Lecture 11

Produced by Dr. Worldwide

Welcome to the 305

Ex: Give-back Weekend

- The Student Government Association (SGA) organizes a recurring event called "Give-Back Weekends" where teams are formed to work on projects for residents in the university community
- This event occurs over four consecutive Saturdays in April
- Coed teams are formed with 3 to 5 students from various dormitory groups, fraternities, sororities, clubs, and organizations
- Residents of the community fill out a form to describe work at their home that needs to be done
- Time to complete a project will vary between teams because of the different number of team members, skills of the team, and physical make-up of the team

Ex: Give-back Weekend

- Time estimates (in hours) submitted by the six teams available to work on 12 different projects for the first Saturday of the event

	Jobs											
Team	1	2	3	4	5	6	7	8	9	10	11	12
1	5	1.5	6	4	3.5	3	6	1.5	5	1	3	3.5
2	4	2	5	5	3	3	5.5	2	4	1.5	4	2.5
3	5	1.5	6.5	3.5	2.5	4	4.5	3	3.5	1	3.5	4
4	3.5	2	5.5	4	3.5	2.5	5	2.5	4	1.5	2.5	4
5	3.5	3	5	3	2	4	5	2	5	2	4	3
6	4	2.5	6	5	3	3	6	3	3	2	3	3.5

- The primary objective of SGA is to complete all 12 projects

Ex: Give-back Weekend

- Teams can work on multiple projects
- Teams cannot work more than 8 hours on Saturday
- Each team should work on at least one project
- Alternative Questions
  - Q1: How can we assign the 6 teams to the 12 projects to maximize the number of jobs completed on Saturday?
  - Q2: How can we assign the 6 teams to the 12 projects and minimize the total time required for all 6 teams to complete all projects?

Ex: Give-back Weekend

- Consider the jobs as the "sources" or "supply
- Consider the teams as the "destinations" or demand
- Decision variables
  - $x_{ij} = \begin{cases} 1 & \text{if job } j \text{ is assigned to team } i \\ 0 & \text{otherwise} \end{cases}$
  - $i \in \{1,2,3,4,5,6\}$
  - $j \in \{1,2,3,\cdots,12\}$
- Objective function for the number of completed jobs

  $Z = \sum_{i=1}^{6} \sum_{j=1}^{12} x_{ij}$
- Objective function for the amount of time for the teams to do all the jobs

  $Z = \sum_{i=1}^{6} \sum_{j=1}^{12} t_{ij} x_{ij}$ where $t_{ij} =$ time required for team i to do job j

Ex: Give-back Weekend

- Constraints
  - Each team cannot work more than 8 hours

$$5x_{11} + 1.5x_{12} + 6x_{13} + 4x_{14} + 3.5x_{15} + 3x_{16} + \\ +6x_{17} + 1.5x_{18} + 5x_{19} + x_{110} + 3x_{111} + 3.5x_{112} \leq 8 \qquad \text{(Team 1)}$$

  - Each project can only be assigned to at most one team, which adds a total of 12 constraints, one for each project $j \in \{1,2,\cdots,12\}$

$$\sum_{i=1}^{6} x_{ij} = x_{1j} + x_{2j} + x_{3j} + x_{4j} + x_{5j} + x_{6j} \leq 1 \qquad \text{(Project } j\text{)}$$

  - Each project is assigned to exactly one team

$$\sum_{i=1}^{6} x_{ij} = x_{1j} + x_{2j} + x_{3j} + x_{4j} + x_{5j} + x_{6j} = 1 \qquad \text{(Project } j\text{)}$$

  - Each decision variable is binary

$$x_{ij} \in \{0,1\}$$

Ex: Give-back Weekend

- Download GiveBack.xlsx from course website from link Sheet 1

- Solution for Q1 on sheet called Max projects
  - Maximize the number of projects that can be completed

Team	Jobs assigned	Time required to complete
1	8	1.5
2	3, 10	6.5
3	9, 12	7.5
4	7, 11	7.5
5	2, 4, 5	8
6	1,6	7
Total time:		38

