Assignment #7 Solutions

due Friday, October 2nd, 2020

Problem 1

Sol 1: Ford-Fulkerson algorithm

The maximum traffic flow the streets can accommodate is 21,000 cars.

Path	Flow
$1 \to 2 \to 5 \to 8$	6
$1 \to 2 \to 3 \to 5 \to 8$	2
$1 \to 3 \to 6 \to 8$	4
$1 \to 4 \to 7 \to 8$	5
$1 \to 4 \to 6 \to 7 \to 8$	3
$1 \to 3 \to 6 \to 7 \to 8$	1

Sol 2: Linear Programming

Assume x_{ij} is the amount of flow (thousands of cars) going through edge (i, j). Let C_{ij} be the flow capacity of edge (i, j). Then the model is as follows:

$$
\begin{aligned}
&\max z = x_{12} + x_{13} + x_{14} \\
&s.t. \\
&x_{12} + x_{32} - x_{23} - x_{25} = 0 \\
&x_{13} + x_{23} + x_{53} - x_{32} - x_{35} - x_{36} = 0 \\
&x_{14} + x_{64} - x_{46} - x_{47} = 0 \\
&x_{25} + x_{35} - x_{53} - x_{58} = 0 \\
&x_{36} + x_{46} + x_{76} - x_{64} - x_{67} - x_{68} = 0 \\
&x_{47} + x_{67} - x_{76} - x_{78} = 0 \\
&x_{ij} \le C_{ij} \\
&x_{ij} \ge 0, \;\; i = 1, 2, ..., 8, \; j = 1, 2, ..., 8 \textit{ and integer.}
\end{aligned}
$$

We can solve the model using excel, shown in Fig 1. From Fig 1, we observe that the maximum traffic flow the streets can accommodate is 21,000 cars. The amount of traffic along each street are shown in excel table A5:A21. The street would be able to handle the expected flow after a game.

1	**Maximum flow problem**			
2				
3				
4	Select branc	Node	Node	Capacity
5	6	1	2	10
6	7	1	3	7
7	8	1	4	8
8	0	2	3	3
9	6	2	5	6
10	0	3	2	5
11	2	3	5	6
12	5	3	6	5
13	4	4	6	4
14	4	4	7	5
15	0	5	3	2
16	8	5	8	8
17	0	6	4	2
18	5	6	7	6
19	4	6	8	4
20	0	7	6	2
21	9	7	8	9
22			Total	21

Flow constraints:

Node	Network Flo	Constraint	Value
2	0	=	0
3	0	=	0
4	0	=	0
5	0	=	0
6	0	=	0
7	0	=	0

Figure 1: Excel in Question 1

Problem 2

Assume x_{ij} is the amount of flow going through edge (i, j), $i < j$. Then the model is as follows:

$$
\begin{aligned}
&\max z = x_{12} + x_{13} + x_{14} + x_{15} \\
&s.t. \\
&x_{12} - x_{26} - x_{29} = 0 \\
&x_{13} - x_{36} - x_{37} = 0 \\
&x_{14} - x_{47} - x_{48} = 0 \\
&x_{15} - x_{58} - x_{5,11} = 0 \\
&x_{26} + x_{36} - x_{6,10} - x_{6,12} = 0 \\
&x_{37} + x_{47} + x_{67} - x_{78} - x_{7,10} = 0 \\
&x_{48} + x_{58} + x_{78} - x_{8,11} - x_{8,13} - x_{8,14} = 0 \\
&x_{29} - x_{9,12} = 0 \\
&x_{6,10} + x_{7,10} - x_{10,12} - x_{10,13} = 0 \\
&x_{5,11} + x_{8,11} - x_{11,14} = 0 \\
&x_{6,12} + x_{9,12} + x_{10,12} - x_{12,15} = 0 \\
&x_{10,13} + x_{8,13} + x_{12,13} - x_{13,15} = 0 \\
&x_{8,14} + x_{11,14} - x_{14,15} = 0 \\
&x_{ij} \geq 0,\ i < j,\ i = 1, 2, ..., 15,\ j = 1, 2, ..., 15 \ \textit{and integer}.
\end{aligned}
$$

We can solve the model using excel, shown in Fig 2. The number of units processed at each work center are shown in Excel A5:A31 and the maximum flow is 250.

