



Lecture 4

Produced by Dr. Worldwide
Welcome to the 305

Special Cases



- Infeasible problem
 - A linear program is **infeasible** if there is no point that satisfies all the constraints
 - Consider the linear program

$$\begin{array}{ll}\text{Maximize} & 5x + 3y \\ \text{Subject to} & 4x + 2y \leq 8 \\ & x \geq 4 \\ & y \geq 6\end{array}$$

- When $x \geq 4$ and $y \geq 6$, $4x + 2y > 8$

$$4(4) + 2(6) = 28 > 8$$

$$4(5) + 2(7) = 34 > 8$$

Special Cases



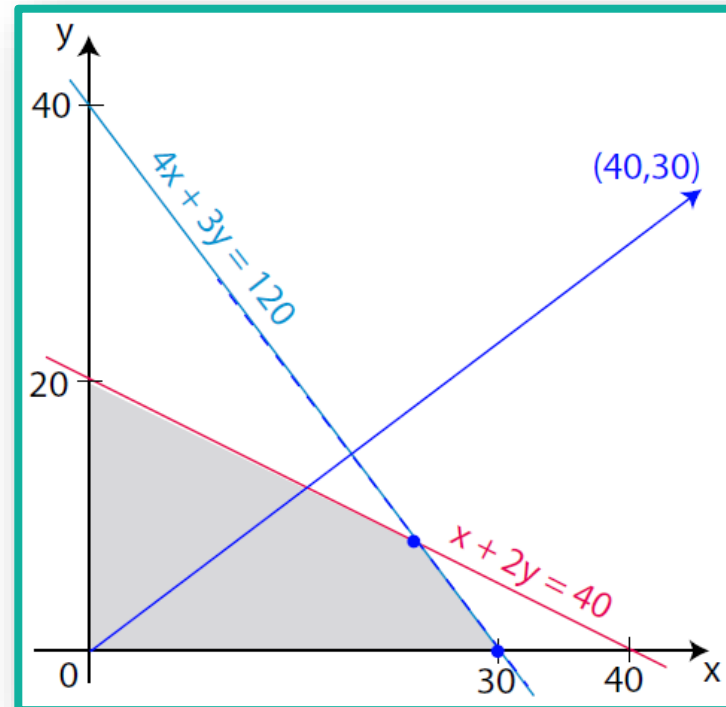
- Multiple optimal solutions
 - A linear program may have **multiple optimal solutions** if there are two or more extreme points along the optimal level curve for the problem
 - Consider the linear program

$$\begin{array}{ll}\text{Maximize} & 40x + 30y \\ \text{Subject to} & x + 2y \leq 40 \\ & x \geq 0 \\ & y \geq 0\end{array}$$

- Optimal points: (24,8) & (30,0)

$$40(24) + 30(8) = 1200$$

$$40(30) + 30(0) = 1200$$



Special Cases



- Unbounded problem
 - A linear program is **unbounded** if the feasible region is not closed and the objective function grows (decreases) indefinitely without bound
 - Similar to the infeasible problem where no solution exists
 - Consider the two linear programs with identical feasible regions

A) Maximize $2x + y$
Subject to $x + y \geq 5$
 $x \leq 4$
 $x \geq 0$
 $y \geq 0$

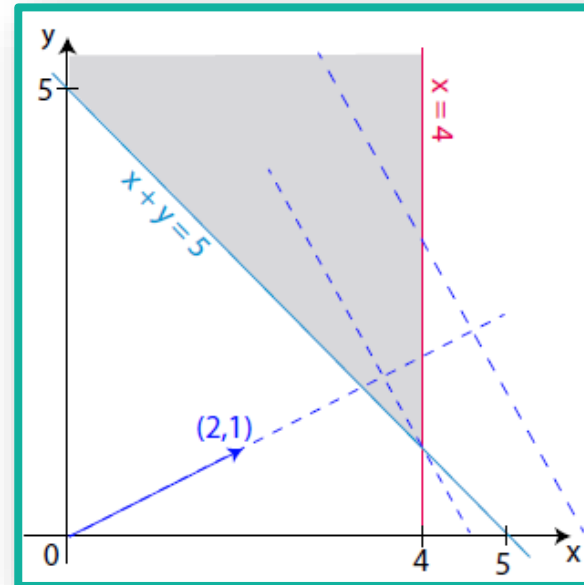
B) Minimize $2x + y$
Subject to $x + y \geq 5$
 $x \leq 4$
 $x \geq 0$
 $y \geq 0$

- Which linear program is unbounded, A or B?

