Assignment #4 Solutions

due Friday, September 20th, 2019

1

(a) Let x_1 = number of salespeople assigned to south.
x_2 = number of salespeople assigned to east.
x_3 = number of salespeople assigned to midwest.
Then the model is as follows:

$$\max z = 600x_1 + 540x_2 + 375x_3$$
$$s.t. \quad x_1 \leq 5$$
$$80x_1 + 70x_2 + 50x_3 \leq 750$$
$$x_1 + x_2 + x_3 = 12$$
$$x_i \geq 0, i = 1, 2, 3 \textit{ and integer.}$$

(b) The results are shown below. The maximum profit is \$5715 with optimal solution = $(1, 6, 5)$.

3	Items:	south	east	midwest			
4	Profit per unit:	600	540	375			
5	Conditions:				Usage	Constraint	Available
6	constrain1	1.00	0.00	0.00	1.00	<=	5.00
7	constrain2	80.00	70.00	50.00	750.00	<=	750.00
8	constrain3	1.00	1.00	1.00	12.00	=	12.00
9							
10	Production:						
11	south=	1.00					
12	east=	6.00					
13	midwest=	5.00					
14	Return =	5715.00					

2

1. Assume the time period is 1, 2, 3, 4, 5 and 6. Let x_1 = number of people beginning to work in period 1.
x_2 = number of people beginning to work in period 2.
x_3 = number of people beginning to work in period 3.
x_4 = number of people beginning to work in period 4.
x_5 = number of people beginning to work in period 5.
x_6 = number of people beginning to work in period 6.
Then the model is as follows:

$$\max z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$
$$s.t. \quad x_1 + x_6 \geq 90$$
$$x_1 + x_2 \geq 215$$
$$x_2 + x_3 \geq 250$$
$$x_3 + x_4 \geq 65$$
$$x_4 + x_5 \geq 300$$
$$x_5 + x_6 \geq 125$$
$$x_i \geq 0, i = 1, 2, 3, 4, 5, 6 \textit{ and integer.}$$

From the table, we can see that the minimum total number employees is 640 with optimal solution = $(90, 250, 0, 175, 125, 0)$.

3	Items:	period1	period2	period3	period4	period5	period6			
4		1.00	1.00	1.00	1.00	1.00	1.00			
5	Conditions:							Usage	Constraint	Available
6	constrain1	1.00	0.00	0.00	0.00	0.00	1.00	90.00	>=	90.00
7	constrain2	1.00	1.00	0.00	0.00	0.00	0.00	340.00	>=	215.00
8	constrain3	0.00	1.00	1.00	0.00	0.00	0.00	250.00	>=	250.00
9	constrain4	0.00	0.00	1.00	1.00	0.00	0.00	175.00	>=	65.00
10	constrain5	0.00	0.00	0.00	1.00	1.00	0.00	300.00	>=	300.00
11	constrain6	0.00	0.00	0.00	0.00	1.00	1.00	125.00	>=	125.00
12										
13	Production:									
14	period1	90.00								
15	period2	250.00								
16	period3	0.00								
17	period4	175.00								
18	period5	125.00								
19	period6	0.00								
20	number of people =	640.00								

3

(a) Let x_1 = number of bracelets produce.
x_2 = number of necklaces produce.
x_3 = number of pins produce.
Then the model is as follows:

$$\max z = 1650x_1 + 850x_2 + 790x_3$$
$$s.t. \quad 6.3x_1 + 3.9x_2 + 3.1x_3 \leq 125$$
$$17x_1 + 10x_2 + 7x_3 \leq 320$$
$$x_i \geq 0, i = 1, 2, 3 \; and \; integer$$

(b) From Fig. 1 and Fig. 2 , we can see that these two optimal solutions are different, so the rounded-down solution of the model with the integer restrictions relaxed is not optimal.

3	Items:	bracelets	necklaces	pins			
4	Profit per unit:	1650	850	790			
5	Conditions:				Usage	Constraint	Available
6	constrain1	6.30	3.90	3.10	125.00	<=	125.00
7	constrain2	17.00	10.00	7.00	320.00	<=	320.00
8							
9	Production:						
10	bracelets=	13.60					
11	necklaces=	0.00					
12	pins=	12.67					
13	Return =	32460.47					

Figure 1: Excel with the integer restrictions relaxed

3	Items:	bracelets	necklaces	pins			
4	Profit per unit:	1650	850	790			
5	Conditions:				Usage	Constraint	Available
6	constrain1	6.30	3.90	3.10	125.00	<=	125.00
7	constrain2	17.00	10.00	7.00	310.00	<=	320.00
8							
9	Production:						
10	bracelets=	10.00					
11	necklaces=	0.00					
12	pins=	20.00					
13	Return =	32300.00					

