



Lecture 6

Produced by Dr. Worldwide
Welcome to the 305

Ex: Quick-Screen Clothing



- Quick-Screen is a clothing manufacturing company specializing in the production of commemorative shirts immediately following major sporting events and they have a contract to produce shirts for winning team of a college football bowl game on New Year's Day between State and Tech
- They will produce two different sweatshirts and two different t-shirts with one of each having writing on front (F) only and the other having writing on both front (F) and back (B)
- All items will be produced by the box where each box contains a dozen items
- Q: How much of each of the items should be produced to maximize profit?



Ex: Quick-Screen Clothing



- Decision Variables
 - $x_1 = \text{Number of Boxes of Sweatshirts} - F$
 - $x_2 = \text{Number of Boxes of Sweatshirts} - B/F$
 - $x_3 = \text{Number of Boxes of T-shirts} - F$
 - $x_4 = \text{Number of Boxes of T-shirts} - B/F$
- Consider the following table showing resource requirements, unit costs, and profit of every dozen (box) of shirts

	Processing time (hr.) per dozen	Cost per dozen	Profit dozen
Sweatshirt - F	0.10	\$36	\$90
Sweatshirt - B/F	0.25	48	125
T-shirt - F	0.08	25	45
T-shirt - B/F	0.21	35	65

Ex: Quick-Screen Clothing



- Objective Function
 - Goal: Maximize profit on shirts
 - $Z = 90x_1 + 125x_2 + 45x_3 + 65x_4$
- Constraints
 - Only have 72 hours of processing time to produce all items:
 $0.1x_1 + 0.25x_2 + 0.08x_3 + 0.21x_4 \leq 72$
 - Company has a budget of \$25,000: $36x_1 + 48x_2 + 25x_3 + 35x_4 \leq 25,000$
 - Trailer truck will pick up shirts and can accommodate 1,200 standard-size boxes where each standard-size box holds 12 T-shirts and a box of 12 sweatshirts is 3 times the size of the standard-size box:
 $3(x_1 + x_2) + x_3 + x_4 \leq 1,200$
 - They have 500 dozens of blank sweatshirts: $x_1 + x_2 \leq 500$
 - They have 500 dozens of blank T-shirts: $x_3 + x_4 \leq 500$
 - Nonnegativity: $x_1, x_2, x_3, x_4 \geq 0$

Ex: Quick-Screen Clothing



- Download [ProductMix.xlsx](#) from website link called [Sheet 1](#)
- Before Excel solver

A product mix								
Products:	Sweatshirt-F (dozen)	Sweatshirt-B/F (dozen)	T-shirt-F (dozen)	T-shirt-B/F (dozen)				
Profit per dozen:	90	125	45	65				
Resources:					Usage	Constraint	Available	Left over
Processing time	0.1	0.25	0.08	0.21	0	<=	72	72
Cost	36	48	25	35	0	<=	25000	25000
Truck capacity	3	3	1	1	0	<=	1200	1200
Blank sweatshirts	1	1	0	0	0	<=	500	500
Blank T-shirts	0	0	1	1	0	<=	500	500
Production:								
Sweatshirts-F =	0							
Sweatshirts-B/F =	0							
T-shirt-F =	0							
T-shirt-B/F =	0							
Profit =	0							

Ex: Quick-Screen Clothing



- After Excel solver

A product mix								
Products:	Sweatshirt-F (dozen)	Sweatshirt-B/F (dozen)	T-shirt-F (dozen)	T-shirt-B/F (dozen)				
Profit per dozen:	90	125	45	65				
Resources:					Usage	Constraint	Available	Left over
Processing time	0.1	0.25	0.08	0.21	72	<=	72	0
Cost	36	48	25	35	21593.333	<=	25000	3406.6667
Truck capacity	3	3	1	1	1200	<=	1200	0
Blank sweatshirts	1	1	0	0	233.33333	<=	500	266.66667
Blank T-shirts	0	0	1	1	500	<=	500	0
Production:								
Sweatshirts-F =	175.55556							
Sweatshirts-B/F =	57.777778							
T-shirt-F =	500							
T-shirt-B/F =	0							
Profit =	45522.222							

