



# Lecture 8

Produced by Dr. Worldwide  
*Welcome to the 305*

# Integer Programming



- Prior linear programs have decision variables that are naturally **integer-valued**
- Optimal solutions are commonly not integer-valued
- Simply rounding up or down could lead to non-optimal solutions or could lie in an infeasible region
- Algorithms exist to handle this common problem
- Models where some/all the variables are required to be integer-valued are known as **integer programming models**



# Integer Programming



- **Total integer models** are linear programming models where all the decision variables must be integer-valued
- **0-1 integer models** are linear programming models where all the decision variables must take the values 0 or 1
- **Mixed integer models** are linear programming models where some of the decision variables must be integer valued while others do not



# Total Integer Model



- Decision variables
  - $x_1 = \text{Dollars invested in municipal bonds}$
  - $x_2 = \text{Dollars invested in CDs}$
  - $x_3 = \text{Dollars invested in Treasury Bills}$
  - $x_4 = \text{Dollars invested in Growth Stock}$

- Linear program

Maximize  $0.085x_1 + 0.05x_2 + 0.065x_3 + 0.13x_4$

Subject to

$$x_1 + x_2 + x_3 + x_4 = 70000$$
$$x_1 / (x_1 + x_2 + x_3 + x_4) \leq 0.2$$
$$x_2 \leq x_1 + x_3 + x_4$$
$$(x_2 + x_3) / (x_1 + x_2 + x_3 + x_4) \leq x_1 + x_3 + x_4$$
$$(x_2 + x_3) / (x_1 + x_4) \geq 1.2$$
$$x_1, x_2, x_3, x_4 \geq 0$$



# 0-1 Integer Model



- Decision variables
  - $x_1 = \text{Dollars invested in municipal bonds}$
  - $x_2 = \text{Dollars invested in CDs}$
  - $x_3 = \text{Dollars invested in Treasury Bills}$
  - $x_4 = \text{Dollars invested in Growth Stock}$

- Linear program

Maximize  $0.085x_1 + 0.05x_2 + 0.065x_3 + 0.13x_4$

Subject to

$$x_1 + x_2 + x_3 + x_4 = 70000$$
$$x_1 / (x_1 + x_2 + x_3 + x_4) \leq 0.2$$
$$x_2 \leq x_1 + x_3 + x_4$$
$$(x_2 + x_3) / (x_1 + x_2 + x_3 + x_4) \leq x_1 + x_3 + x_4$$
$$(x_2 + x_3) / (x_1 + x_4) \geq 1.2$$
$$x_1, x_2, x_3, x_4 \geq 0$$

# Mixed Integer Model



- Decision variables
  - $x_1 = \text{Dollars invested in municipal bonds}$
  - $x_2 = \text{Dollars invested in CDs}$
  - $x_3 = \text{Dollars invested in Treasury Bills}$
  - $x_4 = \text{Dollars invested in Growth Stock}$

- Linear program

Maximize  $0.085x_1 + 0.05x_2 + 0.065x_3 + 0.13x_4$

Subject to

$$x_1 + x_2 + x_3 + x_4 = 70000$$
$$x_1 / (x_1 + x_2 + x_3 + x_4) \leq 0.2$$
$$x_2 \leq x_1 + x_3 + x_4$$
$$(x_2 + x_3) / (x_1 + x_2 + x_3 + x_4) \leq x_1 + x_3 + x_4$$
$$(x_2 + x_3) / (x_1 + x_4) \geq 1.2$$
$$x_1, x_2, x_3, x_4 \geq 0$$

# Ex: Investment



- Linear program in standard form

Maximize  $0.085x_1 + 0.05x_2 + 0.065x_3 + 0.13x_4$

Subject to

$$\begin{aligned}x_1 + x_2 + x_3 + x_4 &= 70000 \\0.8x_1 - 0.2x_2 - 0.2x_3 - 0.2x_4 &\leq 0 \\-x_1 + x_2 - x_3 - x_4 &\leq 0 \\0.3x_1 - 0.7x_2 - 0.7x_3 + 0.3x_4 &\leq 0 \\1.2x_1 - x_2 - x_3 + 1.2x_4 &\leq 0 \\x_1, x_2, x_3, x_4 &\geq 0\end{aligned}$$

- Download [Investment-1.xlsx](#) from course website from link [Sheet 1](#)
- Optimal solution  $(x_1, x_2, x_3, x_4) = (0, 0, 38181, 3181.18)$

