d_\alpha < d_\alpha^{**}, \\ q_\alpha^{**} & \text{for} \quad d_\alpha \geq d_\alpha^{**}, \end{cases} \tag{4}$$

$$\vec{p}_\alpha = \begin{cases} (c_\alpha, d_\alpha) & \text{for} \quad d_\alpha < d_\alpha^*, c_\alpha < c_\alpha^*(d), \\ (c_\alpha^*(d), d_\alpha) & \text{for} \quad d_\alpha < d_\alpha^*, c_\alpha \geq c_\alpha^*(d), \\ (0, d_\alpha) & \text{for} \quad d_\alpha^* \leq d_\alpha < d_\alpha^{**}, \\ (0, d_\alpha^{**}) & \text{for} \quad d_\alpha \geq d_\alpha^{**}, \end{cases} \tag{5}$$

where $d_\alpha^* \equiv \frac{D_\theta(q_\alpha^*)}{R^d}$ and $d_\alpha^{**} \equiv \frac{D_\theta(q_\alpha^{**})}{R^d}$ are the amount of deposits needed to consume q_α^* and q_α^{**} respectively. $c_\alpha^*(d)$ is the amount of extra cash needed to obtain q_α^* after d units of deposits are being paid. What is clear from the payment structure above is that deposits are always preferred to cash. In addition, buyers will only pay with deposits if the amount of deposits held can trade at least q_α^* units of DM goods, as q_α^* is the largest possible DM trade when cash is also used. Figure 3 visualizes equations (4) and (5). The x-axis and y-axis are the amount of cash and deposits carried into the DM by an α-type buyer respectively. When deposits only are not enough to achieve q_α^*, the terms of trade then depends on the buyer's cash holdings. After exhausting the usage of deposits, if the amount of cash is sufficient to obtain q_α^*, then $c_\alpha^*(d)$ units of cash will be paid; otherwise, the buyer will spend her entire portfolio of cash and deposits. However, if the amount of deposits is large enough to achieve at least q_α^*, only deposits will be used. The first-best is attainable when the size of deposits holding is sufficiently large.

3.2 Buyers' optimal portfolio choices

With the terms of trade derived earlier, I am now ready to further characterize buyers' optimal portfolio choices of cash and deposits in the CM. Again, buyers' portfolio choices rely on the seller's acceptance decision over deposits as well as the rates of returns on cash and deposits.

When cash is the only means of payment accepted, buyers will not accumulate any deposits for its liquidity property in the DM, and therefore $\vec{a_\alpha} = (c_\alpha, 0)$. Define function $\lambda(L_\alpha) = \max\{(1-\theta\nu)\frac{\alpha u'(q_\alpha(L_\alpha))}{D'_\theta(q_\alpha(L_\alpha)} - 1, 0\}$, where $q_\alpha(L_\alpha)$ satisfies $D_\theta(q_\alpha(L_\alpha)) = L_\alpha$, and L_α is available effective

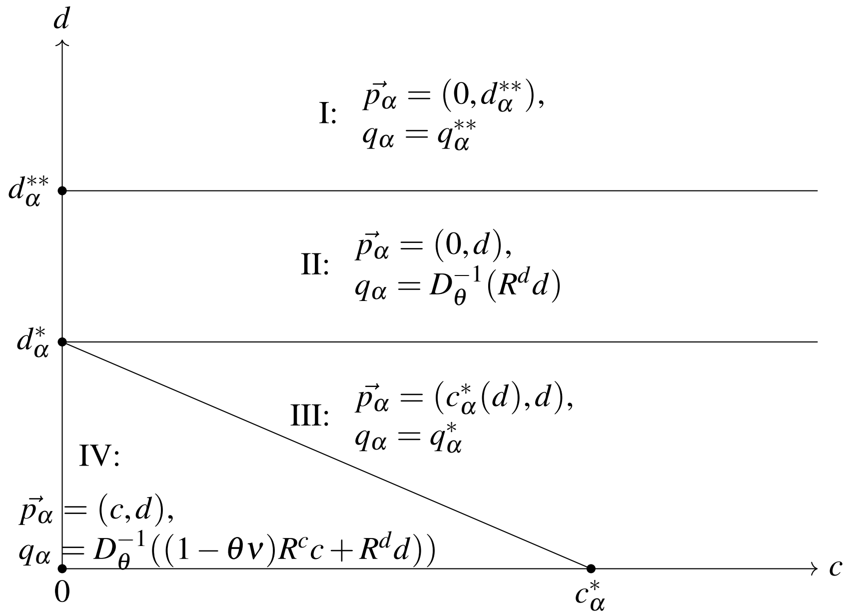

Figure 3: Terms of Trade in Deposit Meetings

liquidity for an α-type buyer. The first-order condition for cash is

$$\lambda((1-\theta\nu)R^c c_\alpha) = \frac{1}{\beta R^c} - 1.$$

An α-type buyer's portfolio can be explicitly expressed as

$$\vec{a_\alpha} = \begin{cases} (c_\alpha \geq c^*_\alpha, 0) & \text{for} \quad R^c = \frac{1}{\beta}, \\ (c_\alpha, 0), \quad c_\alpha = \frac{1}{(1-\theta\nu)R^c} D_\theta(u'^{-1}(\frac{\theta}{\alpha(\beta((1-\theta\nu))R^c+\theta-1)})) & \text{for} \quad R^c < \frac{1}{\beta}. \end{cases} \tag{6}$$

If cash is not costly to hold, buyers will accumulate sufficient cash to achieve q^*_α, and economize on their holdings of cash otherwise.

When deposits are accepted by merchants, the portfolio will depend on the deposit rate in turn:

$$\vec{a_\alpha} = \begin{cases} (0, d_\alpha \geq d^{**}_\alpha) & \text{for} \quad R^d = \frac{1}{\beta}, \\ (0, d_\alpha), \quad d_\alpha = \frac{1}{R^d} D_\theta(u'^{-1}(\frac{\theta}{\alpha(\beta R^d+\theta-1)}))) & \text{for} \quad (1-\theta\nu)R^c < R^d < \frac{1}{\beta}, \\ (c_\alpha, d_\alpha), \quad \text{such that}: c_\alpha + d_\alpha = \frac{1}{R^d} D_\theta(u^{me-1}(\frac{\theta}{\alpha(\beta R^d+\theta-1)}))) & \text{for} \quad (1-\theta\nu)R^c = R^d, \\ (c_\alpha, 0), \quad c_\alpha = \frac{1}{(1-\theta\nu)R^c} D_\theta(u'^{-1}(\frac{\theta}{\alpha(\beta((1-\theta\nu))R^c+\theta-1)})) & \text{for} \quad (1-\theta\nu)R^c > R^d, \end{cases}$$

where $(1-\theta\nu)R^c$ can be interpreted as the effective units of liquid cash facilitating DM trade per unit of cash accumulated in the current CM from buyers' point of view. This adjustment accounts for the proportional loss due to the handling costy, with buyers bearing a share of the loss equal to θ. When it is smaller than R^d, buyers will only accumulate deposits because deposits are a better means of payment in terms of promoting DM trade. Moreover, if deposits are not costly to hold, buyers will accumulate sufficient deposits to obtain q_α^{**}. Cash and deposits will have the same liquidity when $(1-\theta\nu)R^c = R^d$, therefore buyers will be indifferent between them. In the last case, cash is considered to be the better means of payment and deposits will not be held. Thereafter, I assume that $R^d > (1-\theta\nu)R^c$. As a result, as long as deposits can be used in the next DM, buyers will only bring deposits. In other words, I am only considering the first two cases when deriving asset demand.