Ex: Quick-Screen Clothing



- Recommended optimal solution to maximize profit at \$45,522.22

- $x_1 = 175.56$
- $x_2 = 57.78$
- $x_3 = 500$
- $x_4 = 0$

- Sensitivity report for objective function coefficients

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$15	Sweatshirts-F = (dozen)	175.5555556	0	90	11.92307692	40
\$B\$16	Sweatshirts-B/F = (dozen)	57.77777778	0	125	13.21428571	11.92307692
\$B\$17	T-shirt-F = (dozen)	500	0	45	1E+30	4.111111111
\$B\$18	T-shirt-B/F = (dozen)	0	-10.33333333	65	10.33333333	1E+30

Ex: Quick-Screen Clothing



- Sensitivity report for constraint quantities

Constraints						
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$F\$10	Blank sweatshirts Usage	233.3333333	0	500	1E+30	266.6666667
\$F\$11	Blank T-shirts Usage	500	4.111111111	500	185.7142857	500
\$F\$7	Processing time Usage	72	233.3333333	72	26.33333333	8.666666667
\$F\$8	Cost Usage	21593.33333	0	25000	1E+30	3406.666667
\$F\$9	Truck capacity Usage	1200	22.22222222	1200	260	316

Vectors and Matrices



- Linear program with 4 decision variables and 4 constraints requires more time to insert formulas in Excel
- Understanding of **linear algebra** can make this a more efficient process
- The object $\mathbf{x} = [x_1, x_2, x_3, x_4]$ is a **row vector** in \mathbb{R}^4

- The object $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ is a **column vector** in \mathbb{R}^4

- The **transpose** of a vector \mathbf{x} , denoted \mathbf{x}' , transforms a row vector into a column vector and vice versa

Vectors and Matrices



- The object $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$ is a **matrix** in $\mathbb{R}^{4 \times 3}$
- The **dimension** of a matrix, denoted $\dim(\mathbf{A})$, describes its number of rows and number of columns (in that order)
- Based on above example, $\dim(\mathbf{A})$ is 4×3
- A row vector in \mathbb{R}^m is a matrix in $\mathbb{R}^{1 \times m}$
- A column vector in \mathbb{R}^n is a matrix in $\mathbb{R}^{n \times 1}$
- Typically, all vectors are by default column vectors

Vectors and Matrices



- For matrices $\mathbf{A} \in \mathbb{R}^{m \times n}$ and $\mathbf{B} \in \mathbb{R}^{n \times p}$, we can define their **product** $\mathbf{M} = \mathbf{AB}$, which will be a matrix in $\mathbb{R}^{m \times p}$
- For $\mathbf{A} \in \mathbb{R}^{3 \times 4}$ and $\mathbf{B} \in \mathbb{R}^{4 \times 2}$, matrix $\mathbf{M} = \mathbf{AB}$ can be expressed as

$$\mathbf{M} = \mathbf{AB} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \end{bmatrix}$$

where

$$m_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + a_{i4}b_{4j} \text{ for } i = 1,2,3 \text{ and } j = 1,2$$

- In order to compute, $\mathbf{M} = \mathbf{AB}$, the number of columns in \mathbf{A} must equal the number of rows in \mathbf{B}
- In above example, the matrix $\mathbf{M} = \mathbf{BA}$ does not exist

Vectors and Matrices



- If \mathbf{A} and \mathbf{B} are square matrices of the same size, say in $\mathbb{R}^{p \times p}$, we can compute both \mathbf{AB} and \mathbf{BA} , but they may not necessarily be equal

- Let $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 5 \\ 7 & 1 \end{bmatrix}$

$$\mathbf{AB} = \begin{bmatrix} 1 \times 2 + 3 \times 7 & 1 \times 5 + 3 \times 1 \\ -2 \times 2 + 0 \times 7 & -2 \times 5 + 0 \times 1 \end{bmatrix} = \begin{bmatrix} 23 & 8 \\ -4 & -10 \end{bmatrix}$$

$$\mathbf{BA} = \begin{bmatrix} 2 \times 1 + 5 \times -2 & 2 \times 3 + 5 \times 0 \\ 7 \times 1 + 1 \times -2 & 7 \times 3 + 1 \times 0 \end{bmatrix} = \begin{bmatrix} -8 & 6 \\ 5 & 21 \end{bmatrix}$$