  - In general, there can be multiple optimal solutions in assignment problems
  - Could have assigned job 4 to Team 1 instead of Team 5
  - Important is the fact that an optimal solution exists to do all 12 jobs

Ex: Give-back Weekend

- Solution for Q2 on sheet called Min time
  - We are trying to identify if a faster way exists where all jobs will still be completed
  - Minimize the total time required

Team	Jobs assigned	Time required to complete
1	2, 8	3
2	3, 12	7.5
3	5, 7, 10	8
4	6, 11	5
5	1, 4	6.5
6	9	3
Total time:		33

  - Q: Do you notice any differences in the optimal solution?
  - Q: Which solution is better?

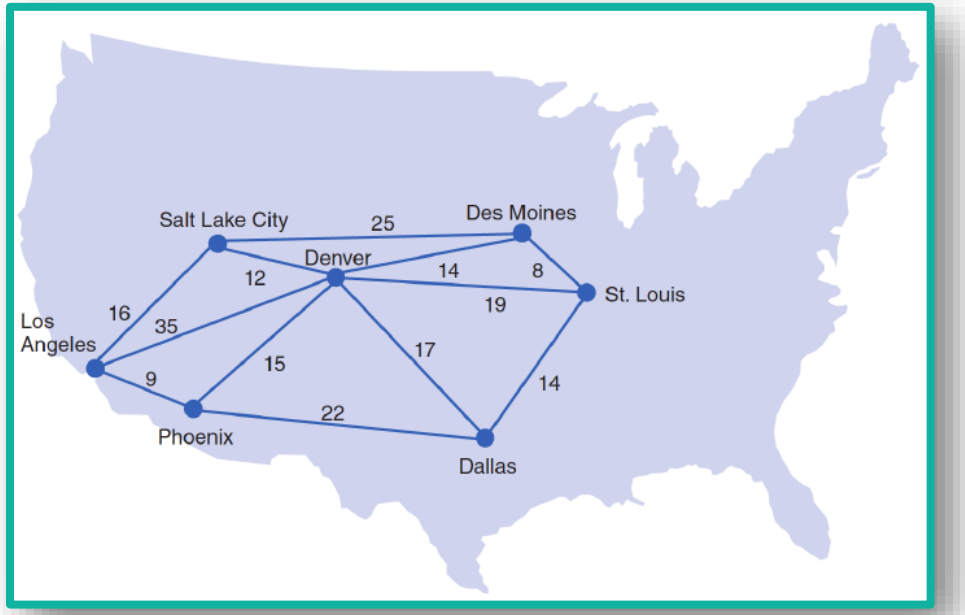
Network Flow Models

- A network is an arrangement of paths connected at various points
- A network has two main components
  - Nodes represent junction points
  - Branches or edges represent routes between nodes
- Diagrams can be utilized to visualize networks
  - Nodes represented by circles
  - Branches represented by lines
- Typical goals of transportation problems
  - Minimize transportation costs
  - Minimize total distance travelled
  - Maximize amount transported

