



Lecture 15

Produced by Dr. Worldwide

Welcome to the 305

Goal Programming



- All prior linear programming problems have had a **single** objective function
- Companies may have **multiple criteria** in consideration for a decision
- Sometimes the multiple objectives conflict
- Company may want to maximize profit and minimize pollution
- **Goal programming** is linear programming for multiple objectives or criteria



Ex: Beaver Creek Pottery



- Trying to choose $x = \text{number of bowls}$ and $y = \text{number of mugs}$ to maximize the profit function

- Recall the original linear program

Maximize $40x + 50y$

Subject to: $x + 2y \leq 40$ (Labor)

$4x + 3y \leq 120$ (Clay)

$x, y \geq 0$

- Objective function reflects a single goal

Ex: Beaver Creek Pottery



- Suppose Beaver Creek wanted to achieve other goals while maximizing profit
- In **order of importance**:
 - To avoid layoffs, they want to use at least 40 hours of labor per day
 - They want to achieve a satisfactory profit level of \$1,600 per day
 - To avoid having clay dry out, they prefer to keep no more than 120 lb of clay on hand each day
 - To avoid overhead costs due to keeping the factory open past normal hours, they want to minimize the amount of overtime
- We reformulate our linear programming model using goal programming
- Transform linear programming model constraints into **goals**

Ex: Beaver Creek Pottery



- Goal 1: Avoid **underutilization** of labor
 - Original constraint $x + 2y \leq 40$
 - Reformulation to a **goal constraint**

$$x + 2y + d_1^- - d_1^+ = 40 \quad (\text{Labor})$$

- Two new variables d_1^- and d_1^+ are nonnegative and represent the **underutilized time** and **overtime**, respectively
- Q: What if the optimal solution had $d_1^- > 0$?
- Q: What if the optimal solution had $d_1^+ > 0$?
- The top priority corresponding to minimization of d_1^-

Minimize $P_1 d_1^-$

- The P_1 indicates the priority of this goal (not a coefficient)

Ex: Beaver Creek Pottery



- Goal 2: **Achieve** daily profit of \$1,600
 - Original objective function $Z = 40x + 50y$
 - Reformulation to a **goal constraint**

$$40x + 50y + d_2^- - d_2^+ = 1600 \quad (\text{Profit})$$

- Two new variables d_2^- and d_2^+ are nonnegative and represent the amount of profit **less than** \$1,600 and **more than** \$1,600
- The second priority corresponding to minimization of d_2^- is added

Minimize $P_1 d_1^-, P_2 d_2^-$

- The comma between the terms indicates that we are minimizing them **sequentially**, not simultaneously
- Q: Why are we not minimizing d_2^+ ?

Ex: Beaver Creek Pottery



- Goal 3: Avoid **waste** of material
 - Original constraint $4x + 3y \leq 120$
 - Reformulation to a **goal constraint**

$$4x + 3y + d_3^- - d_3^+ = 120 \quad (\text{Clay})$$

- Two new variables d_3^- and d_3^+ are nonnegative and represent the amount of clay **less than** 120 lbs and **more than** 120 lbs
- The company cannot keep **more than** 120 lbs in storage
- The third priority corresponds to minimization of d_3^+ is added

Minimize $P_1 d_1^-, P_2 d_2^-, P_3 d_3^+$



Ex: Beaver Creek Pottery



- Goal 4: Avoid overtime costs
 - Recall the modified goal constraint for labor

$$x + 2y + d_1^- - d_1^+ = 40 \quad (\text{Labor})$$

- Already attempting to minimize d_1^-
- To ensure we don't exceed the maximum labor, we involve d_1^+
- Finalization of objective function

Minimize $P_1 d_1^-, P_2 d_2^-, P_3 d_3^+, P_4 d_1^+$

Ex: Beaver Creek Pottery



- Full goal programming model

Minimize $P_1 d_1^-, P_2 d_2^-, P_3 d_3^+, P_4 d_1^+$

Subject to $x + 2y + d_1^- - d_1^+ = 40$ (Labor)
 $40x + 50y + d_2^- - d_2^+ = 1600$ (Profit)
 $4x + 3y + d_3^- - d_3^+ = 120$ (Clay)
 $x, y, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0$

- The variables $\{d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+\}$ are called **deviation** variables
- We minimize the four different objective functions **individually** by **priority**

Ex: Beaver Creek Pottery



- Modification 1: Prefer **no more** than 10 hours of overtime
 - Recall the goal constraint for labor

$$x + 2y + d_1^- - d_1^+ = 40 \quad (\text{Under hours})$$

- Remember that d_1^+ represents overtime
- We want $0 \leq d_1^+ \leq 10$
- Use same strategy as before by adding a goal constraint

$$d_1^+ + d_4^- - d_4^+ = 10 \quad (\text{Over hours})$$

- Possible goal constraint of all **deviation** variables
- Two new variables d_4^- and d_4^+ are nonnegative and represent the amount of overtime hours **less than** 10 hours and **more than** 10 hours
- New objective function

Minimize $P_1 d_1^-, P_2 d_2^-, P_3 d_3^+, P_4 d_4^+$

Ex: Beaver Creek Pottery



- Modification 2: **Maximum** number of bowls and mugs made daily
 - Pottery company has **limited warehouse** space
 - They can only store at most 30 bowls and 20 mugs each day
 - Profit for bowls (\$40) less than profit for mugs (\$50)
 - Consider the new constraints

$$x + d_5^- = 30 \quad (\text{Bowls})$$

$$y + d_6^- = 20 \quad (\text{Mugs})$$

- We want to minimize d_5^- and d_6^-
- Q: Why not include positive deviational variables d_5^+ and d_6^+ ?
- Q: For which item is it more important to achieve this goal?

Ex: Beaver Creek Pottery



- Modification 2: Maximum number of bowls and mugs made daily
 - Positive deviational variables are unnecessary since it is imperative to not exceed the warehouse space
 - We need to achieve the goal for mugs more than the goal for bowls because the profit is higher for mugs
 - If goals were of equal importance, we would minimize

Minimize $P_1 d_1^-, P_2 d_2^-, P_3 d_3^+, P_4 d_4^+, P_5 (d_5^- + d_6^-)$

- We can make the **degree of importance** in proportion to the profit
- The goal for mugs is more important than the goal for bowls by a **ratio** of 5 to 4

Minimize $P_1 d_1^-, P_2 d_2^-, P_3 d_3^+, P_4 d_4^+, P_5 (4d_5^- + 5d_6^-)$

- The coefficients 4 and 5 are referred to as **weights**

Ex: Beaver Creek Pottery



- Full modified goal programming model

Minimize $P_1 d_1^-, P_2 d_2^-, P_3 d_3^+, P_4 d_4^+, 4P_5 d_5^- + 5P_5 d_6^-$

Subject to

$x + 2y + d_1^- - d_1^+ = 40$	(Labor)
$40x + 50y + d_2^- - d_2^+ = 1600$	(Profit)
$4x + 3y + d_3^- - d_3^+ = 120$	(Clay)
$d_1^+ + d_4^- - d_4^+ = 10$	(Overtime)
$x + d_5^- = 30$	(Bowls)
$y + d_6^- = 20$	(Mugs)
$x, y, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+, d_5^-, d_6^- \geq 0$	





The End



Dale

