Due to the existence of first derivative:

$$\begin{split} \vec{g} &:= \mathbb{E}_e\{\nabla f_\gamma(x,e)\} = \\ d\mathbb{E}_e\{\frac{f(x) + \langle \nabla f(x), \gamma e \rangle + o(\|\gamma e\|_2) - (f(x) - \langle \nabla f(x), \gamma e \rangle + o(\|\gamma e\|_2))}{2\gamma}e\} &= \\ &= d\mathbb{E}_e\{\frac{2\langle \nabla f(x), \gamma e \rangle + o(|\gamma|)}{2\gamma}e\} \underset{\gamma \to 0}{=} d\mathbb{E}_e\{\langle \nabla f(x), e \rangle e\} = \nabla f(x) \end{split}$$

Let $\delta(x): |\delta(x)| \leq \Delta$ Oracle's noise, then:

$$egin{aligned} ec{g} &:= \mathbb{E}_e\{
abla f_{\gamma}(x,e)\} = \ d\mathbb{E}_e\{rac{\delta(x+\gamma e) + \langle
abla f(x), \gamma e
angle - (\delta(x-\gamma e) - \langle
abla f(x), \gamma e
angle) + O(\|\gamma e\|_2^2)}{2\gamma}e\} &\leq \ d\mathbb{E}_e\{rac{2\langle
abla f(x), \gamma e
angle + 2\Delta + O(\gamma^2)}{2\gamma}e\} =
abla f(x) + \mathbb{E}_e\{e\}(O(\gamma) + rac{d\Delta}{\gamma}), \end{aligned}$$

More than that, let $ec{r}=x_0-x^*, R=\|ec{r}\|_2$

$$egin{aligned} \langle ec{g},ec{r}
angle &=
abla f(x) + \mathbb{E}_e\{e\}(O(\gamma) + rac{d\Delta}{\gamma}) \leq \langle
abla f(x),ec{r}
angle + \mathbb{E}_e\{\langle e,ec{r}
angle\}(O(\gamma) + rac{d\Delta}{\gamma}) \leq \\ &\leq \langle
abla f(x),ec{r}
angle + rac{R}{\sqrt{d}}(O(\gamma) + rac{d\Delta}{\gamma}) \end{aligned}$$