Step 4

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1 Part A

The emitter is approximately in the northwest direction, II Quadrant.

2 Part B

Relative distance in this coordinate system is defined by TDoA * (speed of sound). Thus, distance is proportional to TDoA. We can use Time Distance of Arrival as our metric. Let TDoA(x) be the Time Difference of Arrival of receiver x. Then,

$$TDoA(C) = 0 < TDoA(B) = 2.63474 * 10^-4 < TDoA(A) = 7.07023 * 10^-4$$

The emitter is closest to C, second closest to B, and farthest from A. Because C is (0,1) and A and B are on the x-axis, the emitter must be in the I or II Quadrant. And since B is (-.5,0) while A is (.5,0) and emitter is closer to B, we conclude the emitter must be in the II Quadrant.

3 Part C

The locus of possible coordinates of the emitter for any two receptors is half of a hyperbola. It is *half* a hyperbola because it is important which distance is greater than the other. In a normal hyperbola, the difference in distances is absolute valued, getting rid of any sign, and giving us two sides of the hyperbola. However, in this case, the emitter is obviously closer to one receptor than it is to another, and thus one distance must be bigger than another, so only one side of a hyperbola is valid.

4 Part D

*Edit: I now know there is an easier solution involving the Distance Formula, but here is my original hardcore solution.

My approach in solving this problem was to choose two pairs of the three points (I chose AC and AB) and find the equations of the full hyperbolas these pairs produce.

It is known that the difference of the distances from any point to each of the foci is equal to the length of the transverse axis: 2a. Also, the distance between the foci is 2c, which we know by the given coordinates. We already know that the distance 2a = TDoA * speed of sound. And a standard form of hyperbola is: $\frac{x^2}{a^2} - \frac{y^2}{c^2-a^2} = 1$

From this, we plug in our values for a and c to get this equation for the hyperbola of receivers A and B. $\frac{x^2}{.1078781114} - \frac{y^2}{.1421218886} = 1$

I tried the same thing for the hyperbola between receivers A and C, but I soon realized that the fact the curve is tilted makes me things much more complicated. In the end, I rotated the coordinate system using the counterclockwise rotation matrix, and got that the rotated coordinates would be:

x' = xcost - ysint y' = xcost + ysint

Knowing that the angle t is equal to $arccosine(\frac{1}{\sqrt{1^2+.5^2}})$ and $arcsine(\frac{.5}{\sqrt{1^2+.5^2}})$ by drawing the right triangle with the origin, A, and C as vertices.

I then plugged in all my known constant values into the standard hyperbola equation, and then applied a transformation, since the center of hyperbola is not at (0,0) but instead at (.25,.5).

$$\frac{((x-.25)\cdot.447213595-(y-.5)\cdot.894427191)^2}{.274105159} - \frac{((x-.25)\cdot.894427191+(y-.5)\cdot.447213595)^2}{.03839484097} = 1$$

Finally, I graphed these equations, and I found the intersection to be (-3.503, 4.004). See the accompanying .png file to view my results.

5 Part E

I would use Newton's Method of Linearization to find the zero of the functions of the two hyperbola subtracted from one another. In this case, the hyperbola equation I used in Part D will not work, because Newton's requires first order derivatives, and my monster equations are impossible to separate the y term on one side, let alone differentiate it.

For this, the Distance Formula comes in handy. Let d = TDoA * speed of sound. Let (x,y) be the coordinates of the emitter, and let (x1,y1) and (x2,y2) be the coordinates of the respective receivers.

$$d = \sqrt{(x-x1)^2+(y-y1)^2} - \sqrt{(x-x2)^2+(y-y1)^2}$$

This is the form of hyperbola I would use to calculate. I would solve for y, which would give either the upper or lower part of the hyperbola, because the expression introduces a plus or minus square root. Since the original function already only accounts for one branch of a hyperbola (can be seen graphically), the squared root function represents only one quarter of a hyperbola. Thus, to represent all possible coordinates of the emitter, the hyperbola curve must be represented by many different "quarter" functions. I would precede the bulk of my computation then with control flow if loops and else if loops, to filter the inputted values and choose which "quarter" functions would yield the solution. Then, the calculation is just multiple iterations of linear approximations, and we check at each iteration if the answer we are approaching gets within a predetermined measure of accuracy, say, perhaps we wish to get within .1 centimeter of the emitter coordinates.