Lecture 101

Produced by Dr. Worldwide

Welcome to the 305

Special Types of Models

- Special linear programming problems
  - Transportation
  - Transshipment
  - Assignment
- Subset of network flow problems

Transportation

- Characteristics of transportation problems
  - Product is being transported from a finite set of sources to a finite set of destinations
  - Sources supply a fixed amount of the product and destinations have a fixed demand for the product

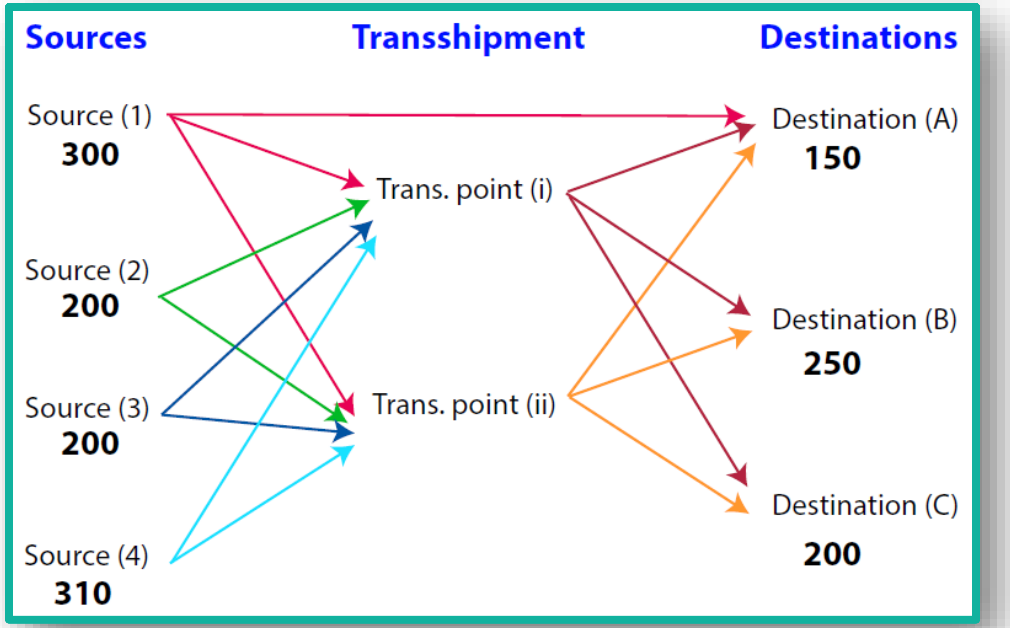
- Balanced when total supply equals total demand

- Unbalanced rule
  - If supply smaller than demand, replace equality demand constraints with $\leq$
  - If supply larger than demand, replace equality supply constraints with $\leq$

- Q: How would we modify the linear program to exclude certain routes that are either prohibited?

Transshipment

- Extension of the transportation model
- Diagram of transshipment problem



- Q: What is the difference between transportation and transshipment?

Transshipment

- Transshipment adds intermediate transshipment points between the sources and the destinations
- Possible routes in transshipment models
  - Sources to transshipment points
  - Transshipment points to destinations
  - Sources to destinations
- Book also states routes can exist between sources and between destinations
- Classic example of transshipment points are warehouses

