Midterm #2

First Name: _____

Last Name: _____

This exam consists of 12 questions. You have 24 hours to complete this exam beginning at the start of class. Follow the instructions on every problem to get the full amount of points. Most questions will require an Excel file to supplement your solution. I will tell you what to name each of the Excel files you submit. If you don't have a working Excel file that shows your work, you may lose the majority or all of your points. I want to see your Excel file after you solve the problem so I can see your work in Excel Solver. Make sure you save your file after using Excel solver.

When you are finished, you can submit your exam to Gradescope and submit your Excel files to Sakai. Your answers can be typed in Microsoft Word or written by hand. Answer each question on a different set of files or images. If your solutions are handwritten you can take photos of your exam with your phone and submit the images to Gradescope. If you use Microsoft Word, make sure you submit one Word document for each question. Make sure you link each of the files you upload to Gradescope to the corresponding question and remember to upload your Excel files to Sakai.

Do not work with other students or get help from any person. You have access to the lectures, textbook, homework, and internet. If you get caught using the answers or excel files of another student, expect a 0% on the exam. If you get caught sending your exam or Excel files to another student, expect a 0% on the exam. If this exam is uploaded to the internet, the next exam will be due in one class period.

1 Information for Questions 1.1 and 1.2

It is Christmas time, and Dr. Worldwide needs to buy each of his children a single toy. Dr. Worldwide has five children who he has conveniently named numbers 1 through 5 so that he would remember the order in which they were birthed.

The children worked together to create a list of 7 unique toys that they all want. Each toy has a different cost. Use the following letters as abbreviations for the 7 unique toys.

Toy	Cost	Abbreviation
Teddy Bear	\$5	A
Fake Baby	\$8	B
Lego Set	\$10	C
Soccer Ball	\$8	D
Box of Paper	\$4	E
Shoes	\$13	F
Tools	\$12	G

Important information for constraints are found in the following bullet points.

- Each child gets a single toy and that toy must be one of the seven toys.
- No two children get the same toy. The children like to share their toys.
- If child 1 gets the Teddy Bear, than child 5 must get the Tools.
- If child 2 gets the Soccer Ball, than child 4 must get the Shoes.
- Child 1 must get either the Teddy Bear or the Tools.
- Child 2 must get either the Soccer Ball or the Lego Set.

1.1 Dr. Worldwide wants to find the optimal solution that maximizes cost. Fill in the table below with the toy each child is getting (do not use letter abbreviations), and the total cost of the optimal solution. Also, submit an Excel file named "Toys.xlsx" to Sakai with your work to get full credit. (10 Points)

Child	Toy
1	Tools (G)
2	Lego Set (C)
3	Fake Baby (B)
4	Shoes (F)
5	Soccer Ball (D)l
Total Cost	$51

1.2 Is the optimal solution unique? Why or why not? Write your answer in complete sentences for full credit. (4 Points)

The optimal solution is not unique because we can switch the toys of child 3 and child 5 since the cost of the fake baby and the soccer ball are the same. Both of these toys cost $8.00. This switch doesn't violate our other constraints.

2 Information for Questions 2.1, 2.2, and 2.3

Medzies produces COVID-19 vaccinations in the following three cities at the quantities listed below:

Location	*Daily Production (Count in Thousands)*
A. Kansas City	100
B. Denver	80
C. Minneapolis	90

Medzies must fulfill the vaccination demands for the following three cities:

Location	*Daily Demand (Count in Thousands)*
F. Brooklyn	120
G. Miami	70
H. Oakland	100

Before shipping the vaccinations from the supply cities to the demand cities, all vaccinations must go to either Austin (D) or Chicago (E) for testing. The shipping costs per thousand vaccinations from the supply cities to the testing locations and the shipping costs per thousand vaccinations from the testing locations to the supply cities are given below:

	to Testing	
From	**D**	**E**
A	\$20	\$15
B	\$22	\$18
C	\$18	\$25

	to Demand		
From	**F**	**G**	**H**
D	\$14	\$18	\$16
E	\$22	\$10	\$18

For this problem, let x_{ij} represent the number of vaccinations (in thousands) sent from city i to city j. Medzies wants to determine the number of vaccinations (in thousands) to send from each of the production cities to each of the testing cities and from each of the testing cities to each of the demand cities that minimizes the total shipping cost.

