

Assignment #9 Solutions

due Friday, November 1st, 2019

1

- (a) Let x_1 be the number of acres of corn.
 x_2 be the number of acres of wheat.
 x_3 be the number of acres of soybeans.
The model is as follows:

$$\min z = P_1 d_1^-, P_2 d_2^+, P_3 d_3^-, P_4 d_1^+, P_5 d_4^-, P_6 (3d_5^- + 4d_6^- + 2d_7^-)$$

s.t.

$$7x_1 + 10x_2 + 8x_3 + d_1^- - d_1^+ = 6000$$

$$100x_1 + 120x_2 + 170x_3 + d_2^- - d_2^+ = 80,000$$

$$30x_1 + 40x_2 + 20x_3 + d_3^- - d_3^+ = 105,000$$

$$x_1 + x_2 + x_3 + d_4^- - d_4^+ = 1,000$$

$$x_1 + d_5^- - d_5^+ = 200$$

$$x_2 + d_6^- - d_6^+ = 500$$

$$x_3 + d_7^- - d_7^+ = 300$$

$$x_i, \text{integer}, i = 1, 2, 3, d_j^-, d_j^+ \geq 0, j = 1, \dots, 7$$

Integer constraint can be dropped.

- (b) We can solve the model using excel, shown in Fig 2, 3, 4, 5, 6, 7. From the result, we can see that

$$x_1 = 2, \quad x_2 = 665, \quad x_3 = 0$$

1	Goal programming for Problem 1																								
2																									
3	Priority 1																								
4	Minimize	d1^~																							
5																									
6																									
7	Objective function	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	Variables	x1	x2	x3	d1^~	d1^+	d2^~	d2^+	d3^~	d3^+	d4^~	d4^+	d5^~	d5^+	d6^~	d6^+	d7^~	d7^+	Used	Constraint	Value				
9	Labor	7	10	8	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	6000	=	6000				
10	Preparation Costs	100	120	170	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	80000	=	80000				
11	Profit	30	40	20	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	105000	=	105000				
12	available acreage	1	1	1	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	1000	=	1000				
13	Sales 1	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	200	=	200				
14	Sales 2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	500	=	500				
15	Sales 3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	300	=	300				
16																									
17																									
18	x1	0																							
19	x2	667																							
20	x3	0																							
21	d1^~	0																							
22	d1^+	670																							
23	d2^~	0																							
24	d2^+	40																							
25	d3^~	78320																							
26	d3^+	0																							
27	d4^~	333																							
28	d4^+	0																							
29	d5^~	200																							
30	d5^+	0																							
31	d6^~	0																							
32	d6^+	167																							
33	d7^~	300																							
34	d7^+	0																							
35		0																							

Figure 1: Excel in Question 1, Priority 1

1	Goal programming for Problem 1																				
2																					
3	Priority 1																				
4	Minimize	d1^~																			
5																					
6																					
7	Objective function	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
8	Variables	x1	x2	x3	d1^~	d1^+	d2^~	d2^+	d3^~	d3^+	d4^~	d4^+	d5^~	d5^+	d6^~	d6^+	d7^~	d7^+	Used	Constraint	Value
9	Labor	7	10	8	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	6000	=	6000
10	Preparation Costs	100	120	170	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	80000	=	80000
11	Profit	30	40	20	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	105000	=	105000
12	available acreage	1	1	1	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	1000	=	1000
13	Sales 1	1	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	200	=	200
14	Sales 2	0	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	500	=	500
15	Sales 3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	300	=	300
16																					
17																					
18	x1	0																			
19	x2	667																			
20	x3	0																			
21	d1^~	0																			
22	d1^+	670																			
23	d2^~	0																			
24	d2^+	40																			
25	d3^~	78320																			
26	d3^+	0																			
27	d4^~	333																			
28	d4^+	0																			
29	d5^~	200																			
30	d5^+	0																			
31	d6^~	0																			
32	d6^+	167																			
33	d7^~	300																			
34	d7^+	0																			
35		0																			

