



Lecture 12

Produced by Dr. Worldwide

Welcome to the 305

Shortest Route Problem



- Dijkstra's algorithm for identifying the shortest route
- Step 1: Select the node with the shortest route from the origin
- Step 2: Create a permanent set that includes the origin and the node chosen in the 1st step
- Step 3: Identify all nodes that are adjacent to the nodes in the permanent set
- Step 4: Select the node with the shortest route from the group of nodes adjacent to the nodes in the permanent set. Add the chosen node to the permanent set
- Step 5: Repeat the 3rd and 4th steps until all nodes are in the permanent set

Shortest Route Problem



- We want to reformulate the problem as a linear program
- The origin in the shortest route problem can be thought of as a single **supply** node
- The other nodes can be thought of as **demand** nodes
- Source has supply equal to the number of nodes in the graph minus one (itself)
- Each demand node requires a single unit
- Distance between nodes corresponds to transportation cost of that edge
- To reduce the number of variables, we assume units only flow in the direction of a higher node number

Ignore x_{ij} if $i \geq j$

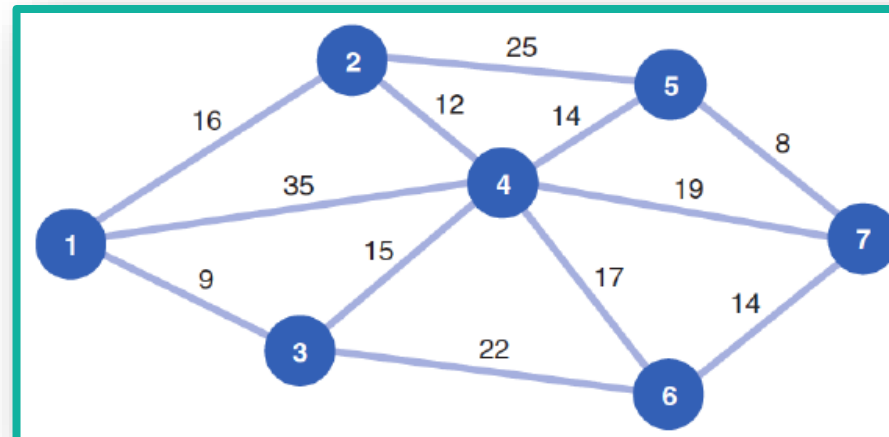
Ex: Shortest Shipping Route



- Decision variables
 - x_{ij} = number of trucks transported along edge (i,j)
 - $i = \{1,2,3,\dots,7\}$
 - $j = \{1,2,3,\dots,7\}$
 - $i < j$

- Objective function

$$Z = 16x_{12} + 35x_{14} + 9x_{13} + 12x_{24} + 15x_{34} + 25x_{25} + 14x_{45} + 17x_{46} + 22x_{36} + 8x_{57} + 19x_{47} + 14x_{67}$$



Ex: Shortest Shipping Route



- Whatever gets into a node leaves the node is known as **flow conservation**
- Origin produces 1 unit of flow and the node with largest index must get 1 unit
- Constraints
 - $x_{12} + x_{13} + x_{14} = 6$ (Out of node 1)
 - $x_{12} = x_{24} + x_{25} + 1$ (Through node 2)
 - $x_{13} = x_{34} + x_{36} + 1$ (Through node 3)
 - $x_{14} + x_{24} + x_{34} = x_{45} + x_{46} + x_{47} + 1$ (Through node 4)
 - $x_{25} + x_{45} = x_{57} + 1$ (Through node 5)
 - $x_{36} + x_{46} = x_{67} + 1$ (Through node 6)
 - $x_{47} + x_{57} + x_{67} = 1$ (Into node 7)
 - x_{ij} is an integer
- Q: Why the **+1** in the constraints?