3.3 Seller's means-of-payment decision

In the current CM, the seller has to make a decision over his acceptance of bank deposits in the following DM when meeting with an α-type customer. Note that the seller's means-of-payment decision is buyers' type-specific. A fixed cost f per meeting is borne by the seller in the following DM if he decides to accept deposits. Moreover, the seller has to commit his acceptance over deposits before trade takes place; therefore f does not appear in the bargaining problem discussed earlier. In the following, I present the seller's CM problem. Let X and V^s denote seller's CM and DM value functions. The seller can accumulate cash and deposits as stores of value, and he might do so when cash and deposits are not costly to hold. When cash is held as a store of value, there is no proportional handling cost incurred by the seller, unlike the situation when cash is used as a means of payment. Therefore, it is important to differentiate between these two types of cash held by the seller. I have two state variables in the seller's CM value function: one for portfolio accumulated by the seller himself and another for payment received by him in the previous DM. The seller chooses his consumption of the CM good x^s, labor h, portfolio $\vec{a_s}'$ carried into the following DM, and buyers' type-specific acceptance over deposits $\mathbb{1}'_{d,\alpha}$ in the next DM. $\mathbb{1}'_{d,\alpha}$ takes the value of 1 if deposits are accepted when trading with an α-type buyer and 0 otherwise. Cash is always accepted. Variables corresponding to the next period are indexed by prime, and by a double prime for two periods after that.

$$
\begin{aligned}
X(\vec{a}_s, \vec{p}_\alpha) = & \max_{x^s, h, \vec{a_s}', \mathbb{1}'_{d,\alpha}} \quad U(x^s) - h + \beta V^s(\vec{a_s}') \\
\text{subject to} \quad & x^s + \nu R^c p_\alpha(c) + \vec{1} \cdot \vec{a_s}' = h + R \cdot (\vec{a}_s + \vec{p}_\alpha), \\
& h \geq 0.
\end{aligned}
$$

where $V^s(\vec{a_s}') = -\mathbb{1}'_{d,\alpha} \times f - q'_\alpha + X(\vec{a_s}', \vec{p'_\alpha})$.

Assuming an interior solution for h and substituting h from the budget constraint and the seller's DM value function, the CM value function can be expressed as:

$$\begin{aligned} X(\vec{a}_s, \vec{p_\alpha}) = \vec{R} \cdot (\vec{a}_s + \vec{p_\alpha}) - \nu R^c p_\alpha(c) + \max_{x^s}\{U(x^s) - x^s\} + \max_{\vec{a_s}'}\{-\vec{1} \cdot \vec{a_s}' + \beta R \cdot \vec{a_s}'\} \\ + \max_{\mathbb{1}'_{d,\alpha}} \Big\{ - \mathbb{1}'_{d,\alpha} \times \beta f - \beta q'_\alpha + \beta(R \cdot \vec{p'_\alpha} - \nu R^c p'_\alpha(c)) \\ + \max_{\mathbb{1}''_{d,\alpha}} \{ - \mathbb{1}''_{d,\alpha} \times \beta f + \max_{\vec{a_s}''}(-\vec{1} \cdot \vec{a_s}'' + \beta U(\vec{a_s}''))\} \Big\} \\ = \max_{\mathbb{1}'_{d,\alpha}}\{-\mathbb{1}'_{d,\alpha} \times \beta f - \beta q'_\alpha + \beta R \cdot \vec{p'_\alpha} - \beta \nu R^c p'_\alpha(c)\} + \Omega. \end{aligned}$$

The seller's portfolio choice $\vec{a_s}'$ is independent of his means-of-payment decision, depending only on the rates of return. Moreover, the seller's acceptance decision over deposits in the current CM determines the terms of trade in the following DM. Ω represents all the remaining terms which are the same whether deposits are accepted or not.

Let $X(\vec{a}_s, \vec{p_\alpha}), X^0(\vec{a}_s, \vec{p_\alpha})$ be the seller's CM value functions when deposits are accepted and are not accepted. I denote variables where deposits are not accepted with a superscript 0.

$$\begin{aligned} X(\vec{a}_s, \vec{p_\alpha}) &= -\beta f + \beta(R \cdot \vec{p_\alpha} - \nu R^c p_\alpha(c) - q_\alpha) + \Omega, \\ X^0(\vec{a}_s, \vec{p_\alpha}) &= \beta(R \cdot \vec{p^0_\alpha} - \nu R^c p^0_\alpha(c) - q^0_\alpha) + \Omega. \end{aligned}$$

The net benefit of accepting deposits instead of cash only is

$$\begin{aligned} \Delta &= X(\vec{a}_s, \vec{p_\alpha}) - X^0(\vec{a}_s, \vec{p_\alpha}) \\ &= -\beta f + \beta\{q^0_\alpha - q_\alpha + R \cdot \vec{p_\alpha} - \nu R^c p_\alpha(c) - R \cdot \vec{p^0_\alpha} + \nu R^c p^0_\alpha(c)\}. \end{aligned} \tag{7}$$

Therefore $\mathbb{1}'_{d,\alpha} = 1$ if $\Delta > 0$. It is clear from expression (7) that the value of the fixed cost and different terms of trade subject to acceptance of deposits are key elements determining the sign of Δ. Since the terms of trade when deposits are accepted are discussed under four cases, I am going to solve the seller's means-of-payment problem under these four cases as well given the portfolio $\vec{a} = (c, d)$ carried into the DM by buyers.

3.3.1 Region 1

In this region, we have abundant deposits and limited amount of cash such that $d \geq d_\alpha^{**}, c < c_\alpha^*$. The terms of trade when deposits are accepted and are not accepted are:

$$\begin{cases} p_\alpha(c) = 0, \quad p_\alpha(d) = d_\alpha^{**}, \quad q_\alpha = q_\alpha^{**}, \\ p_\alpha^0(c) = c, \quad p_\alpha(d) = 0, \qquad q_\alpha^0 = D_\theta^{-1}((1-\theta\nu)R^c c)) < q_\alpha^*. \end{cases}$$

Hence the net benefit of accepting deposits are:

$$\begin{aligned} \Delta_1 &= -\beta f + \beta\{(1-\theta)(\alpha u(q_\alpha^{**}) - q_\alpha^{**}) + D_\theta^{-1}((1-\theta\nu)R^c c)) - (1-\nu)R^c c\} \\ &= \Delta(c,\alpha). \end{aligned}$$

It is shown in appendix A.1 that Δ_1 is decreasing in c, increasing in α and independent of d. These results are intuitive. When the amount of cash held increases, there will be more DM trade using cash only and therefore the net benefit of accepting deposits decreases. When α increases, both quantity traded and payment transferred in the DM rise, which makes the fixed cost of accepting deposits less costly per unit. Hence, the seller is more willing to accept deposits. In this region, the amount of deposits held is already sufficient to achieve the first-best. By having buyers carry one more unit of deposits, the seller's benefit of accepting deposits will remain unchanged and for this reason Δ_1 is independent of d.

Setting $\Delta(c,\alpha) = 0$ gives me a cutoff for c, which is α specific. Let c^α denote this cutoff, below which deposits will be accepted and vice versa. It is shown in appendix A.1 that c^α is increasing in α and decreasing in f. As a buyer's preference for DM consumption goes up, she will consume more and pay more in the DM, which makes deposits more desirable and the acceptance region of deposits will expand as a result. On the other hand, when the fixed cost of accepting deposits rises, the acceptance region of deposits will shrink.