Ex: Shortest Shipping Route

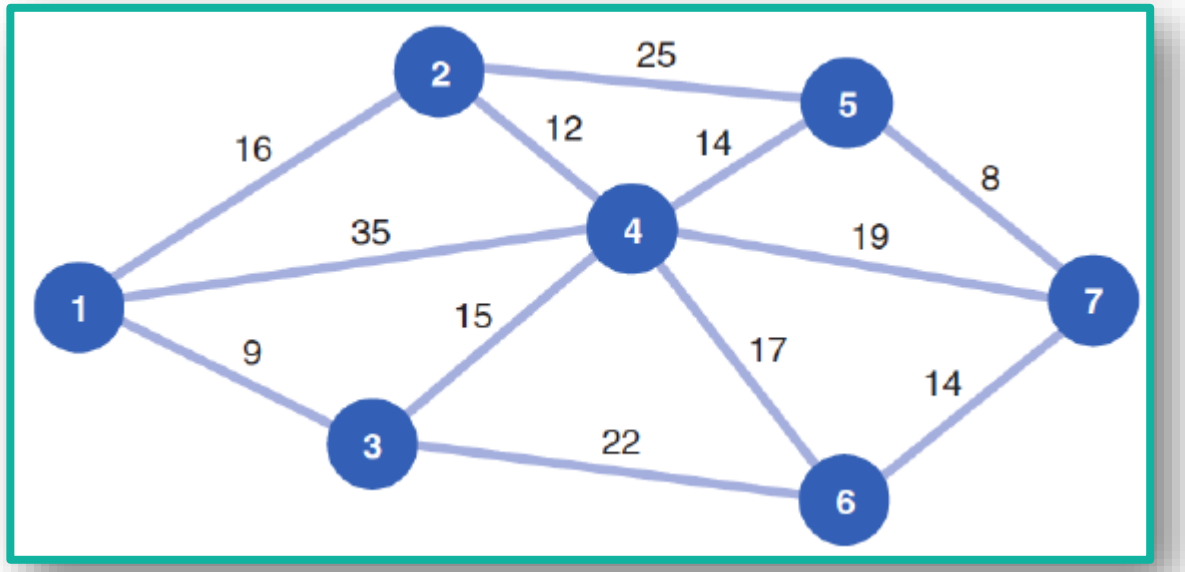
- The shortest route problem is to determine the shortest distance between an originating point and several destination points

- Stagecoach Shipping Company transports oranges from Los Angeles
  - Uses 6 trucks to travel to 6 different cities in the West and Midwest
  - Diagram of network with length of time (hours) between cities for transport



Ex: Shortest Shipping Route

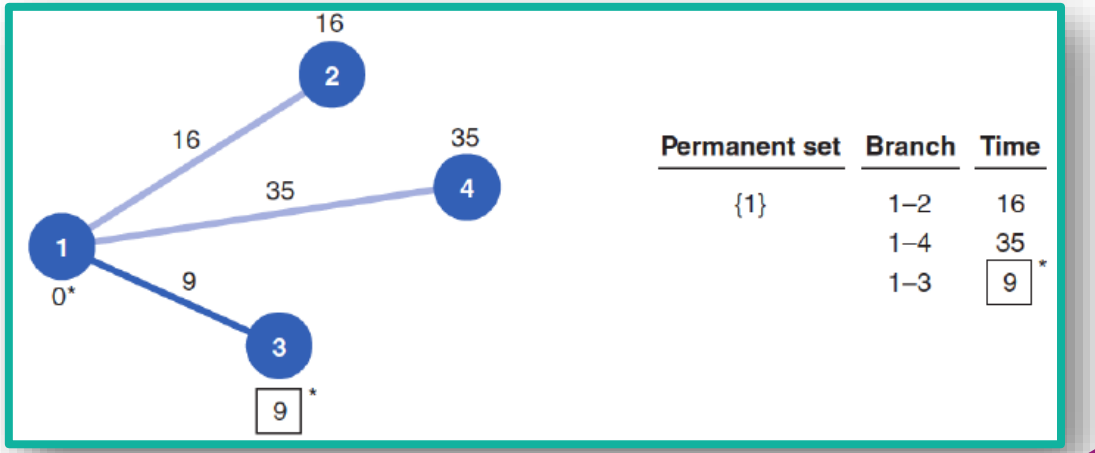
- In this problem, we are looking for the shortest route based on time
- Simplified network



- Q: What is the shortest route from node 1 to each of the other nodes?