Ex: Shortest Shipping Route



- Download [ShortestRoute.xlsx](#) from course website from link [Sheet 1](#)
- Look at tab titled [One-to-All](#)

Units shipped	Node	City	Node	City	Distance (hours)
1	1	Los Angeles	2	Salt Lake City	16
5	1	Los Angeles	3	Phoenix	9
0	1	Los Angeles	4	Denver	35
0	2	Salt Lake City	4	Denver	12
0	2	Salt Lake City	5	Des Moines	25
3	3	Phoenix	4	Denver	15
1	3	Phoenix	6	Dallas	22
1	4	Denver	5	Des Moines	14
0	4	Denver	6	Dallas	17
1	4	Denver	7	St. Louis	19
0	5	Des Moines	7	St. Louis	8
0	6	Dallas	7	St. Louis	14
			Total		161

Ex: Shortest Shipping Route



- Look at tab titled **One-to-One**

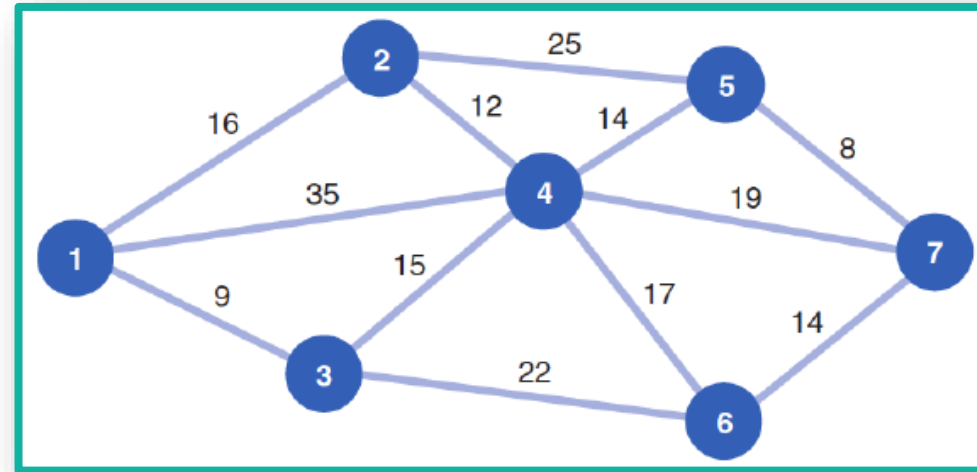
Arcs used	Node	City	Node	City	Distance (hours)
0	1	Los Angeles	2	Salt Lake City	16
1	1	Los Angeles	3	Phoenix	9
0	1	Los Angeles	4	Denver	35
0	2	Salt Lake City	4	Denver	12
0	2	Salt Lake City	5	Des Moines	25
1	3	Phoenix	4	Denver	15
0	3	Phoenix	6	Dallas	22
0	4	Denver	5	Des Moines	14
0	4	Denver	6	Dallas	17
1	4	Denver	7	St. Louis	19
0	5	Des Moines	7	St. Louis	8
0	6	Dallas	7	St. Louis	14
			Total		43

- Q:What is the difference between **One-to-All** and **One-to-One**?

Shortest Route Problem



- Assumed that we had supply at origin (node 1) to fulfill demand at all other nodes



- Suppose we only wanted to get one unit from the origin to one destination node
- We must modify the constraints to reflect a supply and demand of one

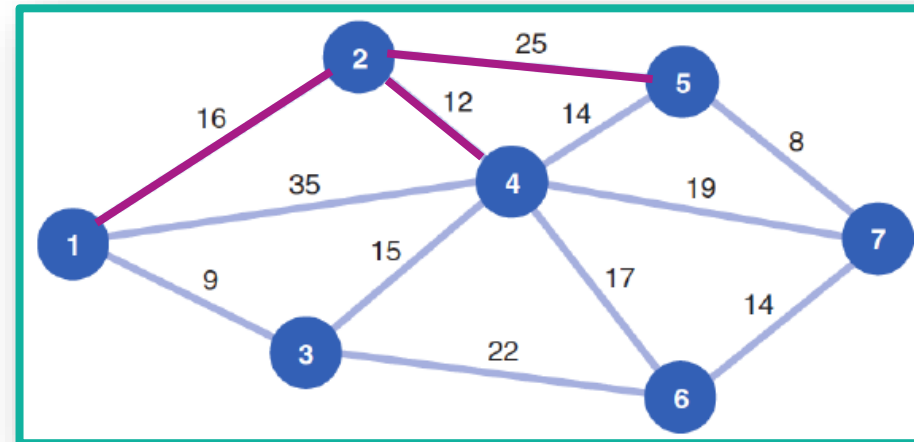
Ex: Shortest Shipping Route



- Modification of constraints
 - One origin to one destination

Flow constraints:

Node	Network Flow	Constraint	Value
1	1	=	1
2	0	=	0
3	0	=	0
4	0	=	0
5	0	=	0
6	0	=	0
7	1	=	1



- One origin to many destinations

Flow constraints:

Node	Network Flow	Constraint	Value
1	6	=	6
2	1	=	1
3	1	=	1
4	1	=	1
5	1	=	1
6	1	=	1
7	1	=	1

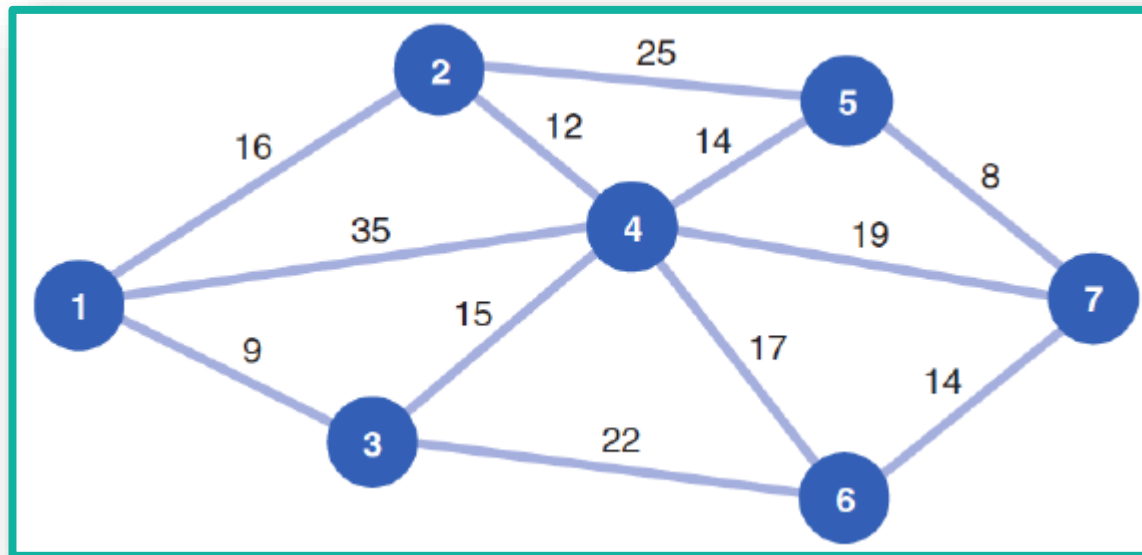
Difference Between In and Out

$$x_{12} - x_{24} - x_{25}$$

Shortest Route Problem



- Assumed that edges had direction from West (node 1) to East (nodes 2-7)

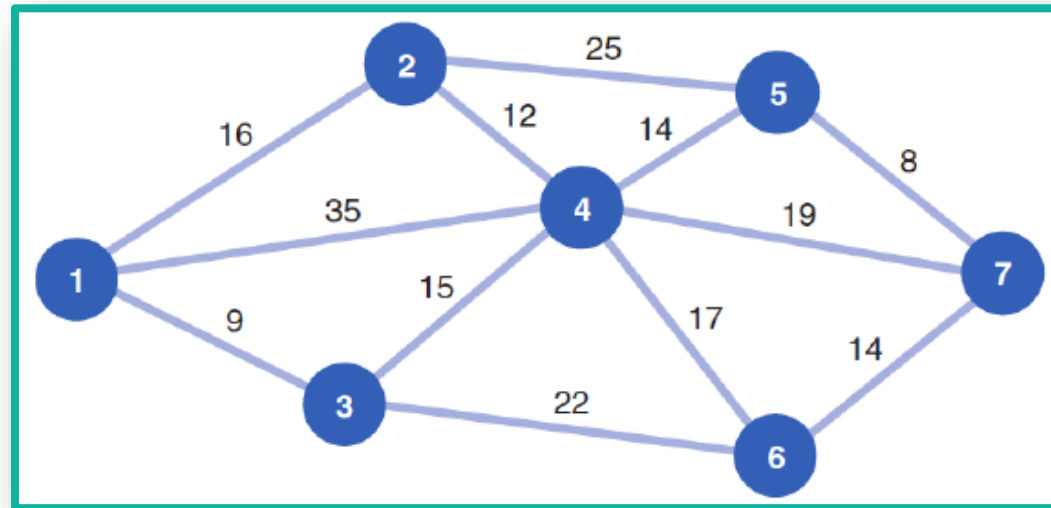


- Suppose direction doesn't matter
- We must consider both directions for each branch

