Midterm #2

First Name: _____

Last Name: _____

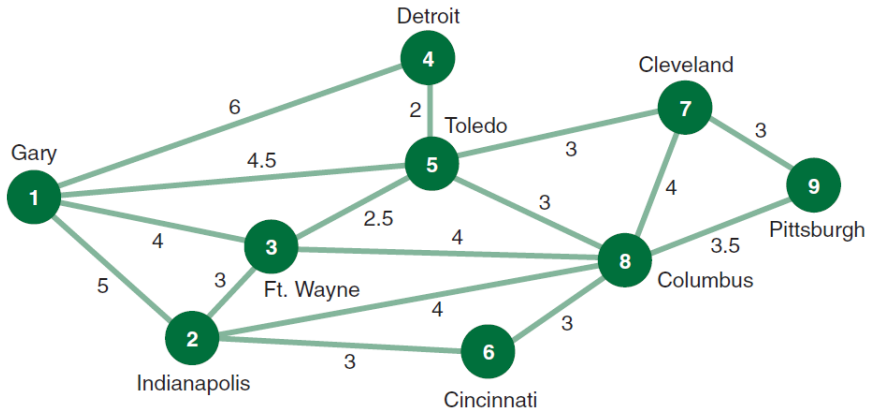
This exam consists of 10 questions. You have 48 hours to complete this exam beginning at the time this exam is emailed to the class. Follow the instructions on every problem to get the full amount of points. Most questions will require an excel file to supplement your solution. I will tell you what to name each of the Excel files you submit. If you don't have a working Excel file that shows your work, you may lose the majority or all of your points. I want to see your Excel file after you solve the problem so I can see your work in Excel Solver. Make sure you save your file after using Excel Solver.

When you are finished, you can submit your exam to Sakai. These answers can be typed in Microsoft Word or written by hand. If your solutions are handwritten, you can write them on a printout of this exam or on a piece of notebook paper. You can take photos of your exam with your phone. Submit digital copies of your solutions to Sakai, but make sure all your solutions are organized in the order found on this exam. If you have to submit your solutions in multiple files, name your documents or images "Q1", "Q2", "Q3" or "Page1", "Page2", "Page3".

Do not work with other students or get help from any person. You have access to the lectures, textbook, homework, and internet. If you get caught using the answers or excel files of another student, expect a 0% on the exam. If you get caught sending your exam or Excel files to another student, expect a 0% on the exam.

1 Information for Questions 1.1 and 1.2

I am from Cleveland, the greatest city on earth, and I want to spread the magic of Cleveland to 8 other cities in the Midwest. The graph below shows the network that exists between all these cities. The travel time, in hours, is shown along each branch. Assume all branches represent undirected paths between the connected cities.



1.1 Use Excel to find the fastest route between Cleveland and all other cities. Fill in the table with the fastest path (use notation 1-3-4-2-5) and time required for that path. Submit an Excel file named "Cleveland.xlsx" with your work to get full credit. You can use an algorithm to check your answer, but you must submit your work Excel. (16 Points)

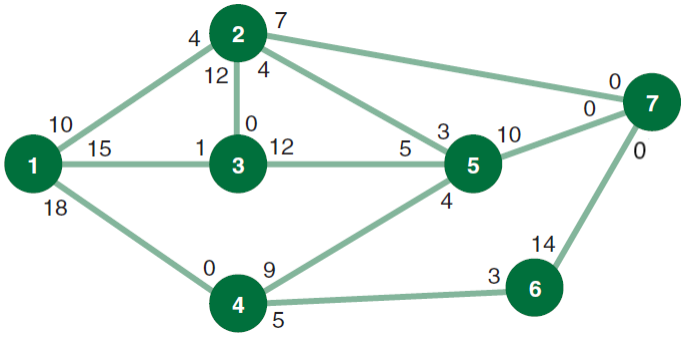
Destination	Shortest Path	Time Required (Hours)
Gary	7-5-1	7.5
Indianapolis	7-8-2	8
Ft. Wayne	7-5-3	5.5
Detroit	7-5-4	5
Toledo	7-5	3
Cincinnati	7-8-6	7
Columbus	7-8	4
Pittsburgh	7-9	3

1.2 In Cleveland, we are raised to despise Pittsburgh. Use Excel to find the slowest route between Cleveland and Pittsburgh where each edge is used at most twice (once in each direction). Fill in the table with the time required to travel the optimal path. Submit an Excel file named "Pittsburgh.xlsx" with your work to get full credit. (6 Points)

Destination	Time Required (Hours)
Pittsburgh	105

2 Information for Questions 2.1, 2.2, and 2.3

Observe the following network with the indicated flow capacities along each branch. We want to investigate the maximum flow from node 1 (source) to node 7 (destination). For this problem, let x_{ij} = amount of flow going through edge (i, j).