# Ex: Investment



- Sensitivity analysis

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$15	Municipal bonds = (\$)	0	-0.045	0.085	0.045	1E+30
\$B\$16	CDs = (\$)	0	-0.015	0.05	0.015	1E+30
\$B\$17	Treasury bills = (\$)	38181.81818	0	0.065	0.065	0.015
\$B\$18	Growth stock = (\$)	31818.18182	0	0.13	1E+30	0.045

Constraints


Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$F\$7	Total investment Usage	70000	0.094545455	70000	1E+30	70000
\$F\$8	Constraint 1 Usage	-14000	0	0	1E+30	14000
\$F\$9	Constraint 2 Usage	-70000	0	0	1E+30	70000
\$F\$10	Constraint 3 Usage	-17181.81818	0	0	1E+30	17181.81818
\$F\$11	Constraint 4 Usage	6.54836E-11	0.029545455	0	37800	70000



# Ex: Investment



- Created variables



Name Manager





Name Manager

New...

Edit...

Delete

Filter ▾

Name	Value	Refers To	Scope	Comment
 A	{"0.8","-0.2","-0.2","-0.2..."}	=Sheet1!\$B\$8:\$E\$11	Workbook	
 b	{"0","0","0","0"}	=Sheet1!\$H\$8:\$H\$11	Workbook	
 obj	{"0.085","0.05","0.065","..."}	=Sheet1!\$B\$5:\$E\$5	Workbook	
 x	{"0","0","38181.81818"..."}	=Sheet1!\$B\$15:\$B\$18	Workbook	

1	1	1	1
0.8	-0.2	-0.2	-0.2
-1	1	-1	-1
0.3	-0.7	-0.7	0.3
1.2	-1	-1	1.2

Edit Name	
Name:	A
Scope:	Workbook ▾
Comment:	
Refers to:	=Sheet1!\$B\$8:\$E\$11
OK Cancel	

# Ex: Investment



- Created variables

Production:		
Municipal bonds =		0
CDs =		0
Treasury bills =		0
Growth stock =		0
		0

Edit Name

Name: x

Scope: Workbook

Comment:

Refers to: =Sheet1!\$B\$15:\$B\$18

OK Cancel

70000
0
0
0
0

Edit Name

Name: b

Scope: Workbook

Comment:

Refers to: =Sheet1!\$H\$8:\$H\$11

OK Cancel

# Ex: Investment



- Usage of variables

Products:	Municipal bonds	CDs	Treasury bills	Growth stock			
	(\$)	(\$)	(\$)	(\$)			
Return:	0.085	0.05	0.065	0.13			
Constraints:					Usage	Constraint	R.H.S.
Total investment	1	1	1	1	0	=	70000
Constraint 1	0.8	-0.2	-0.2	-0.2	=MMULT(A,x)		0
Constraint 2	-1	1	-1	-1	0	<=	0
Constraint 3	0.3	-0.7	-0.7	0.3	0	<=	0
Constraint 4	1.2	-1	-1	1.2	0	<=	0
Production:							
Municipal bonds =	0						
CDs =	0						
Treasury bills =	0						
Growth stock =	0						
Return =	0						

# Ex: Investment



- Usage of variables

Solver Parameters

Set Objective:

To: ☒ Max ☐ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

Add  
Change  
Delete  
Reset All  
Load/Save

- Q: What other variable was created and how is it being used?

# Ex: Transportation



- Best Buy retail chain ships televisions from 3 of its distribution warehouses to three of its retail stores monthly
- Each warehouse has a fixed supply per month and fixed demand per month
- Q: How many TVs should be shipped from each warehouse to each store to minimize the total cost of transportation?
- Supply (700 TVs) and Demand (600 TVs)

Warehouse	Supply (TVs)
1. Cincinnati	300
2. Atlanta	200
3. Pittsburgh	200

Store	Demand (TVs)
A. New York	150
B. Dallas	250
C. Detroit	200



# Ex: Transportation



- Shipping cost per TV for each route

## Warehouse

1. Cincinnati
2. Atlanta
3. Pittsburgh

## Store

- A. New York
- B. Dallas
- C. Detroit

From Warehouse	To Store		
	A	B	C
1	\$16	\$18	\$11
2	14	12	13
3	13	15	17

# Ex: Transportation



- Visual of all routes (supply > demand)

