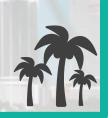
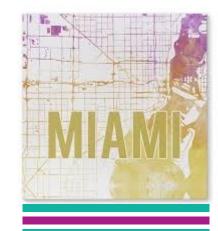


Goal Programming



- All prior linear programming problems have had a single objective function
- Companies may have multiple criteria in consideration for a decision
- Sometimes the multiple objectives conflict
- Company may want to maximize profit and minimize pollution
- Goal programming is linear programming for multiple objectives or criteria





- Trying to choose $x = number\ of\ bowls\$ and $y = number\ of\ mugs\$ to maximize the profit function
- Recall the original linear program

Maximize
$$40x + 50y$$

Subject to:
$$x + 2y \le 40$$
 (Labor)

$$4x + 3y \le 120 \tag{Clay}$$

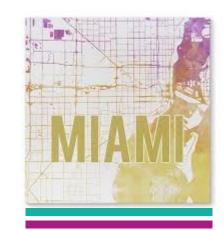
$$x, y \ge 0$$

Objective function reflects a single goal





- Suppose Beaver Creek wanted to achieve other goals while maximizing profit
- In order of importance:
 - To avoid layoffs, they want to use at least 40 hours of labor per day
 - They want to achieve a satisfactory profit level of \$1,600 per day
 - To avoid having clay dry out, they prefer to keep no more than 120 lb of clay on hand each day
 - To avoid overhead costs due to keeping the factory open past normal hours, they want to minimize the amount of overtime
- We reformulate our linear programming model using goal programming
- Transform linear programming model constraints into goals





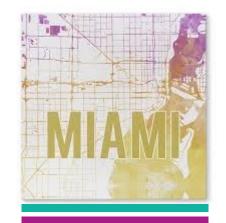
- Goal 1: Avoid underutilization of labor
 - Original constraint $x + 2y \le 40$
 - Reformulation to a goal constraint

$$x + 2y + d_1^- - d_1^+ = 40$$
 (Labor)

- Two new variables d_1^- and d_1^+ are nonnegative and represent the underutilized time and overtime, respectively
- Q: What if the optimal solution had $d_1^- > 0$?
- Q: What if the optimal solution had $d_1^+ > 0$?
- The top priority corresponding to minimization of d_1^-

Minimize
$$P_1d_1^-$$

• The P_1 indicates the priority of this goal (not a coefficient)





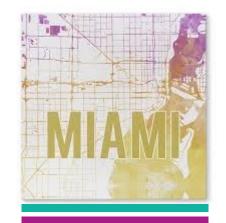
- Goal 2: Achieve daily profit of \$1,600
 - Original objective function Z = 40x + 50y
 - Reformulation to a goal constraint

$$40x + 50y + d_2^- - d_2^+ = 1600$$
 (Profit)

- Two new variables d_2^- and d_2^+ are nonnegative and represent the amount of profit less than \$1,600 and more than \$1,600
- The second priority corresponding to minimization of d_2^- is added

Minimize
$$P_1d_1^-$$
, $P_2d_2^-$

- The comma between the terms indicates that we are minimizing them sequentially, not simultaneously
- Q: Why are we not minimizing d_2^+ ?



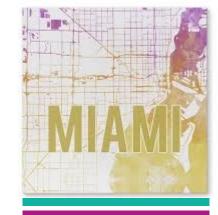


- Goal 3: Avoid waste of material
 - Original constraint $4x + 3y \le 120$
 - Reformulation to a goal constraint

$$4x + 3y + d_3^- - d_3^+ = 120$$
 (Clay)

- Two new variables d_3^- and d_3^+ are nonnegative and represent the amount of clay less than 120 lbs and more than 120 lbs
- The company cannot keep more than 120 lbs in storage
- The third priority corresponds to minimization of d_3^+ is added

Minimize $P_1 d_1^-, P_2 d_2^-, P_3 d_3^+$



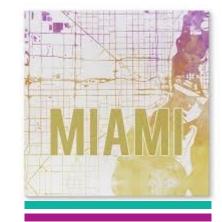


- Goal 4: Avoid overtime costs
 - Recall the modified goal constraint for labor

$$x + 2y + d_1^- - d_1^+ = 40$$
 (Labor)