Figure 2: Excel in Question 1, Priority 1

1	Goal programming for Problem 1																				
2																					
3	Priority 2																				
4	Minimize	d2^+																			
5																					
6																					
7	Objective function	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	
8	Variables	x1	x2	x3	d1^+	d1^+	d2^+	d2^+	d3^+	d3^+	d4^+	d4^+	d5^+	d5^+	d6^+	d6^+	d7^+	d7^+	Used	Constraint	Value
9	Labor	7	10	8	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	6000	=	6000
10	Preparation Costs	100	120	170	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	80000	=	80000
11	Profit	30	40	20	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	105000	=	105000
12	available acreage	1	1	1	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	1000	=	1000
13	Sales 1	1	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	200	=	200
14	Sales 2	0	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	500	=	500
15	Sales 3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	300	=	300
16	Priority 1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	=	0
17																					
18																					
19	x1	0																			
20	x2	666																			
21	x3	0																			
22	d1^+	0																			
23	d1^+	660																			
24	d2^+	80																			
25	d2^+	0																			
26	d3^+	78360																			
27	d3^+	0																			
28	d4^+	334																			
29	d4^+	0																			
30	d5^+	200																			
31	d5^+	0																			
32	d6^+	0																			
33	d6^+	166																			
34	d7^+	300																			
35	d7^+	0																			
36		0																			

[illegible]

Figure 4: Excel in Question 1, Priority 3

1	Goal programming for Problem 1																			
2																				
3	Priority 4																			
4	Minimize d1^+																			
5																				
6																				
7	Objective function																			
8	Variables																			
9	x1	x2	x3	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	d4^-	d4^+	d5^-	d5^+	d6^-	d6^+	d7^-	d7^+	Used	Constraint	Value
10	1	7	10	8	1	-1	0	0	0	0	0	0	0	0	0	0	0	6000	=	6000
11	Preparation Costs	100	120	170	0	0	1	-1	0	0	0	0	0	0	0	0	0	80000	=	80000
12	Profit	30	40	20	0	0	0	0	1	-1	0	0	0	0	0	0	0	105000	=	105000
13	available acreage	1	1	1	0	0	0	0	0	0	1	-1	0	0	0	0	0	1000	=	1000
14	Sales 1	1	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	200	=	200
15	Sales 2	0	1	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	500	=	500
16	Sales 3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	300	=	300
17	Priority 1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	=	0
18	Priority 2	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	=	0
19	Priority 3	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	78340	=	78340
20																				
21	k1	2																		
22	k2	665																		
23	k3	0																		
24	d1^-	0																		
25	d1^+	664																		
26	d2^-	0																		
27	d2^+	0																		
28	d3^-	78340																		
29	d3^+	0																		
30	d4^-	333																		
31	d4^+	0																		
32	d5^-	198																		
33	d5^+	0																		
34	d6^-	0																		
35	d6^+	165																		
36	d7^-	300																		
37	d7^+	0																		
38		664																		

Figure 5: Excel in Question 1, Priority 4

Goal programming for Problem 1																							
2																							
3	Priority 5																						
4	Minimize																			d4^			
5																							
6																							
7	Objective function	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0			
8	Variables	x1	x2	x3	d1^	d1^+	d2^	d2^+	d3^	d3^+	d4^	d4^+	d5^	d5^+	d6^	d6^+	d7^	d7^+	Used	Constraint	Value		
9	Labor	7	10	8	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	6000	=	6000		
10	Preparation Costs	100	120	170	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	80000	=	80000		
11	Profit	30	40	20	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	105000	=	105000		
12	available acreage	1	1	1	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	1000	=	1000		
13	Sales1	1	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	200	=	200		
14	Sales2	0	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	500	=	500		
15	Sales3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	300	=	300		
16	Priority 1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	=	0		
17	Priority 2	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	=	0		
18	Priority 3	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	78450	=	78450		
19	Priority 4	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	664	=	664		
20																							
21																							
22	x1	7																					
23	x2	665																					
24	x3	0																					
25	d1^	0																					
26	d1^+	664																					
27	d2^	0																					
28	d2^+	0																					
29	d3^	78450																					
30	d3^+	110																					
31	d4^	333																					
32	d4^+	0																					
33	d5^	198																					
34	d5^+	0																					
35	d6^	0																					
36	d6^+	165																					
37	d7^	300																					
38	d7^+	0																					
39		333																					