Ex: Shortest Shipping Route



- Decision variables for undirected network
 - x_{ij} = number of trucks transported along edge (i,j)
 - $i = \{1,2,3,\dots,7\}$
 - $j = \{1,2,3,\dots,7\}$
 - $i < j$ or $i > j$



- Objective function for undirected network

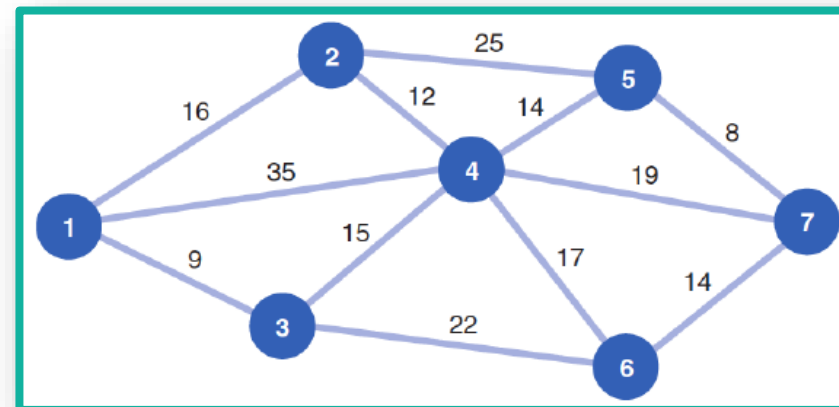
$$\begin{aligned} Z = & 16(x_{12} + x_{21}) + 35(x_{14} + x_{41}) + 9(x_{13} + x_{31}) + 12(x_{24} + x_{42}) \\ & + 15(x_{34} + x_{43}) + 25(x_{25} + x_{52}) + 14(x_{45} + x_{54}) + 17(x_{46} + x_{64}) \\ & + 22(x_{36} + x_{63}) + 8(x_{57} + x_{75}) + 19(x_{47} + x_{74}) + 14(x_{67} + x_{76}) \end{aligned}$$

Ex: Shortest Shipping Route



- Constraints for undirected network

- $x_{21} + x_{31} + x_{41} - x_{12} - x_{13} - x_{14} = -6$ (Node 1)
- $x_{12} + x_{52} + x_{42} - x_{21} - x_{24} - x_{25} = 1$ (Node 2)
- $x_{13} + x_{43} + x_{63} - x_{31} - x_{34} - x_{36} = 1$ (Node 3)
- $x_{14} + x_{24} + x_{34} + x_{54} + x_{64} + x_{74} - x_{41} - x_{42} - x_{43} - x_{45} - x_{46} - x_{47} = 1$ (Node 4)
- $x_{25} + x_{45} + x_{75} - x_{52} - x_{54} - x_{57} = 1$ (Node 5)
- $x_{36} + x_{46} + x_{76} - x_{63} - x_{64} - x_{67} = 1$ (Node 6)
- $x_{47} + x_{57} + x_{67} - x_{74} - x_{75} - x_{76} = 1$ (Node 7)
- x_{ij} is an integer



Ex: Shortest Shipping Route



- Download [ShortestRoute-1.xlsx](#) from course website from link [Sheet 2](#)
- The tabs [One-to-All-Directed](#) and [One-to-One-Directed](#) contain previous results
- Closely examine the tabs [One-to-All-Undirected](#) and [One-to-One-Undirected](#)
- Q: What do you notice is the same between [directed](#) and [undirected](#) problems?
- Q: What do you notice is different between [directed](#) and [undirected](#) problems?
- Q: What is the purpose of the number [1,000](#) in the [distance matrix](#)?





The End



Dale