Ex: Shortest Shipping Route

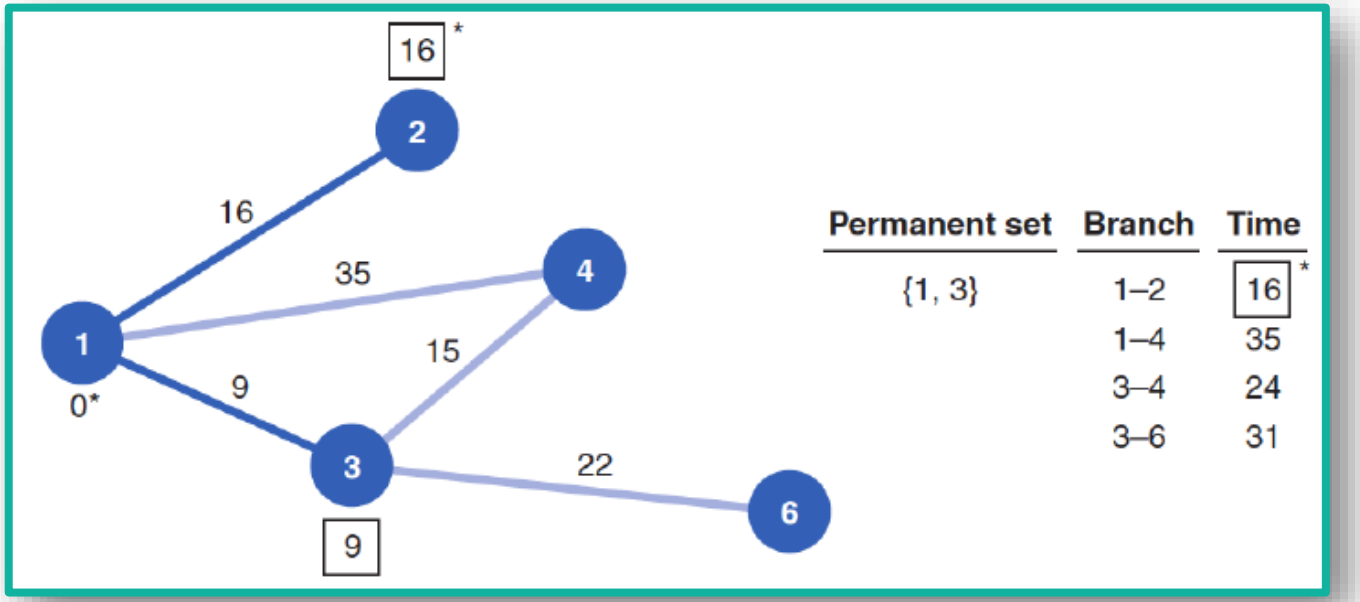
- We will keep track of the paths we choose by defining, and then updating a subset of the nodes called the permanent set
- To start, define the permanent set to be the origin, node 1
- The shortest path from node 1 to any of its adjacent nodes
- Node 3 is the closest to node 1
- Add node 3 to the permanent set