Figure 2: Excel with integer restrictions

4

(a) We can split 24 hour into six 4-hour periods. Assume the time period is 1, 2, 3, 4, 5, and 6. Then we assume the average fare made by the driver in period i, $i = 1, ..., 6$, is c_i .
Based on the question, we know:

$$\begin{aligned} c_1 &= 80 \\ c_2 + c_3 &= 500 \\ c_3 + c_4 &= 420 \\ c_4 + c_5 &= 300 \\ c_5 + c_6 &= 270 \\ c_6 + c_1 &= 210 \end{aligned}$$

Then we can get

$$\begin{aligned} c_1 &= 80 \\ c_2 &= 240 \\ c_3 &= 260 \\ c_4 &= 160 \\ c_5 &= 140 \\ c_6 &= 130 \end{aligned}$$

So, the average fare made by drivers who start their 8-hour shift at midnight is $c_1 + c_2 = 320$.

(b) Assume the time period is 1, 2, 3, 4, 5, and 6. Define x_i as the number of people beginning work in period $i, i = 1, ..., 6$.
Then the model is as follows:

$$\begin{aligned} &\max z = 320x_1 + 500x_2 + 420x_3 + 300x_4 + 270x_5 + 210x_6 \\ &s.t. \quad x_1 + x_6 \geq 10 \\ &x_1 + x_2 \geq 12 \\ &x_2 + x_3 \geq 20 \\ &x_3 + x_4 \geq 25 \\ &x_4 + x_5 \geq 32 \\ &x_5 + x_6 \geq 18 \\ &x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 70 \\ &x_i \geq 0, i = 1, \cdots, 6 \; and \; integer \end{aligned}$$

The maximum revenue is \$25380 with optimal solution $= (10, 17, 11, 14, 18, 0)$.

3	Items:	period1	period2	period3	period4	period5	period6			
4		320.00	500.00	420.00	300.00	270.00	210.00			
5	Conditions:							Usage	Constraint	Available
6	constrain1	1.00	0.00	0.00	0.00	0.00	1.00	10.00	>=	10.00
7	constrain2	1.00	1.00	0.00	0.00	0.00	0.00	27.00	>=	12.00
8	constrain3	0.00	1.00	1.00	0.00	0.00	0.00	28.00	>=	20.00
9	constrain4	0.00	0.00	1.00	1.00	0.00	0.00	25.00	>=	25.00
10	constrain5	0.00	0.00	0.00	1.00	1.00	0.00	32.00	>=	32.00
11	constrain6	0.00	0.00	0.00	0.00	1.00	1.00	18.00	>=	18.00
12	constrain7	1.00	1.00	1.00	1.00	1.00	1.00	70.00	=	70.00
13										
14	Production:									
15	period1	10.00								
16	period2	17.00								
17	period3	11.00								
18	period4	14.00								
19	period5	18.00								
20	period6	0.00								
21	revenue=	25380.00								

(c) Add constraint $x_1 \leq 15$. The model is as follows:

$$
\begin{aligned}
&\max z = 320x_1 + 500x_2 + 420x_3 + 300x_4 + 270x_5 + 210x_6 \\
&s.t. \quad x_1 + x6 \geq 10 \\
&x_1 + x_2 \geq 12 \\
&x_2 + x_3 \geq 20 \\
&x_3 + x_4 \geq 25 \\
&x_4 + x_5 \geq 32 \\
&x_5 + x_6 \geq 18 \\
&x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 70 \\
&x_1 \leq 15 \\
&x_i \geq 0, i = 1, \cdots, 6 \ and \ integer
\end{aligned}
$$

The maximum revenue is \$25380 with optimal solution $= (10, 17, 11, 14, 18, 0)$.

3	Items:	period1	period2	period3	period4	period5	period6			
4		320.00	500.00	420.00	300.00	270.00	210.00			
5	Conditions:							Usage	Constraint	Available
6	constrain1	1.00	0.00	0.00	0.00	0.00	1.00	10.00	>=	10.00
7	constrain2	1.00	1.00	0.00	0.00	0.00	0.00	27.00	>=	12.00
8	constrain3	0.00	1.00	1.00	0.00	0.00	0.00	28.00	>=	20.00
9	constrain4	0.00	0.00	1.00	1.00	0.00	0.00	25.00	>=	25.00
10	constrain5	0.00	0.00	0.00	1.00	1.00	0.00	32.00	>=	32.00
11	constrain6	0.00	0.00	0.00	0.00	1.00	1.00	18.00	>=	18.00
12	constrain7	1.00	1.00	1.00	1.00	1.00	1.00	70.00	=	70.00
13	constrain8	1.00	0.00	0.00	0.00	0.00	0.00	10.00	<=	15.00
14										
15	Production:									
16	period1	10.00								
17	period2	17.00								
18	period3	11.00								
19	period4	14.00								
20	period5	18.00								
21	period6	0.00								
22	revenue=	25380.00								