- Let $\mathbf{x} = [x_1, x_2, x_3, x_4]'$ and $\mathbf{y} = [y_1, y_2, y_3, y_4]'$ be column vectors in \mathbb{R}^4

$$\mathbf{x}'\mathbf{y} = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4 = \mathbf{y}'\mathbf{x}$$

Excel: Matrix Multiplication



- MMULT Function in Excel
 - The **MMULT** Function in Excel is used to multiply arrays (matrices) that have compatible dimensions and returns an array (matrix)
 - Syntax: **MMULT**(array1,array2)
 - Example: Vector Multiplication

	A	B	C	D	E	F
1	Vector a	1	2	3		Vector b
2						3
3						2
4						1
5						
6	MMULT(a,b)	10				
7	MMULT(b,a)	3	6	9		
8		2	4	6		
9		1	2	3		

Excel: Matrix Multiplication



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	A	B	C	D	E	F
1	Vector a	1	2	3		Vector b
2						3
3						2
4						1
5						
6	MMULT(a,b)	10	=MMULT(B1:D1,F2:F4)			
7	MMULT(b,a)	3	6	9		
8		2	4	6		
9		1	2	3		

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	A	B	C	D	E	F
1	Vector a	1	2	3		Vector b
2						3
3						2
4						1
5						
6	MMULT(a,b)	10				
7	MMULT(b,a)	3	6	9	=MMULT(F2:F4, B1:D1)	
8		2	4	6		
9		1	2	3		



Excel: Matrix Multiplication



- MMULT Function in Excel
 - Example: Matrix Multiplication

	A	B	C	D	E	F	G
1	Matrix A	1	2	3		Matrix B	
2		4	5	6		1	2
3						3	4
4							
5	MMULT(A,B)	#VALUE!					
6	MMULT(B,A)	9	12	15			
7		19	26	33			

Excel: Matrix Multiplication



- MMULT Function in Excel
 - Example: Matrix Multiplication

	A	B	C	D	E	F	G
1	Matrix A	1	2	3		Matrix B	
2		4	5	6		1	2
3						3	4
4							
5	MMULT(A,B)	#VALUE!	=MMULT(B1:D2,F2:G3)				
6	MMULT(B,A)	9	12	15			
7		19	26	33			

Excel: Matrix Multiplication



- MMULT Function in Excel
 - Example: Matrix Multiplication

	A	B	C	D	E	F	G
1	Matrix A	1	2	3		Matrix B	
2		4	5	6		1	2
3						3	4
4							
5	MMULT(A,B)	#VALUE!					
6	MMULT(B,A)	9	12	15	=MMULT(F2:G3,B1:D2)		
7		19	26	33			

Excel: Matrix Multiplication



- SUMPRODUCT Function in Excel
 - The **SUMPRODUCT** function in Excel is used to multiply arrays (matrices) element-wise and then returns the sum of their products
 - In mathematics, this is often referred to as a **dot-product** or **vector-product** when the arrays are vectors
 - Syntax: **SUMPRODUCT(array1,array2)**
 - Ex: Usage on Vectors

	A	B	C	D	E	F
1	Vector a	1	2	3		Vector c
2	Vector b	3	2	1		4
3						5
4						6
5						
6	SUMPRODUCT(a,b)	10				
7	SUMPRODUCT(a,c)	#VALUE!				

Excel: Matrix Multiplication



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 - Syntax: **SUMPRODUCT(array1,array2)**
 - Ex: Usage on Vectors

	A	B	C	D	E	F
1	Vector a	1	2	3		Vector c
2	Vector b	3	2	1		4
3						5
4						6
5						
6	SUMPRODUCT(a,b)	10	=SUMPRODUCT(B1:D1,B2:D2)			
7	SUMPRODUCT(a,c)	#VALUE!				