3.3.2 Region 2

In this region, we have intermediate amount of deposits and limited amount of cash such that $d_\alpha^* < d < d_\alpha^{**}, c < c_\alpha^*$. The terms of trade when deposits are accepted and are not accepted are:

$$\begin{cases} p_\alpha(c) = 0, \quad p_\alpha(d) = d, \quad q_\alpha = D_\theta^{-1}(R^d d) \in (q_\alpha^*, q_\alpha^{**}), \\ p_\alpha^0(c) = c, \quad p_\alpha(d) = 0, \quad q_\alpha^0 = D_\theta^{-1}((1-\theta\nu)R^c c)) < q_\alpha^*. \end{cases}$$

Hence the net benefit of accepting deposits are:

$$\begin{aligned}\Delta_2 &= -\beta f + \beta\{D_\theta^{-1}((1-\theta\nu)R^c c)) - D_\theta^{-1}(R^d d) + R^d d - (1-\nu)R^c c\} \\ &= \Delta(c,d,\alpha).\end{aligned}$$

Same with the previous region, Δ_2 is decreasing in c, increasing in α, and additionally, increasing in d. The seller will be more willing to accept deposits if buyers carry more deposits in this region.

3.3.3 Region 3

In this region, we have small amount of deposits and limited but not too small amount of cash such that $d \leq d_\alpha^*, c_\alpha^*(d) \leq c < c_\alpha^*$. In this region, buyers carry enough cash to supplement whatever is needed to pay for q_α^* after using up deposits. The terms of trade when deposits are accepted and are not accepted are:

$$\begin{cases} p_\alpha(c) = c_\alpha^*(d), & p_\alpha(d) = d, \quad q_\alpha = q_\alpha^*, \\ p_\alpha^0(c) = c, & p_\alpha(d) = 0, \quad q_\alpha^0 = D_\theta^{-1}((1-\theta\nu)R^c c)) < q_\alpha^*. \end{cases}$$

Hence the net benefit of accepting deposits are:

$$\begin{aligned}\Delta_3 = &-\beta f + \beta\{D_\theta^{-1}((1-\theta\nu)R^c c)) - (1-\nu)R^c c \\ &+ \frac{(1-\theta)\nu}{1-\theta\nu}R^d d + \frac{(1-\theta)(1-\nu)}{1-\theta\nu}(\alpha u(q_\alpha^*) - \frac{1}{1-\nu}q_\alpha^*)\} \\ &= \Delta(c,d,\alpha).\end{aligned}$$

3.3.4 Region 4

In this region, we have small amount of deposits and small amount of cash such that $d \leq d_\alpha^*, c < c_\alpha^*(d)$. In this region, buyers do not carry enough cash and deposits to pay for q_α^*. The terms of trade when deposits are accepted and are not accepted are:

$$\begin{cases} p_\alpha(c) = c, & p_\alpha(d) = d, \quad q_\alpha = D_\theta^{-1}((1-\theta\nu)R^c c) + R^d d) < q_\alpha^*, \\ p_\alpha^0(c) = c, & p_\alpha(d) = 0, \quad q_\alpha^0 = D_\theta^{-1}((1-\theta\nu)R^c c) < q_\alpha^*. \end{cases}$$

Hence the net benefit of accepting deposits are:

$$\begin{aligned}\Delta_4 &= -\beta f + \beta\{D_\theta^{-1}((1-\theta\nu)R^c c) - D_\theta^{-1}((1-\theta\nu)R^c c) + R^d d) + R^d d\} \\ &= \Delta(c,d,\alpha).\end{aligned}$$

$\Delta_2, \Delta_3, \Delta_4$ all are functions of c, d, α. The only difference is the range of values that c and d can take. Setting them to be zero gives me d_α^c, which expresses d as a function of c and α. When buyers carry $d \geq d_\alpha^c$, the seller will accept deposits. By continuity, not surprisingly, d_α^c is reduced to c_α in region 1 where the amount of deposits are large. The four regions for an α-type meeting are summarized by Figure 4 below, where on the horizontal axis I have the amount of cash carried by the buyer into the DM and deposits on the vertical axis.

Lemma 1. *The red curve represents all the combinations of c and d such that $\Delta = 0$. Therefore, for (c,d)s sit above the curve, deposits are accepted and are rejected otherwise. $d_\alpha^{c=0}$ is the minimum amount of deposits that should be carried into the DM for the seller to accept deposit payment when $c = 0$. It is shown in appendix that the red curve will move to the right when α increases.*

Meanwhile, when α increases, the horizontal lines for d_α^* and d_α^{**} will shift upwards, and the vertical line for c_α^* will shift rightwards. In the specific case illustrated by Figure 4, the deposits acceptance region will become larger in a higher-α meeting.

Proposition 1. *There exists a cutoff, α_D, above which deposits will be accepted by the seller and rejected vice versa. It it such that*

$$-f + \{D_\theta(u'^{-1}(\frac{\theta}{\alpha_D(\beta R^d + \theta - 1)})) - u'^{-1}(\frac{\theta}{\alpha_D(\beta R^d + \theta - 1)})\} = 0, \tag{8}$$

Proof. *See the Appendix.*

where the terms in curly brackets are the seller's trade surplus when deposits are used as a means of payment. Equation (8) implicitly defines α_D as a function of f and R^d. It is shown in appendix A.3 that this cutoff is strictly increasing in f and decreasing in R^d. This is because when deposits become more expensive and yields lower return, the seller will be able to afford it in fewer meetings.

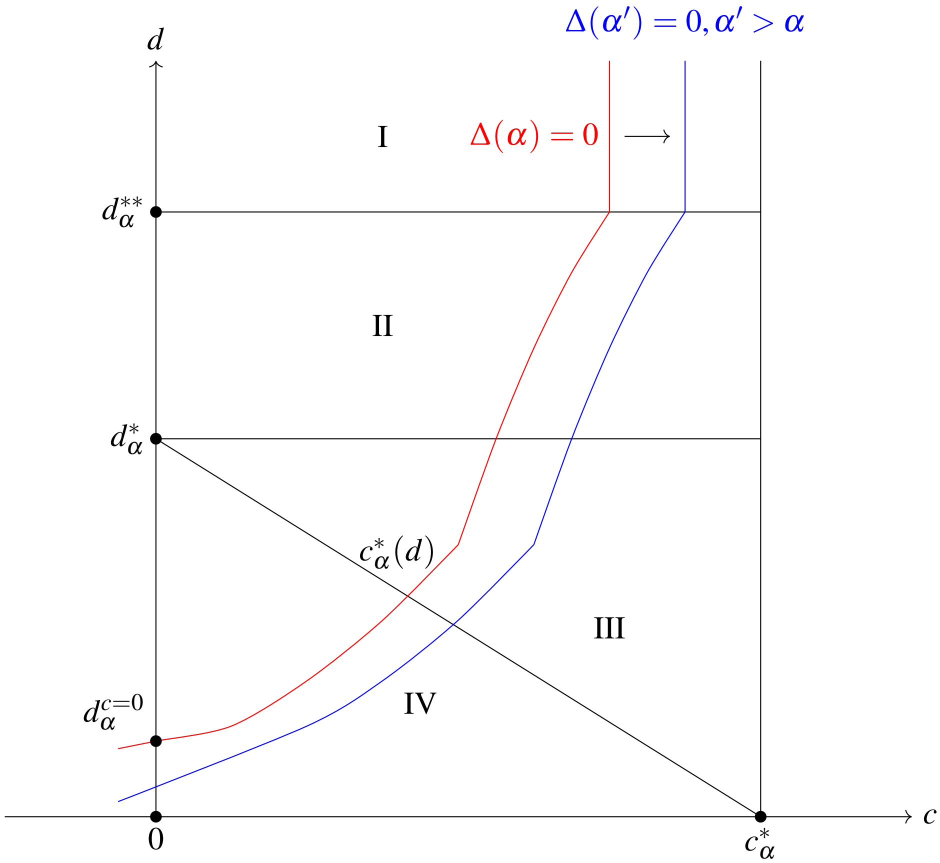

Figure 4: Seller's Acceptance Region

3.4 Asset supply

3.4.1 Bank deposits

As in Keister and Sanches (2022), the banking sector is perfectly competitive and all banks charge a single deposit rate. Let $\hat{\gamma}$ denote the productivity of the cutoff bank whose pledgeable future income is just enough to cover the promised repayment on deposits. The pledgeability restriction is only placed on returns of project investment. Banks' asset holding of central bank reserves is fully pledgeable. And it is assumed that bankers' ability to issue deposits is restricted by this plegeability constraint. Thus I have

