Lecture 6

Produced by Dr. Worldwide

Welcome to the 305

Ex: Quick-Screen Clothing

- Quick-Screen is a clothing manufacturing company specializing in the production of commemorative shirts immediately following major sporting events and they have a contract to produce shirts for winning team of a college football bowl game on New Year's Day between State and Tech

- They will produce two different sweatshirts and two different t-shirts with one of each having writing on front (F) only and the other having writing on both front (F) and back (B)

- All items will be produced by the box where each box contains a dozen items

- Q: How much of each of the items should be produced to maximize profit?

Ex: Quick-Screen Clothing

- Decision Variables
  - $x_1 = Number\ of\ Boxes\ of\ Sweatshirts\ - F$
  - $x_2 = Number\ of\ Boxes\ of\ Sweatshirts - B/F$
  - $x_3 = Number\ of\ Boxes\ of\ T - shirts - F$
  - $x_4 = Number\ of\ Boxes\ of\ T - shirts - B/F$

- Consider the following table showing resource requirements, unit costs, and profit of every dozen (box) of shirts

	Processing time (hr.) per dozen	Cost per dozen	Profit dozen
Sweatshirt - F	0.10	\$36	\$90
Sweatshirt - B/F	0.25	48	125
T-shirt - F	0.08	25	45
T-shirt - B/F	0.21	35	65

Ex: Quick-Screen Clothing

- Objective Function
  - Goal: Maximize profit on shirts
  - $Z = 90x_1 + 125x_2 + 45x_3 + 65x_4$

- Constraints
  - Only have 72 hours of processing time to produce all items: $0.1x_1 + 0.25x_2 + 0.08x_3 + 0.21x_4 \leq 72$
  - Company has a budget of $25,000: $36x_1 + 48x_2 + 25x_3 + 35x_4 \leq 25{,}000$
  - Trailer truck will pick up shirts and can accommodate 1,200 standard-size boxes where each standard-size box holds 12 T-shirts and a box of 12 sweatshirts is 3 times the size of the standard-size box: $3(x_1 + x_2) + x_3 + x_4 \leq 1{,}200$
  - They have 500 dozens of blank sweatshirts: $x_1 + x_2 \leq 500$
  - They have 500 dozens of blank T-shirts: $x_3 + x_4 \leq 500$
  - Nonnegativity: $x_1, x_2, x_3, x_4 \geq 0$

Ex: Quick-Screen Clothing

- Download ProductMix.xlsx from website link called Sheet 1

- Before Excel solver

A product mix								
Products:	Sweatshirt-F (dozen)	Sweatshirt-B/F (dozen)	T-shirt-F (dozen)	T-shirt-B/F (dozen)				
Profit per dozen:	90	125	45	65				
Resources:					Usage	Constraint	Available	Left over
Processing time	0.1	0.25	0.08	0.21	0	<=	72	72
Cost	36	48	25	35	0	<=	25000	25000
Truck capacity	3	3	1	1	0	<=	1200	1200
Blank sweatshirts	1	1	0	0	0	<=	500	500
Blank T-shirts	0	0	1	1	0	<=	500	500
Production:								
Sweatshirts-F =	0							
Sweatshirts-B/F =	0							
T-shirt-F =	0							
T-shirt-B/F =	0							
Profit =	0							

Ex: Quick-Screen Clothing

- After Excel solver

A product mix								
Products:	Sweatshirt-F (dozen)	Sweatshirt-B/F (dozen)	T-shirt-F (dozen)	T-shirt-B/F (dozen)				
Profit per dozen:	90	125	45	65				
Resources:					Usage	Constraint	Available	Left over
Processing time	0.1	0.25	0.08	0.21	72	<=	72	0
Cost	36	48	25	35	21593.333	<=	25000	3406.6667
Truck capacity	3	3	1	1	1200	<=	1200	0
Blank sweatshirts	1	1	0	0	233.33333	<=	500	266.66667
Blank T-shirts	0	0	1	1	500	<=	500	0
Production:								
Sweatshirts-F =	175.55556							
Sweatshirts-B/F =	57.777778							
T-shirt-F =	500							
T-shirt-B/F =	0							
Profit =	45522.222							

Ex: Quick-Screen Clothing

- Recommended optimal solution to maximize profit at $45,522.22
  - $x_1 = 175.56$
  - $x_2 = 57.78$
  - $x_3 = 500$
  - $x_4 = 0$