Maximum flow problem

Select branch	Node	Node	Capacity
70	1	2	90
50	1	3	120
60	1	4	75
70	1	5	85
25	2	6	25
45	2	9	45
30	3	6	30
20	3	7	20
35	4	7	35
25	4	8	25
40	5	8	40
30	5	11	30
55	6	10	65
0	6	12	75
15	7	8	50
40	7	10	40
0	8	11	35
0	8	13	60
65	8	14	70
45	9	12	50
30	10	12	55
65	10	13	65
30	11	14	60
60	12	13	60
15	12	15	90
125	13	15	190
95	14	15	110
			250

Flow constraints:

Node	Network Flo	Constraint	Value
2	0	=	0
3	0	=	0
4	0	=	0
5	0	=	0
6	0	=	0
7	0	=	0
8	0	=	0
9	0	=	0
10	0	=	0
11	0	=	0
12	0	=	0
13	0	=	0
14	0	=	0

Figure 2: Excel in Question 2

Problem 3

Sol 1: Ford-Fulkerson algorithm

The maximum number of flights the ariline can schedule per day from Chicago to Los Angeles is 16.

Path	Flow
$8 \to 6 \to 2 \to 1$	3
$8 \to 4 \to 1$	9
$8 \to 7 \to 4 \to 2 \to 1$	3
$8 \to 6 \to 5 \to 3 \to 1$	1

Sol 2: Linear Programming

Assume x_{ij} is the amount of flow going through edge (i, j). Let c_{ij} be the capacity of edge (i, j). Then the model is as follows:

$$
\begin{aligned}
&\min z = x_{84} + x_{86} + x_{87} \\
&s.t. \\
&x_{42} + x_{62} - x_{21} - x_{24} - x_{26} = 0 \\
&x_{53} - x_{31} - x_{34} - x_{35} = 0 \\
&x_{24} + x_{34} + x_{64} + x_{74} + x_{84} - x_{42} - x_{41} - x_{46} - x_{47} = 0 \\
&x_{35} + x_{75} - x_{53} - x_{57} = 0 \\
&x_{86} + x_{26} + x_{46} - x_{62} - x_{64} = 0 \\
&x_{87} + x_{47} + x_{57} - x_{74} - x_{75} = 0 \\
&x_{84} + x_{86} + x_{87} - x_{21} - x_{31} - x_{41} = 0 \\
&0 \leq x_{ij} \leq c_{ij}, \quad i = 1, 2, ..., 8, \; j = 1, 2, ..., 8 \textit{ and integer.}
\end{aligned}
$$

The maximum number of flights the airline can schedule per day from Chicago to Los Angeles is 16. The number of flights along each route is shown in A5:A29.

1	**Maximum flow problem**			
2				
3				
4	Select branch	Node	Node	Capacity
5	6	2	1	10
6	0	2	4	7
7	0	2	6	8
8	1	3	1	2
9	0	3	4	4
10	0	3	5	6
11	9	4	1	9
12	4	4	2	4
13	0	4	6	5
14	0	4	7	7
15	0	4	8	4
16	1	5	3	1
17	0	5	7	5
18	2	6	2	8
19	1	6	4	2
20	3	7	4	3
21	1	7	5	1
22	9	8	4	9
23	3	8	6	3
24	4	8	7	5
25				16

Flow constraints:

Node	Network Flow	Constraint	Value
2	0	=	0
3	0	=	0
4	0	=	0
5	0	=	0
6	0	=	0
7	0	=	0
Flow out from 8 = Flow in to 1	0	=	0

Figure 3: Excel in Question 3