Assignment # 3 Solutions

due Friday, September 13th, 2019

1 Since it takes 55 minutes to bake 4 pies, it still requires 55 minutes of oven time to bake pies that are less than 4. Similarly, it takes 15 minutes to bake a dozen cookies, it still requires 15 minutes of oven time to bake cookies that are less than 2 dozens. Taking this into consideration, we define variables as follows:

Let x_1 = number of 4 pecan pies sold.
x_2 = number of packages of 2 dozens cookies sold.
x_3 = number of 1-pound bags of shelled pecans sold.
x_4 = number of 5-pound bags of unshelled pecans sold.
Note that 1 pound = 16o.z.. The model is as follows:

$$
\begin{aligned}
&\max z = 20x_1 + 6x_2 + 7x_3 + 16x_4 \\
&s.t. \quad \frac{(2)(4)(4x_1)}{16} + \frac{(2)(2)(6x_2)}{16} + 2x_3 + 5x_4 \leq 5000 \\
&55x_1 + 15x_2 \leq 120 \times 60 = 7200 \\
&24x_1 + 8x_2 + 10x_3 + x_4 \leq 300 \times 60 = 18000 \\
&x_i \geq 0, \quad \forall i = 1, 2, 3, 4.
\end{aligned}
$$

If ignoring that baking less than 4 pies still requires 55 minutes of oven time and baking less than 2 dozen cookies still requires 15 minutes of oven time, then the model is formulated as follows:

Let x_1 = number of pecan pies sold.
x_2 = number of packages of a dozen cookies sold.
x_3 = number of 1-pound bags of shelled pecans sold.
x_4 = number of 5-pound bags of unshelled pecans sold.
Note that 1 pound = 16o.z.. The model is as follows:

$$
\begin{aligned}
&\max z = 5x_1 + 3x_2 + 7x_3 + 16x_4 \\
&s.t. \quad \frac{(2)(4x_1)}{16} + \frac{(2)(6x_2)}{16} + 2x_3 + 5x_4 \leq 5000 \\
&\frac{55x_1}{4} + \frac{15x_2}{2} \leq 120 \times 60 = 7200 \\
&6x_1 + 4x_2 + 10x_3 + x_4 \leq 300 \times 60 = 18000 \\
&x_i \geq 0, \quad \forall i = 1, 2, 3, 4.
\end{aligned}
$$

2

(a) The results are shown below. We observe that there are no extra resource available.

3	Items:	4 pecan pies	2 dozens of cookie	bags of shelled pecans	bags of unshelled pecans			
4	Profit per unit:	20	6	7	16			
5	Conditions:					Usage	Constraint	Available
6	unshelled pecans constrain	2.00	1.50	2.00	5.00	5000.00	<=	5000.00
7	baking time constrain	55.00	15.00	0.00	0.00	7200.00	<=	7200.00
8	family members time constrain	24.00	8.00	10.00	1.00	18000.00	<=	18000.00
9								
10	Production:							
11	pecan pies=	130.91						
12	dozens of cookies=	0.00						
13	bags of shelled pecans=	1449.02						
14	bags of unshelled pecans=	368.03						
15	Return =	18649.77						

(b) Below is the Sensitivity Report. The answer will varies depends on how you define units of additional resources.

6	Variable Cells						
7			Final	Reduced	Objective	Allowable	Allowable
8	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9	B11	pecan pies= 4 pecan pies	130.9090909	0	20	1E+30	9.489583333
10	B12	dozens of cookies= 4 pecan pies	0	-2.588068182	6	2.588068182	1E+30
11	B13	bags of shelled pecans= 4 pecan pies	1449.015152	0	7	4.93220339	0.6
12	B14	bags of unshelled pecans= 4 pecan pies	368.030303	0	16	1.5	15.3
13							
14	Constraints						
15			Final	Shadow	Constraint	Allowable	Allowable
16	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
17	F6	unshelled pecans constrain Usage	5000	3.1875	5000	69552.72727	1766.545455
18	F7	baking time constrain Usage	7200	0.220454545	7200	32418.64407	7200
19	F8	family members time constrain Usage	18000	0.0625	18000	8832.727273	13910.54545

Assume that the unit of increase in unshelled pecans constraint is pounds and the units of increase in baking time and available time for shelling and packaging are hours.