## Warehouse

Cincinnati (1)  
**300**

Atlanta (2)  
**200**

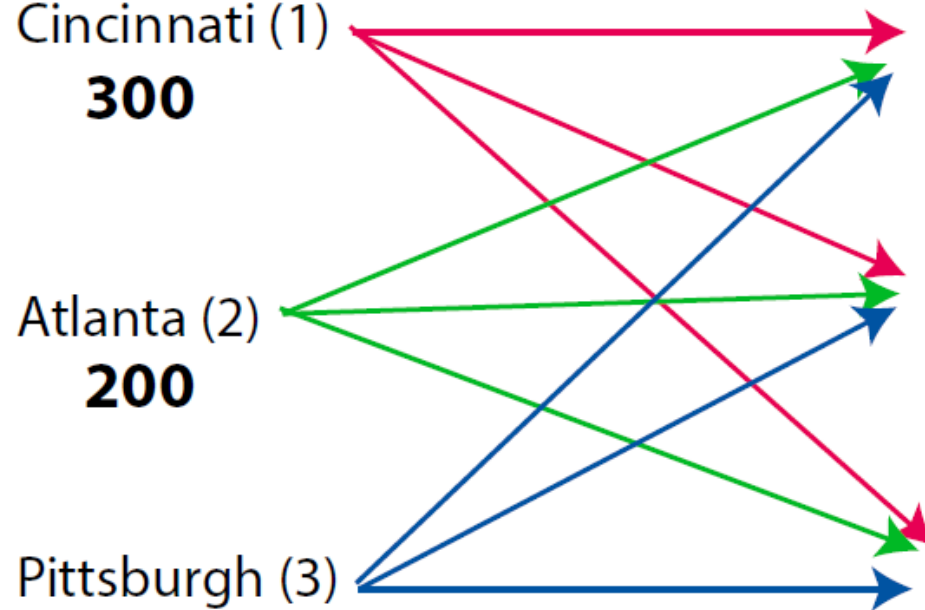
Pittsburgh (3)  
**200**

## Store

New York (A)  
**150**

Dallas (B)  
**250**

Detroit (C)  
**200**



# Ex: Transportation



- Decision variables
  - Need to have one for each of the 9 routes
  - $x_{ij}$  = number of televisions from warehouse  $i$  to store  $j$
  - $i = 1, 2, 3$  &  $j = A, B, C$
- Linear program in standard form

Minimize  $16x_{1A} + 18x_{1B} + 11x_{1C} + 14x_{2A} + 12x_{2B} + 13x_{2C} + 13x_{3A} + 15x_{3B} + 17x_{3C}$

Subject to

$$\begin{aligned} x_{1A} + x_{1B} + x_{1C} &\leq 300 && \text{(Cincinnati supply)} \\ x_{2A} + x_{2B} + x_{2C} &\leq 200 && \text{(Atlanta supply)} \\ x_{3A} + x_{3B} + x_{3C} &\leq 200 && \text{(Pittsburgh supply)} \end{aligned}$$
$$\begin{aligned} x_{1A} + x_{2A} + x_{3A} &\geq 150 && \text{(New York demand)} \\ x_{1B} + x_{2B} + x_{3B} &\geq 250 && \text{(Dallas demand)} \\ x_{1C} + x_{2C} + x_{3C} &\geq 200 && \text{(Detroit demand)} \end{aligned}$$

# Ex: Transportation



- Download [Transportation-1.xlsx](#) from course website from link [Sheet 2](#)
- Sheet called [Standard](#) contains the standard linear program format and the sheet called [Alternative](#) contains a more compact form of the same linear program
- Focus on [Alternative](#) sheet

	A	B	C	D	E	F	G
4	<b>Warehouse</b>	New York	Dallas	Detroit	<b>TV sets shipped</b>	<b>Constraint</b>	<b>Supply</b>
5	Cincinnati	0	0	200	200	<=	300
6	Atlanta	0	200	0	200	<=	200
7	Pittsburgh	150	50	0	200	<=	200
8	<b>TV sets shipped</b>	150	250	200			
9	<b>Constraint</b>	>=	>=	>=			
10	<b>Demand</b>	150	250	200			
11	<b>Cost (\$)</b>	7300					
12							
13							
14	<b>Warehouse</b>	New York	Dallas	Detroit			
15	Cincinnati	16	18	11			
16	Atlanta	14	12	13			
17	Pittsburgh	13	15	17			