2.1 Write the constraints to the linear program in standard form. Use the letters to represent the 8 different cities. (10 Points)

$$x_{AD} + x_{AE} = 100$$
$$x_{BD} + x_{BE} = 80$$
$$x_{CD} + x_{CE} = 90$$

$$x_{DF} + x_{EF} \leq 120$$
$$x_{DG} + x_{EG} \leq 70$$
$$x_{DH} + x_{EH} \leq 100$$

$$x_{AD} + x_{BD} + x_{CD} - x_{DF} - x_{DG} - x_{DH} = 0$$
$$x_{AE} + x_{BE} + x_{CE} - x_{EF} - x_{EG} - x_{EH} = 0$$

$$x_{ij} \in \{0, 1, 2, \cdots\}$$

2.2 Fill in the following tables with the optimal solution for the number of vaccinations (in thousands) to send along each of the routes. Also, submit an Excel file named "Vaccination.xlsx" to Sakai with your work to get full credit. (10 Points)

	to Testing	
From	**D**	**E**
A	0	100
B	10	70
C	90	0

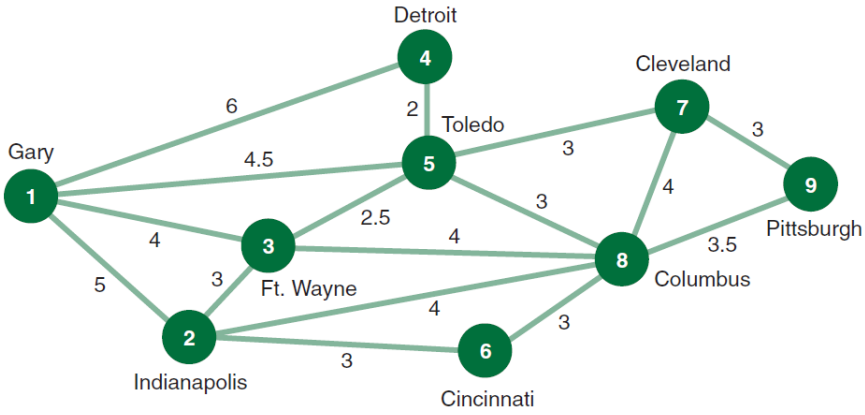
	to Demand		
From	**F**	**G**	**H**
D	100	0	0
E	0	70	100

2.3 Suppose that government policy states that each of the testing facilities must test the exact same number of vaccinations. Write in standard form, one additional constraint, that ensures that Austin and Chicago receive the same number of COVID-19 vaccinations to test. (2 Points)

$$x_{AD} + x_{BD} + x_{CD} - x_{AE} - x_{BE} - x_{CE} = 0$$

3 Information for Questions 3.1 and 3.2

Kroger is a major grocery company based in Cincinnati, OH. They have to create shipping routes to get food from Cincinnati to 7 other cities. The image below shows the time it takes in hours to travel between all 8 cities in the network. All roads can be traveled in both directions.



3.1 Use Excel to find the fastest route between Cincinnati and all other cities. Fill in the table with the fastest path (use notation 1-3-4-2-5) and time required for that path. Submit an Excel file named "Cincinnati.xlsx" with your work to get full credit. You can use an algorithm to check your answer, but you must submit your work Excel. (12 Points)

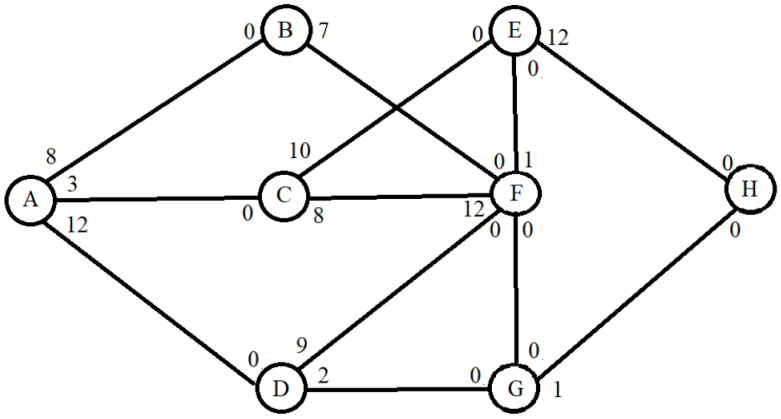
Destination	Shortest Path	Time Required (Hours)
Gary	6-2-1	8
Indianapolis	6-2	3
Ft. Wayne	6-2-3	6
Detroit	6-8-5-4	8
Toledo	6-8-5	5
Cleveland	6-8-7	7
Columbus	6-8	3
Pittsburgh	6-8-9	6.5

3.2 You found out that the road between Cincinnati and Columbus has been closed for the year. In complete sentences, describe how you can quickly modify your existing Excel file by either changing a single cell or adding a constraint in solver to ensure that your optimal solution ignores the route between Cincinnati and Columbus. (4 Points)

We can add a constraint to ensure that the decision variables for the number of passes going from Cincinnati to Columbus equals 0. Also, we could modify the matrix of times between the cities and change the 3 for Cincinatti to Columbus to a really large number like 10,000.

4 Information for Questions 4.1 and 4.2

Observe the following network with the indicated flow capacities along each branch. We want to investigate the maximum flow from node A (source) to node H (destination). For this problem, let x_{ij} = amount of flow going from node i to node j where $i, j \in \{A, B, C, D, E, F, G, H\}$.