Ex: Transporting Grain Again

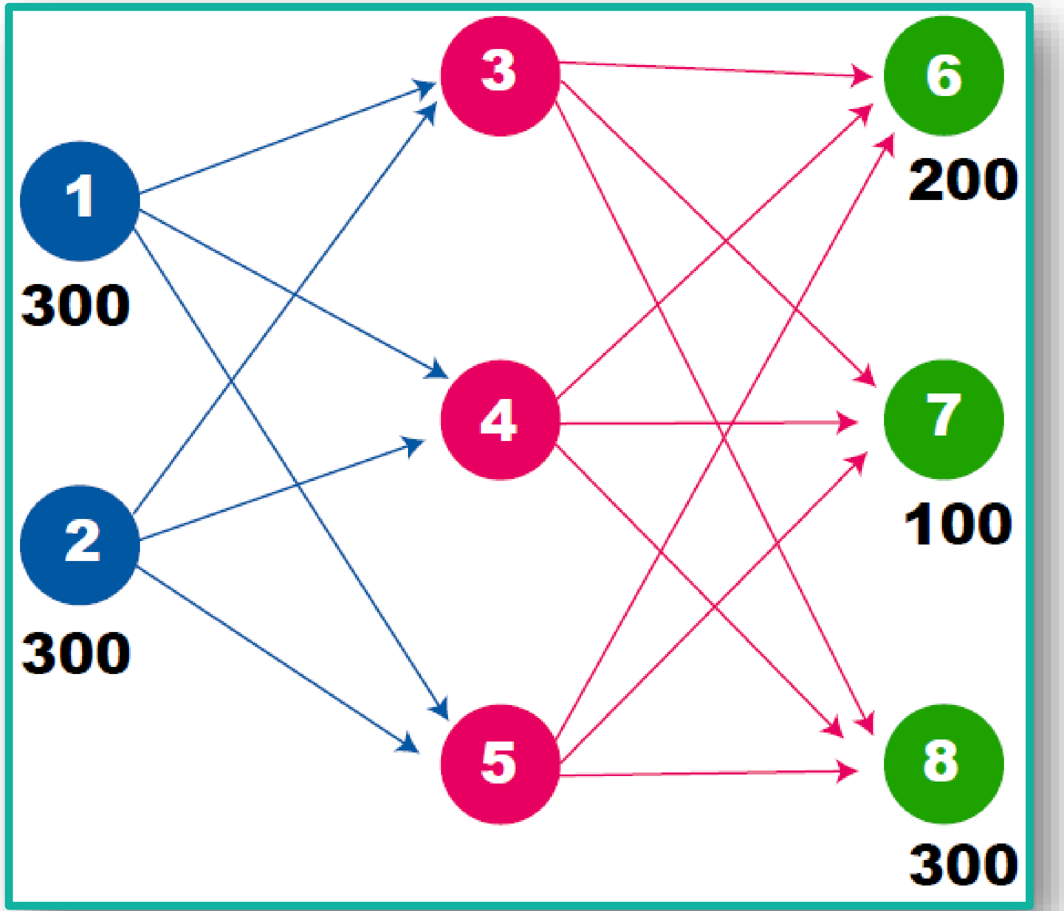
- Farms to grain elevators to flour mills
- Table of locations

Farms	Grain Elevator	Flour Mills
1. Nebraska	3. Kansas City	6. Chicago
2. Colorado	4. Omaha	7. St. Louis
	5. Des Moines	8. Cincinnati

- Nebraska and Colorado have become the sources of the wheat
- Each of the two farms produces 300 tons of wheat
- Kansas City, Omaha, and Des Moines have become our transshipment points

Ex: Transporting Grain Again

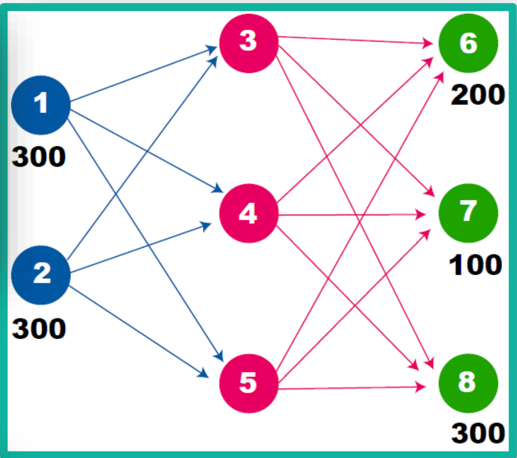
- General diagram of transshipment problem



Ex: Transporting Grain Again

- Shipping costs from farms to the grain elevators

	Grain elevator		
Farm	3. Kansas City	4. Omaha	5. Des Moines
1. Nebraska	\$16	\$10	\$12
2. Colorado	15	14	17