# Ex: Transportation



- Use of **SUMPRODUCT** in creation of objective function

	A	B	C	D
4	<b>Warehouse</b>	New York	Dallas	Detroit
5	Cincinatti	0	0	200
6	Altanta	0	200	0
7	Pittsburgh	150	50	0
8	<b>TV sets shipped</b>	150	250	200
9	<b>Constraint</b>	>=	>=	>=
10	<b>Demand</b>	150	250	200
11	<b>Cost (\$)</b>	7300		
12		<b>=SUMPRODUCT(B5:D7,B15,D17)</b>		
13				
14	<b>Warehouse</b>	New York	Dallas	Detroit
15	Cincinatti	16	18	11
16	Altanta	14	12	13
17	Pittsburgh	13	15	17



# Ex: Scheduling



- Download [Multischedule-1.xlsx](#) from course website from link [Sheet 3](#)









# Ex: Quick-Screen Clothing



- Decision Variables
  - $x_1 = \text{Number of Boxes of Sweatshirts} - F$
  - $x_2 = \text{Number of Boxes of Sweatshirts} - B/F$
  - $x_3 = \text{Number of Boxes of T-shirts} - F$
  - $x_4 = \text{Number of Boxes of T-shirts} - B/F$
- Consider the following table showing resource requirements, unit costs, and profit of every dozen (box) of shirts

	Processing time (hr.) per dozen	Cost per dozen	Profit dozen
Sweatshirt - F	0.10	\$36	\$90
Sweatshirt - B/F	0.25	48	125
T-shirt - F	0.08	25	45
T-shirt - B/F	0.21	35	65



# Ex: Quick-Screen Clothing



- Objective Function
  - Goal: Maximize profit on shirts
  - $Z = 90x_1 + 125x_2 + 45x_3 + 65x_4$
- Constraints
  - Only have 72 hours of processing time to produce all items:  
 $0.1x_1 + 0.25x_2 + 0.08x_3 + 0.21x_4 \leq 72$
  - Company has a budget of \$25,000:  $36x_1 + 48x_2 + 25x_3 + 35x_4 \leq 25,000$
  - Trailer truck will pick up shirts and can accommodate 1,200 standard-size boxes where each standard-size box holds 12 T-shirts and a box of 12 sweatshirts is 3 times the size of the standard-size box:  
 $3(x_1 + x_2) + x_3 + x_4 \leq 1,200$
  - They have 500 dozens of blank sweatshirts:  $x_1 + x_2 \leq 500$
  - They have 500 dozens of blank T-shirts:  $x_3 + x_4 \leq 500$
  - Nonnegativity:  $x_1, x_2, x_3, x_4 \geq 0$

# Ex: Quick-Screen Clothing



- Download [ProductMix.xlsx](#) from website link called [Sheet 2](#)
- Before Excel solver

A product mix								
Products:	Sweatshirt-F (dozen)	Sweatshirt-B/F (dozen)	T-shirt-F (dozen)	T-shirt-B/F (dozen)				
Profit per dozen:	90	125	45	65				
Resources:					Usage	Constraint	Available	Left over
Processing time	0.1	0.25	0.08	0.21	0	<=	72	72
Cost	36	48	25	35	0	<=	25000	25000
Truck capacity	3	3	1	1	0	<=	1200	1200
Blank sweatshirts	1	1	0	0	0	<=	500	500
Blank T-shirts	0	0	1	1	0	<=	500	500
Production:								
Sweatshirts-F =	0							
Sweatshirts-B/F =	0							
T-shirt-F =	0							
T-shirt-B/F =	0							
Profit =	0							

# Ex: Quick-Screen Clothing



- After Excel solver

A product mix								
Products:	Sweatshirt-F (dozen)	Sweatshirt-B/F (dozen)	T-shirt-F (dozen)	T-shirt-B/F (dozen)				
Profit per dozen:	90	125	45	65				
Resources:					Usage	Constraint	Available	Left over
Processing time	0.1	0.25	0.08	0.21	72	<=	72	0
Cost	36	48	25	35	21593.333	<=	25000	3406.6667
Truck capacity	3	3	1	1	1200	<=	1200	0
Blank sweatshirts	1	1	0	0	233.33333	<=	500	266.66667
Blank T-shirts	0	0	1	1	500	<=	500	0
Production:								
Sweatshirts-F =	175.55556							
Sweatshirts-B/F =	57.777778							
T-shirt-F =	500							
T-shirt-B/F =	0							
Profit =	45522.222							