- Sensitivity report for objective function coefficients

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B15	Sweatshirts-F = (dozen)	175.5555556	0	90	11.92307692	40
B16	Sweatshirts-B/F = (dozen)	57.77777778	0	125	13.21428571	11.92307692
B17	T-shirt-F = (dozen)	500	0	45	1E+30	4.111111111
B18	T-shirt-B/F = (dozen)	0	-10.33333333	65	10.33333333	1E+30

Ex: Quick-Screen Clothing

- Sensitivity report for constraint quantities

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F10	Blank sweatshirts Usage	233.3333333	0	500	1E+30	266.6666667
F11	Blank T-shirts Usage	500	4.111111111	500	185.7142857	500
F7	Processing time Usage	72	233.3333333	72	26.33333333	8.666666667
F8	Cost Usage	21593.33333	0	25000	1E+30	3406.666667
F9	Truck capacity Usage	1200	22.22222222	1200	260	316

Vectors and Matrices

- Linear program with 4 decision variables and 4 constraints requires more time to insert formulas in Excel

- Understanding of linear algebra can make this a more efficient process

- The object $\mathbf{x} = [x_1, x_2, x_3, x_4]$ is a row vector in $\mathbb{R}^4$

- The object $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ is a column vector in $\mathbb{R}^4$

- The transpose of a vector $\mathbf{x}$, denoted $\mathbf{x}'$, transforms a row vector into a column vector and vice versa

Vectors and Matrices

- The object $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$ is a matrix in $\mathbb{R}^{4\times 3}$
- The dimension of a matrix, denoted $\dim(\mathbf{A})$, describes its number of rows and number of columns (in that order)
- Based on above example, $\dim(\mathbf{A})$ is 4×3
- A row vector in $\mathbb{R}^m$ is a matrix in $\mathbb{R}^{1\times m}$
- A column vector in $\mathbb{R}^n$ is a matrix in $\mathbb{R}^{n\times 1}$
- Typically, all vectors are by default column vectors

Vectors and Matrices

- For matrices $\mathbf{A} \in \mathbb{R}^{m\times n}$ and $\mathbf{B} \in \mathbb{R}^{n\times p}$, we can define their product $\mathbf{M} = \mathbf{AB}$, which will be a matrix in $\mathbb{R}^{m\times p}$

- For $\mathbf{A} \in \mathbb{R}^{3\times 4}$ and $\mathbf{B} \in \mathbb{R}^{4\times 2}$, matrix $\mathbf{M} = \mathbf{AB}$ can be expressed as

$$\mathbf{M} = \mathbf{AB} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \end{bmatrix}$$

where

$$m_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{j3} + a_{i4}b_{4j} \text{ for } i = 1,2,3 \text{ and } j = 1,2$$

- In order to compute, $\mathbf{M} = \mathbf{AB}$, the number of columns in $\mathbf{A}$ must equal the number of rows in $\mathbf{B}$

- In above example, the matrix $\mathbf{M} = \mathbf{BA}$ does not exist

Vectors and Matrices

- If $\mathbf{A}$ and $\mathbf{B}$ are square matrices of the same size, say in $\mathbb{R}^{p\times p}$, we can compute both $\mathbf{AB}$ and $\mathbf{BA}$, but they may not necessarily be equal

- Let $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 5 \\ 7 & 1 \end{bmatrix}$

$$\mathbf{AB} = \begin{bmatrix} 1\times 2+3\times 7 & 1\times 5+3\times 1 \\ -2\times 2+0\times 7 & -2\times 5+0\times 1 \end{bmatrix} = \begin{bmatrix} 23 & 8 \\ -4 & -10 \end{bmatrix}$$

$$\mathbf{BA} = \begin{bmatrix} 2\times 1+5\times -2 & 2\times 3+5\times 0 \\ 7\times 1+1\times -2 & 7\times 3+1\times 0 \end{bmatrix} = \begin{bmatrix} -8 & 6 \\ 5 & 21 \end{bmatrix}$$

- Let $\mathbf{x} = [x_1, x_2, x_3, x_4]'$ and $\mathbf{y} = [y_1, y_2, y_3, y_4]'$ be column vectors in $\mathbb{R}^4$

$$\mathbf{x}'\boldsymbol{y} = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4 = \mathbf{y}'\mathbf{x}$$

Excel: Matrix Multiplication

- MMULT Function in Excel
  - The MMULT Function in Excel is used to multiply arrays (matrices) that have compatible dimensions and returns an array (matrix)
  - Syntax: MMULT(array1,array2)
  - Example: Vector Multiplication

	A	B	C	D	E	F
1	**Vector a**	1	2	3		**Vector b**
2						3
3						2
4						1
5						
6	MMULT(a,b)	10				
7	MMULT(b,a)	3	6	9		
8		2	4	6		
9		1	2	3		