(d) Add constraint $x_3 \leq 20$. The model is as follows:

$$
\begin{aligned}
&\max z = 320x_1 + 500x_2 + 420x_3 + 300x_4 + 270x_5 + 210x_6\\
&s.t.\quad x_1 + x6 \geq 10\\
&x_1 + x_2 \geq 12\\
&x_2 + x_3 \geq 20\\
&x_3 + x_4 \geq 25\\
&x_4 + x_5 \geq 32\\
&x_5 + x_6 \geq 18\\
&x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 70\\
&x_1 \leq 15\\
&x_3 \leq 20\\
&x_i \geq 0, i = 1, \cdots, 6 \ and\ integer
\end{aligned}
$$

The maximum revenue is \$25380 with optimal solution $= (10, 17, 11, 14, 18, 0)$.

3	Items:	period1	period2	period3	period4	period5	period6			
4		320.00	500.00	420.00	300.00	270.00	210.00			
5	Conditions:							Usage	Constraint	Available
6	constrain1	1.00	0.00	0.00	0.00	0.00	1.00	10.00	>=	10.00
7	constrain2	1.00	1.00	0.00	0.00	0.00	0.00	27.00	>=	12.00
8	constrain3	0.00	1.00	1.00	0.00	0.00	0.00	28.00	>=	20.00
9	constrain4	0.00	0.00	1.00	1.00	0.00	0.00	25.00	>=	25.00
10	constrain5	0.00	0.00	0.00	1.00	1.00	0.00	32.00	>=	32.00
11	constrain6	0.00	0.00	0.00	0.00	1.00	1.00	18.00	>=	18.00
12	constrain7	1.00	1.00	1.00	1.00	1.00	1.00	70.00	=	70.00
13	constrain8	1.00	0.00	0.00	0.00	0.00	0.00	10.00	<=	15.00
14	constrain9	0.00	0.00	1.00	0.00	0.00	0.00	11.00	<=	20.00
15										
16	Production:									
17	period1	10.00								
18	period2	17.00								
19	period3	11.00								
20	period4	14.00								
21	period5	18.00								
22	period6	0.00								
23	revenue =	25380.00								

5

(a) **Interpretation 1**

Let Rugby fields, Football fields, Soccer fields, Dog park, Playground, Walking/running trails, Softball fields and Baseball fields are ith $(i = 1, ..., 8)$ project.

Assume

$$x_i = \begin{cases} 1 & \text{if ith project is constructed, } i = 1, ..., 8 \\ 0 & \text{otherwise} \end{cases}$$

Then the model is as follows:

$$
\begin{aligned}
&\max z = 4700x_1 + 12500x_2 + 32000x_3 + 7500x_4 + 41000x_5 + 47000x_6 + 23000x_7 + 16000x_8\\
&s.t.\quad 7x_1 + 12x_2 + 20x_3 + 6x_4 + 3x_5 + 25x_6 + 5x_7 + 8x_8 \leq 55\\
&75000x_1 + 180000x_2 + 350000x_3 + 45000x_4 + 120000x_5 + 80000x_6 + 115000x_7 + 210000x_8 \leq 550000\\
&3x_1 + 2x_2 + x_3 + 3x_4 + 2x_5 + x_6 + 2x_7 + 3x_8 \leq 1.75(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8)\\
&0 \leq x_i \leq 1, i = 1, \cdots, 8, \text{ integer}
\end{aligned}
$$

After simplification, the model becomes:

$$\max z = 4700x_1 + 12500x_2 + 32000x_3 + 7500x_4 + 41000x_5 + 47000x_6 + 23000x_7 + 16000x_8$$
$$s.t. \quad 7x_1 + 12x_2 + 20x_3 + 6x_4 + 3x_5 + 25x_6 + 5x_7 + 8x_8 \le 55$$
$$75000x_1 + 180000x_2 + 350000x_3 + 45000x_4 + 120000x_5 + 80000x_6 + 115000x_7 + 210000x_8 \le 550000$$
$$1.25x_1 + 0.25x_2 - 0.75x_3 + 1.25x_4 + 0.25x_5 - 0.75x_6 + 0.25x_7 + 1.25x_8 \le 0$$
$$0 \le x_i \le 1, i = 1, \cdots, 8, \text{ integer}$$

