Due to the existence of first derivative:

$$
\begin{aligned}
&\vec{g} := \mathbb{E}_e\{\nabla f_\gamma(x,e)\} = \\
&d\mathbb{E}_e\{\frac{f(x)+\langle\nabla f(x),\gamma e\rangle + o(\|\gamma e\|_2) - (f(x)-\langle\nabla f(x),\gamma e\rangle + o(\|\gamma e\|_2))}{2\gamma}e\} = \\
&= d\mathbb{E}_e\{\frac{2\langle\nabla f(x),\gamma e\rangle + o(|\gamma|)}{2\gamma}e\} \underset{\gamma\to 0}{=} d\mathbb{E}_e\{\langle\nabla f(x),e\rangle e\} = \nabla f(x)
\end{aligned}
$$

Let $\delta(x) : |\delta(x)| \le \Delta$ Oracle's noise, then:

$$
\begin{aligned}
&\vec{g} := \mathbb{E}_e\{\nabla f_\gamma(x,e)\} = \\
&d\mathbb{E}_e\{\frac{\delta(x+\gamma e)+\langle\nabla f(x),\gamma e\rangle - (\delta(x-\gamma e)-\langle\nabla f(x),\gamma e\rangle) + O(\|\gamma e\|_2^2)}{2\gamma}e\} \le \\
&\le d\mathbb{E}_e\{\frac{2\langle\nabla f(x),\gamma e\rangle + 2\Delta + O(\gamma^2)}{2\gamma}e\} = \nabla f(x) + \mathbb{E}_e\{e\}(O(\gamma)+\frac{d\Delta}{\gamma}),
\end{aligned}
$$

More than that, let $\vec{r} = x_0 - x^*, R = \|\vec{r}\|_2$

$$
\begin{aligned}
&\langle\vec{g},\vec{r}\rangle = \nabla f(x) + \mathbb{E}_e\{e\}(O(\gamma)+\frac{d\Delta}{\gamma}) \le \langle\nabla f(x),\vec{r}\rangle + \mathbb{E}_e\{\langle e,\vec{r}\rangle\}(O(\gamma)+\frac{d\Delta}{\gamma}) \le \\
&\le \langle\nabla f(x),\vec{r}\rangle + \frac{R}{\sqrt{d}}(O(\gamma)+\frac{d\Delta}{\gamma})
\end{aligned}
$$