Excel: Matrix Multiplication

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  - Example: Vector Multiplication

	A	B	C	D	E	F
1	**Vector a**	1	2	3		**Vector b**
2						3
3						2
4						1
5						
6	MMULT(a,b)	10	=MMULT(B1:D1,F2:F4)			
7	MMULT(b,a)	3	6	9		
8		2	4	6		
9		1	2	3		

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	A	B	C	D	E	F
1	**Vector a**	1	2	3		**Vector b**
2						3
3						2
4						1
5						
6	MMULT(a,b)	10				
7	MMULT(b,a)	3	6	9	=MMULT(F2:F4,	
8		2	4	6		B1:D1)
9		1	2	3		

Excel: Matrix Multiplication

- MMULT Function in Excel
  - Example: Matrix Multiplication

	A	B	C	D	E	F	G
1	Matrix A	1	2	3		Matrix B	
2		4	5	6		1	2
3						3	4
4							
5	MMULT(A,B)	#VALUE!					
6	MMULT(B,A)	9	12	15			
7		19	26	33			

Excel: Matrix Multiplication

- MMULT Function in Excel
  - Example: Matrix Multiplication

	A	B	C	D	E	F	G
1	Matrix A	1	2	3		Matrix B	
2		4	5	6		1	2
3						3	4
4							
5	MMULT(A,B)	#VALUE!	=MMULT(B1:D2,F2:G3)				
6	MMULT(B,A)	9	12	15			
7		19	26	33			

Excel: Matrix Multiplication

- MMULT Function in Excel
  - Example: Matrix Multiplication

	A	B	C	D	E	F	G
1	Matrix A	1	2	3		Matrix B	
2		4	5	6		1	2
3						3	4
4							
5	MMULT(A,B)	#VALUE!					
6	MMULT(B,A)	9	12	15	=MMULT(F2:G3,B1:D2)		
7		19	26	33			

Excel: Matrix Multiplication

- SUMPRODUCT Function in Excel
  - The SUMPRODUCT function in Excel is used to multiply arrays (matrices) element-wise and then returns the sum of their products
  - In mathematics, this is often referred to as a dot-product or vector-product when the arrays are vectors
  - Syntax: SUMPRODUCT(array1,array2)
  - Ex: Usage on Vectors

	A	B	C	D	E	F
1	**Vector a**	1	2	3		**Vector c**
2	**Vector b**	3	2	1		4
3						5
4						6
5						
6	SUMPRODUCT(a,b)	10				
7	SUMPRODUCT(a,c)	#VALUE!				

Excel: Matrix Multiplication

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  - The SUMPRODUCT function in Excel is used to multiply arrays (matrices) element-wise and then returns the sum of their products
  - In mathematics, this is often referred to as a dot-product or vector-product when the arrays are vectors
  - Syntax: SUMPRODUCT(array1,array2)
  - Ex: Usage on Vectors

	A	B	C	D	E	F
1	**Vector a**	1	2	3		**Vector c**
2	**Vector b**	3	2	1		4
3						5
4						6
5						
6	SUMPRODUCT(a,b)	10	=SUMPRODUCT(B1:D1,B2:D2)			
7	SUMPRODUCT(a,c)	#VALUE!				

Excel: Matrix Multiplication

- SUMPRODUCT Function in Excel
  - The SUMPRODUCT function in Excel is used to multiply arrays (matrices) element-wise and then returns the sum of their products
  - In mathematics, this is often referred to as a dot-product or vector-product when the arrays are vectors
  - Syntax: SUMPRODUCT(array1,array2)
  - Ex: Usage on Vectors

	A	B	C	D	E	F
1	**Vector a**	1	2	3		**Vector c**
2	**Vector b**	3	2	1		4
3						5
4						6
5						
6	SUMPRODUCT(a,b)	10				
7	SUMPRODUCT(a,c)	#VALUE!	=SUMPRODUCT(B1:D1,F2:F4)			

Excel: Application of Matrices

- Both MMULT and SUMPRODUCT can be used in Excel to make the creation of formulas and constraints of linear programs considerably easier

