Assignment #8 Solutions

due Friday, October 25th, 2019

1 We can apply the minimal spanning tree algorithm to derive the minimal spanning tree as follows:

Node 1 is adjacent to 4
Node 2 is adjacent to 4
Node 3 is adjacent to 6
Node 4 is adjacent to 6
Node 5 is adjacent to 7
Node 6 is adjacent to 7
Node 7 is adjacent to 8

The total length of the path way is 1160 yards.

2 We can apply the minimal spanning tree algorithm to derive the minimal spanning tree as follows:

Node 1 is adjacent to 2 and 4
Node 3 is adjacent to 6
Node 4 is adjacent to 6
Node 5 is adjacent to 8
Node 6 is adjacent to 9
Node 7 is adjacent to 20
Node 9 is adjacent to 10 and 11
Node 10 is adjacent to 12 and 13
Node 12 is adjacent to 14

The total number of refurbished sidewalks is 1086 feet.

3

The model is as follows:

$$
\begin{aligned}
&\min z = P_1 d_1^-, P_2 d_2^-, P_3 d_1^+, P_4 d_3^+ \\
&s.t. \\
&5x_1 + 2x_2 + 4x_3 + d_1^- - d_1^+ = 240 \\
&3x_1 + 5x_2 + 2x_3 + d_2^- - d_2^+ = 500 \\
&4x_1 + 6x_2 + 3x_3 + d_3^- - d_3^+ = 400 \\
&x_i, d_i^-, d_i^+ \geq 0, i = 1, 2, 3.
\end{aligned}
$$

We can solve the model using excel, shown in Fig 2, **??**, 3, 4. From the result, we can see that

$$x_1 = 10.5263, \quad x_2 = 93.6842, \quad x_3 = 0, \quad d_1^- = 0, \quad d_1^+ = 0$$
$$d_2^- = 0, \quad d_2^+ = 0, \quad d_3^- = 0, \quad d_3^+ = 204.2105$$

1	**Goal programming for Problem 3**													
2														
3	*Priority 1*													
4	Minimize	d1^-												
5														
6														
7	Objective function	0	0	0	1	0	0	0	0	0				
8	Variables	x1	x2	x3	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	Used	Constraint	Value	
9	Labor	5	2	4	1	-1	0	0	0	0	240	=	240	
10	Profit	3	5	2	0	0	1	-1	0	0	500	=	500	
11	Material	4	6	3	0	0	0	0	1	-1	400	=	400	
12														
13														
14	x1	29.0909091												
15	x2	47.2727273												
16	x3	0												
17	d1^-	0												
18	d1^+	0												
19	d2^-	176.363636												
20	d2^+	0												
21	d3^-	0												
22	d3^+	0												
23														
24	Objective function	0												

Figure 1: Excel in Question 3, Priority 1

1	**Goal programming for Problem 3**													
2														
3	*Priority 2*													
4	Minimize	d2^-												
5														
6														
7	Objective function	0	0	0	0	0	1	0	0	0				
8	Variables	x1	x2	x3	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	Used	Constraint	Value	
9	Labor	5	2	4	1	-1	0	0	0	0	240	=	240	
10	Profit	3	5	2	0	0	1	-1	0	0	500	=	500	
11	Material	4	6	3	0	0	0	0	1	-1	400	=	400	
12	Goal 1	0	0	0	1	0	0	0	0	0	0	=	0	
13														
14														
15	x1	10.5263158												
16	x2	93.6842105												
17	x3	0												
18	d1^-	0												
19	d1^+	0												
20	d2^-	0												
21	d2^+	0												
22	d3^-	0												
23	d3^+	204.210526												
24														
25	Objective function	0												