# Ex: Quick-Screen Clothing



- Recommended optimal solution to maximize profit at \$45,522.22

- $x_1 = 175.56$
- $x_2 = 57.78$
- $x_3 = 500$
- $x_4 = 0$

- Sensitivity report for objective function coefficients

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$15	Sweatshirts-F = (dozen)	175.5555556	0	90	11.92307692	40
\$B\$16	Sweatshirts-B/F = (dozen)	57.77777778	0	125	13.21428571	11.92307692
\$B\$17	T-shirt-F = (dozen)	500	0	45	1E+30	4.111111111
\$B\$18	T-shirt-B/F = (dozen)	0	-10.33333333	65	10.33333333	1E+30

# Ex: Quick-Screen Clothing



- Sensitivity report for constraint quantities

Constraints						
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$F\$10	Blank sweatshirts Usage	233.3333333	0	500	1E+30	266.6666667
\$F\$11	Blank T-shirts Usage	500	4.111111111	500	185.7142857	500
\$F\$7	Processing time Usage	72	233.3333333	72	26.33333333	8.666666667
\$F\$8	Cost Usage	21593.33333	0	25000	1E+30	3406.666667
\$F\$9	Truck capacity Usage	1200	22.22222222	1200	260	316



# Vectors and Matrices



- Linear program with 4 decision variables and 4 constraints requires more time to insert formulas in Excel
- Understanding of **linear algebra** can make this a more efficient process
- The object  $\mathbf{x} = [x_1, x_2, x_3, x_4]$  is a **row vector** in  $\mathbb{R}^4$

- The object  $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$  is a **column vector** in  $\mathbb{R}^4$

- The **transpose** of a vector  $\mathbf{x}$ , denoted  $\mathbf{x}'$ , transforms a row vector into a column vector and vice versa

# Vectors and Matrices



- The object  $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$  is a **matrix** in  $\mathbb{R}^{4 \times 3}$
- The **dimension** of a matrix, denoted  $\dim(\mathbf{A})$ , describes its number of rows and number of columns (in that order)
- Based on above example,  $\dim(\mathbf{A})$  is  $3 \times 4$
- A row vector in  $\mathbb{R}^m$  is a matrix in  $\mathbb{R}^{1 \times m}$
- A column vector in  $\mathbb{R}^n$  is a matrix in  $\mathbb{R}^{n \times 1}$
- Typically, all vectors are by default column vectors

# Vectors and Matrices



- For matrices  $\mathbf{A} \in \mathbb{R}^{m \times n}$  and  $\mathbf{B} \in \mathbb{R}^{n \times p}$ , we can define their **product**  $\mathbf{M} = \mathbf{AB}$ , which will be a matrix in  $\mathbb{R}^{m \times p}$
- For  $\mathbf{A} \in \mathbb{R}^{3 \times 4}$  and  $\mathbf{B} \in \mathbb{R}^{4 \times 2}$ , matrix  $\mathbf{M} = \mathbf{AB}$  can be expressed as

$$\mathbf{M} = \mathbf{AB} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \end{bmatrix}$$

where

$$m_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + a_{i4}b_{4j} \text{ for } i = 1,2,3 \text{ and } j = 1,2$$

- In order to compute,  $\mathbf{M} = \mathbf{AB}$ , the number of columns in  $\mathbf{A}$  must equal the number of rows in  $\mathbf{B}$
- In above example, the matrix  $\mathbf{M} = \mathbf{BA}$  does not exist

# Vectors and Matrices



- If  $\mathbf{A}$  and  $\mathbf{B}$  are square matrices of the same size, say in  $\mathbb{R}^{p \times p}$ , we can compute both  $\mathbf{AB}$  and  $\mathbf{BA}$ , but they may not necessarily be equal

- Let  $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$  and  $\mathbf{B} = \begin{bmatrix} 2 & 5 \\ 7 & 1 \end{bmatrix}$

$$\mathbf{AB} = \begin{bmatrix} 1 \times 2 + 3 \times 7 & 1 \times 5 + 3 \times 1 \\ -2 \times 2 + 0 \times 7 & -2 \times 5 + 0 \times 1 \end{bmatrix} = \begin{bmatrix} 23 & 8 \\ -4 & -10 \end{bmatrix}$$

$$\mathbf{BA} = \begin{bmatrix} 2 \times 1 + 5 \times -2 & 2 \times 3 + 5 \times 0 \\ 7 \times 1 + 1 \times -2 & 7 \times 3 + 1 \times 0 \end{bmatrix} = \begin{bmatrix} -8 & 6 \\ 5 & 21 \end{bmatrix}$$

