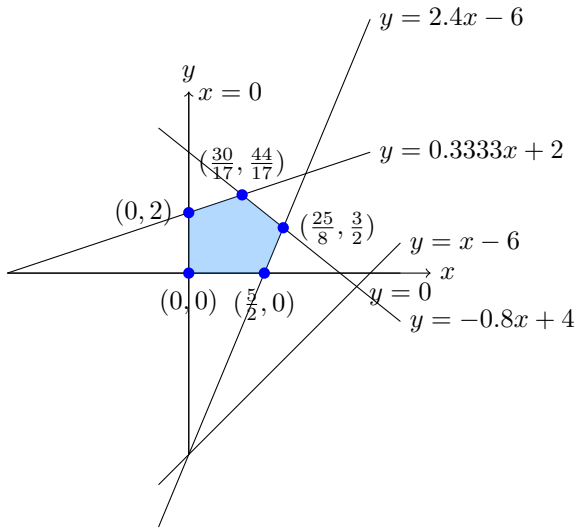
Assignment # 2 Solutions

due Friday, February 5th, 2021

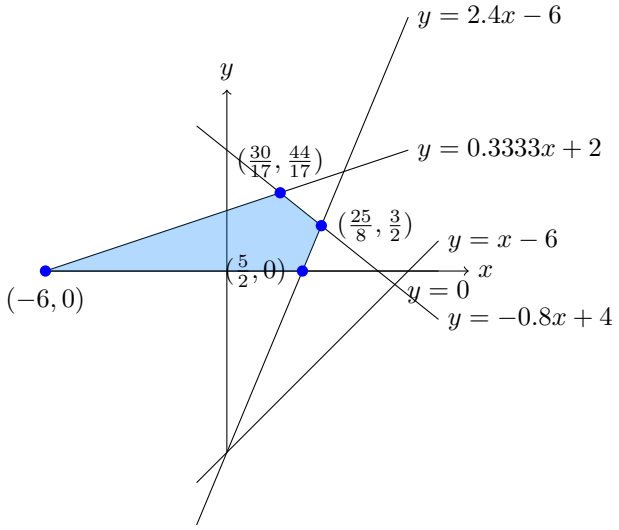
1

The feasible region is the blue (shaded) part of the graph.

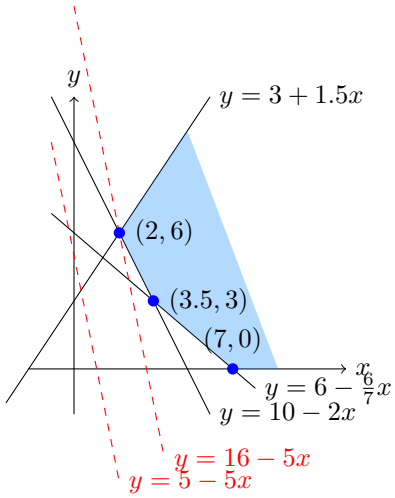


(a) There are 5 extreme points in the feasible region.

(b) The feasible region is bounded. The feasible region is still bounded if removing the constraint $x \geq 0$.



2 First, draw the feasible region (shaded area). Then, find the minimum value using level curves. The optimal solution is 32 with $(x, y) = (2, 6)$.



3

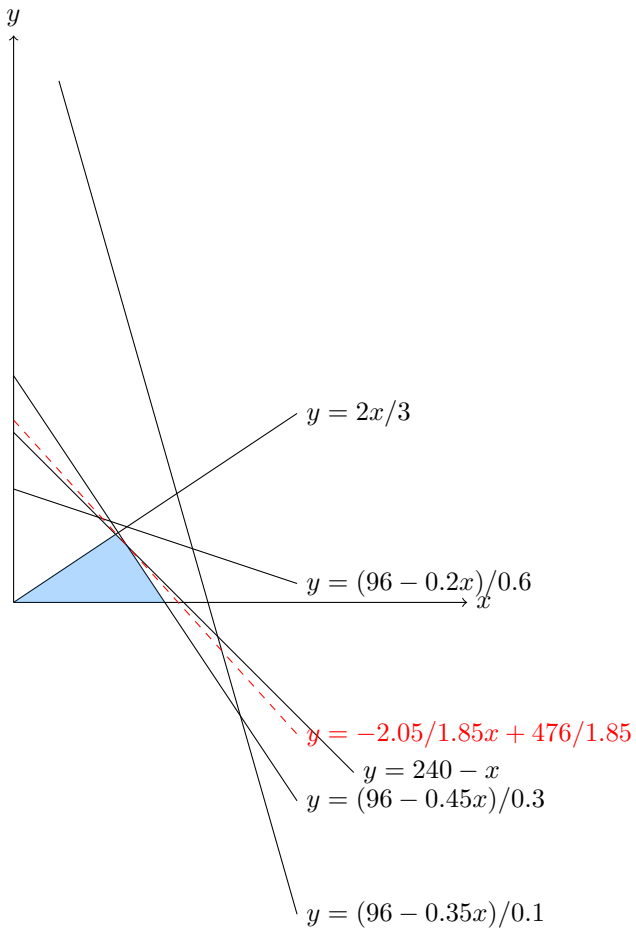
(a) Let

x = number of 16-ounce cups of Pomona sold each day.
y = number of 16-ounce cups of Coastal sold each day.