Figure 2: Excel in Question 3, Priority 2

1	**Goal programming for Problem 3**													
2														
3	*Priority 3*													
4	Minimize	d1^+												
5														
6														
7	Objective function	0	0	0	0	0	1	0	0	0				
8	Variables	x1	x2	x3	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	Used	Constraint	Value	
9	Labor	5	2	4	1	-1	0	0	0	0	240	=	240	
10	Profit	3	5	2	0	0	1	-1	0	0	500	=	500	
11	Material	4	6	3	0	0	0	0	1	-1	400	=	400	
12	Goal 1	0	0	0	1	0	0	0	0	0	0	=	0	
13	Goal 2	0	0	0	0	0	1	0	0	0	0	=	0	
14														
15														
16	x1	10.5263158												
17	x2	93.6842105												
18	x3	0												
19	d1^-	0												
20	d1^+	0												
21	d2^-	0												
22	d2^+	0												
23	d3^-	0												
24	d3^+	204.210526												
25														
26	Objective function	0												

Figure 3: Excel in Question 3, Priority 3

Goal programming for Problem 3													
Priority 4													
Minimize	d3^+												
Objective function	0	0	0	0	0	0	0	0	0	1			
Variables	x1	x2	x3	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	Used	Constraint	Value	
Labor	5	2	4	1	-1	0	0	0	0	240	=	240	
Profit	3	5	2	0	0	1	-1	0	0	500	=	500	
Material	4	6	3	0	0	0	0	1	-1	400	=	400	
Goal 1	0	0	0	1	0	0	0	0	0	0	=	0	
Goal 2	0	0	0	0	0	1	0	0	0	0	=	0	
Goal 3	0	0	0	0	1	0	0	0	0	0	=	0	
x1	10.5263158												
x2	93.6842105												
x3	0												
d1^-	0												
d1^+	0												
d2^-	0												
d2^+	0												
d3^-	0												
d3^+	204.210526												
Objective function	204.210526												

Figure 4: Excel in Question 3, Priority 4

4

(a) Assume the number of hours the GP, nurse and internist needed to work is x_1, x_2 and x_3, respectively.

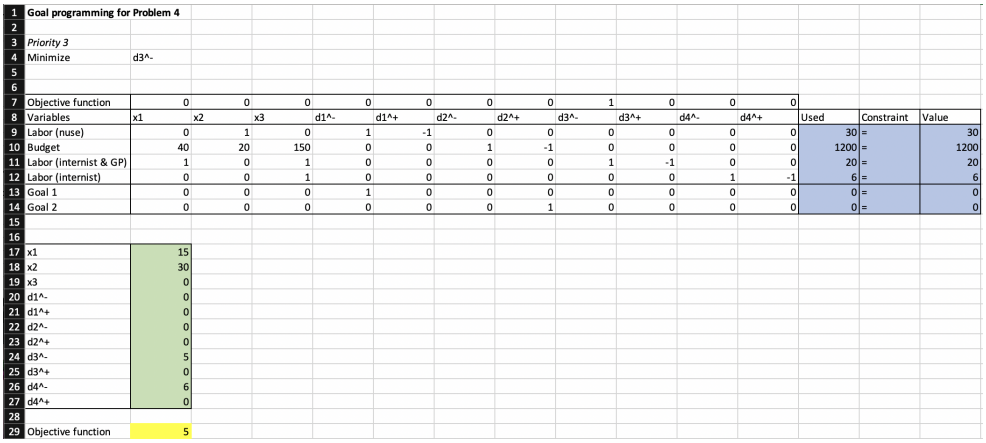
$$
\begin{aligned}
&\min z = P_1 d_1^-, P_2 d_2^+, P_3 d_3^-, P_4 d_4^- \\
&s.t. \\
&x_2 + d_1^- - d_1^+ = 30 \\
&40x_1 + 20x_2 + 150x_3 + d_2^- - d_2^+ = 1200 \\
&x_1 + x_3 + d_3^- - d_3^+ = 20 \\
&x_3 + d_4^- - d_4^+ = 6 \\
&x_i \geq 0, i = 1, ..., 3, d_j^-, d_j^+, j = 1, ..., 4.
\end{aligned}
$$