- Let  $\mathbf{x} = [x_1, x_2, x_3, x_4]'$  and  $\mathbf{y} = [y_1, y_2, y_3, y_4]'$  be column vectors in  $\mathbb{R}^4$

$$\mathbf{x}'\mathbf{y} = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4 = \mathbf{y}'\mathbf{x}$$

# Excel: Matrix Multiplication



- MMULT Function in Excel
  - The **MMULT** Function in Excel is used to multiply arrays (matrices) that have compatible dimensions and returns an array (matrix)
  - Syntax: **MMULT**(array1,array2)
  - Example: Vector Multiplication

	A	B	C	D	E	F
1	<b>Vector a</b>	1	2	3		<b>Vector b</b>
2						3
3						2
4						1
5						
6	MMULT(a,b)	10				
7	MMULT(b,a)	3	6	9		
8		2	4	6		
9		1	2	3		

# Excel: Matrix Multiplication



- MMULT Function in Excel
  - The **MMULT** function in Excel is used to multiply arrays (matrices) that have compatible dimensions and returns an array (matrix)
  - Syntax: **MMULT**(array1,array2)
  - Example: Vector Multiplication

	A	B	C	D	E	F
1	<b>Vector a</b>	1	2	3		<b>Vector b</b>
2						3
3						2
4						1
5						
6	MMULT(a,b)	10	=MMULT(B1:D1,F2:F4)			
7	MMULT(b,a)	3	6	9		
8		2	4	6		
9		1	2	3		



# Excel: Matrix Multiplication



- MMULT Function in Excel
  - The **MMULT** Function in Excel is used to multiply arrays (matrices) that have compatible dimensions and returns an array (matrix)
  - Syntax: **MMULT**(array1,array2)
  - Example: Vector Multiplication

	A	B	C	D	E	F
1	<b>Vector a</b>	1	2	3		<b>Vector b</b>
2						3
3						2
4						1
5						
6	MMULT(a,b)	10				
7	MMULT(b,a)	3	6	9	=MMULT(F2:F4, B1:D1)	
8		2	4	6		
9		1	2	3		



# Excel: Matrix Multiplication



- MMULT Function in Excel
  - Example: Matrix Multiplication

	A	B	C	D	E	F	G
1	Matrix A	1	2	3		Matrix B	
2		4	5	6		1	2
3						3	4
4							
5	MMULT(A,B)	#VALUE!					
6	MMULT(B,A)	9	12	15			
7		19	26	33			

# Excel: Matrix Multiplication



- MMULT Function in Excel
  - Example: Matrix Multiplication

	A	B	C	D	E	F	G
1	Matrix A	1	2	3		Matrix B	
2		4	5	6		1	2
3						3	4
4							
5	MMULT(A,B)	#VALUE!	=MMULT(B1:D2,F2:G3)				
6	MMULT(B,A)	9	12	15			
7		19	26	33			

# Excel: Matrix Multiplication



- MMULT Function in Excel
  - Example: Matrix Multiplication

	A	B	C	D	E	F	G
1	Matrix A	1	2	3		Matrix B	
2		4	5	6		1	2
3						3	4
4							
5	MMULT(A,B)	#VALUE!					
6	MMULT(B,A)	9	12	15	=MMULT(F2:G3,B1:D2)		
7		19	26	33			

# Excel: Matrix Multiplication



- SUMPRODUCT Function in Excel
  - The **SUMPRODUCT** function in Excel is used to multiply arrays (matrices) element-wise and then returns the sum of their products
  - In mathematics, this is often referred to as a **cross-product** or **vector-product** when the arrays are vectors
  - Syntax: **SUMPRODUCT(array1,array2)**
  - Ex: Cross-product

	A	B	C	D	E	F
1	<b>Vector a</b>	1	2	3		<b>Vector c</b>
2	<b>Vector b</b>	3	2	1		4
3						5
4						6
5						
6	SUMPRODUCT(a,b)	10				
7	SUMPRODUCT(a,c)	#VALUE!				

# Excel: Matrix Multiplication



- SUMPRODUCT Function in Excel
  - The **SUMPRODUCT** function in Excel is used to multiply arrays (matrices) element-wise and then returns the sum of their products
  - In mathematics, this is often referred to as a **cross-product** or **vector-product** when the arrays are vectors
  - Syntax: **SUMPRODUCT(array1,array2)**
  - Ex: Cross-product

	A	B	C	D	E	F
1	Vector a	1	2	3		Vector c
2	Vector b	3	2	1		4
3						5
4						6
5						
6	SUMPRODUCT(a,b)	10	=SUMPRODUCT(B1:D1,B2:D2)			
7	SUMPRODUCT(a,c)	#VALUE!				