3	Items:	project1	project2	project3	project4	project5	project6	project7	project8			
4		4700.00	12500.00	32000.00	7500.00	41000.00	47000.00	23000.00	16000.00			
5	Conditions:									Usage	Constraint	Available
6	constrain1	7.00	12.00	20.00	6.00	3.00	25.00	5.00	8.00	53.00	<=	55.00
7	constrain2	75000.00	180000.00	350000.00	45000.00	120000.00	80000.00	115000.00	210000.00	495000.00	<=	550000.00
8	constrain3	1.25	0.25	-0.75	1.25	0.25	-0.75	0.25	1.25	0.00	<=	0.00
9												
10												
11	Production:											
12	project1	0.00										
13	project2	1.00										
14	project3	0.00										
15	project4	0.00										
16	project5	1.00										
17	project6	1.00										
18	project7	1.00										
19	project8	0.00										
20	usage=	123500.00										

The maximum annual usage is 123500 with $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 1, 0, 0, 1, 1, 1, 0)$.

Interpretation 2

Assume x_i = number of project 1 are constructed, $i = 1, ..., 8$.
Replace $0 \le x_i \le 1, i = 1, \cdots, 8$, integer by $x_i \ge 0, i = 1, \cdots, 8$, integer. Then the model is as follows:

3	Items:	project1	project2	project3	project4	project5	project6	project7	project8			
4		4700.00	12500.00	32000.00	7500.00	41000.00	47000.00	23000.00	16000.00			
5	Conditions:									Usage	Constraint	Available
6	constrain1	7.00	12.00	20.00	6.00	3.00	25.00	5.00	8.00	42.00	<=	55.00
7	constrain2	75000.00	180000.00	350000.00	45000.00	120000.00	80000.00	115000.00	210000.00	440000.00	<=	550000.00
8	constrain3	1.25	0.25	-0.75	1.25	0.25	-0.75	0.25	1.25	0.00	<=	0.00
9												
10												
11	Production:											
12	project1	0.00										
13	project2	0.00										
14	project3	0.00										
15	project4	0.00										
16	project5	3.00										
17	project6	1.00										
18	project7	0.00										
19	project8	0.00										
20	usage=	170000.00										

The maximum annual usage is 170000 with $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 0, 0, 0, 3, 1, 0, 0)$.

(b) **Interpretation 1**

In this case, the model is as follows:

$$\max z = 3x_1 + 2x_2 + x_3 + 3x_4 + 2x_5 + x_6 + 2x_7 + 3x_8$$
$$s.t. \quad 7x_1 + 12x_2 + 20x_3 + 6x_4 + 3x_5 + 25x_6 + 5x_7 + 8x_8 \le 55$$
$$75000x_1 + 180000x_2 + 350000x_3 + 45000x_4 + 120000x_5 + 80000x_6 + 115000x_7 + 210000x_8 \le 550000$$
$$4700x_1 + 12500x_2 + 32000x_3 + 7500x_4 + 41000x_5 + 47000x_6 + 23000x_7 + 16000x_8 \ge 120000$$
$$0 \le x_i \le 1, i = 1, \cdots, 8, \text{ integer}$$

3	Items:	project1	project2	project3	project4	project5	project6	project7	project8			
4		3.00	2.00	1.00	3.00	2.00	1.00	2.00	3.00			
5	Conditions:									Usage	Constraint	Available
6	constrain1	7.00	12.00	20.00	6.00	3.00	25.00	5.00	8.00	48.00	<=	55.00
7	constrain2	75000.00	180000.00	350000.00	45000.00	120000.00	80000.00	115000.00	210000.00	550000.00	<=	550000.00
8	constrain3	4700.00	12500.00	32000.00	7500.00	41000.00	47000.00	23000.00	16000.00	120000.00	>=	120000.00
9												
10												
11	Production:											
12	project1	0.00										
13	project2	0.00										
14	project3	1.00										
15	project4	0.00										
16	project5	1.00										
17	project6	1.00										
18	project7	0.00										
19	project8	0.00										
20	averagepriority=	4.00										

The minimum sum of priority is 4 with with $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 0, 1, 0, 1, 1, 0, 0)$.

