Lecture 16

Produced by Dr. Worldwide
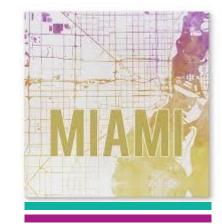
Welcome to the 305

Excel for Goal Programming

- Builds off linear programming using Excel Solver

- Solve the linear program multiple times with different objective functions

- Go in order of priority

- After finding the optimal solution, we add the optimal value attained in the first objective function as a new constraint and move on to the next objective function

- Possible that while solving for a given priority, we simultaneously optimize other lower ranked priorities
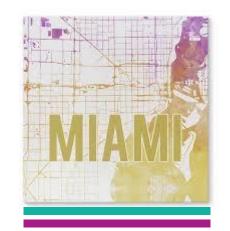
Ex: Beaver Creek Pottery

- Download GoalProgramming.xlsx from link Sheet 1 on course website

- See tab Priority 1 for minimization of d_1^-
  - Optimal solution

  $$x = 15 \quad y = 20 \quad d_1^+ = 15 \quad d_4^+ = 5 \quad d_5^- = 15$$
  $$d_1^-, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_6^- = 0$$

  - It is optimal to set $d_1^- = 0$
  - In our system of linear constraints, we have employees working at least 40 hr

- Move on to P2 for minimization of d_2^-
  - Notice from last solution $d_2^- = 0$
  - Optimal solution from P1 minimizes objective function from P2

- Unnecessary to consider P3 since $d_3^+ = 0$ under optimal solution of P1
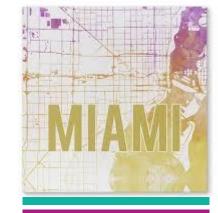
Ex: Beaver Creek Pottery

- See tab Priority 4 for minimization of d_4^+
  - To ensure none of the optimal values achieved thus far change when we attempt to minimize d_4^+, we add the values attained as constraints
  - We add one constraint for each goal we have already attained

Minimize $\qquad d_4^+$

Subject to
$$x + 2y + d_1^- - d_1^+ = 40 \qquad \text{(Labor)}$$
$$40x + 50y + d_2^- - d_2^+ = 1600 \qquad \text{(Profit)}$$
$$4x + 3y + d_3^- - d_3^+ = 120 \qquad \text{(Clay)}$$
$$d_1^+ + d_4^- - d_4^+ = 10 \qquad \text{(Overtime)}$$
$$x + d_5^- = 30 \qquad \text{(Bowls)}$$
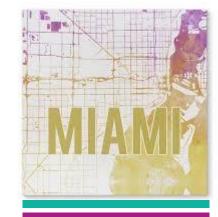$$y + d_6^- = 20 \qquad \text{(Mugs)}$$
$$d_1^-, d_2^-, d_3^+ = 0$$
$$x, y, d_1^+, d_2^+, d_3^-, d_4^-, d_4^+, d_5^-, d_6^- \geq 0$$

Ex: Beaver Creek Pottery

- See tab Priority 4 for minimization of d_4^+
  - Optimal solution

    $$x = 15 \quad y = 20 \quad d_1^+ = 15 \quad d_4^+ = 5 \quad d_5^- = 15$$
    $$d_1^-, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_6^- = 0$$

  - Solution did not change and $d_4^+ = 5$ in both cases
  - Not possible to reduce the value of d_4^+ without violating the optimal solutions for the three goals that have higher priority
  - This indicates that the overtime must be exceed by 5 hours to fulfill other constraints from higher priority goals

Ex: Beaver Creek Pottery

- See tab Priority 5 for minimization of $4d_5^- + 5d_6^-$
  - Add result from previous priority rank as a constraint