# Excel: Matrix Multiplication



- SUMPRODUCT Function in Excel
  - The **SUMPRODUCT** function in Excel is used to multiply arrays (matrices) element-wise and then returns the sum of their products
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	A	B	C	D	E	F
1	<b>Vector a</b>	1	2	3		<b>Vector c</b>
2	<b>Vector b</b>	3	2	1		4
3						5
4						6
5						
6	SUMPRODUCT(a,b)	10				
7	SUMPRODUCT(a,c)	#VALUE!	=SUMPRODUCT(B1:D1,F2:F4)			



# Excel: Application of Matrices

- Both MMULT and CROSSPRODUCT can be used in Excel to make the creation of formulas and constraints of linear programs considerably easier
- Made up example for practice

	A	B	C	D	E	F
1	Profit	3	4			
2				Total	Constraint	Max
3	Metal	2	1	3	<=	30
4	Plastic	0	4	4	<=	50
5						
6						
7	Swag 1	1				
8	Swag 2	1				
9	Total	7				

Solver

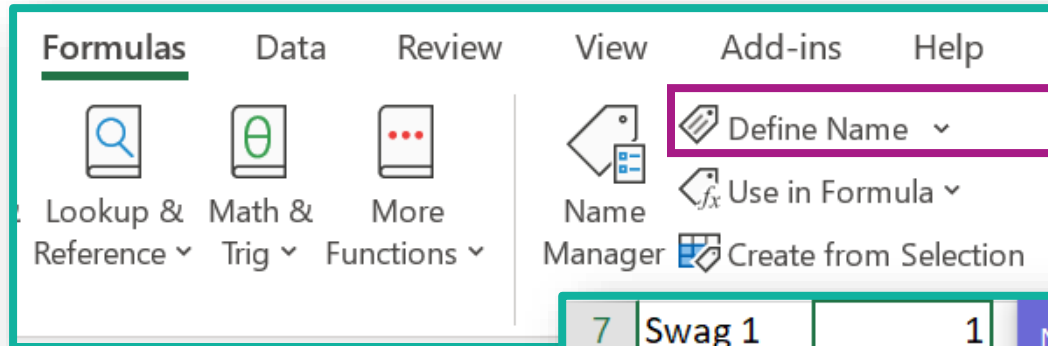
	A	B	C	D	E	F
1	Profit	3	4			
2				Total	Constraint	Max
3	Metal	2	1	30	<=	30
4	Plastic	0	4	50	<=	50
5						
6						
7	Swag 1	8.75				
8	Swag 2	12.5				
9	Total	76.25				



# Excel: Application of Matrices



- Creating EXCEL variable for easy referencing



7	Swag 1	1
8	Swag 2	1
9	Total	7
10		
11		
12		
13		
14		
15		
16		
17		
18		

### New Name

Name:

Scope:

Comment:

Refers to:

OK Cancel



# Excel: Application of Matrices

- Usage of created variable in establishing constraints and objective function

	A	B	C	D	E	F
1	Profit	3	4			
2				Total	Constraint	Max
3	Metal	2	1	30	<=	30
4	Plastic	0	4	50	<=	50
5				=MMULT(B4:C4,x)		
6						
7	Swag 1	8.75				
8	Swag 2	12.5				
9	Total	76.25	=MMULT(B1:C1,x)			