- Made up example for practice

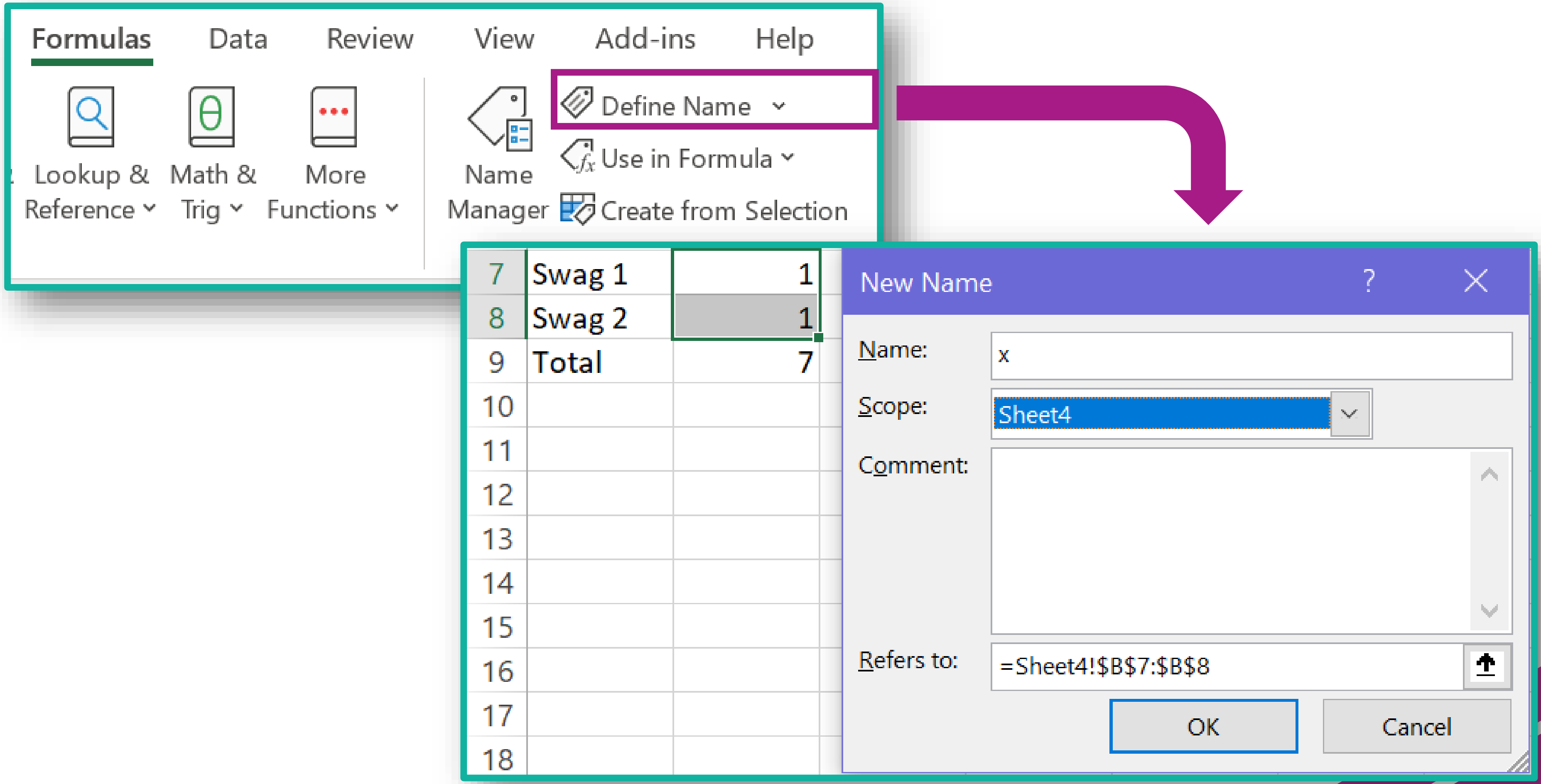
	A	B	C	D	E	F
1	Profit	3	4			
2				Total	Constraint	Max
3	Metal	2	1	3	<=	30
4	Plastic	0	4	4	<=	50
5						
6						
7	Swag 1	1				
8	Swag 2	1				
9	Total	7				

Solver

	A	B	C	D	E	F
1	Profit	3	4			
2				Total	Constraint	Max
3	Metal	2	1	30	<=	30
4	Plastic	0	4	50	<=	50
5						
6						
7	Swag 1	8.75				
8	Swag 2	12.5				
9	Total	76.25				

Excel: Application of Matrices

- Creating EXCEL variable for easy referencing



Excel: Application of Matrices

- Usage of created variable in establishing constraints and objective function

	A	B	C	D	E	F
1	Profit	3	4			
2				Total	Constraint	Max
3	Metal	2	1	30	<=	30
4	Plastic	0	4	50	<=	50
5				=MMULT(B4:C4,x)		
6						
7	Swag 1	8.75				
8	Swag 2	12.5				
9	Total	76.25	=MMULT(B1:C1,x)			