Excel: Matrix Multiplication



- SUMPRODUCT Function in Excel
 - The **SUMPRODUCT** function in Excel is used to multiply arrays (matrices) element-wise and then returns the sum of their products
 - In mathematics, this is often referred to as a **dot-product** when the arrays are vectors
 - Syntax: **SUMPRODUCT(array1,array2)**
 - Ex: Usage on Vectors

	A	B	C	D	E	F
1	Vector a	1	2	3		Vector c
2	Vector b	3	2	1		4
3						5
4						6
5						
6	SUMPRODUCT(a,b)	10				
7	SUMPRODUCT(a,c)	#VALUE!	=SUMPRODUCT(B1:D1,F2:F4)			

Excel: Application of Matrices

- Both MMULT and SUMPRODUCT can be used in Excel to make the creation of formulas and constraints of linear programs considerably easier
- Made up example for practice

	A	B	C	D	E	F
1	Profit	3	4			
2				Total	Constraint	Max
3	Metal	2	1	3	<=	30
4	Plastic	0	4	4	<=	50
5						
6						
7	Swag 1	1				
8	Swag 2	1				
9	Total	7				

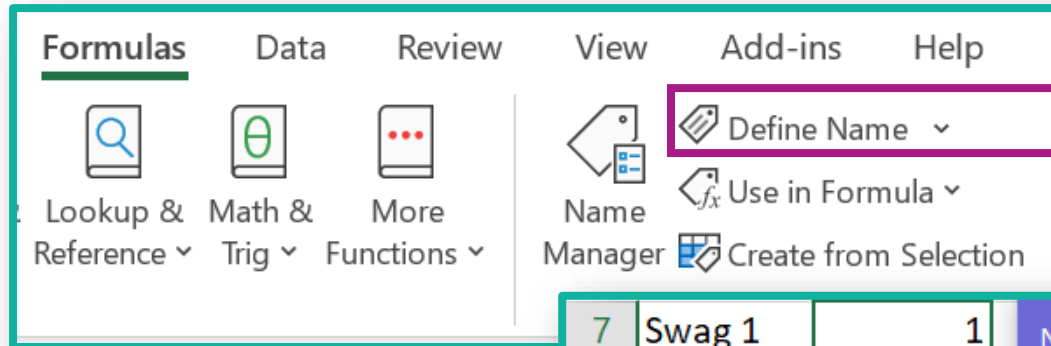
Solver

	A	B	C	D	E	F
1	Profit	3	4			
2				Total	Constraint	Max
3	Metal	2	1	30	<=	30
4	Plastic	0	4	50	<=	50
5						
6						
7	Swag 1	8.75				
8	Swag 2	12.5				
9	Total	76.25				



Excel: Application of Matrices

- Creating EXCEL variable for easy referencing



7	Swag 1	1
8	Swag 2	1
9	Total	7
10		
11		
12		
13		
14		
15		
16		
17		
18		

New Name

Name:

Scope:

Comment:

Refers to:



Excel: Application of Matrices

- Usage of created variable in establishing constraints and objective function

	A	B	C	D	E	F
1	Profit	3	4			
2				Total	Constraint	Max
3	Metal	2	1	30	<=	30
4	Plastic	0	4	50	<=	50
5				=MMULT(B4:C4,x)		
6						
7	Swag 1	8.75				
8	Swag 2	12.5				
9	Total	76.25	=MMULT(B1:C1,x)			