$$R^d = (1-\mu)\varepsilon\hat{\gamma} + \frac{\mu}{\pi}. \tag{9}$$

The right-hand side is the cutoff bank's pledgeable income, which has two components. For every unit of deposits aresued, $(1-\mu)$ units is invested in the project which generates $(1-\mu)\hat{\gamma}$ units of CM goods next period and only a fraction ε of this return can be pledged. This explains the first term. The remaining μ units of issued deposit is invested in reserves with a rate of return of $\frac{1}{\pi}$, which is the second term.

3.4.2 Currency

I focus on stationary equilibria where the supply of money grows at a constant rate π such that $\frac{M'}{M} = \frac{\phi}{\phi'} = \pi > \beta$, where ϕ is the price of money in terms of CM good in current period and ϕ' for the next CM. The central bank's budget constraint is

$$\phi'(M' + R') = \phi'(M + R) + T',$$

where R' and T' are the amount of reserves outstanding, and the real lump-sum transfers received by buyers in the CM at period $t+1$. Here I assume that R^c, the gross real rate of return of cash, is bounded away from the Friedman rule. Hence, cash will never be held as a store of value by households.

3.5 Market clearing and equilibrium

Demand for cash comes from transactional need of buyers with $\alpha < \alpha_D$. The market clearing condition for cash is

$$\int_0^{\alpha_D} c_\alpha dF(\alpha) = \phi M.$$

Similarly, because only buyers with $\alpha \geq \alpha_D$ will accumulate bank deposits, the market clearing condition for deposits is

$$\int_{\alpha_D}^{\bar{\alpha}} d_\alpha dF(\alpha) = \eta(\bar{\gamma} - \hat{\gamma}), \tag{10}$$

where I have buyers' aggregate demand for bank deposits on the left-hand side and total deposits supply from the banking sector on the right-hand side. Again, only banks with productivity above $\hat{\gamma}$ can issue deposits to finance their projects.

The market clearing condition for reserves is

$$R = R'.$$

Substituting the expression for $\hat{\gamma}$ implied by equation (9), equation (10) can be rearranged as a function of α_D and R^d:

$$R^d = -\frac{(1-\mu)\varepsilon}{\eta} \int_{\alpha_D}^{\bar{\alpha}} d_\alpha dF(\alpha) + (1-\mu)\varepsilon\bar{\gamma} + \frac{\mu}{\pi}. \tag{11}$$

I can solve for α_D and R^d from equations (8) and (11). Then I can obtain $\hat{\gamma}$ from (9).

Definition 1. *A stationary equilibrium is a list of portfolios accumulated by buyers $\vec{a_\alpha}$, terms of trade $\{\vec{p_\alpha}, q_\alpha\}$, deposit rate R^d and two cutoffs $\{\alpha_D, \hat{\gamma}\}$ that satisfy equation* (1)*, equations* (2)-(5) *dictating terms of trade, and conditions* (8)-(10).

To guarantee the existence and uniqueness of an equilibrium where deposits are used as medium of exchange, I make the following two assumptions:

A1: $\frac{1}{R^d} D_\theta(u'^{-1}(\frac{\theta}{\alpha(\beta R^d + \theta - 1)}))$ is strictly increasing in R^d;

A2: $f < \bar{f} = (1-\theta)(\bar{\alpha} u(q^{\bar{\alpha}**}) - q^{\bar{\alpha}**})$.

Proof. *See the Appendix.*

The first assumption ensures that an α-type buyer's demand for deposits are strictly increasing in the deposit rate, which is a sufficient but not necessary condition for the equilibrium to exist.[3] The necessary condition is that when the deposit rate increases, if an α-type buyer's demand for deposits were to drop, this decrease is smaller than the reduction in fund demand from the banking sector due to the higher borrowing cost. As a result, α_D goes up to further bring aggregate fund supply down. In this case, intensive margin effect and extensive margin effect work together to lower households' aggregate demand for deposits. $\bar{f}$ in the second assumption is the fixed cost in an equilibrium where $R^d = \frac{1}{\beta}$ and $\alpha_D = \bar{\alpha}$. In this particular equilibrium, deposits are not costly to carry and yet it is not accepted and used as a medium of exchange in the DM due to the high fixed cost. Hence, an equilibrium where bank deposits are used as means of payment exists whenever $f < \bar{f}$. Intuitively, $(1-\theta)(\bar{\alpha}u(q^{\bar{\alpha}**}) - q^{\bar{\alpha}**})$ is the seller's largest possible gain from trade when deposits are used, so if the fixed cost for accepting deposits is smaller than that, the seller will always be willing to accept deposits in some meetings. The next proposition presents some comparative static results:

Proposition 2. *In the equilibrium where cash is used in meetings with $\alpha < \alpha_D$ and deposits are used in meetings with $\alpha \geq \alpha_D$:*

1. *By lowering the fixed cost f of accepting deposits, $\alpha_D, R^d, \hat{\gamma}$ all decrease;*

2. *By having a higher rate of inflation, α_D and $\hat{\gamma}$ increase while R^d decreases.*

Proof. *See the Appendix.*

Firstly, when deposits becomes less costly to accept, the seller will accept deposits in more meetings. This increases buyers' aggregate demand for deposits. As a result, the deposit rate decreases and more bankers are getting financed. Secondly, when inflation is higher, bankers earn lower profits from their investments in reserves. Consequently, the banking sector can only afford a lower deposit rate. With a lower deposit rate, buyers will bring less deposits into the DM in meetings where deposits are used and the gains from trade will drop, putting pressure on the seller to raise α_D. Less bankers are getting financed in response to this reduction in fund supply[4]. In order to

[3] If u takes the form of CRRA, a sufficient but not necessary condition for this assumption to be satisfied is that the coefficient of relative risk aversion is $\in (0, \frac{3}{4})$ given $\theta \in (0,1)$. When ρ is greater than 1, this assumption is always violated but the equilibrium may still exist.

[4] However, this monetary policy shock can have a different impact on the banking sector if Assumption $A1$ is violated but the equilibrium still exists. Now suppose an individual buyer holds more deposits in her portfolio following a decrease in R^c and R^d, and this increase in individual deposits holdings outweighs the decline in the fraction of meetings where deposits are accepted, then $\hat{\gamma}$ decreases in response to the rise in aggregate fund supply. In this case, higher inflation benefits the financial sector since more projects are being invested with a lower funding cost.

have an increase in α_D when R^c decreases, what is necessary is that the decrease in the deposit rate R^d is larger than the decrease in the cost of accepting deposits. Therefore, the second mechanism can still hold true when I relax the fixed cost assumption for accepting deposits.