From the report, if they can get 1 pound of unshelled pecans, the sales revenue will increase by \$3.1875. If the baking time is increase by 1 hour, the sales revenues will increase by $0.220454545 * 1 * 60 = \$13.22723$. If the available time for shelling and packaging is increase by 1 hour, the sales revenue will increase by $0.0625 * 1 * 60 = 3.75$.

Hence, increase of oven times would be of most value. The Wisham family would be willing to pay at most \$13.22723.

(c) If they get an additional 500 pounds of pecans, the profit will increase by $3.1875 \times 500 = \$1593.75$. By contrast, if they can get an additional 30 hours of oven time, the profit will increase by $30 \times 60 \times 0.22045 = \$396.81$. So, they should choose an additional 500 pounds of pecans.

(d) In this case, let x_1 = number of 5 pecan pies sold.
x_2 = number of packages of 3 dozens cookies sold.
x_3 = number of 1-pound bags of shelled pecans sold.
x_4 = number of 5-pound bags of unshelled pecans sold.
Note that 1 pound = 16o.z.. The model is as follows:

$$
\begin{aligned}
&\max z = 25x_1 + 9x_2 + 7x_3 + 16x_4 \\
&s.t. \quad \frac{(2)(5)(4x_1)}{16} + \frac{(2)(3)(6x_2)}{16} + 2x_3 + 5x_4 \leq 5000 \\
&55x_1 + 15x_2 \leq 120 \times 60 = 7200 \\
&30x_1 + 12x_2 + 10x_3 + x_4 \leq 300 \times 60 = 18000 \\
&x_i \geq 0, \quad \forall i = 1, 2, 3, 4
\end{aligned}
$$

Then using Excel Solver, we find that the optimal profit will become \$19046.59. The profit is increase by $19046.59 - 18649.77 = 396.82$, which is less than the cost \$3000. So, they should not buy the oven.

3	Items:	5 pecan pies	3 dozens of cookie	bags of shelled pecans	bags of unshelled pecans			
4	Profit per unit:	25	9	7	16			
5	Conditions:					Usage	Constraint	Available
6	unshelled pecans constrain	2.50	2.25	2.00	5.00	5000.00	<=	5000.00
7	baking time constrain	55.00	15.00	0.00	0.00	7200.00	<=	7200.00
8	family members time constrain	30.00	12.00	10.00	1.00	18000.00	<=	18000.00
9								
10	Production:							
11	pecan pies=	130.91						
12	dozens of cookies=	0.00						
13	bags of shelled pecans=	1368.56						
14	bags of unshelled pecans=	387.12						
15	Return =	19046.59						

3

(a) Changing the coefficient of objective function might affect the shadow price. Hence, if the objective function changes, we should not use the same sensitivity analysis from the original model. Since we do not know how profit is formulated, there may be different answers.

(Interpretation 1) The processing time reduces by 10% and cost per item increases by 10%. If the profit is related to cost incurred from processing time and cost of each item, the effect cancel out so that the profit remains the same. Hence we can use the original model to perform sensitivity analysis.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B15	Sweatshirts-F = (dozen)	175.5555556	0	90	11.92307692	40
B16	Sweatshirts-B/F = (dozen)	57.77777778	0	125	13.21428571	11.92307692
B17	T-shirt-F = (dozen)	500	0	45	1E+30	4.111111111
B18	T-shirt-B/F = (dozen)	0	-10.33333333	65	10.33333333	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F10	Blank sweatshirts Usage	233.3333333	0	500	1E+30	266.6666667
F11	Blank T-shirts Usage	500	4.111111111	500	185.7142857	500
F7	Processing time Usage	72	233.3333333	72	26.33333333	8.666666667
F8	Cost Usage	21593.33333	0	25000	1E+30	3406.666667
F9	Truck capacity Usage	1200	22.22222222	1200	260	316

The profit will increase by $8 * 233.333 = \$18,666.64$. We should take this alternative.

(Interpretation 2) If the profit is affected only by the cost of each item, then the objective function changes. We can not use sensitivity analysis from the original model. Reformulate the problem, we can see that the profit decreases, so we should not undertake this alternative.