Special Cases



- Unbounded problem
 - In both linear programs, the feasible region is unbounded



- The maximization linear program is **unbounded**
- The minimization linear program has a single optimal solution at (0,5)

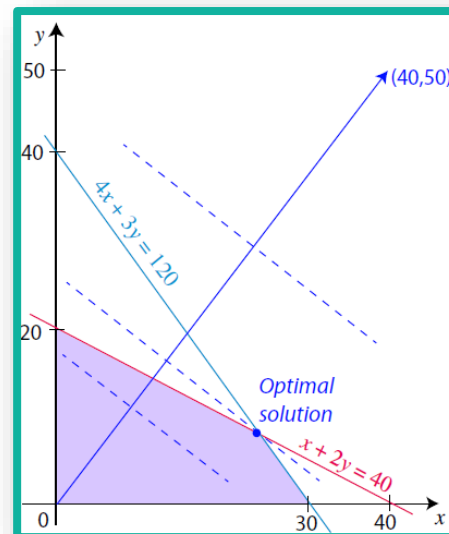
Computer Solution



- Majority of linear programs solved using a computer
- Excel's built-in tool called **Solver** is capable of handling linear optimization using the **simplex algorithm** from George Dantzig
- Recall: Beaver Creek Pottery Company from Lecture 2
 - Download **BeaverCreek.xlsx** from website link called **Sheet 1**
 - Linear Program

$$\begin{array}{ll}\text{Maximize} & 40x + 50y \\ \text{Subject to} & x + 2y \leq 40 \\ & 4x + 3y \leq 120 \\ & x \geq 0 \\ & y \geq 0\end{array}$$

- Optimal: 24 Bowls and 8 Mugs



Computer Solution



- Preview of Spreadsheet

	A	B	C	D	E	F	G
1	The Beaver Creek Pottery Company						
2							
3	Products:	Bowl	Mug				
4	Profit per unit:	40	50				
5	Resources:			Usage	Constraint	Available	Left over
6	Labor (hr/unit)	1	2	0	<=	40	40
7	Clay (lb/unit)	4	3	0	<=	120	120
8							
9							
10	Production:						
11	Bowls =	0					
12	Mugs =	0					
13	Profit =	0					

Computer Solution



- Preview of Spreadsheet

	A	B	C	D	E	F	G
1	The Beaver Creek Pottery Company						
2							
3	Products:	Bowl	Mug				
4	Profit per unit:	40	50				
5	Resources:			Usage	Constraint	Available	Left over
6	Labor (hr/unit)	1	2	0	<=	40	40
7	Clay (lb/unit)	4	3	0	<=	120	120
8							
9							
10	Production:						
11	Bowls =	0					
12	Mugs =	0					
13	Profit =	0		Objective Function: $B_4 * B_{11} + C_4 * B_{12}$			

Computer Solution



- Preview of Spreadsheet

	A	B	C	D	E	F	G
1	The Beaver Creek Pottery Company						
2							
3	Products:	Bowl	Mug				
4	Profit per unit:	40	50	Labor Used: $B6*B11+C6*B12$			
5	Resources:			Usage	Constraint	Available	Left over
6	Labor (hr/unit)	1	2	0	<=	40	40
7	Clay (lb/unit)	4	3	0	<=	120	120
8				Clay Used: $B7*B11+C7*B12$			
9							
10	Production:						
11	Bowls =	0					
12	Mugs =	0					
13	Profit =	0					

Computer Solution



- Preview of Spreadsheet

	A	B	C	D	E	F	G
1	The Beaver Creek Pottery Company						
2							
3	Products:	Bowl	Mug				
4	Profit per unit:	40	50				
5	Resources:			Usage	Constraint	Available	Left over
6	Labor (hr/unit)	1	2	0	<=	40	40
7	Clay (lb/unit)	4	3	0	<=	120	120
8						Capacity	
9							
10	Production:						
11	Bowls =	0					
12	Mugs =	0					
13	Profit =	0					