- Already attempting to minimize d_1^-
- To ensure we don't exceed the maximum labor, we involve d_1^+
- Finalization of objective function

Minimize
$$P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_1^+$$



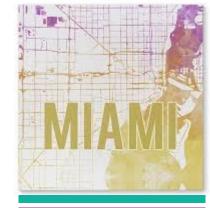


Full goal programming model

Minimize
$$P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_1^+$$

Subject to $x + 2y + d_1^- - d_1^+ = 40$ (Labor) $40x + 50y + d_2^- - d_2^+ = 1600$ (Profit) $4x + 3y + d_3^- - d_3^+ = 120$ (Clay) $x, y, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \ge 0$

- The variables $\{d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+\}$ are called deviational variables
- We minimize the four different objective functions individually by priority





- Modification 1: Prefer no more than 10 hours of overtime
 - Recall the goal constraint for labor

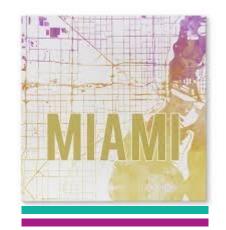
$$x + 2y + d_1^- - d_1^+ = 40$$
 (Under hours)

- Remember that d_1^+ represents overtime
- We want $0 \le d_1^+ \le 10$
- Use same strategy as before by adding a goal constraint

$$d_1^+ + d_4^- - d_4^+ = 10$$
 (Over hours)

- Possible goal constraint of all deviational variables
- Two new variables d_4^- and d_4^+ are nonnegative and represent the amount of overtime hours less than 10 hours and more than 10 hours
- New objective function

Minimize
$$P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_4^+$$

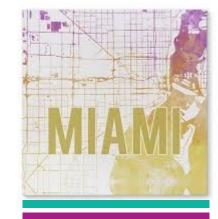




- Modification 2: Maximum number of bowls and mugs made daily
 - Pottery company has limited warehouse space
 - They can only store at most 30 bowls and 20 mugs each day
 - Profit for bowls (\$40) less than profit for mugs (\$50)
 - Consider the new constraints

$$x + d_5^- = 30$$
 (Bowls)
 $y + d_6^- = 20$ (Mugs)

- We want to minimize d_5^- and d_6^-
- Q: Why not include positive deviational variables d_5^+ and d_6^+ ?
- Q: For which item is it more important to achieve this goal?





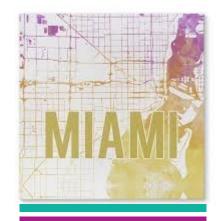
- Modification 2: Maximum number of bowls and mugs made daily
 - Positive deviational variables are unnecessary since it is imperative to not exceed the warehouse space
 - We need to achieve the goal for mugs more than the goal for bowls because the profit is higher for mugs
 - If goals were of equal importance, we would minimize

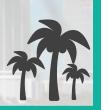
Minimize
$$P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_4^+, P_5(d_5^- + d_6^-)$$

- We can make the degree of importance in proportion to the profit
- The goal for mugs is more important than the goal for bowls by a ratio of 5 to 4

Minimize
$$P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_4^+, P_5(4d_5^- + 5d_6^-)$$

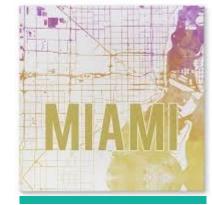
• The coefficients 4 and 5 are referred to as weights





• Full modified goal programming model

Minimize	$P_1d_1^-, P_2d_2^-, P_3d_3^+, P_4d_4^+, 4P_5d_5^- + 5P_5d_6^-$	
Subject to	$x + 2y + d_1^ d_1^+ = 40$ $40x + 50y + d_2^ d_2^+ = 1600$ $4x + 3y + d_3^ d_3^+ = 120$ $d_1^+ + d_4^ d_4^+ = 10$ $x + d_5^- = 30$ $y + d_6^- = 20$ $x, y, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+, d_5^-, d_6^-$	(Labor)(Profit)(Clay)(Overtime)(Bowls)(Mugs)≥ 0









The End