Figure 6: Excel in Question 1, Priority 5

Goal programming for Problem 1																					
2																					
3	Priority 6																				
4	Minimize $3d5^+ + 4d6^- + 2d7^+$																				
5																					
6																					
7	Objective function																				
8	Variables	x1	x2	x3	d1 ⁻	d1 ⁺	d2 ⁻	d2 ⁺	d3 ⁻	d3 ⁺	d4 ⁻	d4 ⁺	d5 ⁻	d5 ⁺	d6 ⁻	d6 ⁺	d7 ⁻	d7 ⁺	Used	Constraint	Value
9	Labor	7	10	8	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	6000	=	6000
10	Preparation Costs	100	120	170	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	80000	=	80000
11	Profit	30	40	20	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	105000	=	105000
12	available acreage	1	1	1	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	1000	=	1000
13	Sales 1	1	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	200	=	200
14	Sales 2	0	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	500	=	500
15	Sales 3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	300	=	300
16	Priority 1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	=	0
17	Priority 2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	=	0
18	Priority 3	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	=	0
19	Priority 4	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	78450	=	78450
20	Priority 5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	664	=	664
21											1	0	0	0	0	0	0	0	333	=	333
22																					
23	x1	2																			
24	x2	665																			
25	x3	0																			
26	d1 ⁻	0																			
27	d1 ⁺	664																			
28	d2 ⁻	0																			
29	d2 ⁺	0																			
30	d3 ⁻	78450																			
31	d3 ⁺	110																			
32	d4 ⁻	333																			
33	d4 ⁺	0																			
34	d5 ⁻	198																			
35	d5 ⁺	0																			
36	d6 ⁻	0																			
37	d6 ⁺	165																			
38	d7 ⁻	300																			
39	d7 ⁺	0																			
40		1194																			

Figure 7: Excel in Question 1, Priority 6

2

- (a) Let x_i be the number of workers begin their 5-day workweek on day i , $i = 1, \dots, 7$.
The model is as follows:

$$\min z = P_1 d_1^+, P_2(d_2^- + d_3^-), P_3 d_4^-, P_4(d_5^- + d_6^- + d_7^- + d_8^-)$$

s.t.

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + d_1^- - d_1^+ = 60$$

$$x_2 + x_3 + x_4 + x_5 + x_6 + d_2^- - d_2^+ = 53$$

$$x_3 + x_4 + x_5 + x_6 + x_7 + d_3^- - d_3^+ = 47$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + d_4^- - d_4^+ = 43$$

$$x_1 + x_4 + x_5 + x_6 + x_7 + d_5^- - d_5^+ = 22$$

$$x_1 + x_2 + x_5 + x_6 + x_7 + d_6^- - d_6^+ = 28$$

$$x_1 + x_2 + x_3 + x_6 + x_7 + d_7^- - d_7^+ = 35$$

$$x_1 + x_2 + x_3 + x_4 + x_7 + d_8^- - d_8^+ = 34$$

$$x_i, \text{integer}, i = 1, \dots, 7, d_j^-, d_j^+ \geq 0, j = 1, \dots, 8$$

- (b) We can solve the model using excel, shown in Fig 9. When solve for Priority 1, all of the other deviation variables needed to minimize are all minimized to zero. From the result, we can see that

$$\begin{aligned} x_1 &= 0, & x_2 &= 7, & x_3 &= 25 & x_4 &= 1 \\ x_5 &= 10, & x_6 &= 10, & x_7 &= 1 \end{aligned}$$