2.1 Formulate a linear program, in standard form, to find the maximal flow through this network. (28 Points)

$$\text{Maximize } Z = x_{12} + x_{13} + x_{14}$$

$$\begin{aligned}
\text{Subject to } & x_{12} + x_{52} - x_{21} - x_{23} - x_{25} - x_{27} = 0 \\
& x_{13} + x_{23} + x_{53} - x_{31} - x_{35} = 0 \\
& x_{14} + x_{54} + x_{64} - x_{45} - x_{46} = 0 \\
& x_{25} + x_{35} + x_{45} - x_{31} - x_{35} = 0 \\
& x_{46} - x_{64} - x_{67} = 0 \\
& x_{12} + x_{13} + x_{14} - x_{27} - x_{57} - x_{67} = 0 \\
& x_{12} \leq 10,\ x_{13} \leq 15,\ x_{14} \leq 18 \\
& x_{21} \leq 4,\ x_{23} \leq 12,\ x_{25} \leq 4,\ x_{27} \leq 7 \\
& x_{31} \leq 1,\ x_{35} \leq 12, \\
& x_{45} \leq 9,\ x_{46} \leq 5 \\
& x_{52} \leq 3,\ x_{53} \leq 5,\ x_{54} \leq 4,\ x_{57} \leq 10 \\
& x_{64} \leq 3,\ x_{67} \leq 14 \\
& x_{ij} \geq 0 \text{ and is integer}
\end{aligned}$$

2.2 Use Excel to solve the previously stated linear program to find the maximal flow. In the table below, write the maximal flow of this network. Submit an Excel file named "MaxFlow.xlsx" with your work to get full credit. You can use an algorithm to check your answer, but you must submit your solved linear program in Excel. (6 Points)

Maximal Flow	22

2.3 Is the solution for maximal flow unique or are there multiple optimal solutions. Explain in complete sentences why or why not? Reference your actual solution in your explanation. You may sketch pictures to prove your point. (4 Points)

My solution is not unique.

In my solution, we would send 10 units from node 1 to node 2, 7 units from node 1 to node 3, and 5 units form node 1 to node 4. This is how we get a maximal flow of $22 = 10 + 7 + 5$. We could have gotten the same solution from sending only 7 units from node 1 to node 2 to node 7, then 10 units to go from node 1 to node 3 to node 5 to node 7, and then 5 units from node 1 to node 4 to node 6 to node 7. We get the same maximal flow of 22.

3 Information for Questions 3.1, 3.2, and 3.3

Airborne is a company that produces 4 different airplane parts from fabricated sheet metal for several major aircraft companies. The manufacturing process consists of four operations – stamping, assembly, finishing, and packaging. The processing times per unit for each operation and total available hours per year to produce these parts are as follows:

	Part (hr./unit)				
Operation	1	2	3	4	Total Hours/Year
Stamping	0.06	0.17	0.10	0.14	700
Assembly	0.18	0.20	—	0.14	700
Finishing	0.07	0.20	0.08	0.12	800
Packaging	0.09	0.12	0.07	0.15	600

The sheet metal required for each part, the estimated annual demand, and the profit per part are as follows:

Part	Sheet Metal ($ft.^2$)	Estimated Annual Demand	Profits
1	2.6	2,600	$ 90
2	1.4	1,800	100
3	2.5	4,100	80
4	3.2	1,200	120

The company ordered 20,000 square feet of fabricated metal for the year. The company has the following prioritized production goals:

(1) Avoid overtime, which would cause a decrease in profit levels.

(2) At minimum, meet the demand for each of the parts.

(3) Achieve an annual profit of $700,000.