1	**A product mix**								
2									
3	Products:	Sweatshirt-F	Sweatshirt-B/F	T-shirt-F	T-shirt-B/F				
4		(dozen)	(dozen)	(dozen)	(dozen)				
5	Profit per dozen:	86.4	120.2	42.5	61.5				
6	Resources:					Usage	Constraint	Available	Left over
7	Processing time	0.09	0.225	0.072	0.189	72	<=	72	0
8	Cost	39.6	52.8	27.5	38.5	24456.6667	<=	25000	543.333333
9	Truck capacity	3	3	1	1	1200	<=	1200	0
10	Blank sweatshirts	1	1	0	0	233.333333	<=	500	266.666667
11	Blank T-shirts	0	0	1	1	500	<=	500	0
12									
13									
14	Production:								
15	Sweatshirts-F =	122.222222							
16	Sweatshirts-B/F =	111.111111							
17	T-shirt-F =	500							
18	T-shirt-B/F =	0							
19	Profit =	45165.5556							

(b) No. If Quick- Screen decided to acquire extra T-shirts, could the company expect to earn an additional \$4.11 for each extra T-shirt it acquires above 500, up to the sensitivity range limit of

T-shirts. But since the profit consists of fixed costs and variable costs, the profit increases non-linearly when quantity increases. Hence, the company will not earn \$4.11 for each extra T-shirt it acquires above 500, up to the sensitivity range limit of T-shirts.

(c) If Quick-Screen produced equal numbers of each of the four shirts, we need to add the constraints in the original model such that $x_1 = x_2 = x_3 = x_4$. The optimal profit is \$36562.5 with $(x_1, x_2, x_3, x_4) = (112.5, 112.5, 112.5, 112.5)$.

1	**A product mix**								
2									
3	Products:	Sweatshirt-F	Sweatshirt-B/F	T-shirt-F	T-shirt-B/F				
4		(dozen)	(dozen)	(dozen)	(dozen)				
5	Profit per dozen:	90	125	45	65				
6	Resources:					Usage	Constraint	Available	Left over
7	Processing time	0.1	0.25	0.08	0.21	72	<=	72	0
8	Cost	36	48	25	35	16200	<=	25000	8800
9	Truck capacity	3	3	1	1	900	<=	1200	300
10	Blank sweatshirts	1	1	0	0	225	<=	500	275
11	Blank T-shirts	0	0	1	1	225	<=	500	275
12	x1 = x2	1	-1	0	0	0	=	0	0
13	x1 = x3	1	0	-1	0	0	=	0	0
14	x1 = x4	1	0	0	-1	0	=	0	0
15									
16									
17	Production:								
18	Sweatshirts-F =	112.5							
19	Sweatshirts-B/F =	112.5							
20	T-shirt-F =	112.5							
21	T-shirt-B/F =	112.5							
22	Profit =	36562.5							

4

(a) Let x_1 = amount of money invested in Job training program.
x_2 = amount of money invested in Parks program.
x_3 = amount of money invested in Sanitation program.
x_4 = amount of money invested in Mobile library program.
The model is as follows:

$$\begin{aligned}
&\max z = 0.02x_1 + 0.09x_2 + 0.06x_3 + 0.04x_4 \\
&s.t. \quad x_i \leq 0.4 * 4,000,000 = 1,600,000, \quad \forall i = 1,2,3,4 \\
&x_2 - x_3 - x_4 \leq 0 \\
&-x_1 + x_3 \leq 0 \\
&x_1 + x_2 + x_3 + x_4 = 4,000,000 \\
&x_i \geq 0, \quad \forall i = 1,2,3,4.
\end{aligned}$$

(b) The optimal value is 240000 with $(x_1, x_2, x_3, x_4) = (800000, 1600000, 800000, 800000, 800000)$.

3	Projects:	Job Training	Parks	Sanitation	Mobile Library				
4	Voles/Dollars	0.02	0.09	0.06	0.04				
5	Constraints:					Usage	Constraint	R.H.S.	Left over
6	Constraint 1	1	0	0	0	800000	<=	1600000	800000
7	Constraint 2	0	1	0	0	1600000	<=	1600000	0
8	Constraint 3	0	0	1	0	800000	<=	1600000	800000
9	Constraint 4	0	0	0	1	800000	<=	1600000	800000
10	Constraint 5	0	1	-1	-1	0	<=	0	0
11	Constraint 6	-1	0	1	0	0	<=	0	0
12	Constraint 7	1	1	1	1	4000000	=	4000000	0
13									
14									
15	Investments:								
16	Job Training =	800000							
17	Parks =	1600000							
18	Sanitation =	800000							
19	Mobile Library =	800000							
20	Votes =	240000							