4 Welfare analysis

In this section, I will look into welfare of the seller, banks and buyers in turn. First of all, the representative seller's welfare, denoted as $\mathscr{W}^s$, is derived from aggregating his trade surplus from all types of meetings minus fixed costs of accepting deposits.

$$\mathscr{W}^s = \int_0^{\alpha_D} (-q_\alpha + (1-\nu)R^c p_\alpha(c)) f(\alpha) d\alpha \\ + \int_{\alpha_D}^{\bar{\alpha}} (-q_\alpha + R^d p_\alpha(d) - f) f(\alpha) d\alpha.$$

The banking sector's welfare sums up old bankers' profits:

$$\mathscr{W}^e = \eta \int_{\hat{\gamma}}^{\bar{\gamma}} ((1-\mu)\gamma + \frac{\mu}{\pi} - R^d) d\gamma,$$

where the integrand represents the profit for the banker whose project return is γ and there is a measure η of them.

Next, I assume that the merchant's business and the whole banking sector are owned by buyers. The merchant and banks can be interpreted as entrepreneurs in the model. The number of shares owned by an α-type buyer is dictated by Φ^α, which is assumed to be strictly increasing in α. That is, buyers with a higher α are holding more shares of entrepreneurs' businesses. Correspondingly, the welfare of an α-type buyer is expressed as:

$$\mathscr{W}^\alpha = \alpha u(q_\alpha) - R^d p_\alpha(d) - R^c p_\alpha(c) + \Phi^\alpha(\mathscr{W}^s + \mathscr{W}^e).$$

It has two components: one is gains from trade in the DM, which are referred to as labor income thereafter, and another is capital income.

Proposition 3. *When there is higher inflation, $\mathscr{W}^s$ decreases, and $\mathscr{W}^e$ also decreases if $\varepsilon > \frac{1}{2}$. Moreover, the whole distribution of buyers lose.*

Proof. *See the Appendix.*

The seller experiences a welfare loss because there is less gain from trade in the DM. The banking sector loses because there is less investment.

I now elaborate on the welfare changes for buyers with different types of α. Aside from the lower capital income, buyers also have lower trade surplus from DM trade, but their exposure varies. Following the higher inflation, the seller will accept deposits in fewer meetings, i.e., α_D increases. I denote the new cutoff as $\tilde{\alpha_D}$. Buyers with $\alpha \in (0, \alpha_D)$, who are at the lower end of the distribution, carry cash both before and after the inflation change. Their exposure is moderate, as cash provides no protection against inflation. Buyers with $\alpha \in (\alpha_D, \tilde{\alpha_D})$, who are at the intermediate part of the distribution, carry deposits before the change but use cash afterwards. They suffer the most, as they are switched from the good hedging device to the bad one against infla tion. For high-income buyers with $\alpha \in (\tilde{\alpha_D}, \bar{\alpha})$, they are able to use deposits both before and after the inflation change. Their exposure is the smallest, as they have consistent access to the good hedging device.

5 Quantitative analysis

5.1 Calibration

I consider an annual model. CM and DM utility functions take the logarithmic form $U(x) = Alog(x)$ and CRRA form $u(q) = \frac{(q+B)^{1-\rho} - B^{1-\rho}}{1-\rho}$ respectively. Hence, I have that the efficient CM consumption is $x^* = A$. The parameter B is set to 0.001, which ensures that $u(0) = 0$. α follows the beta distribution with two shape parameters denoted as $\alpha_{dist}, \beta_{dist}$. $\bar{\alpha}$ can be normalized to one without loss of generality. It is the ratio between DM and CM consumption that matters, which can be effectively captured by the parameters α and A. Hence, there are 14 parameters to calibrate: $(\beta, \mu, \nu, \pi, \varepsilon, \Phi^{\alpha}, f, \theta, A, \rho, \alpha_{dist}, \beta_{dist}, \bar{\gamma}, \eta)$. There is a direct match for the first six parameters: $(\beta, \mu, \nu, \pi, \varepsilon, \Phi^{\alpha})$. The rest has to be calibrated internally. I calibrate $(\mu, \pi, \Phi^{\alpha}, f, \theta, A, \rho, \alpha_{dist}, \beta_{dist}, \bar{\gamma}, \eta)$ using data from 2017 to 2021. ν is set to match the data in 2014.

The data used in my calibration exercise come from seven sources: (1) data from the Diary of Consumer Payment Choice (DCPC) from the Federal Reserve Bank of Atlanta; (2) data from the survey of costs of payments in Canada; (3) distribution of personal consumption expenditures from U.S. Bureau of Labor Statistics; (4) distribution of household capital income from U.S. Bureau of Economic Analysis; (5) haircuts applied to high-quality liquid assets (HQLA) in the Basel Framework; (6) call report data from the Federal Financial Institutions Examination Council; (7)

several time series on macro variables and reserves from Federal Reserve Economic Data (FRED). In what follows, I will elaborate on calibration details.

5.1.1 Direct targets

Firstly, I will explain the direct targets for $(\beta, \mu, \nu, \pi, \varepsilon, \Phi^{\alpha})$. β is set to 0.96 as standard in literature. I choose the reserve requirement μ to match the average ratio of required reserves to the total transaction in deposits in 2017-2020. Similarly, π is chosen to match the average inflation rate in 2017-2021. The proportional handling cost of accepting cash, ν, is targeted to Canadian retailers' average private cost per transaction value when handling cash. In 2014, the total value of point-of-sale cash transactions in Canada amounted to \$145,951 million Canadian dollars, and the private cost of cash incurred by retailers is \$2,384 million Canadian dollars as calculated in Kosse et al. (2017). Hence, $\nu = \frac{2{,}384}{145{,}951} = 0.016$. Since U.S. data is not available in this regard to the best of my knowledge, I use Canada's data as a close substitute. I target banks' plegeability constraint ε to the 15% haircut applied to Level 2A assets held in the stock of HQLA as stipulated by Basel III. Level 2A assets include commercial paper and covered bonds. In this sense, ε can be interpreted as the fraction of banks' project returns that can be liquidated by depositsors. Lastly, Φ^{α} is chosen to match the average distribution of household capital income share from 2017 to 2020. However, since the data is only available at the decile level, I need to impute the capital income share for each α-type consumer. To this end, I assume that the capital income share function for α is an upward-sloping linear function within each decile. The slope of this function is estimated so that the predicted capital income share for each decile matches the observed data. Next, I will explain how the rest of parameters are calibrated internally.

5.1.2 Indirect targets

I use card acceptance data from DCPC, consumption expenditure share for households divided into ten deciles, the real interest rate on transactional deposits, and a retail markup of 1.2 to jointly calibrate the rest eight parameters: $(f, \theta, A, \rho, \alpha_{dist}, \beta_{dist}, \bar{\gamma}, \eta)$. In DCPC, there is a transaction-level variable which records the fraction of transactions where a credit or debit card would have been accepted for this transaction. Therefore I can calculate the percentage of transactions where card is rejected, which I regard as the card rejection rate. The card rejection rate in my model is the sum of densities of meetings where deposits are not accepted by the seller. Consumption expenditure for an α-type consumer is $C^{\alpha} = R^d p_{\alpha}(d) + R^c p_{\alpha}(c) + A + \Phi^{\alpha}(\mathscr{W}^s + \mathscr{W}^e)$, where $R^d p_{\alpha}(d) + R^c p_{\alpha}(c)$ is expenditure in the DM and $A + \Phi^{\alpha}(\mathscr{W}^s + \mathscr{W}^e)$ is expenditure in the CM.