Permanent set	Branch	Time
{1}	1–2	16
	1–4	35
	1–3	9*

Ex: Shortest Shipping Route

- Next, explore all the nodes adjacent to nodes in the permanent set {1,3}

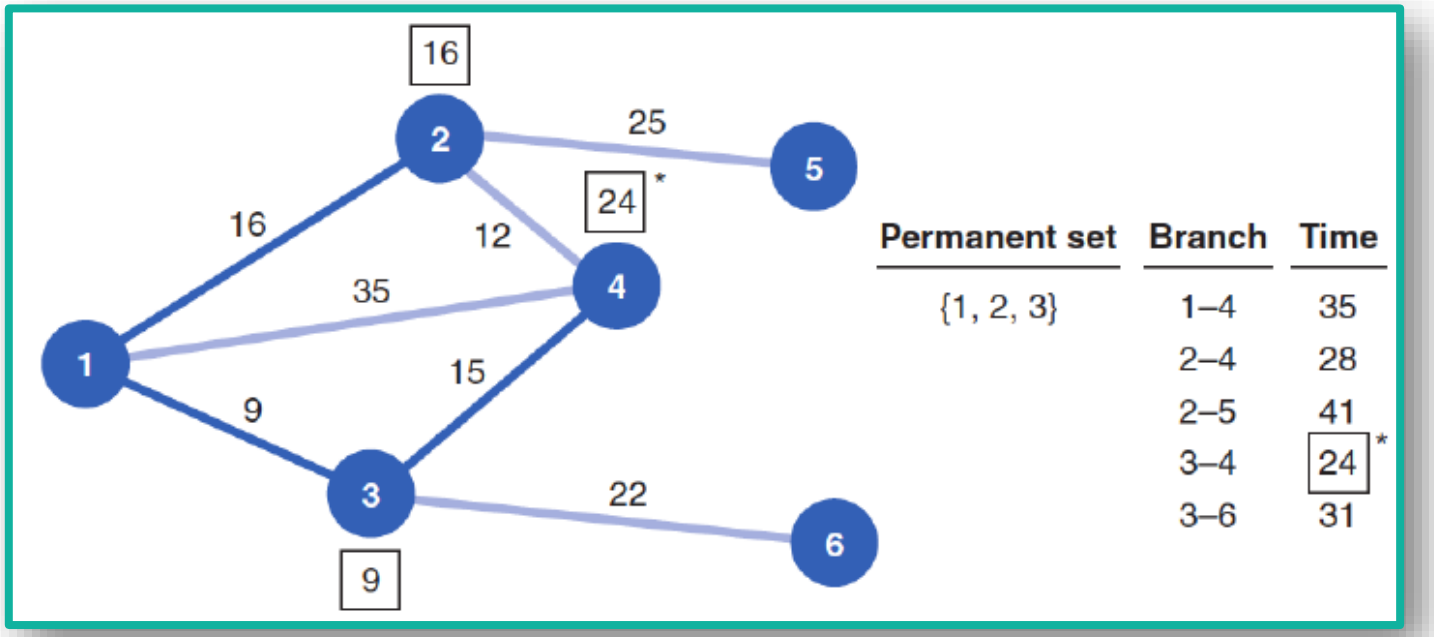


Permanent set	Branch	Time
{1, 3}	1–2	16*
	1–4	35
	3–4	24
	3–6	31

- Node 2 is the closest to the origin, so it is added to the permanent set

Ex: Shortest Shipping Route

- Next, explore all the nodes adjacent to nodes in the permanent set {1,2,3}

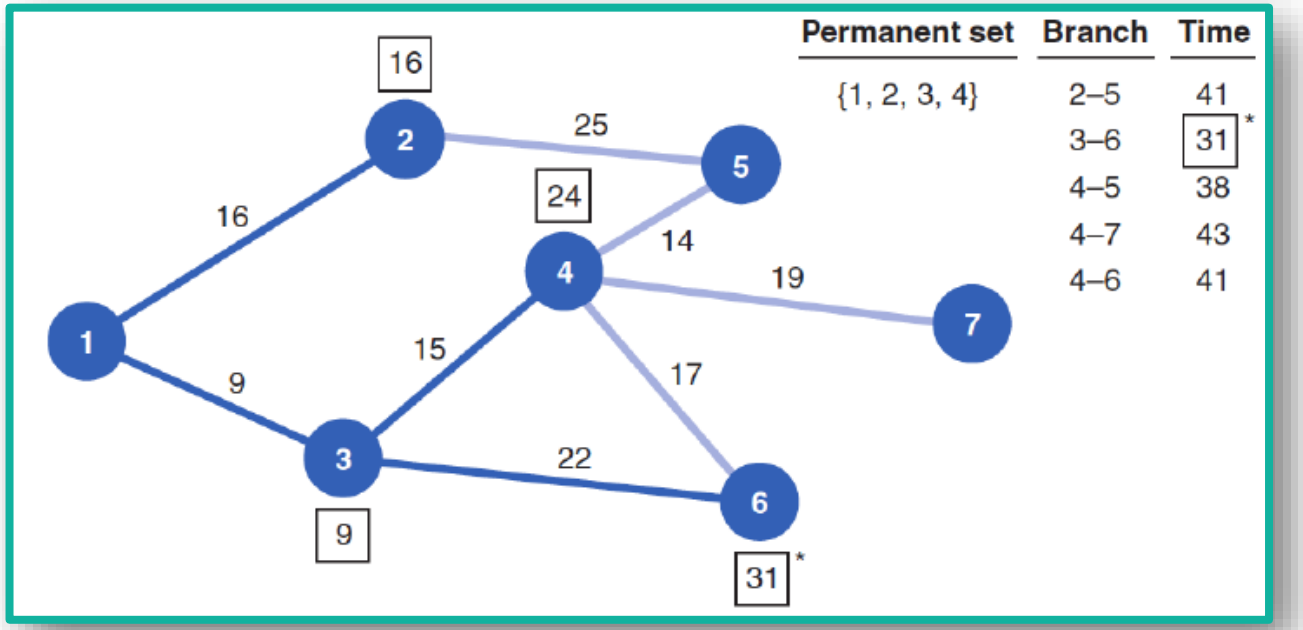


Permanent set	Branch	Time
{1, 2, 3}	1–4	35
	2–4	28
	2–5	41
	3–4	24*
	3–6	31

- Node 4 is the closest to the origin among {4,5,6} and added to the permanent set