Excel: Application of Matrices



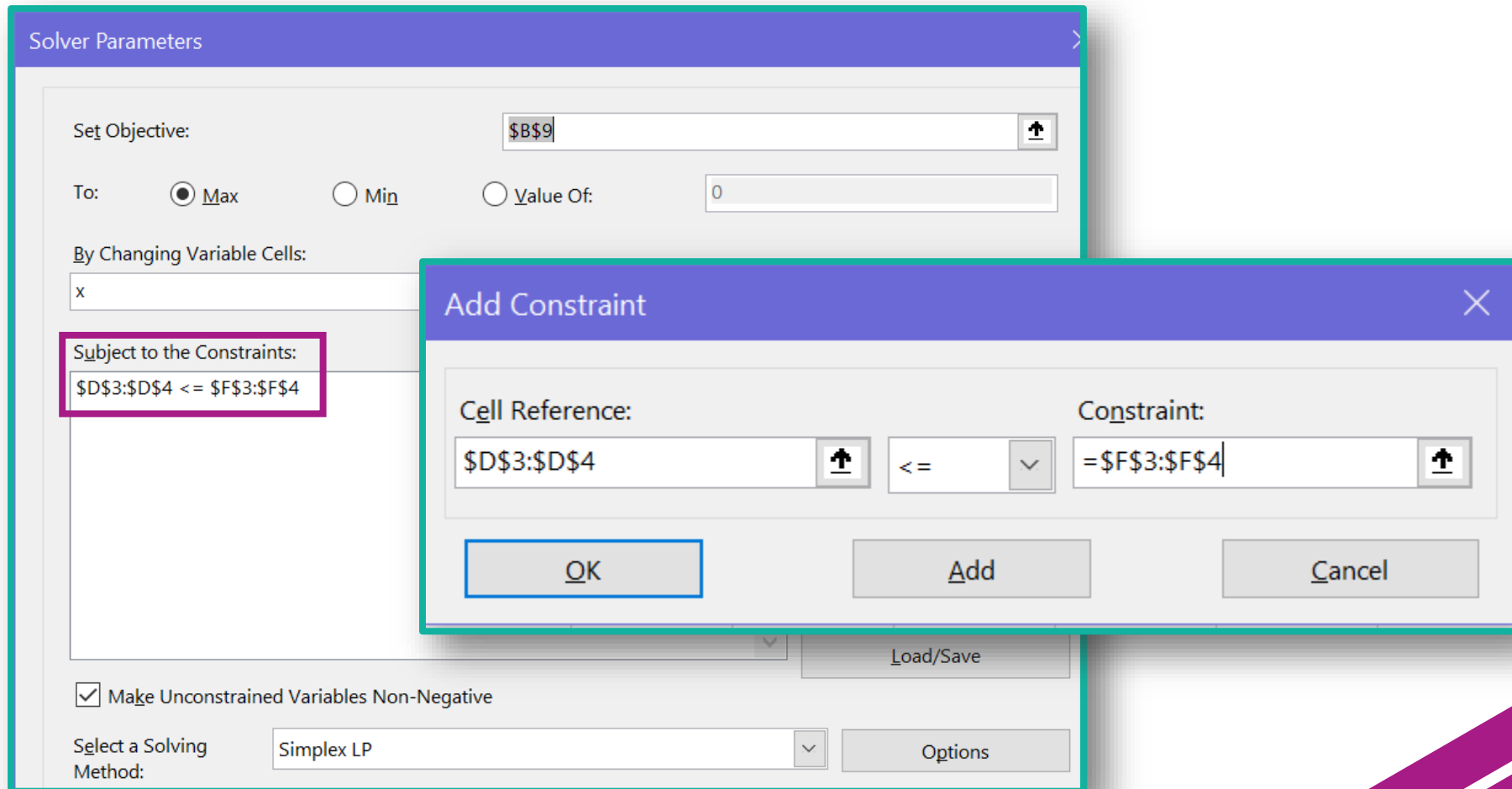
- Another option for specification

SUM		:	✕	✓	f_x	=MMULT(B3:C4,B7:B8)	
	A	B	C	D	E	F	
1	Profit	3	4				
2				Total	Constraint	Max	
3	Metal	2	1	B8)	<=	30	
4	Plastic	0	4	4	<=	50	
5							
6							
7	Swag 1	1					
8	Swag 2	1					
9	Total	7					



Excel: Application of Matrices

- Another option for specification



The image shows the Excel Solver interface. The 'Solver Parameters' task pane is open, showing the following settings:

- Set Objective:**
- To:** ☒ Max ☐ Min ☐ Value Of:
- By Changing Variable Cells:**
- Subject to the Constraints:**
- ☒ Make Unconstrained Variables Non-Negative
- Select a Solving Method:** Simplex LP

The 'Add Constraint' dialog box is also open, showing the following details:

- Cell Reference:**
- Constraint:**
- Buttons: OK, Add, Cancel



Excel: Application of Matrices

- Try using MMULT and SUMPRODUCT in cell formulas
- Try using vectors/matrices in specification of constraints
- Download [Lecture6WS.xlsx](#) from course website from link [Sheet 2](#) for all examples seen in this lecture





The End



Dale