The overall markup predicted by the model is $\int_0^{\bar{\alpha}} \frac{\vec{R}\cdot\vec{p}_\alpha}{q_\alpha} dF(\alpha)$. The loss function to be minimized is defined as the sum of squared errors for these 13 moments with equal weights.

Table 1 lists all calibrated parameter values and their corresponding calibration targets. Figure 5 shows the match on the average consumption expenditure distribution from 2017 to 2021.

Table 1: Calibration Results

Parameters	Notation	Value	Calibration Targets
Calibrated externally			
Discount factor	β	0.96	Standard in literature
Reserve requirement	μ	6.1%	2017-20 avg. required reserves/trans. balances
Proportional cost of handling cash	ν	1.6%	2014 Canadian retailers' private cost of using cash per transaction value
Inflation rate	π	1.0246	2017-21 avg. annual inflation
Bank's plegeability constraint	ε	0.85	Haircut applied to level 2A assets in Basel III
Capital income share[5]	Φ^α	-	2017-21 avg. capital income distribution
Calibrated internally			
Fixed cost of accepting deposits	f	0.016	2017-20 avg. card rejection rate in DCPC
Buyers' bargaining power	θ	0.897	Retailer markup 20%
Coefficient for CM consumption	A	0.0097	
Curvature of DM consumption	ρ	0.767	
Shape parameters for $F(\alpha)$	α_{dist}	9.088	2017-21 avg. consumption expenditure distribution,
Shape parameters for $F(\alpha)$	β_{dist}	32.206	2017-20 avg. real interest rate on transactional deposits
Upper bound of bank's productivity	$\bar{\gamma}$	1.226	
Measure of banks	η	169	

5.2 Counterfactual analysis

5.2.1 Aggregate welfare cost of inflation

Figure 6 documents how aggregate welfare in this calibrated economy changes with respect to inflation. Here, the optimal monetary policy is unsurprisingly at the Friedman rule. The aggregate welfare with gross inflation rate π is given by

$$\mathscr{W}(\pi) = \frac{1}{1-\beta}\{\int_0^1 (\alpha u(q_\alpha) - R^d p_\alpha(d) - R^c p_\alpha(c))dF(\alpha) + U(A + \mathscr{W}^s + \mathscr{W}^e) - A\},$$

where the first integral term on the right-hand side represents welfare in the DM, while the last two terms correspond to welfare in the CM.

[5](0.21%, 0.41%, 0.71%, 1.27%, 2.03%, 2.23%, 4.62%, 6.77%, 11.30%, 69.44%)

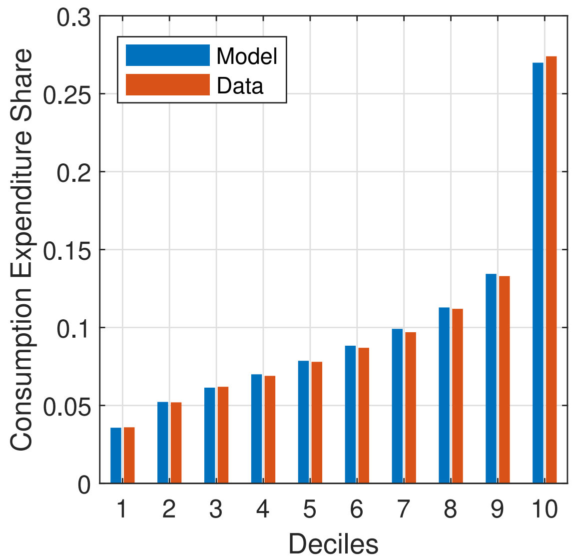

Figure 5: Consumption Expenditure Distribution under Calibrated Parameters

5.2.2 Distributional welfare costs of inflation

To calculate inflation costs, I ask how much consumption buyers would be willing to forgo to achieve zero inflation instead of a gross inflation rate of π. For any π, the steady state lifetime utility for an α-type buyer is

$$(1-\beta)V^{\alpha}(\pi) = \alpha u(q_{\alpha}(\pi)) - R^{d}p_{\alpha}(d) - R^{c}p_{\alpha}(c) + U(A + \Phi^{\alpha}(\mathscr{W}^{\int}(\pi) + \mathscr{W}^{\rceil}(\pi))) - A.$$

When there is no inflation and the α-type buyer's consumption for DM and CM goods both decrease by a factor of $1-\Delta_0$, her utility becomes

$$(1-\beta)V^{\alpha}_{\Delta_0}(0) = \alpha u(q_{\alpha}(0)\Delta_0) - R^{d}p_{\alpha}(d) - R^{c}p_{\alpha}(c) + U((A + \Phi^{\alpha}(W^{s}(0) + W^{e}(0)))\Delta_0) - A.$$

$1-\Delta_0^{\alpha}$ represents the welfare cost for an α-type buyer when inflation increases from zero to π such that $V^{\alpha}(\pi) = V^{\alpha}_{\Delta_0}(0)$. This means that an α-type consumer is willing to reduce her total consumption by $1-\Delta_0^{\alpha}$ to remain in a zero-inflation environment rather than living in a world with inflation π.

The counterfactual analysis is carried out at the decile level. To achieve this, Δ_0 for a given decile in the distribution of α is determined by the following aggregation:

$$\Delta_0 = 10\int_{x}^{y} \Delta_0^{\alpha} dF(\alpha),$$

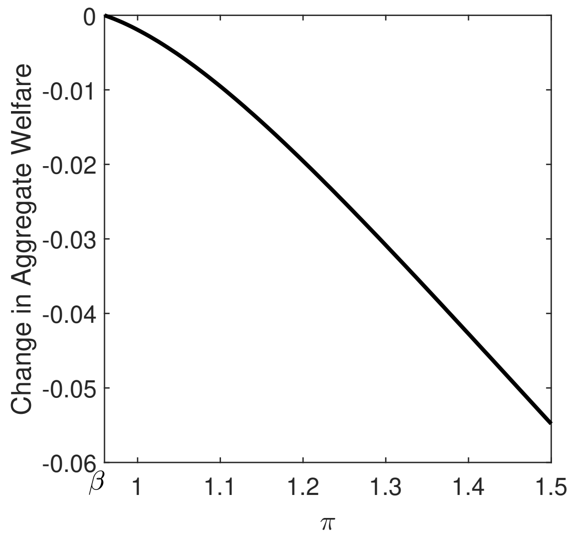

Figure 6: Aggregate Welfare Change Against Inflation

Note: This figure plots the percentage change in aggregate welfare when inflation increases. The baseline is the aggregate welfare at the Friedman rule

where x and y are the corresponding lower and upper bounds for the decile, and $F(\alpha)$ is the cumulative density function. Figure 7 displays welfare costs for the ten deciles when $\pi = 1.1$. The inflation cost exhibits a downward sloping trend as we move from the bottom 10% to the top 10% in the upper figure. Given the equilibrium cutoff α_D, the poorest 10% buyers and a majority of the second-decile buyers use only cash in DM meetings. For the rest of their peers, deposits are instead used. Hence, it is no surprise that inflation inflicts the greatest pain on the first decile, to the extent that they would be willing to trade around 1% of their consumption for zero inflation. The inflation cost for the second decile is slightly less, at just under 0.85%, closely following the first decile. This is because only one-fifth of the second decile consistently uses deposits. The numbers are much smaller for the rest of the distribution, with an average of 0.008%.