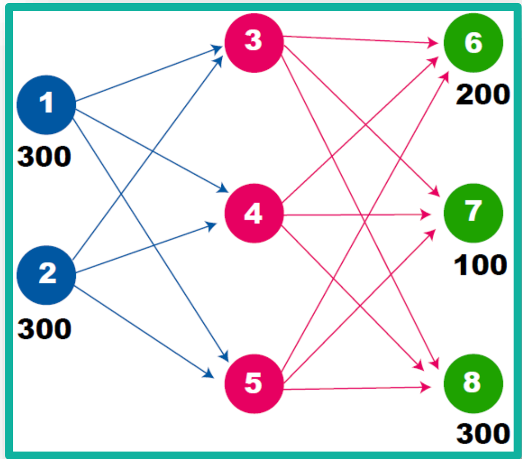
- Shipping costs from grain elevators to flour mills

	Mill		
Grain elevator	6. Chicago	7. St. Louis	8. Cincinnati
3. Kansas City	\$6	\$8	\$10
4. Omaha	7	11	11
5. Des Moines	4	5	12

Ex: Transporting Grain Again

- Demand from flour mills

Mill	Demand
6. Chicago	200
7. St. Louis	100
8. Cincinnati	300
Total	600 tons



- Q: How to transport grain (in tons) from farms to flour mills with minimal costs?

- Decision variables
  - x_{ij} = number of tons of grain to ship from i to j
  - $i, j \in \{1,2,3,4,5,6,7,8\}$ and $i \neq j$

Ex: Transporting Grain Again

- Objective function

$$Z = 16x_{13} + 10x_{14} + 12x_{15} + 15x_{23} + 14x_{24} + 17x_{25}$$
$$+ 6x_{36} + 8x_{37} + 10x_{38} + 7x_{46} + 11x_{47} + 11x_{48} + 4x_{56} + 5x_{57} + 12x_{58}$$

- In this problem, total supply (600) equals total demand (600)

- Supply constraints

$x_{13} + x_{14} + x_{15} = 300$ (Nebraska Supply)
$x_{23} + x_{24} + x_{25} = 300$ (Colorado Supply)

- Demand constraints

$x_{36} + x_{37} + x_{38} = 200$ (Chicago Demand)
$x_{46} + x_{47} + x_{48} = 100$ (St. Louis Demand)
$x_{56} + x_{57} + x_{58} = 300$ (Cincinnati Demand)

Ex: Transporting Grain Again

- Transshipment points have constraints that express equality between what goes in and what goes out

- Transshipment constraints

$x_{13} + x_{23} = x_{36} + x_{37} + x_{38}$ (Kansas City Transshipment)
$x_{14} + x_{24} = x_{46} + x_{47} + x_{48}$ (Omaha Transshipment
$x_{15} + x_{25} = x_{56} + x_{57} + x_{58}$ (Des Moines Transshipment

- Integer constraints

$x_{ij} \in \{0,1,2,\cdots\}$

- Download MillsTransship.xlsx from course website from link Sheet 1

- Try to find the solution using Excel Solver

$x_{15} = 300 \ \& \ x_{24} = 300 \ \& \ x_{48} = 300 \ \& \ x_{56} = 200 \ \& \ x_{57} = 100$

Assignment

- Similar to the transportation model with slight difference
- In the assignment model, the supply at each source and demand at each destination is exactly one
- Think of the sources as unique units that need to be assigned to specific recipients
- There is cost associated to each pair of source and destination

Ex: ACC Officials

- Four basketball games in the Atlantic Coast Conference (ACC) on a night
- Conference wants to assign four teams of officials to the four games
- Supply is always one team of officials
- Demand is always requiring only one team of officials
- Q: How should we assign the four teams of officials so that distance is minimized?

	Game Sites			
Officials	1. Raleigh	2. Atlanta	3. Durham	4. Clemson
A	201	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Ex: ACC Officials

- Decision variables
  - x_{ij} = indicator of whether official team i is assigned to game in city j
  - $i \in \{A, B, C, D\}$
  - $j \in \{1,2,3,4\}$

- Objective function

$$
\begin{aligned}
Z = {} & 200x_{A1} + 90x_{A2} + 180x_{A3} + 160x_{A4} \\
& +100x_{B1} + 70x_{B2} + 130x_{B3} + 200x_{B4} \\
& +175x_{C1} + 105x_{C2} + 140x_{C3} + 170x_{C4} \\
& +80x_{D1} + 65x_{D2} + 105x_{D3} + 120x_{D4}
\end{aligned}
$$