Ex: Shortest Shipping Route

- Next, explore all the nodes adjacent to nodes in the permanent set {1,2,3,4}

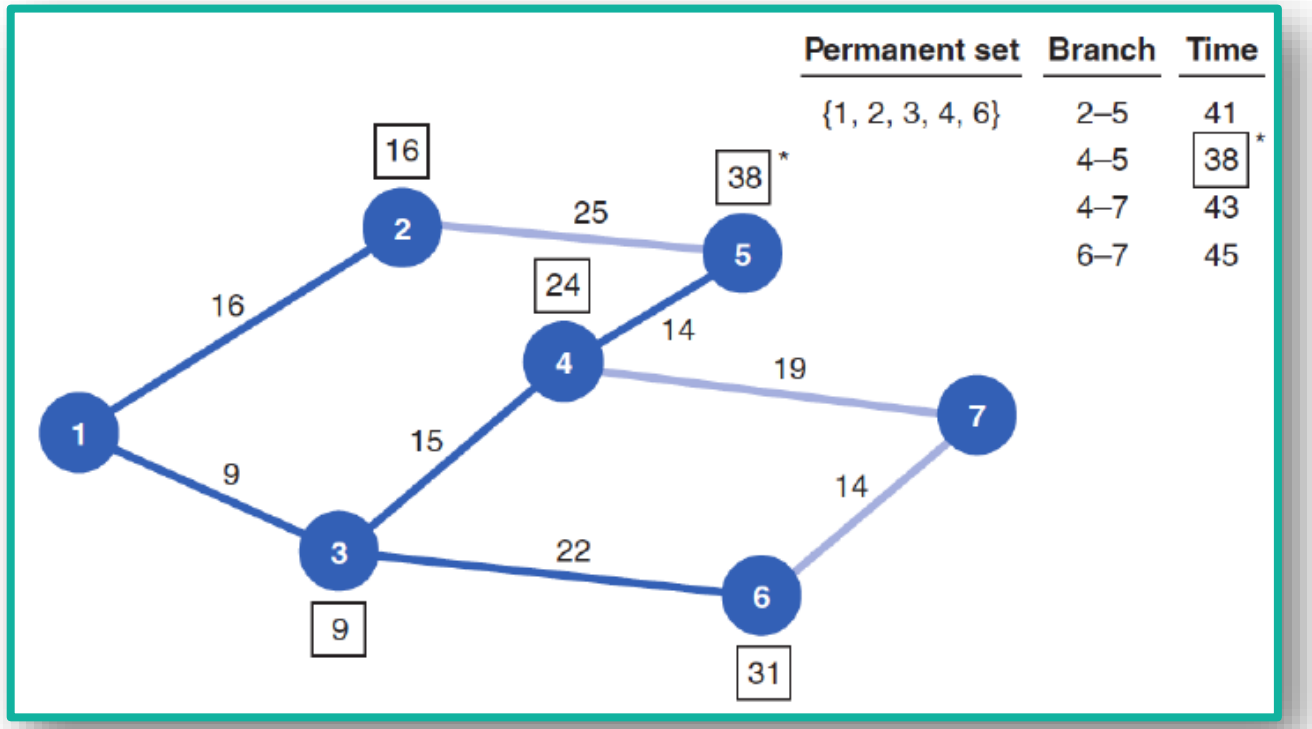


Permanent set	Branch	Time
{1, 2, 3, 4}	2–5	41
	3–6	31*
	4–5	38
	4–7	43
	4–6	41