Interpretation 2

Assume x_i = number of project 1 are constructed, $i = 1, ..., 8$.
Replace $0 \le x_i \le 1, i = 1, \cdots, 8$, integer by $x_i \ge 0, i = 1, \cdots, 8$, integer. Then the model is as follows:

3	Items:	project1	project2	project3	project4	project5	project6	project7	project8			
4		3.00	2.00	1.00	3.00	2.00	1.00	2.00	3.00			
5	Conditions:									Usage	Constraint	Available
6	constrain1	7.00	12.00	20.00	6.00	3.00	25.00	5.00	8.00	53.00	<=	55.00
7	constrain2	75000.00	180000.00	350000.00	45000.00	120000.00	80000.00	115000.00	210000.00	280000.00	<=	550000.00
8	constrain3	4700.00	12500.00	32000.00	7500.00	41000.00	47000.00	23000.00	16000.00	135000.00	>=	120000.00
9												
10												
11	Production:											
12	project1	0.00										
13	project2	0.00										
14	project3	0.00										
15	project4	0.00										
16	project5	1.00										
17	project6	2.00										
18	project7	0.00										
19	project8	0.00										
20	averagepriority=	4.00										

The minimum sum of priority is 4 with with $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 0, 0, 0, 1, 2, 0, 0)$.

(c) **Interpretation 1**

In this case, the model is as follows:

$$
\begin{aligned}
&\max z = 7x_1 + 12x_2 + 20x_3 + 6x_4 + 3x_5 + 25x_6 + 5x_7 + 8x_8 \\
&s.t. \quad 7x_1 + 12x_2 + 20x_3 + 6x_4 + 3x_5 + 25x_6 + 5x_7 + 8x_8 \le 55 \\
&75000x_1 + 180000x_2 + 350000x_3 + 45000x_4 + 120000x_5 + 80000x_6 + 115000x_7 + 210000x_8 \le 550000 \\
&1.25x_1 + 0.25x_2 - 0.75x_3 + 1.25x_4 + 0.25x_5 - 0.75x_6 + 0.25x_7 + 1.25x_8 \le 0 \\
&0 \le x_i \le 1, i = 1, \cdots, 8, \text{ integer}
\end{aligned}
$$

3	Items:	project1	project2	project3	project4	project5	project6	project7	project8			
4	acreage per project	7.00	12.00	20.00	6.00	3.00	25.00	5.00	8.00			
5	usage per project	4700.00	12500.00	32000.00	7500.00	41000.00	47000.00	23000.00	16000.00			
6	Conditions:									Usage	Constraint	Available
7	constrain1	7.00	12.00	20.00	6.00	3.00	25.00	5.00	8.00	52.00	<=	55.00
8	constrain2	75000.00	180000.00	350000.00	45000.00	120000.00	80000.00	115000.00	210000.00	505000.00	<=	550000.00
9	constrain3	1.25	0.25	-0.75	1.25	0.25	-0.75	0.25	1.25	-0.25	<=	0.00
10												
11												
12	Production:											
13	project1	1.00										
14	project2	0.00										
15	project3	1.00										
16	project4	0.00										
17	project5	0.00										
18	project6	1.00										
19	project7	0.00										
20	project8	0.00										
21	acreage=	52.00										
22	usage=	83700.00										

The maximum acreage used is 52 with $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (1, 0, 1, 0, 0, 1, 0, 0)$. The annual usage with these facilities is $4,700 + 32,000 + 47,000 = 83,700$.

Interpretation 2

Assume x_i = number of project 1 are constructed, $i = 1, ..., 8$.
Replace $0 \le x_i \le 1, i = 1, \cdots, 8$, integer by $x_i \ge 0, i = 1, \cdots, 8$, integer. Then the model is as follows:

3	Items:	project1	project2	project3	project4	project5	project6	project7	project8			
4	acreage per project	7.00	12.00	20.00	6.00	3.00	25.00	5.00	8.00			
5	usage per project	4700.00	12500.00	32000.00	7500.00	41000.00	47000.00	23000.00	16000.00			
6	Conditions:									Usage	Constraint	Available
7	constrain1	7.00	12.00	20.00	6.00	3.00	25.00	5.00	8.00	55.00	<=	55.00
8	constrain2	75000.00	180000.00	350000.00	45000.00	120000.00	80000.00	115000.00	210000.00	275000.00	<=	550000.00
9	constrain3	1.25	0.25	-0.75	1.25	0.25	-0.75	0.25	1.25	-1.25	<=	0.00
10												
11												
12	Production:											
13	project1	0.00										
14	project2	0.00										
15	project3	0.00										
16	project4	0.00										
17	project5	0.00										
18	project6	2.00										
19	project7	1.00										
20	project8	0.00										
21	acreage=	55.00										
22	usage=	117000.00										

The maximum acreage used is 55 with $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (0, 0, 0, 0, 0, 2, 1, 0)$. The annual usage with these facilities is $2 * 47,000 + 23,000 = 117,000$.