- Use multiple choice constraints to ensure supply fulfills demand

Ex: ACC Officials

- Constraints

$x_{A1} + x_{A2} + x_{A3} + x_{A4} = 1$ (Official Team A)
$x_{B1} + x_{B2} + x_{B3} + x_{B4} = 1$ (Official Team B)
$x_{C1} + x_{C2} + x_{C3} + x_{C4} = 1$ (Official Team C)
$x_{D1} + x_{D2} + x_{D3} + x_{D4} = 1$ (Official Team D)

$x_{A1} + x_{B1} + x_{C1} + x_{D1} = 1$ (City 1)
$x_{A2} + x_{B2} + x_{C2} + x_{D2} = 1$ (City 2)
$x_{A3} + x_{B3} + x_{C3} + x_{D3} = 1$ (City 3)
$x_{A4} + x_{B4} + x_{C4} + x_{D4} = 1$ (City 4)

$x_{ij} \in \{0,1\}$

Ex: Give-back Weekend

- The Student Government Association (SGA) organizes a recurring event called "Give-Back Weekends" where teams are formed to work on projects for residents in the university community
- This event occurs over four consecutive Saturdays in April
- Coed teams are formed with 3 to 5 students from various dormitory groups, fraternities, sororities, clubs, and organizations
- Residents of the community fill out a form to describe work at their home that needs to be done
- Time to complete a project will vary between teams because of the different number of team members, skills of the team, and physical make-up of the team

Ex: Give-back Weekend

MIAMI

- Time estimates (in hours) submitted by the six teams available to work on 12 different projects for the first Saturday of the event

	Jobs											
Team	1	2	3	4	5	6	7	8	9	10	11	12
1	5	1.5	6	4	3.5	3	6	1.5	5	1	3	3.5
2	4	2	5	5	3	3	5.5	2	4	1.5	4	2.5
3	5	1.5	6.5	3.5	2.5	4	4.5	3	3.5	1	3.5	4
4	3.5	2	5.5	4	3.5	2.5	5	2.5	4	1.5	2.5	4
5	3.5	3	5	3	2	4	5	2	5	2	4	3
6	4	2.5	6	5	3	3	6	3	3	2	3	3.5

- The primary objective of SGA is to complete all 12 projects

Ex: Give-back Weekend

- Teams can work on multiple projects
- Teams cannot work more than 8 hours on Saturday
- Each team should work on at least one project
- Alternative Questions
  - Q: How can we assign the 6 teams to the 12 projects to maximize the number of jobs completed on Saturday?
  - Q: How can we assign the 6 teams to the 12 projects to minimize the total time required for all 6 teams?

Ex: Give-back Weekend

- Consider the jobs as the "sources" or "supply
- Consider the teams as the "destinations" or demand
- Decision variables
  - $x_{ij} = \begin{cases} 1 & \text{if job } j \text{ is assigned to team } i \\ 0 & \text{otherwise} \end{cases}$
  - $i \in \{1,2,3,4,5,6\}$
  - $j \in \{1,2,3,\cdots,12\}$
- Objective function for the number of completed jobs

  $Z = \sum_{i=1}^{6} \sum_{j=1}^{12} x_{ij}$
- Objective function for the amount of time for the teams to do the jobs

  $Z = \sum_{i=1}^{6} \sum_{j=1}^{12} t_{ij} x_{ij}$ where $t_{ij} =$ time required for team i to do job j

Ex: Give-back Weekend

- Constraints
  - Each team cannot work more than 8 hours

$$5x_{11} + 1.5x_{12} + 6x_{13} + 4x_{14} + 3.5x_{15} + 3x_{16} + \\ +6x_{17} + 1.5x_{18} + 5x_{19} + x_{110} + 3x_{111} + 3.5x_{112} \leq 8 \qquad \text{(Team 1)}$$

  - Each project can only be assigned to at most one team, which adds a total of 12 constraints, one for each project $j \in \{1,2,\cdots,12\}$

$$\sum_{i=1}^{6} x_{ij} = x_{1j} + x_{2j} + x_{3j} + x_{4j} + x_{5j} + x_{6j} \leq 1$$

  - Binary constraints for each decision variable

$$x_{ij} \in \{0,1\}$$

- See GiveBack.xlsx from link Sheet 2 on course website

The End

Dale