Excel: Application of Matrices

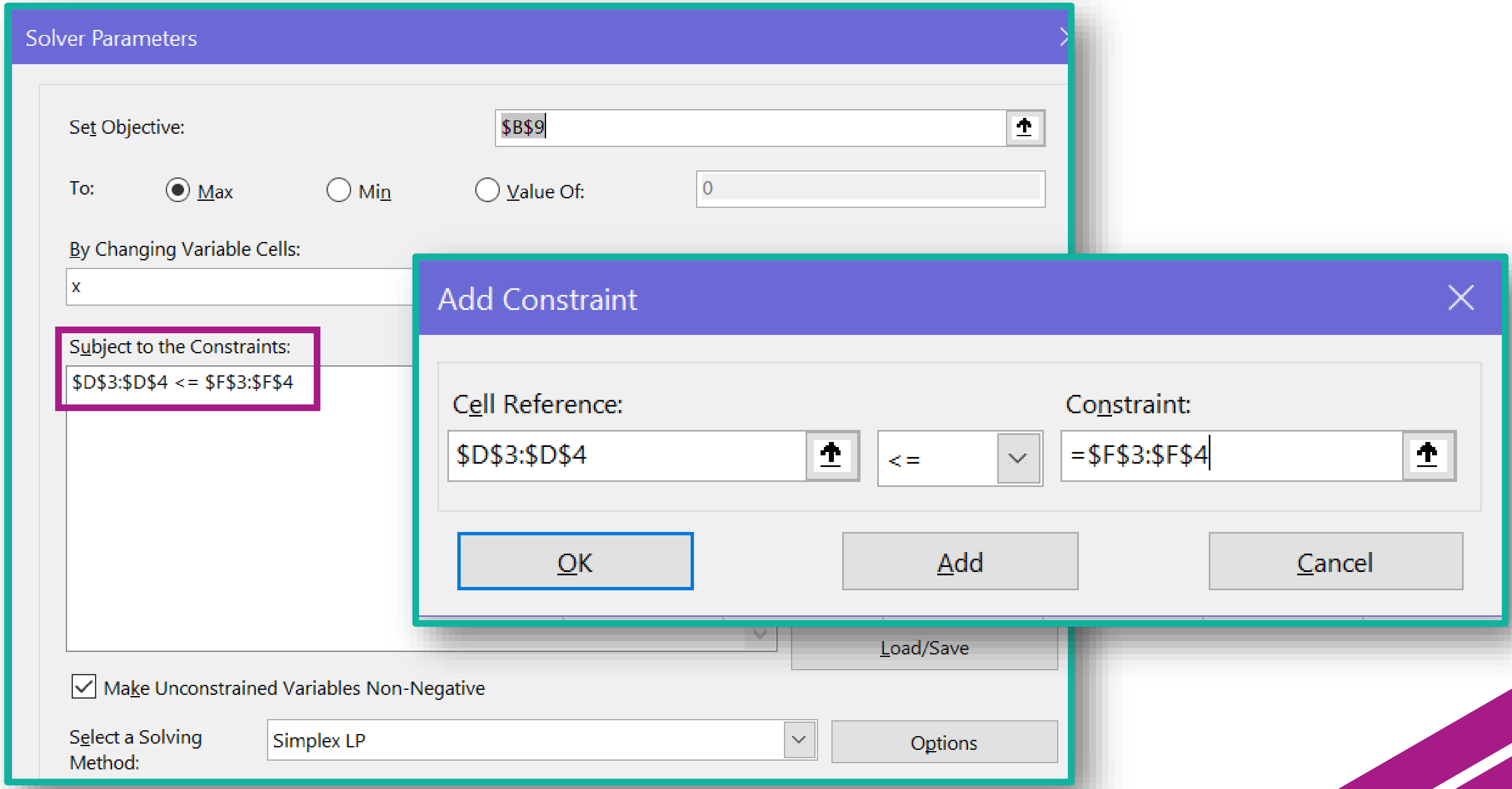
- Another option for specification

SUM | =MMULT(B3:C4,B7:B8)

	A	B	C	D	E	F
1	Profit	3	4			
2				Total	Constraint	Max
3	Metal	2	1	B8)	<=	30
4	Plastic	0	4	4	<=	50
5						
6						
7	Swag 1	1				
8	Swag 2	1				
9	Total	7				

Excel: Application of Matrices

- Another option for specification



Excel: Application of Matrices

- Try using MMULT and SUMPRODUCT in cell formulas
- Try using vectors/matrices in specification of constraints
- Download Lecture6WS.xlsx from course website from link Sheet 2 for all examples seen in this lecture

The End

Dale