Midterm #1

First Name: _____

Last Name: _____

This exam consists of 13 questions. You have 24 hours to complete this exam beginning at the start of class. Follow the instructions on every problem to get the full amount of points. Most questions will require an Excel file to supplement your solution. I will tell you what to name each of the Excel files you submit. If you don't have a working Excel file that shows your work, you may lose the majority or all of your points. I want to see your Excel file after you solve the problem so I can see your work in Excel Solver. Make sure you save your file after using Excel solver.

When you are finished, you can submit your exam to Gradescope and submit your Excel files to Sakai. Your answers can be typed in Microsoft Word or written by hand. Answer each question on a different set of files or images. If your solutions are handwritten you can take photos of your exam with your phone and submit the images to Gradescope. If you use Microsoft Word, make sure you submit one Word document for each question. Make sure you link each of the files you upload to Gradescope to the corresponding question and remember to upload your Excel files to Sakai.

Do not work with other students or get help from any person. You have access to the lectures, textbook, homework, and internet. If you get caught using the answers or excel files of another student, expect a 0% on the exam. If you get caught sending your exam or Excel files to another student, expect a 0% on the exam. If this exam is uploaded to the internet, the next exam will be due in one class period.

1 Information for Questions 1.1 and 1.2

UNISWAG produces mugs with UNC's logo to sell at the university bookstore. Every day it costs \$500 to operate their business and an additional \$6.00 per mug that they produce. Assume that every mug that UNISWAG produces gets purchased by someone.

1.1 If they sell each mug for \$12.00, what is the minimum number of mugs they need to produce each day to ensure that they break-even? (4 Points)

$$x^* = \frac{500}{12-6} = 83.33 \approx 84 \text{ mugs}$$

1.2 Suppose that their business can only produce a maximum of 60 mugs per day. If they produce the maximum number of mugs each day, what should the price of each mug be to ensure that they break-even? Remember that US currency is rounded to 2 decimal places. (4 Points)

$$60 = \frac{500}{p-6} \rightarrow p = 500/60 + 6 = 14.333 \approx \$14.34$$

2 Information for Questions 2.1 and 2.2

Consider the following set of linear constraints:

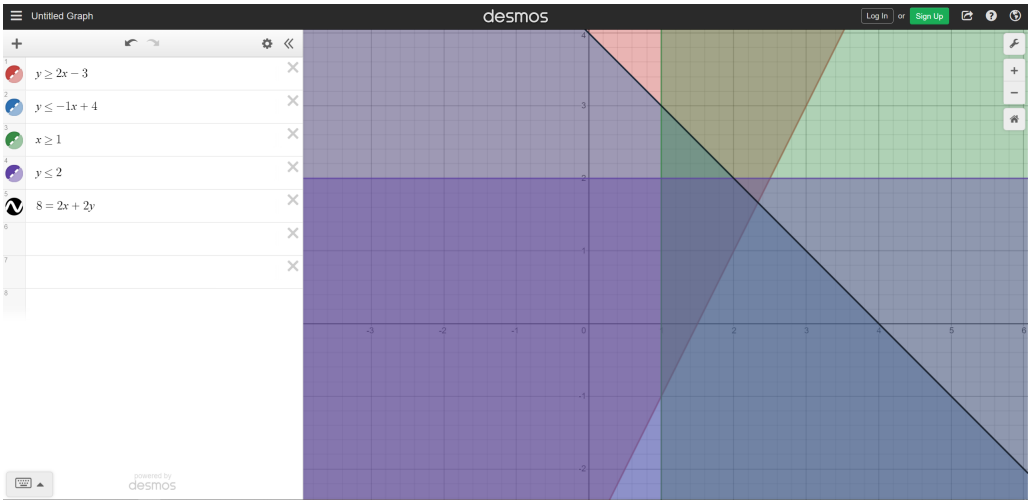
$$\begin{aligned} -2x + y &\geq -3 \\ -x - y &\geq -4 \\ x &\geq 1 \\ y &\leq 2 \end{aligned}$$

2.1 List all of the corners/extreme points of the feasible region. If necessary, round your answer to 2 decimal places based off classic rounding rules. (8 Points)

x	y
1	2
1	-1
2.33	1.67
2	2

2.2 Suppose we wanted to maximize the objective function $Z = 2x + 2y$. How many optimal solutions exist? (4 Points)

Infinite number of optimal solutions.