Minimize	$4d_5^- + 5d_6^-$

Subject to		
	$x + 2y + d_1^- - d_1^+ = 40$	(Labor)
	$40x + 50y + d_2^- - d_2^+ = 1600$	(Profit)
	$4x + 3y + d_3^- - d_3^+ = 120$	(Clay)
	$d_1^+ + d_4^- - d_4^+ = 10$	(Overtime)
	$x + d_5^- = 30$	(Bowls)
	$y + d_6^- = 20$	(Mugs)
	$d_1^-, d_2^-, d_3^+ = 0$	(New Constraints)
	$d_4^+ = 5$	(New Constraints)
	$x, y, d_1^+, d_2^+, d_3^-, d_4^-, d_5^-, d_6^- \geq 0$	

Ex: Beaver Creek Pottery

- See tab Priority 5 for minimization of $4d_5^- + 5d_6^-$
  - Optimal solution

    $$x = 15 \quad y = 20 \quad d_1^+ = 15 \quad d_4^+ = 5 \quad d_5^- = 15$$
    $$d_1^-, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_6^- = 0$$

  - Solution still did not change
  - Optimal solution stays optimal

- Final Solution
  - Produce 15 bowls and 20 mugs
  - Hours of work: $15 + 2(20) = 55$ (Over by 15 hours)
  - Profit: $40(15) + 50(20) = 1600$
  - Pounds of clay: $4(15) + 3(20) = 120$
  - Overtime beyond 10 hours: $d_4^+ = 5$
  - Slack for bowls below 30: $d_5^- = 15$
  - Slack for mugs below 20: $d_4^+ = 0$

Ex: Beaver Creek Pottery

- Full modified goal programming model

Minimize $\qquad P_1 d_1^-, P_2 d_2^-, P_3 d_3^+, P_4 d_4^+, 4P_5 d_5^- + 5P_5 d_6^-$

Subject to

$$x + 2y + d_1^- - d_1^+ = 40 \qquad \text{(Labor)}$$
$$40x + 50y + d_2^- - d_2^+ = 1600 \qquad \text{(Profit)}$$
$$4x + 3y + d_3^- - d_3^+ = 120 \qquad \text{(Clay)}$$
$$d_1^+ + d_4^- - d_4^+ = 10 \qquad \text{(Overtime)}$$
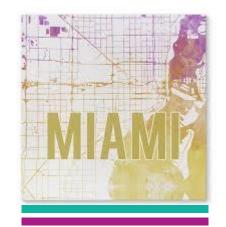$$x + d_5^- = 30 \qquad \text{(Bowls)}$$
$$y + d_6^- = 20 \qquad \text{(Mugs)}$$

$$x, y, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+, d_5^-, d_6^- \geq 0$$

Ex: Oakdale School Busing

- Oakdale County school board met in a special session

- Federal Judge Barry ordered them to present an acceptable busing plan for racially balancing the 4 high schools in Oakdale County

- Racial balance of the 4 high schools
  - West High School: 500 Black & 500 White
  - North High School: 300 Black & 1,000 White
  - East High School: 400 Black & 1,050 White
  - South High School: 800 Black & 450 White

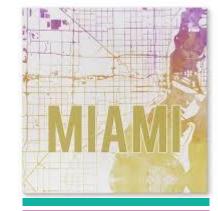- Overall 5,000 Students: 40% Black & 60% White

Ex: Oakdale School Busing

- Act I
  - "Rather than starting off by trying to shift students from one district to another, why don't we try to establish what we want to accomplish?" – JC
  - "Good idea, Lil John" – FH
  - "Sixty percent of our students are white and 40% are black, so that's what we need our schools to be, 60% and 40%." – JC
  - "That's okay for you to say, Lil John, because your district (East) is already close to those proportions. My district in the North is a long way from that ratio, and we would have to bus a lot of our students." – BP
  - "I'm not saying it, Betty; Judge Barry has been saying it for 6 months." – JC
  - "John's right, Betty, and we're not busing students yet; we're just putting down our objectives. I think that must be our highest-priority." – FH
  - *They all node in agreement like a bunch of bobbleheads*

- Intermission I
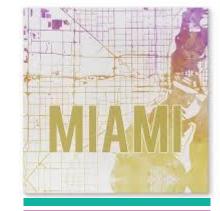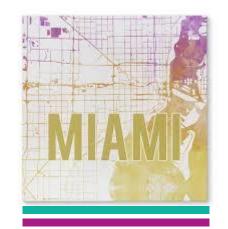
Ex: Oakdale School Busing