# Excel: Application of Matrices



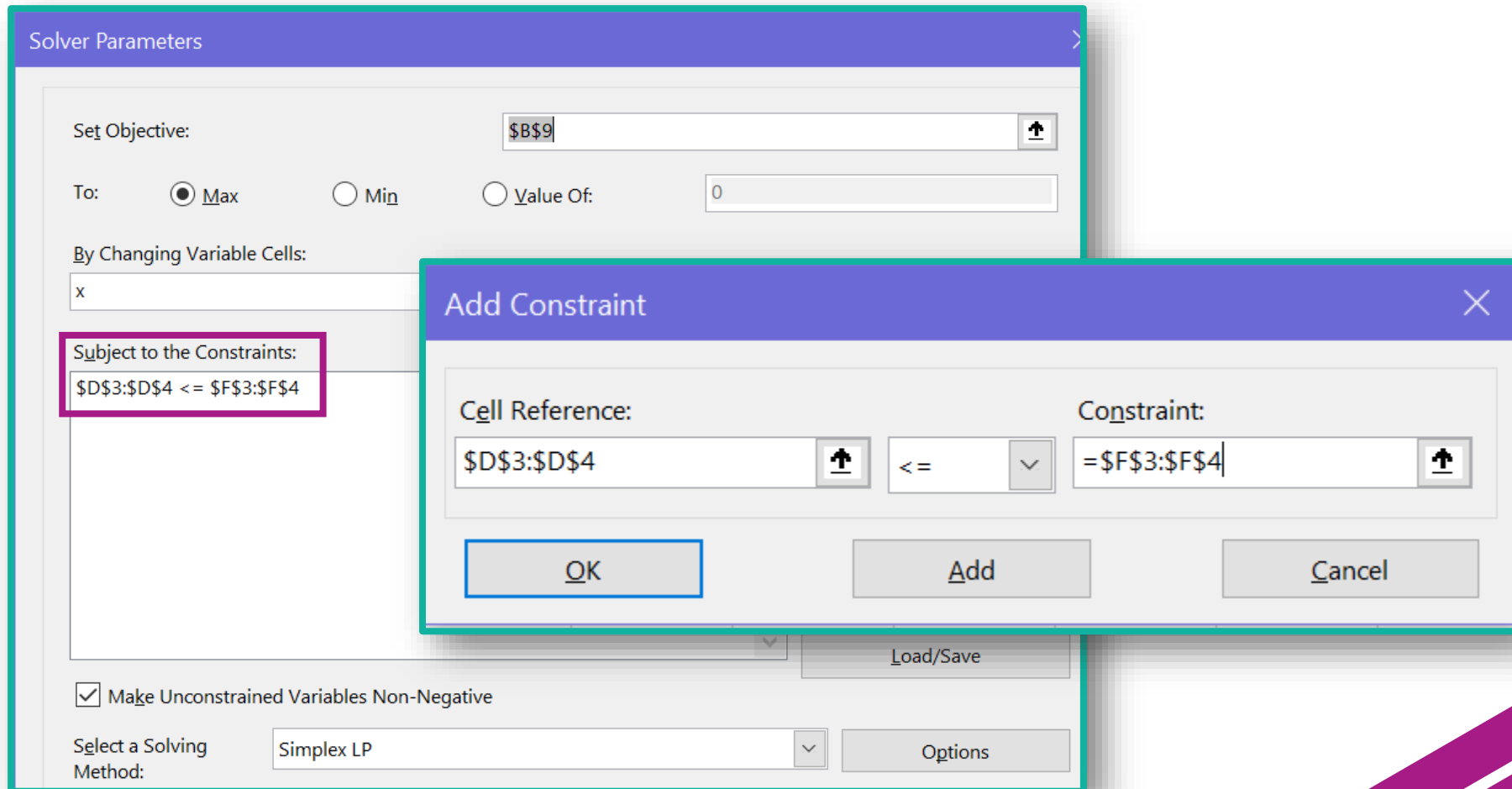
- Another option for specification

SUM		:	✕	✓	<i>fx</i>	=MMULT(B3:C4,B7:B8)	
	A	B	C	D	E	F	
1	Profit	3	4				
2				Total	Constraint	Max	
3	Metal	2	1	B8)	<=	30	
4	Plastic	0	4	4	<=	50	
5							
6							
7	Swag 1	1					
8	Swag 2	1					
9	Total	7					



# Excel: Application of Matrices

- Another option for specification



The image shows the Excel Solver interface. The 'Solver Parameters' task pane is open, showing the following settings:

- Set Objective:**
- To:** ☒ Max ☐ Min ☐ Value Of:
- By Changing Variable Cells:**
- Subject to the Constraints:**
- ☒ Make Unconstrained Variables Non-Negative
- Select a Solving Method:** Simplex LP

The 'Add Constraint' dialog box is also open, showing the following details:

- Cell Reference:**
- Constraint:**
- Buttons: OK, Add, Cancel



# Excel: Application of Matrices

- Try using MMULT and SUMPRODUCT in cell formulas
- Try using vectors/matrices in specification of constraints
- Download [Lecture6WS.xlsx](#) from course website from link [Sheet 2](#) for all examples seen in this lecture







# The End



# Dale