3 Information for Questions 3.1, 3.2, and 3.3

I recently decided to quit my job at UNC to pursue my dream of manufacturing jewelry to get enough money to finally afford a home within 40 miles of this town. I currently have designs for three different rings that I named the King, the Queen, and the Butler. I want to know how many Kings, Queens, and Butlers to produce each day to maximize profit. Now consider the information below:

- Each King brings \$80 in profit, each Queen brings \$100 in profit, and each Butler brings \$60 in profit.
- Each King requires 2 oz of yellow gold and 1 oz of white gold. Each Queen requires 2 oz of yellow gold, 2 oz of white gold, and 1 oz of sterling silver. Each Butler requires 3 oz of sterling silver and 0.5 oz of white gold. Each day we receive 15 oz of yellow gold, 10 oz of white gold, and 20 oz of sterling silver. (oz stands for ounce).
- I plan on working 24 hours each day for the love of the game. Each King requires 2 hours of work, each Queen requires 4 hours of work, and each Butler requires 1 hour of work.

Let x_1 represent the number of Kings to produce each day, let x_2 represent the number of Queens to produce each day, and let x_3 represent the number of Butlers to produce each day.

3.1 Write the linear programming model for this problem in standard form. (8 Points)

$$\begin{aligned}
&\text{Maximize } 80x_1 + 100x_2 + 60x_3 \\
&\text{Subject to} \\
&\qquad 2x_1 + 2x_2 \leq 15 \\
&\qquad x_1 + 2x_2 + 0.5x_3 \leq 10 \\
&\qquad x_2 + 3x_3 \leq 20 \\
&\qquad 2x_1 + 4x_2 + x_3 \leq 24 \\
&\qquad x_1, x_2, x_3 \in \{0, 1, 2, 3, \cdots\}
\end{aligned}$$

$$\begin{aligned}
&\text{Maximize } 80x_1 + 100x_2 + 60x_3 \\
&\text{Subject to} \\
&\qquad 2x_1 + 2x_2 \leq 15 \\
&\qquad x_1 + 2x_2 + 0.5x_3 \leq 10 \\
&\qquad x_2 + 3x_3 \leq 20 \\
&\qquad 2x_1 + 4x_2 + x_3 = 24 \\
&\qquad x_1, x_2, x_3 \in \{0, 1, 2, 3, \cdots\}
\end{aligned}$$

3.2 Use Excel to find the optimal number of Kings, Queens, and Butlers to produce each day. What is the optimal solution and what is the maximum profit? Fill in the table below with your answer. Also, submit an Excel file named "Jewelry.xlsx" to Sakai with your work to get full credit. (12 Points)

x_1	x_2	x_3	Profit
7	0	6	920

x_1	x_2	x_3	Profit
NA	NA	NA	NA

3.3 Which of our daily resource(s) are we wasting each day? Wasting means that we are acquiring more resources than we need to satisfy the optimal solution. (4 Points)

We are wasting yellow gold, sterling silver, and our hours.

Since no solution exists for our linear program, we are wasting all of our resources each day.

4 Information for Questions 4.1, 4.2, 4.3, and 4.4

Dr. Worldwide has two businesses to share his passions with the world, a karaoke company and a board game store. Each hour he dedicates to the karaoke company, he makes \$40 in profit, and each hour he dedicates to the board game store, he makes \$45 in profit. Each week, Dr. Worldwide cannot dedicate more than 20 hours at the karaoke company and cannot dedicate more than 15 hours at the board game store. Furthermore, Dr. Worldwide cannot dedicate more than 30 hours of work to both of these businesses combined.

Every hour that Dr. Worldwide dedicates to the board game business, the value of his karaoke company drops by \$5.25. Every hour that Dr. Worldwide dedicates to the karaoke company, the value of his board game store drops by \$4.35. Dr. Worldwide doesn't want the combined value of his companies to lose more than \$100 each week.

Let x represent the number hours dedicated to the karaoke company each week. Let y represent the number of hours dedicated to the board game store each week. How much time should Dr. Worldwide dedicate to each of these businesses each week to maximize profit?

4.1 Write the linear programming model for this problem in standard form. Furthermore, use Excel to solve the linear program and obtain a sensitivity report. Submit an Excel file named "Business.xlsx" to Sakai with the solved linear program and the sensitivity report to get full credit. (12 Points)

$$
\begin{aligned}
&\text{Maximize } 40x + 45y \\
&\text{Subject to} \\
&\qquad x \leq 20 \\
&\qquad y \leq 15 \\
&\qquad x + y \leq 30 \\
&\qquad 4.35x + 5.25y \leq 100 \\
&\qquad x, y \geq 0
\end{aligned}
$$

4.2 Which of the constraints are binding at the optimal solution? (6 Points)

Only the karaoke hours constraint and the value constraints are binding at the optimal solution.