(b) We can solve the model using excel, shown in Fig 2, **??**, 3, 4. From the result, we can see that

$$
\begin{array}{ccccccc}
x_1 = 15, & x_2 = 30, & x_3 = 4.44 \times 10^{-16}, & d_1^- = 0, & d_1^+ = 0 & d_2^- = 0 \\
d_2^+ = 0, & d_3^- = 5, & d_3^+ = 0 & d_4^- = 6 & d_4^+ = 0 &
\end{array}
$$

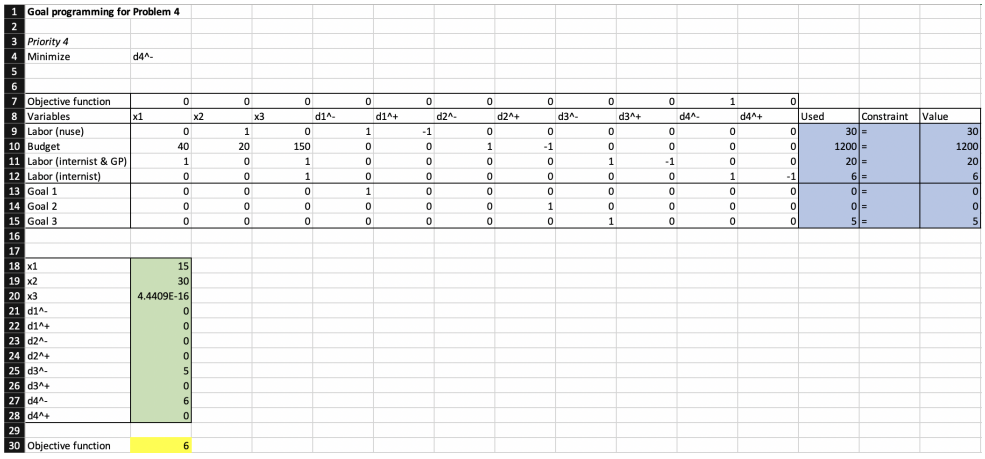
Goal programming for Problem 4														
Priority 1														
Minimize	d1^-													
Objective function	0	0	0	1	0	0	0	0	0	0	0			
Variables	x1	x2	x3	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	d4^-	d4^+	Used	Constraint	Value
Labor (nuse)	0	1	0	1	-1	0	0	0	0	0	0	30	=	30
Budget	40	20	150	0	0	1	-1	0	0	0	0	1200	=	1200
Labor (internist & GP)	1	0	1	0	0	0	0	1	-1	0	0	20	=	20
Labor (internist)	0	0	1	0	0	0	0	0	0	1	-1	6	=	6
x1	0													
x2	30													
x3	4													
d1^-	0													
d1^+	0													
d2^-	0													
d2^+	0													
d3^-	16													
d3^+	0													
d4^-	2													
d4^+	0													
Objective function	0													

Figure 5: Excel in Question 4, Priority 1



1	**Goal programming for Problem 4**															
2																
3	*Priority 3*															
4	Minimize	d3^-														
5																
6																
7	Objective function	0	0	0	0	0	0	0	1	0	0	0				
8	Variables	x1	x2	x3	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	d4^-	d4^+	Used	Constraint	Value	
9	Labor (nuse)	0	1	0	1	-1	0	0	0	0	0	0	30	=	30	
10	Budget	40	20	150	0	0	1	-1	0	0	0	0	1200	=	1200	
11	Labor (internist & GP)	1	0	1	0	0	0	0	1	-1	0	0	20	=	20	
12	Labor (internist)	0	0	1	0	0	0	0	0	0	1	-1	6	=	6	
13	Goal 1	0	0	0	1	0	0	0	0	0	0	0	0	=	0	
14	Goal 2	0	0	0	0	0	0	1	0	0	0	0	0	=	0	
15																
16																
17	x1	15														
18	x2	30														
19	x3	0														
20	d1^-	0														
21	d1^+	0														
22	d2^-	0														
23	d2^+	0														
24	d3^-	5														
25	d3^+	0														
26	d4^-	6														
27	d4^+	0														
28																
29	Objective function	5														