(4) Avoid ordering more material because a surcharge is required by the supplier to increase the annual order.

Define the decision variables x_i as the number of airplane part i to produce per year. We want to help Airborne make the best decision in choosing values for x_1, x_2, x_3, and x_4.

3.1 Formulate a goal programming model to determine the quantity of each part to produce to achieve the company's objectives. Don't include a constraint requiring integer solutions. (15 Points)

$$
\begin{aligned}
\text{Minimize } & P_1(d_1^+ + d_2^+ + d_3^+ + d_4^+) \\
& P_2(d_5^- + d_6^- + d_7^- + d_8^-) \\
& P_3(d_9^-) \\
& P_4(d_{10}^+)
\end{aligned}
$$

$$
\begin{aligned}
\text{Subject to } & 0.06x_1 + 0.17x_2 + 0.10x_3 + 0.14x_4 + d_1^- - d_1^+ = 700 \\
& 0.18x_1 + 0.2x_2 + 0.14x_4 + d_2^- - d_2^+ = 700 \\
& 0.07x_1 + 0.2x_2 + 0.08x_3 + 0.12x_4 + d_3^- - d_3^+ = 800 \\
& 0.09x_1 + 0.12x_2 + 0.07x_3 + 0.15x_4 + d_4^- - d_4^+ = 600 \\
& x_1 + d_5^- - d_5^+ = 2600 \\
& x_2 + d_6^- - d_6^+ = 1800 \\
& x_3 + d_7^- - d_7^+ = 4100 \\
& x_4 + d_8^- - d_8^+ = 1200 \\
& 90x_1 + 100x_2 + 80x_3 + 120x_4 + d_9^- - d_9^+ = 700000 \\
& 2.6x_1 + 1.4x_2 + 2.5x_3 + 3.2x_4 + d_{10}^- - d_{10}^+ = 20000 \\
& d_i^- d_i^+ \geq 0 \quad \forall i \in \{1, 2, 3, \cdots, 10\} \\
& x_i \geq 0
\end{aligned}
$$

3.2 Use Excel to solve the goal programming model. Fill in the table below with your optimal solution according to the list of prioritized goals. Submit an Excel file named "Airborne.xlsx" with your work to get a full credit. If necessary, make sure you have multiple tabs to solve this goal programming problem. Also, make sure the optimal solution can be found at every step when optimizing according to the different goals. (8 Points)

	Number to Produce Per Year
Part 1	2600
Part 2	658.33 or 658
Part 3	4100
Part 4	6.82E-13 or 0

3.3 How would the solution be affected if the first two priorities were reversed? Reverse the first two priorities and fill in the table below with the new optimal solution according to the new list of prioritized goals. Submit an Excel file named "Airborne2.xlsx" with your work for this problem. (8 Points)

	Number to Produce Per Year
Part 1	2600
Part 2	1800
Part 3	4100
Part 4	1200

4 Information for Questions 4.1 and 4.2

Dr. 305 is looking to get into the scrapbook making business. It is such a hot business right now. In order to start his business, he needs to an initial investment of $20,000 for heavy duty scrapbook equipment. We going big here. The cost to produce each scrapbook will be around $15. Notice the following linear relationship between the monthly sales volume v and price per scrapbook p.

$$v = 5,000 - 70p$$

4.1 Profit is a function of price. Develop the nonlinear profit function $Z(p)$ where $p = price$. Use algebra to simplify your function, and write your answer below. (6 Points)

$$\begin{aligned} Z(p) &= Revenue - Cost \\ &= vp - (20,000 + 15v) \\ &= vp - 20,000 - 15v \\ &= (5,000 - 70p)p - 20,000 - 15(5,000 - 70p) \\ &= 5,000p - 70p^2 - 20,000 - 75,000 + 1,050p \\ &= -70p^2 + 6,050p - 95,000 \end{aligned}$$

4.2 In Excel, determine the price that will maximize profit. In the table below, give the optimal price, the optimal volume, and the maximum profit per month. Submit an Excel file named "Scrapbook.xlsx" that shows your work and solution for this problem. (6 Points)

	Result Based on Optimal Solution
Optimal Price	$43.21
Optimal Volume	1975.00 units
Maximum Profit	$35,723.21