- Node 6 is the closest to the origin and added to the permanent set

Ex: Shortest Shipping Route

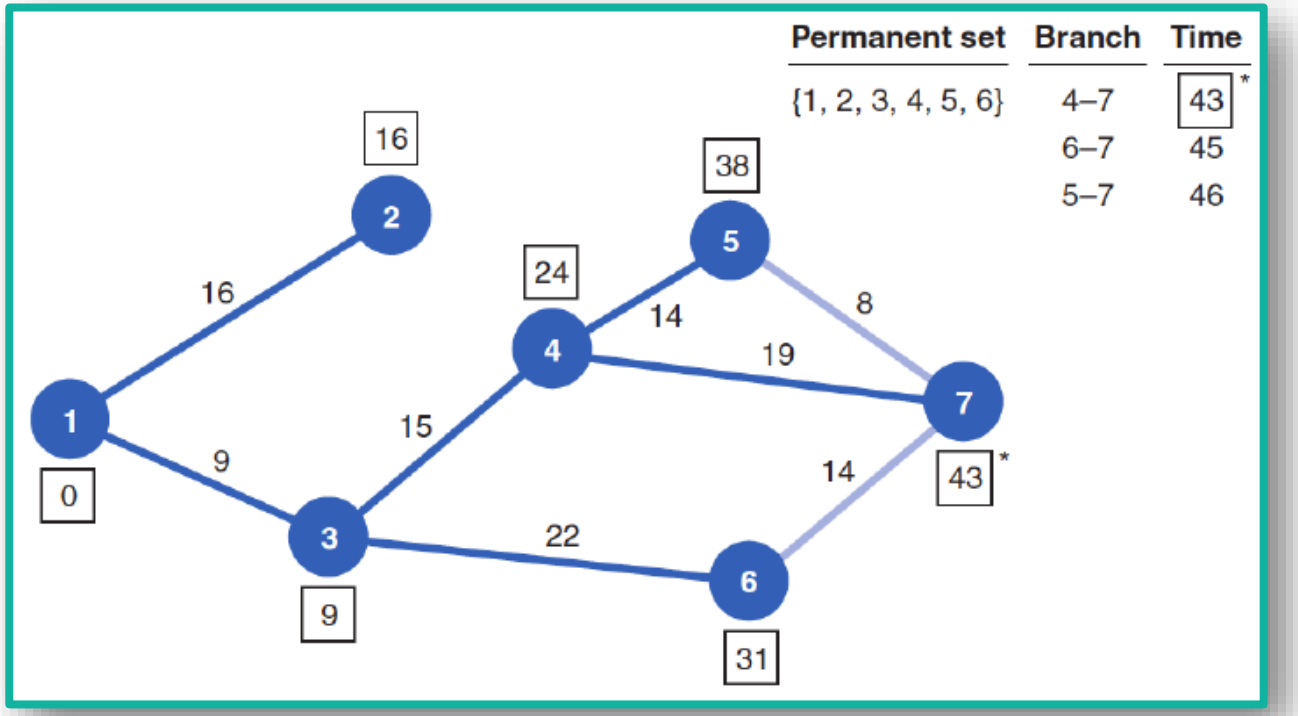
- Next, explore all the nodes adjacent to nodes in the permanent set {1,2,3,4,6}