Moreover, it is interesting to decompose the inflation cost for the second decile since there are three distinct groups within it. The result is displayed in the lower part of Figure 7. Group one buyers consistently use cash in both inflation regimes, while group three buyers consistently use deposits. However, for buyers in group two, they use deposits when there is no inflation and switch to cash when $\pi = 1.1$. The result shows that group two buyers suffer the most from a ten-percent increase in inflation, with the cost being 2.7% of consumption. This is precisely because they are deposits users when there is no inflation and become cash users when there is 10% net inflation.

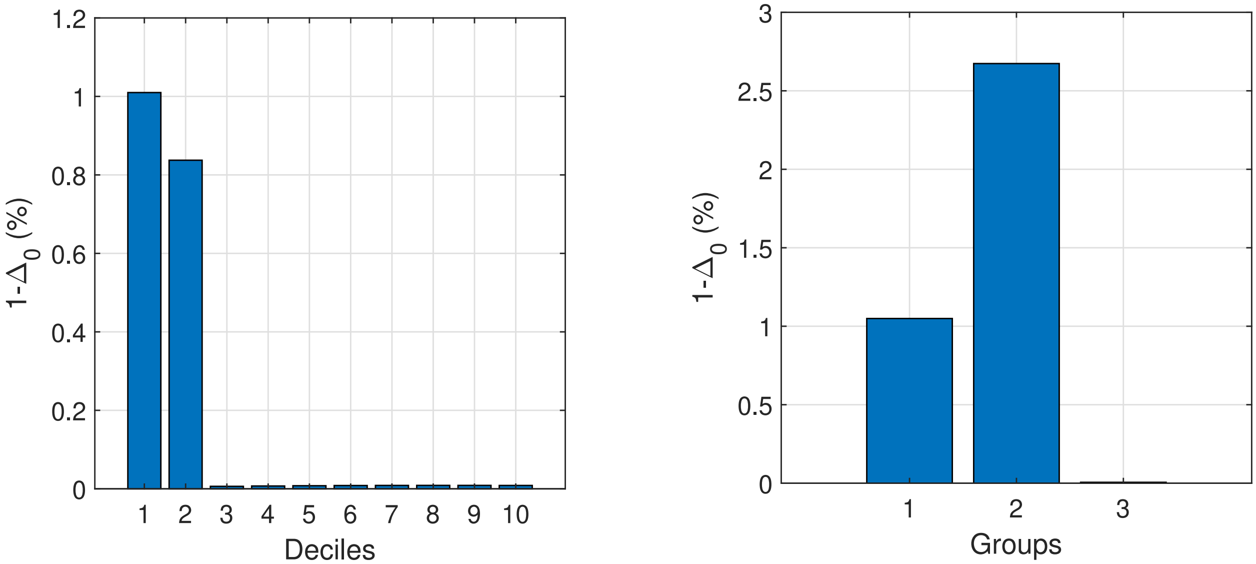

Figure 7: Distributional Welfare Costs Across Deciles and Within the Second Decile when $\pi = 1.1$

5.2.3 Sensitivity analysis

In this section, I vary the size of the proportional cost of handling cash, ν, by decreasing and increasing it by 10% relative to the baseline calibration. For each case, I re-calibrate the model and compute the resulting distributional welfare costs under a ten-percent inflation rate. Figure 8 displays the welfare costs across income deciles under these alternative values of ν. The trend follows the baseline pattern, although the magnitudes differ. Low-income individuals experience the effects of inflation to a degree much greater than high-income individuals.

6 Conclusion

A micro-founded model of money has been constructed and a quantitative exercise has been conducted to examine the long-run real distributional effects of inflation. The results highlight the importance of considering the distribution of household payment portfolios, both types and quantities, when evaluating the welfare costs of inflation. First, inflation inflicts greater pain on households holding cash compared to those with deposits. Thus a representative agent model with only one means of payment can be misleading. Second, both subsidies lowering the merchant's cost of accepting deposits and a lower inflation rate can help increase the proportion of households holding deposits.

This paper focuses only on agents' means-of-payment decisions in offline markets where cash and deposits are used. There are other types of currencies beginning to occupy a position in house-

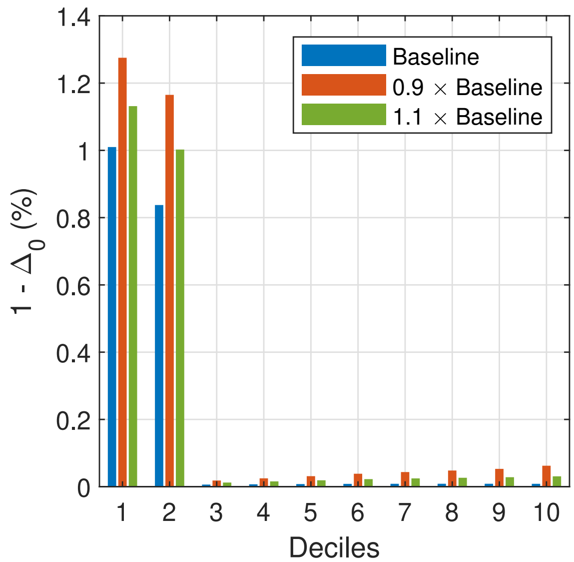

Figure 8: Distributional Welfare Costs when $\pi = 1.1$ with Different ν_s

holds' payment portfolios. Typical examples include cryptocurrencies and central bank digital currencies. Therefore, incorporating these payment options into the model, along with an online market, generates a framework suitable for studying questions in areas of payment and digital currencies.

A Proofs of Propositions and Lemmas

A.1 Proof 1

1. Strict increasingness in α: $\frac{\partial \Delta_1}{\partial \alpha} = \beta((1-\theta)u(q_\alpha^{**}) + \frac{\partial q_\alpha^0}{\partial \alpha})$
$= \beta(1-\theta)(u(q_\alpha^{**}) - \frac{u(q_\alpha^0)}{(1-\theta)\alpha u'(q_\alpha^0)+\theta}) > 0$ because $q_\alpha^0 < q_\alpha^{**}$;

2. Strict decreasingness in c: $\frac{\Delta_1}{\partial c} = \frac{\Delta_2}{\partial c} = \frac{\Delta_3}{\partial c} = \beta R^c(\frac{1-\theta\nu}{(1-\theta)\alpha u'(q_\alpha^0)+\theta} - (1-\nu)) < 0$
because $\alpha u'(q_\alpha^0) > \frac{1}{1-\nu}$;

3. As Δ_1 decreases in c and f, and increases in α, when α increases, c^α increases to balance $\Delta_1 = 0$; when f increases, c^α decreases to balance $\Delta_1 = 0$.

A.2 Proof of Lemma 1

To show that the red curve $\Delta(c,d,\alpha)$ shifts to the right when α increases, I only need to show that $\Delta(c,d,\alpha)$ is strictly increasing in α and strictly decreasing in c in all the four regions. Hence, when α increases, for the same value of d, c has to increase in order to make Δ stays at zero.