Figure 6: Excel in Question 4, Priority 3



1	**Goal programming for Problem 4**															
2																
3	*Priority 4*															
4	Minimize	d4^-														
5																
6																
7	Objective function	0	0	0	0	0	0	0	0	0	1	0				
8	Variables	x1	x2	x3	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	d4^-	d4^+	Used	Constraint	Value	
9	Labor (nuse)	0	1	0	1	-1	0	0	0	0	0	0	30	=	30	
10	Budget	40	20	150	0	0	1	-1	0	0	0	0	1200	=	1200	
11	Labor (internist & GP)	1	0	1	0	0	0	0	1	-1	0	0	20	=	20	
12	Labor (internist)	0	0	1	0	0	0	0	0	0	1	-1	6	=	6	
13	Goal 1	0	0	0	1	0	0	0	0	0	0	0	0	=	0	
14	Goal 2	0	0	0	0	0	0	1	0	0	0	0	0	=	0	
15	Goal 3	0	0	0	0	0	0	0	1	0	0	0	5	=	5	
16																
17																
18	x1	15														
19	x2	30														
20	x3	4.4409E-16														
21	d1^-	0														
22	d1^+	0														
23	d2^-	0														
24	d2^+	0														
25	d3^-	5														
26	d3^+	0														
27	d4^-	6														
28	d4^+	0														
29																
30	Objective function	6														

Figure 7: Excel in Question 4, Priority 4

5 Assume whether we invested project i (i=1,2,...,8) is x_i. When x_i is 1, it means we invest project i, otherwise, we should not invest project i.

Data:

b_i = development budget for project i ($1,000,000), i = 1,...,8.
p_i = number of research personnel for project i, i = 1,...,8.
s_i = expected annual sales for project i ($1,000,000), i = 1,...,8.

The model is as follows:

$$\min z = P_1 d_1^+ + P_2 d_2^+ + P_3 d_3^- + P_4(d_4^- + d_5^-) + P_5 d_6^+ + P_6 d_7^-$$

$$s.t.$$

$$\sum_{i=1}^{8} b_i x_i + d_1^- - d_1^+ = 5$$

$$\sum_{i=1}^{8} p_i x_i + d_2^- - d_2^+ = 27$$

$$\sum_{i=1}^{8} s_i x_i + d_3^- - d_3^+ = 6.5$$

$$x_1 + x_3 + x_4 + x_6 + d_4^- - d_4^+ = 2$$

$$x_2 + x_5 + x_7 + x_8 + d_5^- - d_5^+ = 2$$

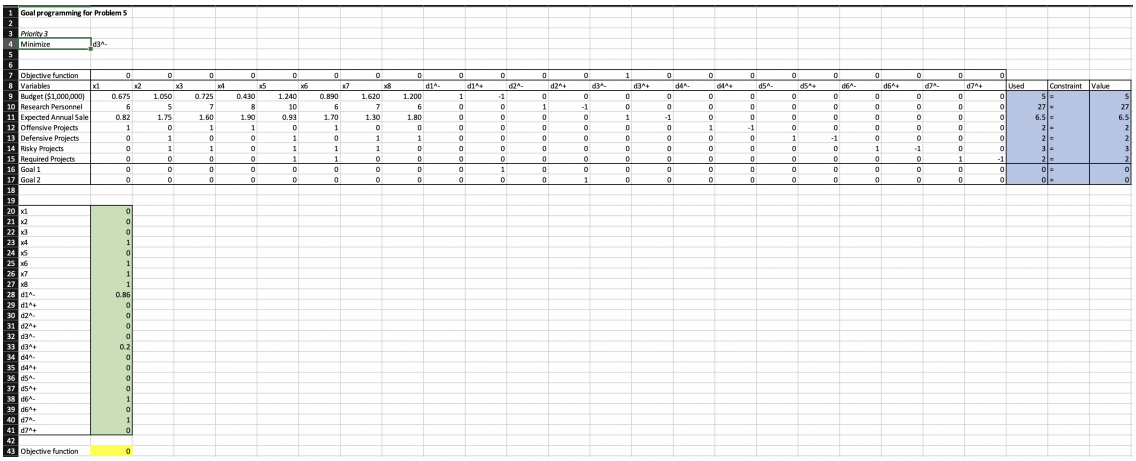
$$x_2 + x_3 + x_5 + x_6 + x_7 + d_6^- - d_6^+ = 3$$