4.1 Formulate a linear program, in standard form, to find the maximal flow through this network. (12 Points)

$$
\begin{aligned}
\text{Maximize } & Z = x_{AB} + x_{AC} + x_{AD} \\
\text{or } & Z = x_{EH} + x_{GH} \\
\text{Subject to } & x_{AB} - x_{BF} = 0 \\
& x_{AC} - x_{CE} - x_{CF} = 0 \\
& x_{AD} - x_{DF} - x_{DG} = 0 \\
& x_{CE} + x_{FE} - x_{EH} = 0 \\
& x_{BF} + x_{CF} + x_{DF} - x_{FC} - x_{FE} = 0 \\
& x_{DG} - x_{DH} = 0 \\
& x_{AB} + x_{AC} + x_{AD} - x_{EH} - x_{GH} = 0 \\
& x_{AB} \leq 8,\ x_{AC} \leq 3,\ x_{AD} \leq 12 \\
& x_{BF} \leq 7,\ x_{CE} \leq 10,\ x_{CF} \leq 8,\ x_{DF} \leq 9,\ x_{DG} \leq 2 \\
& x_{EH} \leq 12,\ x_{FC} \leq 12,\ x_{FE} \leq 1,\ x_{GH} \leq 1 \\
& x_{ij} \geq 0 \text{ and is integer}
\end{aligned}
$$

4.2 Use Excel to solve the previously stated linear program to find the maximal flow. In the table below, write the maximal flow of this network. Submit an Excel file named "MaxFlow.xlsx" with your work to get full credit. You can use an algorithm to check your answer, but you must submit your solved linear program in Excel. (6 Points)

Maximal Flow	12

5 Information for Questions 5.1 and 5.2

The Hickory Cabinet and Furniture Company produces sofas, tables, and chairs at its plant in Greensboro, NC. Each sofa earns $400 in profit, each table earns $200 in profit, and each chair earns $100 in profit. This company produces furniture on a weekly basis and stores the furniture in a warehouse. The plant uses three main resources to make furniture – wood, upholstery, and labor. The resource requirements for each piece of furniture are as follows:

	Resource Requirements		
	Wood (board ft.)	Upholstery (yd.)	Labor (hr.)
Sofa	7	12	6
Table	5	—	9
Chair	4	7	5

Furthermore, the company has 5 employees who each work 40 hours per week. Each employee is willing to work an additional 5 hours of overtime each week if necessary. The customers have a weekly demand of 200 sofas, 300 tables, and 400 chairs. The company has a warehouse that can comfortably hold 650 pieces of furniture.

The company wants to achieve the following goals listed in order of priority:

(1) They want to use at least 200 hours of labor per week.

(2) They would prefer to not exceed their warehouse capacity.

(3) They get 2,250 board feet of wood per week and would prefer to not order more.

(4) They get 1,000 yards of upholstery per week and would prefer to not order more.

(5) The company doesn't want to go below or above their weekly demand. The degree of importance for each type of furniture is proportional to the amount of profit.

(6) The company wants to achieve a weekly profit of at least $100,000.

Define the decision variables as follows:

- x_1 = Number of sofas produced each week
- x_2 = Number of tables produced each week
- x_3 = Number of chairs produced each week

5.1 Formulate a goal programming model to determine the quantity of each part to produce to achieve the company's objectives. (15 Points)

$$
\begin{aligned}
\text{Minimize } & P_1(d_1^-) \\
& P_2(d_2^+) \\
& P_3(d_3^+) \\
& P_4(d_4^+) \\
& P_5(400(d_5^- + d_5^+) + 200(d_6^- + d_6^+) + 100(d_7^- + d_7^+)) \\
& P_6(d_8^-)
\end{aligned}
$$

$$
\begin{aligned}
\text{Subject to } & 6x_1 + 9x_2 + 5x_3 + d_1^- - d_1^+ = 200 \\
& x_1 + x_2 + x_3 + d_2^- - d_2^+ = 650 \\
& 7x_1 + 5x_2 + 4x_3 + d_3^- - d_3^+ = 2250 \\
& 12x_1 + 7x_3 + d_4^- - d_4^+ = 1000 \\
& x_1 + d_5^- - d_5^+ = 200 \\
& x_2 + d_6^- - d_6^+ = 300 \\
& x_3 + d_7^- - d_7^+ = 400 \\
& 400x_1 + 200x_2 + 100x_3 + d_8^- - d_8^+ = 100000 \\
& x_i \in \{0, 1, 2, 3, 4, \cdots\} \\
& d_i^-, d_i^+ \geq 0
\end{aligned}
$$

5.2 Use Excel to solve the goal programming model. Fill in the table below with your optimal solution according to the list of prioritized goals. Submit an Excel file named "Furniture.xlsx" with your work to get a full credit. If necessary, make sure you have multiple tabs to solve this goal programming problem. Also, make sure the optimal solution can be found at every step when optimizing according to the different goals. (10 Points)

	Number to Produce Per Week
Sofas	83
Tables	300
Chairs	0

5.3 We want to modify the first priority. The company's new first priority goal is to not exceed the maximum amount of overtime that the 5 employees want to work. Rewrite the first objective function and modify the appropriate constraint to reflect this change. (4 Points)

New Objective for Priority 1: $P_1(d_1^+)$

Modified Constraint: $6x_1 + 9x_2 + 5x_3 + d_1^- - d_1^+ = 225$