1. Strict increasingness in α: $\frac{\partial\Delta_1}{\partial\alpha} = \beta((1-\theta)u(q_\alpha^{**}) + \frac{\partial q_\alpha^0}{\partial\alpha})$
$= \beta(1-\theta)(u(q_\alpha^{**}) - \frac{u(q_\alpha^0)}{(1-\theta)\alpha u'(q_\alpha^0)+\theta}) > 0$ because $q_\alpha^0 < q_\alpha^{**}$; $\frac{\partial\Delta_2}{\partial\alpha} = \frac{\partial\Delta_4}{\partial\alpha} = \beta(\frac{\partial q_\alpha^0}{\partial\alpha} - \frac{\partial q_\alpha}{\partial\alpha}) =$
$\beta(1-\theta)(\frac{u(q_\alpha)}{(1-\theta)\alpha u'(q_\alpha)+\theta} - \frac{u(q_\alpha^0)}{(1-\theta)\alpha u'(q_\alpha^0)+\theta}) > 0$ because $q_\alpha^0 < q_\alpha$; $\frac{\partial\Delta_3}{\partial\alpha} = \beta(\frac{\partial q_\alpha^0}{\partial\alpha} + \frac{(1-\theta)(1-\nu)}{1-\theta\nu}u(q_\alpha^*)) =$
$\beta(1-\theta)(\frac{1-\nu}{1-\theta\nu}u(q_\alpha^*) - \frac{u(q_\alpha^0)}{(1-\theta)\alpha u'(q_\alpha^0)+\theta}) > 0$ because $q_\alpha^0 < q_\alpha^*$;

2. Strict decreasingness in c: $\frac{\Delta_1}{\partial c} = \frac{\Delta_2}{\partial c} = \frac{\Delta_3}{\partial c} = \beta R^c(\frac{1-\theta\nu}{(1-\theta)\alpha u'(q_\alpha^0)+\theta} - (1-\nu)) < 0$
because $\alpha u'(q_\alpha^0) > \frac{1}{1-\nu}$; $\frac{\Delta_4}{\partial c} = \beta(1-\theta\nu)R^c(\frac{1}{(1-\theta)\alpha u'(q_\alpha^0)+\theta} - \frac{1}{(1-\theta)\alpha u'(q_\alpha)+\theta}) < 0$
because $q_\alpha^0 < q_\alpha$.

A.3 Proof of Proposition 1

$$\Delta_4 = -f + \beta\{D_\theta^{-1}((1-\theta\nu)R^c c) - D_\theta^{-1}((1-\theta\nu)R^c c) + R^d d) + R^d d\}.$$

By having $c = 0$ and $\Delta_4 = 0$, I have

$$-f + \beta\{R^d d_\alpha^{c=0} - D_\theta^{-1}(R^d d_\alpha^{c=0})\} = 0. \tag{12}$$

Deposits will be accepted if buyers bring at least $d_\alpha^{c=0}$ units of deposits into the DM. An α_D-type buyer will bring $d = \frac{1}{R^d}D_\theta(u'^{-1}(\frac{\theta}{\alpha_D(\beta R^d+\theta-1)}))$ units of deposits conditional on acceptance. Now suppose $d = d_\alpha^{c=0}$ and substitute the expression of d into equation (12), I arrive at condition (8). When α is greater than α_D, the left-hand side of equation (12) goes up and becomes greater than 0, which means deposits will be accepted by the seller in the meeting. In addition, as shown in A.1, Δ decreases in f and increases in α, so α_D increases in f to balance $\Delta = 0$; when R^d increases,Δ increases as sellers' trade surplus increases, so α_D decreases in R^d to balance $\Delta = 0$.

A.4 Proof of Proposition 2

1. As shown in A.3, Δ decreases in f, and increases in α and R^d. Moreover, γ increases in R^d.

2. Differentiate both sides of equation (11) with respect to R^c
$\Longrightarrow \frac{dR^d}{dR^c} = \frac{\mu}{1-\frac{(1-\mu)\varepsilon}{\eta}d_{\alpha_D}f(\alpha_D)\frac{\partial\alpha_D}{\partial R^d} + \frac{(1-\mu)\varepsilon}{\eta}\int_{\alpha_D}^{\bar{\alpha}}\frac{\partial d_\alpha}{\partial R^d}f(\alpha)d\alpha} > 0$. Condition (8) defines $\alpha_D(f, R^d(\alpha_D, R^c))$
$\Longrightarrow \frac{\partial\alpha_D}{\partial R^c} = \frac{\partial\alpha_D}{\partial R^d}(\frac{\partial R^d}{\partial\alpha_D}\frac{\partial\alpha_D}{\partial R^c} + \frac{\partial R^d}{\partial R^c}) \Longrightarrow \frac{\partial\alpha_D}{\partial R^c} = \frac{\partial\alpha_D}{\partial R^d}\frac{dR^d}{dR^c} < 0.$

A.5 Proof of Proposition 3

1. As $\frac{dR^d}{dR^c} > 0$, when R^c decreases, the seller's DM trade surplus in both cash and deposit meetings decrease, so $\frac{\partial \mathscr{W}^s}{\partial R^c} > 0$.

2. $\frac{\partial \mathscr{W}^e}{\partial R^c} = \frac{\eta}{\varepsilon}(\frac{dR^d}{dR^c} - \mu)(-\varepsilon\bar{\gamma} + (2\varepsilon - 1)\hat{\gamma})$¿0 $\Leftrightarrow \hat{\gamma} < \frac{\varepsilon\bar{\gamma}}{2\varepsilon-1}$, which is true when $\varepsilon > \frac{1}{2}$.

A.6 Proof of Existence of Equilibrium

α_D and R^d are pinned down by equations (8) and (11).

1.
$$-f + \beta\{D_\theta(u'^{-1}(\frac{\theta}{\alpha_D(\beta R^d + \theta - 1)})) - u'^{-1}(\frac{\theta}{\alpha_D(\beta R^d + \theta - 1)})\} = 0.$$

   differentiate both sides of equation (8) with respect to α_D, when $q_\alpha < q_\alpha^{**}$ and $\theta < 1$:

$$\begin{aligned}
&(D'_\theta(q_\alpha) - 1)\frac{1}{u''}\frac{-\theta(\frac{\partial\alpha_D}{\partial R^d}(\beta R^d + \theta - 1) + \alpha_D\beta)}{(\alpha_D(\beta R^d + \theta - 1))^2} = 0 \\
&\Longrightarrow \frac{\partial\alpha_D}{\partial R^d}(\beta R^d + \theta - 1) + \alpha_D\beta = 0 \\
&\Longrightarrow \frac{\partial\alpha_D}{\partial R^d} = \frac{-\alpha_D\beta}{\beta R^d + \theta - 1} < 0.
\end{aligned}$$

2.
$$R^d = -\frac{(1-\mu)\varepsilon}{\eta}\int_{\alpha_D}^{\bar{\alpha}} d_\alpha f(\alpha)d\alpha + (1-\mu)\varepsilon\bar{\gamma} + \frac{\mu}{\pi}.$$

   differentiate both sides of equation (11) with respect to α_D:

$$\begin{aligned}
\frac{\partial R^d}{\partial\alpha_D} &= \frac{(1-\mu)\varepsilon}{\eta}\{d_{\alpha_D}f(\alpha_D) + \int_{\bar{\alpha}}^{\alpha_D}\frac{\partial d_\alpha}{\partial R^d}\frac{\partial R^d}{\partial\alpha_D}f(\alpha)d\alpha\} \\
&= \frac{\frac{(1-\mu)\varepsilon}{\eta}d_{\alpha_D}f(\alpha_D)}{1 - \frac{(1-\mu)\varepsilon}{\eta}\int_{\bar{\alpha}}^{\alpha_D}\frac{\partial d_\alpha}{\partial R^d}f(\alpha)d\alpha} \\
&= \frac{\frac{(1-\mu)\varepsilon}{\eta}d_{\alpha_D}f(\alpha_D)}{1 + \frac{(1-\mu)\varepsilon}{\eta}\int_{\alpha_D}^{\bar{\alpha}}\frac{\partial d_\alpha}{\partial R^d}f(\alpha)d\alpha} > 0.
\end{aligned}$$

With assumption A2, I have the existence and uniqueness of an equilibrium where deposits are used as medium of exchange.

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