$$x_5 + x_6 + d_7^- - d_7^+ = 2$$

$$x_i \in \{0, 1\}, i = 1, ..., 8, d_j^-, d_j^+, j = 1, ..., 7.$$

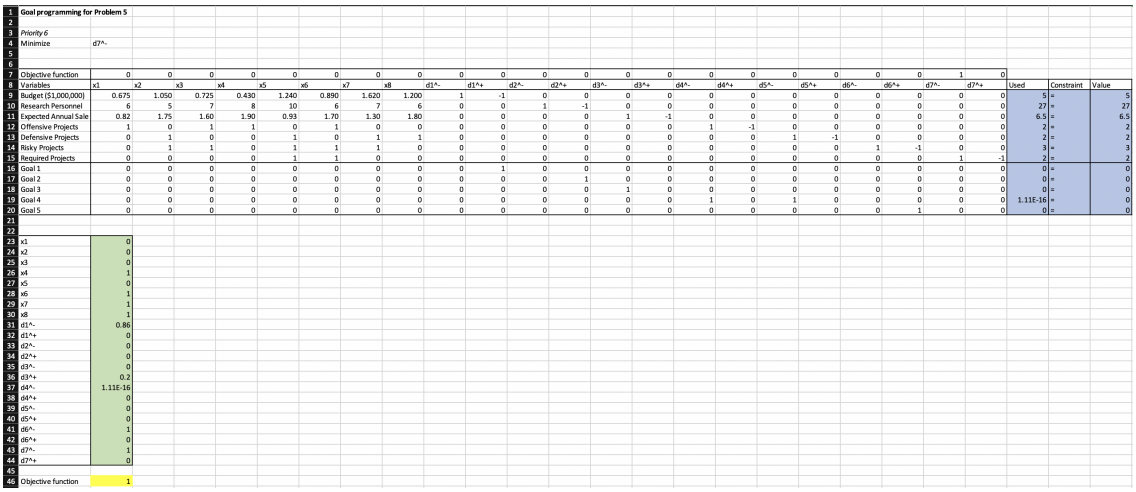
1	**Goal programming for Problem 5**																												
2																													
3	*Priority 1*																												
4	Minimize	d1^+																											
5																													
6																													
7	Objective function	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0					
8	Variables	x1	x2	x3	x4	x5	x6	x7	x8	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	d4^-	d4^+	d5^-	d5^+	d6^-	d6^+	d7^-	d7^+	Used	Constraint	Value				
9	Budget ($1,000,000)	0.675	1.05	0.725	0.43	1.24	0.89	1.62	1.2	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	5	=	5			
10	Research Personnel	6	5	7	8	10	6	7	6	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	27	=	27			
11	Expected Annual Sale	0.82	1.75	1.6	1.9	0.93	1.7	1.3	1.8	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	6.5	=	6.5			
12	Offensive Projects	1	0	1	1	0	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	2	=	2			
13	Defensive Projects	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	2	=	2			
14	Risky Projects	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	3	=	3			
15	Required Projects	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	2	=	2			
16																													
17																													
18	x1	0																											
19	x2	0																											
20	x3	0																											
21	x4	1																											
22	x5	0																											
23	x6	1																											
24	x7	1																											
25	x8	0																											
26	d1^-	2.06																											
27	d1^+	0																											
28	d2^-	6																											
29	d2^+	0																											
30	d3^-	1.6																											
31	d3^+	0																											
32	d4^-	0																											
33	d4^+	1.11E-16																											
34	d5^-	1																											
35	d5^+	0																											
36	d6^-	1																											
37	d6^+	0																											
38	d7^-	1																											
39	d7^+	0																											
40																													
41	Objective function	0																											