Computer Solution



- Preview of Spreadsheet

	A	B	C	D	E	F	G
1	The Beaver Creek Pottery Company						
2							
3	Products:	Bowl	Mug				
4	Profit per unit:	40	50			Labor Waste: F6-D6	
5	Resources:			Usage	Constraint	Available	Left over
6	Labor (hr/unit)	1	2	0	<=	40	40
7	Clay (lb/unit)	4	3	0	<=	120	120
8						Clay Waste: F6-D6	
9							
10	Production:						
11	Bowls =	0					
12	Mugs =	0					
13	Profit =	0					

Computer Solution



- Optimizing with Excel
 - Select **Data** and then select **Solver** by the **Analyze** section
 - Observe window


The image shows the 'Solver Parameters' dialog box in Microsoft Excel. The 'Set Objective:' field is set to '\$B\$13'. The 'To:' section has three radio buttons: 'Max' (selected), 'Min', and 'Value Of:'. The 'By Changing Variable Cells:' field is set to '\$B\$11:\$B\$12'. The 'Subject to the Constraints:' list contains two constraints: '\$D\$6 <= \$F\$6' and '\$D\$7 <= \$F\$7'. There are buttons for 'Add', 'Change', 'Delete', 'Reset All', and 'Load/Save' next to the constraints list. The 'Make Unconstrained Variables Non-Negative' checkbox is checked. The 'Select a Solving Method:' dropdown is set to 'Simplex LP', with an 'Options' button next to it. A 'Solving Method' section at the bottom provides instructions on selecting the GRG Nonlinear engine, LP Simplex engine, or Evolutionary engine. At the bottom of the dialog are 'Help', 'Solve', and 'Close' buttons.



Computer Solution



- Optimizing with Excel
 - Cell you are trying to optimize with objective function

Set Objective: 

- Maximize or minimize

To: ☒ Max ☐ Min

- Choose your decision variables

10	Production:			
11	Bowls =		0	By Changing Variable Cells: <input type="text" value="\$B\$11:\$B\$12"/>
12	Mugs =		0	

Computer Solution



- Optimizing with Excel
 - Create your constraints using **Add**
 - You can type or click to select cell
 - You will see your constraints in the **Subject to the Constraints**
 - Notice box for nonnegativity

☒ Make Unconstrained Variables Non-Negative

B	C	D	E	F
Lottery Company				
Bowl	Mug			
40	50			
		Usage	Constraint	Available
1	2	0	<=	40
4	3	0	<=	120

Add Constraint
×

Cell Reference:

\$D\$6
↑

<=
↓

Constraint:

=\$F\$6
↑

OK

Add

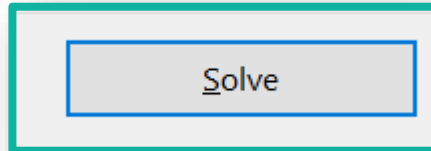
Cancel

5	Resources:			Usage	Constraint	Available	Subject to the Co
6	Labor (hr/unit)	1	2	0	<=	40	\$D\$6 <= \$F\$6
7	Clay (lb/unit)	4	3	0	<=	120	\$D\$7 <= \$F\$7

Computer Solution

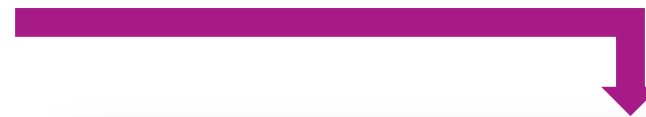


- Optimizing with Excel
 - Select **Solve**



- Optimal solution can be found in decision variables

Production:	
Bowls =	0
Mugs =	0
Profit =	0



Production:	
Bowls =	24
Mugs =	8
Profit =	1360

Ex: *Annabelle Invests*



- Annabelle Sizemore has a massive amount of money (AKA stacks) from numerous sources that she needs to do something with (AKA make it rain)
- After researching the market, she has decided to split her money to 2 places
 - S&P index fund from Shield Securities
 - Internet stock fund from Madison Funds, Inc.
- Q: How should Annabelle split her money in these two funds?
- Decision Variables
 - x = Amount Invested in S&P Index Fund
 - y = Amount Invested in Internet Stock Fund
 - u = Number of S&P Index Fund Shares
 - v = Number of Internet Stock Fund Shares