Note that 1 gallon = 128 ounces and 1 pound = 16 ounces. Then the model is the following:

$$\begin{aligned} \max z &= 2.05x + 1.85y \\ s.t.\ x + y &\le 30\frac{128}{16} = 240 \\ 0.2x + 0.6y &\le 96 \\ 0.35x + 0.1y &\le 96 \\ 0.45x + 0.3y &\le 96 \\ x - 1.5y &\ge 0 \\ x, y &\ge 0 \end{aligned}$$

(b) The graph is as follows. From the figure, we can see that the maximum value is 476 with $(x, y) = (160, 80)$.



(c) Using Excel Solver, we find that the optimal value and optimal solutions are the same as part(b).

1	**Q3(c)**						
2	Products:	Pomona	Coastal				
3	Profit per unit:	2.05	1.85				
4	Condition:			Usage	Constraint	Available	Left over
5	Capacity constraint	1.00	1.00	240.00	<=	240.00	0.00
6	Colombia constraint	0.20	0.60	80.00	<=	96.00	16.00
7	Kenya constraint	0.35	0.10	64.00	<=	96.00	32.00
8	Indonesia constrain	0.45	0.30	96.00	<=	96.00	0.00
9	Sells 1.5 times more Pomona than Coastal each day.	1.00	-1.50	40	>=	0.00	40.00
10							
11	Poduction:						
12	Pomona =	160.00					
13	Coastal =	80.00					
14	Return =	476.00					

4

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D10	Amounts Sold Usage	40	0	0	40	1E+30
D6	Colombian Usage	80	0	96	1E+30	16
D7	Kenyan Usage	64	0	96	1E+30	32
D8	Indonesian Usage	96	1.333333333	96	12	2.4
D9	Brewing Cap Usage	240	1.45	240	6.153846154	26.66666667

(a) If we examine the sensitivity report above, we see that our Colombian and Kenyan coffee supplies are not being fully utilized and thus have shadow prices of 0, meaning that an increase in the RHS of their respective constraints will have no effect on the optimal solution/objective value. This leaves Indonesian coffee, which is being fully utilized and has a positive shadow price, as the best choice of coffee for which to increase supply. However, since the allowable increase corresponding to its shadow price is only 12 ounces and a pound is 16 ounces, we will need to re-solve the problem with the updated supply to fully understand its effect on the optimal solution. The new solution is seen below—one additional pound of coffee leads to a $16 increase in daily revenue.

1	**Q5(a)**						
2	Products:	Pomona	Coastal				
3	Profit per unit:	2.05	1.85				
4	Condition:			Usage	Constraint	Available	Left over
5	Capacity constraint	1.00	1.00	240.00	<=	240.00	0.00
6	Colombia constraint	0.20	0.60	48.00	<=	96.00	48.00
7	Kenya constraint	0.35	0.10	84.00	<=	96.00	12.00
8	Indonesia constrain	0.45	0.30	108.00	<=	112.00	4.00
9	Sells 1.5 times more Pomona than Coastal each day.	1.00	-1.50	240	>=	0.00	240.00
10							
11	Poduction:						
12	Pomona =	240.00					
13	Coastal =	0.00					
14	Return =	492.00					

(b) Again consulting the sensitivity report above, we see that all brewing capacity is currently being used and that upping it will have a positive effect on sales. Once again, the allowable increase associated with the shadow price for brewing capacity (a little more than 6 cups) is too low for us to come to a conclusion using the sensitivity report alone, meaning we must re-solve the problem with the updated capacity. When we do, we see that the upped brewing capacity leads to an $8.92 increase in daily revenue, which would not be enough to offset the $15 a day cost of the increase.

1	**Q5(a)**						
2	Products:	Pomona	Coastal				
3	Profit per unit:	2.05	1.85				
4	Condition:			Usage	Constraint	Available	Left over
5	Capacity constraint	1.00	1.00	246.15	<=	320.00	73.85
6	Colombia constraint	0.20	0.60	88.62	<=	96.00	7.38
7	Kenya constraint	0.35	0.10	61.54	<=	96.00	34.46
8	Indonesia constrain	0.45	0.30	96.00	<=	96.00	0.00
9	Sells 1.5 times more Pomona than Coastal each day.	1.00	-1.50	0	>=	0.00	0.00
10							
11	Poduction:						
12	Pomona =	147.69					
13	Coastal =	98.46					
14	Return =	484.92					

(c) If the shop spent $20 per day on advertising that would increase the relative demand for Pomona to twice that of Coastal, the fifth constraint would become $x - 2y \geq 0$. Note that this constraint is

already met by our current optimal brewing policy (160 cups Pomona, 80 cups Coastal), so there is no reason to spend the additional \$20. This can also be seen by updating and re-solving the model, as shown below.

1	**Q5(b)**						
2	Products:	Pomona	Coastal				
3	Profit per unit:	2.05	1.85				
4	Condition:			Usage	Constraint	Available	Left over
5	Capacity constraint	1.00	1.00	240.00	<=	240.00	0.00
6	Colombia constraint	0.20	0.60	80.00	<=	96.00	16.00
7	Kenya constraint	0.35	0.10	64.00	<=	96.00	32.00
8	Indonesia constrain	0.45	0.30	96.00	<=	96.00	0.00
9	Sells 1.5 times more Pomona than Coastal each day.	1.00	-2.00	8.1185E-06	>=	0.00	0.00
10							
11	Poduction:						
12	Pomona =	160.00					
13	Coastal =	80.00					
14	Return =	476.00					