4.3 Dr. Worldwide's son, Nationwide, wants to dedicate 1 hour a week for $5. Should Dr. Worldwide give Nationwide a job, and if so, where should he send Nationwide to work? Defend your answer using the sensitivity report in Excel. If you get the answer correct, but do not use the sensitivity report to defend your answer, you will receive 1 point. (4 Points)

If Nationwide would work anywhere, he should work for the Karaoke company; however, because the shadow price is $2.71 and Nationwide wants $5.00 per hour, Dr. Worldwide should not hire his son.

4.4 Dr. Worldwide's daughter, Global, wants to dedicate 5 hours a week for $1. What would you advise Dr. Worldwide to do with his daughter? Defend your answer using the sensitivity report in Excel. If you get the answer correct, but do not use the sensitivity report to defend your answer, you will receive 1 point. (4 Points)

Dr. Worldwide should send Global to work at the Karaoke company, but only let her work for 2.989 hours. If she works any more than that, her work will be wasted since the constraint will no longer be binding.

5 Information for Questions 5.1 and 5.2

It is Christmas time, and Dr. Worldwide needs to buy each of his children a single toy. Dr. Worldwide has five children who he has conveniently named numbers 1 through 5 so that he would remember the order in which they were birthed.

The children worked together to create a list of 7 unique toys that they all want. Each toy has a different cost. Use the following letters as abbreviations for the 7 unique toys.

Toy	Cost	Abbreviation
Teddy Bear	\$5	A
Fake Baby	\$8	B
Lego Set	\$10	C
Soccer Ball	\$8	D
Box of Paper	\$4	E
Shoes	\$13	F
Tools	\$12	G

The important information you need for a constraints are found in the following bullet points.

- Each child gets a single toy and that toy must be one of the seven toys.
- No two children get the same toy. The children like to share their toys.
- If child 1 gets the Teddy Bear, than child 5 must get the Tools.
- If child 2 gets the Soccer Ball, than child 4 must get the Shoes.
- Child 1 must get either the Teddy Bear or the Tools.
- Child 2 must get either the Soccer Ball or the Lego Set.

5.1 Use appropriate mathematical notation to describe your decision variables based on representing the children as numbers and the toys as letters. (6 Points)

$$x_{ij} = \begin{cases} 1 & \text{if child i gets toy j} \\ 0 & \text{if child i doesn't get toy j} \end{cases}$$

where $i \in \{1, 2, 3, 4, 5\}$ and $j \in \{A, B, C, D, E, F, G\}$

5.2 Write your constraints below based on your representation of the decision variables in the previous question.
(18 Points)

$$
\begin{aligned}
x_{1A} + x_{1B} + x_{1C} + \cdots + x_{1G} &= 1 \\
x_{2A} + x_{2B} + x_{2C} + \cdots + x_{2G} &= 1 \\
x_{3A} + x_{3B} + x_{3C} + \cdots + x_{3G} &= 1 \\
x_{4A} + x_{4B} + x_{4C} + \cdots + x_{4G} &= 1 \\
x_{5A} + x_{5B} + x_{5C} + \cdots + x_{5G} &= 1
\end{aligned}
$$

$$
\begin{aligned}
x_{1A} + x_{2A} + x_{3A} + \cdots + x_{5A} &\leq 1 \\
x_{1B} + x_{2B} + x_{3B} + \cdots + x_{5B} &\leq 1 \\
x_{1C} + x_{2C} + x_{3C} + \cdots + x_{5C} &\leq 1 \\
x_{1D} + x_{2D} + x_{3D} + \cdots + x_{5D} &\leq 1 \\
x_{1E} + x_{2E} + x_{3E} + \cdots + x_{5E} &\leq 1 \\
x_{1F} + x_{2F} + x_{3F} + \cdots + x_{5F} &\leq 1 \\
x_{1G} + x_{2G} + x_{3G} + \cdots + x_{5G} &\leq 1
\end{aligned}
$$

$$
\begin{aligned}
x_{1A} &\leq x_{5G} \\
x_{2D} &\leq x_{4F} \\
x_{1A} + x_{1G} &= 1 \\
x_{2C} + x_{2D} &= 1
\end{aligned}
$$

x_{ij} is binary