Goal programming for Problem 2																																																									
1																																																									
2																																																									
3	Priority 1																																																								
4	Minimize	d1*																																																							
5																																																									
6																																																									
7																																																									
8	Objective function	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0																											
9	Variables	x1	x2	x3	x4	x5	x6	x7	d1~	d1*	d2~	d2*	d3~	d3*	d4~	d4*	d5~	d5*	d6~	d6*	d7~	d7*	d8~	d8*	Used	Constraint	Value																														
10	Labor	1	1	1	1	1	1	1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	60	=	60																														
11	Preparation Costs	0	1	1	1	1	1	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	53	=	53																														
12	Profit	0	0	1	1	1	1	1	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	47	=	47																														
13	available acreage	1	1	1	1	1	1	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	43	=	43																														
14	Sales 1	1	0	0	1	1	1	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	22	=	22																														
15	Sales 2	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	28	=	28																														
16		1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	35	=	35																														
17	Sales 3	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	34	=	34																														
18																																																									
19	x1	0																																																							
20	x2	7																																																							
21	x3	25																																																							
22	x4	1																																																							
23	x5	10																																																							
24	x6	10																																																							
25	x7	1																																																							
26	d1~	0																																																							
27	d1*	0																																																							
28	d2~	0																																																							
29	d2*	0																																																							
30	d3~	0																																																							
31	d3*	0																																																							
32	d4~	0																																																							
33	d4*	0																																																							
34	d5~	0																																																							
35	d5*	0																																																							
36	d6~	0																																																							
37	d6*	0																																																							
38	d7~	0																																																							
39	d7*	8																																																							
40	d8~	0																																																							
41	d8*	0																																																							
42		0																																																							

Figure 8: Excel in Question 2, Priority 1

(a) Let x_1 be the number of yards of denim (in 10,000).
 x_2 be the number of yards of brush-cotton.
The model is as follows:

$s.t.$

$$d_1^+ - d_1^- + d_2^- - d_2^+ = 10$$

$$x_2 + d_4^- - d_4^+ = 35$$

$$x_i, integer, i = 1, 2. d_j^-, d_j^+ \geq 0, j = 1, \dots, 4$$

$$x_1 = 60, \quad x_2 = 30$$

Figure 9: Excel in Question 2, Priority 1

Figure 10: Excel in Question 3, Priority 1

1	Goal programming for Problem 3													
2														
3	Priority 3													
4	Minimize													
5	3d3^- + 2d4^+													
6														
7	Objective function	0	0	0	0	0	0	3	0	2	0			
8	Variables	x1	x2	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	d4^-	d4^+	Used	Constraint	Value
9	Labor	1	1	1	-1	0	0	0	0	0	0	80	=	80
10	Preparation Costs	0	0	-1	1	1	-1	0	0	0	0	10	=	10
11	Profit	1	0	0	0	0	0	1	-1	0	0	60	=	60
12	available acreage	0	1	0	0	0	0	0	0	1	-1	35	=	35
13	Priority 1	0	0	1	0	0	0	0	0	0	0	0	=	0
14	Priority 2	0	0	0	0	0	1	0	0	0	0	0	=	0
15														
16														
17	x1	60												
18	x2	30												
19	d1^-	0												
20	d1^+	10												
21	d2^-	0												
22	d2^+	0												
23	d3^-	0												
24	d3^+	0												
25	d4^-	5												
26	d4^+	0												
27		10												

Figure 11: Excel in Question 3, Priority 3

1	Goal programming for Problem 3													
2														
3	Priority 4													
4	Minimize													
5	d1^+													
6														
7	Objective function	0	0	0	1	0	0	0	0	0	0			
8	Variables	x1	x2	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	d4^-	d4^+	Used	Constraint	Value
9	Labor	1	1	1	-1	0	0	0	0	0	0	80	=	80
10	Preparation Costs	0	0	-1	1	1	-1	0	0	0	0	10	=	10
11	Profit	1	0	0	0	0	0	1	-1	0	0	60	=	60
12	available acreage	0	1	0	0	0	0	0	0	1	-1	35	=	35
13	Priority 1	0	0	1	0	0	0	0	0	0	0	0	=	0
14	Priority 2	0	0	0	0	0	1	0	0	0	0	0	=	0
15	Priority 3	0	0	0	0	0	0	3	0	2	0	10	=	10
16														
17														
18	x1	60												
19	x2	30												
20	d1^-	0												
21	d1^+	10												
22	d2^-	0												
23	d2^+	0												
24	d3^-	5.3291E-15												
25	d3^+	0												
26	d4^-	5												
27	d4^+	0												
28		10												

Figure 12: Excel in Question 3, Priority 4

4

The objective function $z = vp - 7500 - 40v = (400 - 1.2p)(p - 40) - 7500 = -1.2p^2 + 448p - 23500$, then we can take the derivative of z over p and set it to zero, namely,

$$\frac{\partial z}{\partial p} = -2.4p + 448 = 0$$

Then we have $p^* = 186.67$ and the optimal profit $z^* = 18313.33$.

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