Figure 8: Excel in Question 5, Priority 1



1	**Goal programming for Problem 5**																														
2																															
3	*Priority 3*																														
4	Minimize	d3^-																													
5																															
6																															
7	Objective function	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0					
8	Variables	x1	x2	x3	x4	x5	x6	x7	x8	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	d4^-	d4^+	d5^-	d5^+	d6^-	d6^+	d7^-	d7^+	Used	Constraint	Value					
9	Budget ($1,000,000)	0.675	1.050	0.725	0.430	1.240	0.890	1.620	1.200	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	5	=	5					
10	Research Personnel	6	5	7	8	10	6	7	6	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	27	=	27					
11	Expected Annual Sale	0.82	1.75	1.60	1.90	0.93	1.70	1.30	1.80	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	6.5	=	6.5					
12	Offensive Projects	1	0	1	1	0	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	2	=	2					
13	Defensive Projects	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	2	=	2					
14	Risky Projects	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	3	=	3					
15	Required Projects	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	2	=	2					
16	Goal 1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	=	0					
17	Goal 2	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	=	0					
18																															
19																															
20	x1	0																													
21	x2	0																													
22	x3	0																													
23	x4	1																													
24	x5	0																													
25	x6	1																													
26	x7	1																													
27	x8	1																													
28	d1^-	0.86																													
29	d1^+	0																													
30	d2^-	0																													
31	d2^+	0																													
32	d3^-	0																													
33	d3^+	0.2																													
34	d4^-	0																													
35	d4^+	0																													
36	d5^-	0																													
37	d5^+	0																													
38	d6^-	1																													
39	d6^+	0																													
40	d7^-	1																													
41	d7^+	0																													
42																															
43	Objective function	0																													

Figure 9: Excel in Question 5, Priority 3



1	**Goal programming for Problem 5**																														
2																															
3	*Priority 6*																														
4	Minimize	d7^-																													
5																															
6																															
7	Objective function	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0							
8	Variables	x1	x2	x3	x4	x5	x6	x7	x8	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	d4^-	d4^+	d5^-	d5^+	d6^-	d6^+	d7^-	d7^+	Used	Constraint	Value						
9	Budget ($1,000,000)	0.675	1.050	0.725	0.430	1.240	0.890	1.620	1.200	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	5	=	5					
10	Research Personnel	6	5	7	8	10	6	7	6	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	27	=	27					
11	Expected Annual Sale	0.82	1.75	1.60	1.90	0.93	1.70	1.30	1.80	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	6.5	=	6.5					
12	Offensive Projects	1	0	1	1	0	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	2	=	2					
13	Defensive Projects	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	2	=	2					
14	Risky Projects	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	3	=	3					
15	Required Projects	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	2	=	2					
16	Goal 1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	=	0					
17	Goal 2	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	=	0					
18	Goal 3	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	=	0					
19	Goal 4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1.11E-16	=	0					
20	Goal 5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	=	0					
21																															
22																															
23	x1	0																													
24	x2	0																													
25	x3	0																													
26	x4	1																													
27	x5	0																													
28	x6	1																													
29	x7	1																													
30	x8	1																													
31	d1^-	0.86																													
32	d1^+	0																													
33	d2^-	0																													
34	d2^+	0																													
35	d3^-	0																													
36	d3^+	0.2																													
37	d4^-	1.11E-16																													
38	d4^+	0																													
39	d5^-	0																													
40	d5^+	0																													
41	d6^-	1																													
42	d6^+	0																													
43	d7^-	1																													
44	d7^+	0																													
45																															
46	Objective function	1																													

Figure 10: Excel in Question 5, Priority 6

We can solve the model using excel, shown in Fig 8, 9, 10. To best achieve its goals, project 4 6 7 8 are taken. (or project 2 4 6 7)