Ex: *Annabelle Invests*



- Average annual return over the last 3 years for the S&P index fund was 17% and 28% for the internet stock fund
 - Goal: Maximize return on her investment
 - Objective function: $Z = 0.17x + 0.28y$
 - Price per share of S&P index fund is \$175 and \$208 for internet stock fund
 - Objective function: $Z = 0.17(175u) + 0.28(208v)$
- Constraints
 - Must invest nonnegative amounts : $x \geq 0$ & $y \geq 0$ ($u \geq 0$ & $v \geq 0$)
 - Only has \$120,000 to invest: $x + y \leq 120,000$ ($175u + 208v \leq 120,000$)
 - The proportion of the dollar amount she invests in the index fund relative to the internet fund should be at least one-third: $x/y \geq 1/3$ or $3x - y \geq 0$ ($3(175u) - 208v \geq 0$)
 - Amount invested in index fund no more than twice the amount invested in the internet fund: $x \leq 2y$ or $x - 2y \geq 0$ ($175u - 2(208v) \geq 0$)

Ex: *Annabelle Invests*

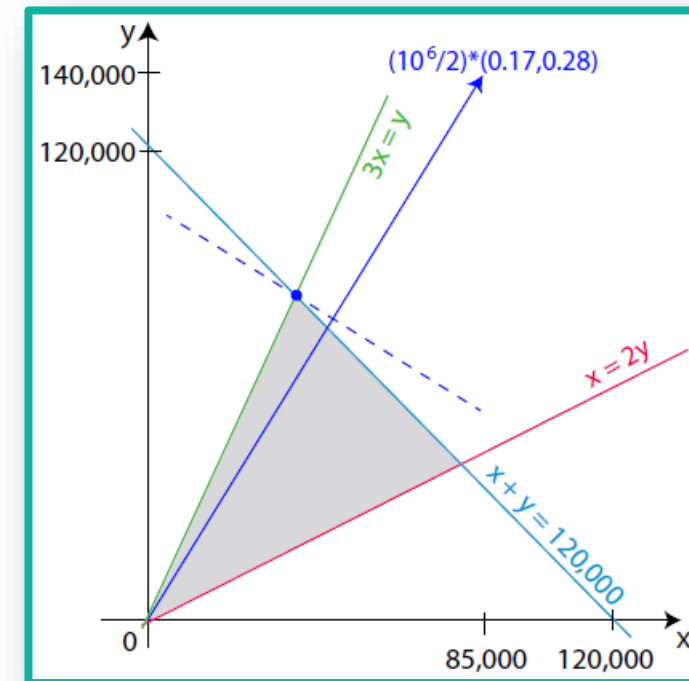


- Full linear program

Maximize $0.17x + 0.28y$
Subject to
 $x + y \leq 120,000$
 $3x - y \geq 0$
 $x - 2y \leq 0$
 $x \geq 0$
 $y \geq 0$



Maximize $0.17(175u) + 0.28(208v)$
Subject to
 $175u + 208v \leq 120,000$
 $3(175u) - 208v \geq 0$
 $175u - 2(208v) \geq 0$
 $u \geq 0$
 $v \geq 0$



Ex: *Annabelle Invests*



- Alternative approach: use growth vector and level curves (contours)
 - Download [AnnabelleInvest.xlsx](#) from website link called [Sheet 2](#)
 - Try to use Excel [Solver](#) to find the optimal solution
 - Solution
 - $x = \$30,000$
 - $y = \$90,000$
- Return is \$30,300

The image shows the 'Solver Parameters' dialog box in Microsoft Excel. The 'Set Objective:' field contains '\$B\$14'. The 'To:' section has 'Max' selected. The 'By Changing Variable Cells:' field contains '\$B\$12:\$B\$13'. The 'Subject to the Constraints:' list contains three constraints: '\$D\$6 <= \$F\$6', '\$D\$7 >= \$F\$7', and '\$D\$8 <= \$F\$8'. The 'Make Unconstrained Variables Non-Negative' checkbox is checked. The 'Select a Solving Method:' dropdown is set to 'Simplex LP'. On the right side, there are buttons for 'Add', 'Change', 'Delete', 'Reset All', 'Load/Save', and 'Options'.





The End



Dale