- Act II
  - "Since we're going to have to bus students to achieve this ratio at each school, I think we ought to try to minimize the amount of traveling."– MG
  - *Fred Harvey noted page 10 of their handout with a chart showing the average mileage a student in one district would have to travel by bus to the high school in each of the other districts*

District/School	Distance (mi.)			
	North	South	East	West
North	–	30	12	20
South	30	–	18	26
East	12	18	–	24
West	20	26	24	–

Ex: Oakdale School Busing

- Act II
  - "Why not set reasonable objectives for total busing miles, for the students' sake and for budgeting reasons? I would suggest about 30,000 miles per day, based on the miles we bus students now. If we get much higher than that we're not going to have the money to pay for it, and it means we'll be busing students all over the place." – CW
  - *They all node in agreement like a bunch of bobbleheads*
  - "Okay, that'll be our number two goal." – FH

- Intermission II

Ex: Oakdale School Busing

- Act III
  - "I'll tell you another thing I don't want to see happen, and that's more overcrowding at North High School. We have 100 students more than capacity now." – BP
  - "You think you have problems! In East, we have 1,450 students and a capacity of 1,000. I think no overcrowding is a great idea!" – BW
  - "I agree. We're 250 over our capacity at South High School" – MG
  - "That's a nice idea, and I realize that we have 200 students less than our capacity at West High School. However, let's face it, in the county we have capacity for 4,400, not 5,000, students, so there's going to be some overcrowding. I think our objective should be that all 4 schools should share in the overcrowding proportionally." – JC
  - "That sounds reasonable to me. How about the rest of you? Okay to say our number three goal is to be as close to capacity at each school as possible but share proportionally in the overcrowding"
  - *They voice their approval by joining hands and singing*

Ex: Oakdale School Busing

- Act III
  - "Well, I think we have identified the things we want to accomplish in our plan. Now, if we could just use some magic trick to find for busing students between the districts that would achieve all these goals." – JC
  - *The others nodded and frowned because they don't know math.*

- The End and Credits
  - John Connor   *Mario Giacomazzo*
  - Fred Harvey   *Mario Giacomazzo*
  - Betty Philips   *Mario Giacomazzo*
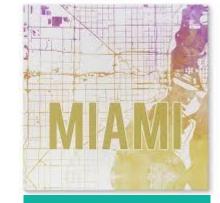 - Mickey Gibbony   *Mario Giacomazzo*
  - Cassandra Watkins   *Mario Giacomazzo*
  - Bob Wilson   *Mario Giacomazzo*
  - Narration   *Mario Giacomazzo*

Ex: Oakdale School Busing

- Goals listed in order based on priority
  - Achieve a 60%/40% ratio of white to black students at each of the schools
  - Minimize the amount of traveling that students will have to do, ideally no more than 30,000 miles per day
  - Keep all schools close to capacity and minimize overcrowding proportionally allocating the excess among the schools

- Q: How can we formulate and solve a goal programming model to help the board with its dilemma?

- Decision variables
  - x_{ij} = Number of white students from district i assigned to district j
  - y_{ij} = Number of black students from district i assigned to district j
  - $i, j \in \{N, S, E, W\}$ and $i \neq j$
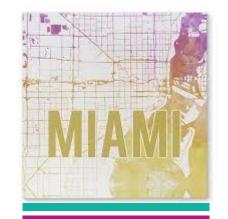
Ex: Oakdale School Busing

- Helpful tables of key information
  - Current amounts of white and black students along with capacity

District/School	# White students	# Black students	Capacity
North	1000	300	1200
South	450	800	1000
East	1050	400	1000
West	500	500	1200

  - Distances each student travels between districts

District/School	Distance (mi.)			
	North	South	East	West
North	–	30	12	20
South	30	–	18	26
East	12	18	–	24
West	20	26	24	–

The End

Dale