- Node 5 is the closest to the origin and added to the permanent set

Ex: Shortest Shipping Route

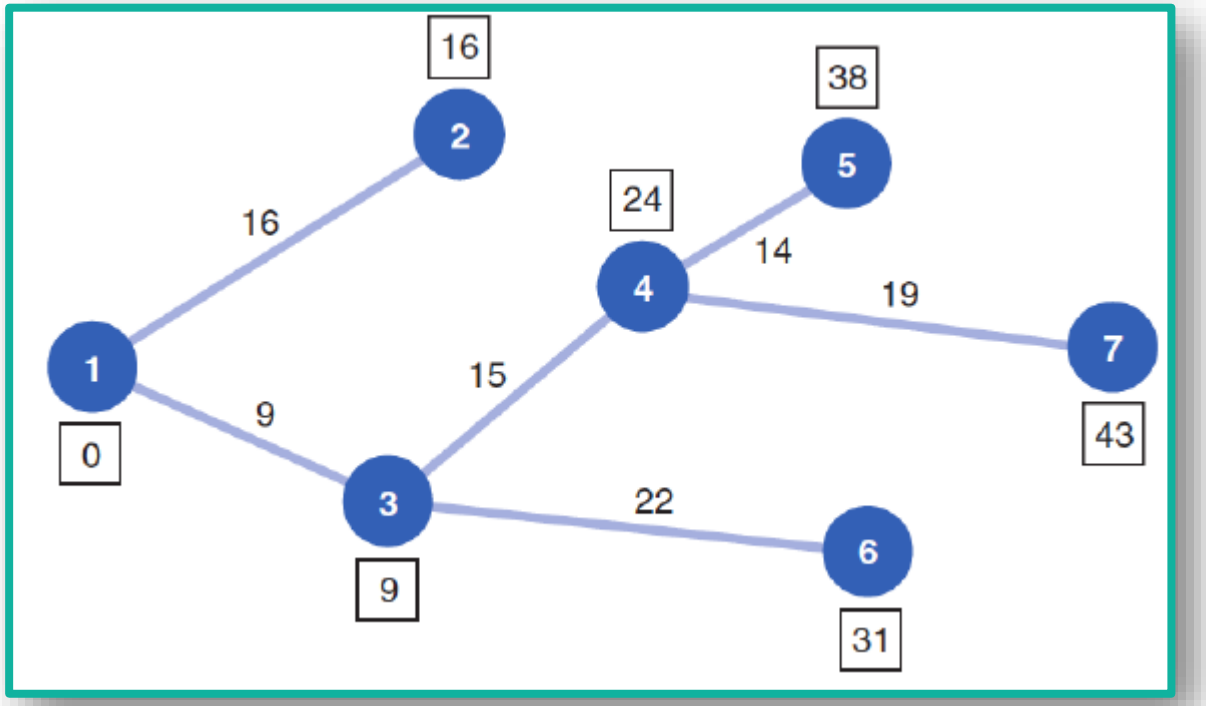
- Last, explore all the nodes adjacent to nodes in the permanent set {1,2,3,4,6,5}



- Node 7 is the last one added
- Minimum time from origin to node 7 is 43 (19+15+9)

Ex: Shortest Shipping Route

- Final solution represented graphically using a subset of the edges



- Minimum time travelled from node 1 to all other nodes

$$9 + 16 + 24 + 31 + 38 + 43 = 161$$

Ex: Shortest Shipping Route

- Optimal routes to reach each of the 7 cities

From L.A. to:	Route	Total hours
Salt Lake City (node 2)	1-2	16
Phoenix (node 3)	1-3	9
Denver (node 4)	1-3-4	24
Des Moines (node 5)	1-3-4-5	38
Dallas (node 6)	1-3-6	31
St. Louis (node 7)	1-3-4-7	43

Routes can be expressed using an ordered list of nodes

The End

Dale