Lecture 3

Produced by Dr. Worldwide

Welcome to the 305

Linear Programming

- Inequality constraints in a linear in a linear program with 2-variables usually lead to a feasible region in the shape of a polygon
- The feasible region can be bounded or unbounded
- The corners of the polygon are called extreme points
- In problems with $d \geq 3$ decision variables, the feasible region is a d −dimensional polytope, which can be bounded or unbounded
- The corners of the polytope are called extreme points
- Unusual Cases
    - Multiple optimal solutions
    - Infeasible problem
    - Unbounded problem

Simplex Algorithm

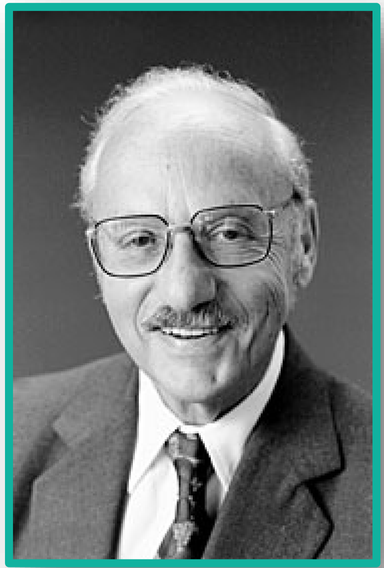
- Theorem: If a linear program has an optimal solution, then it always has an optimal solution which is an extreme point

- The simplex algorithm was designed by George Dantzig to solve linear programs
  - Intelligently explores the feasible region to find extreme points
  - Useful for linear programs in standard form

Maximize $\boldsymbol{c}^T\boldsymbol{x}$

Subject to $\boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b}$

$\boldsymbol{x} \geq \boldsymbol{0}$



  - First an extreme point must be identified
  - If this point is not optimal, then an edge exists to Another extreme point where the objective function becomes closer to optimal

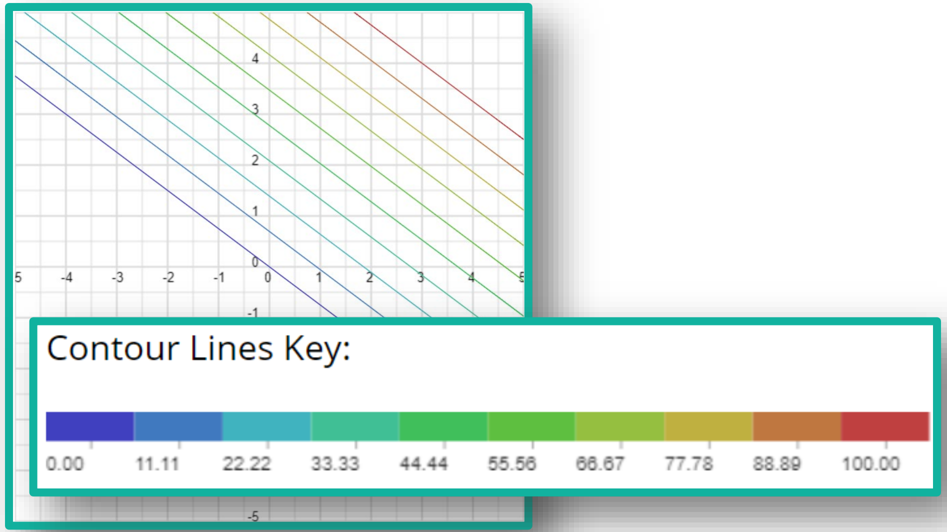
Ex: *The Possibility* Restaurant

- Angela Fox and Zooey Caulfield studied food and nutrition at UNC
- They want to open a French restaurant in Chapel Hill called *The Possibility*
- Unaware of the local customer's tastes, they decide to serve only 2 full-course meals around beef and fish
- Chef Pierre plans to experiment with different appetizers, soups, salads, deserts, etc. to identify the best selection of menu items
- Q: What considerations exist for Angela and Zooey to optimize their business?

Ex: *The Possibility* Restaurant

- Decision Variables:
  - $x = Number\ of\ Fish\ Meals\ Each\ Night$
  - $y = Number\ of\ Beef\ Meals\ Each\ Night$

- They plan to profit $12 from each fish dinner and $16 from each beef dinner
  - Goal: Maximize their nightly profit
  - Objective function: $f(x, y) = Z = 12x + 16y$



Ex: *The Possibility* Restaurant

- Constraints
  - Number of dinners is nonnegative: $x \geq 0 \quad \& \quad y \geq 0$
  - Angela and Zooey estimate that they will sell a maximum of 60 meals each night: $x + y \leq 60$
  - Each fish dinner requires 15 minutes to prepare, each beef dinner takes twice as long, and there is a total of 20 hours of kitchen staff labor available each day: $15x + 30y \leq 1200$ (or $x + 2y \leq 80$)
  - Based on the health consciousness of their potential clientele, they will sell at least three fish dinners for every two beef dinners: $\frac{x}{y} \geq \frac{3}{2}$ (or $2x - 3y \geq 0$)
  - They also believe a minimum of 10% of their customers will order beef dinners: $\mathrm{y} \geq 0.1(x + y)$ (or $x - 9y \leq 0$)

Ex: *The Possibility* Restaurant

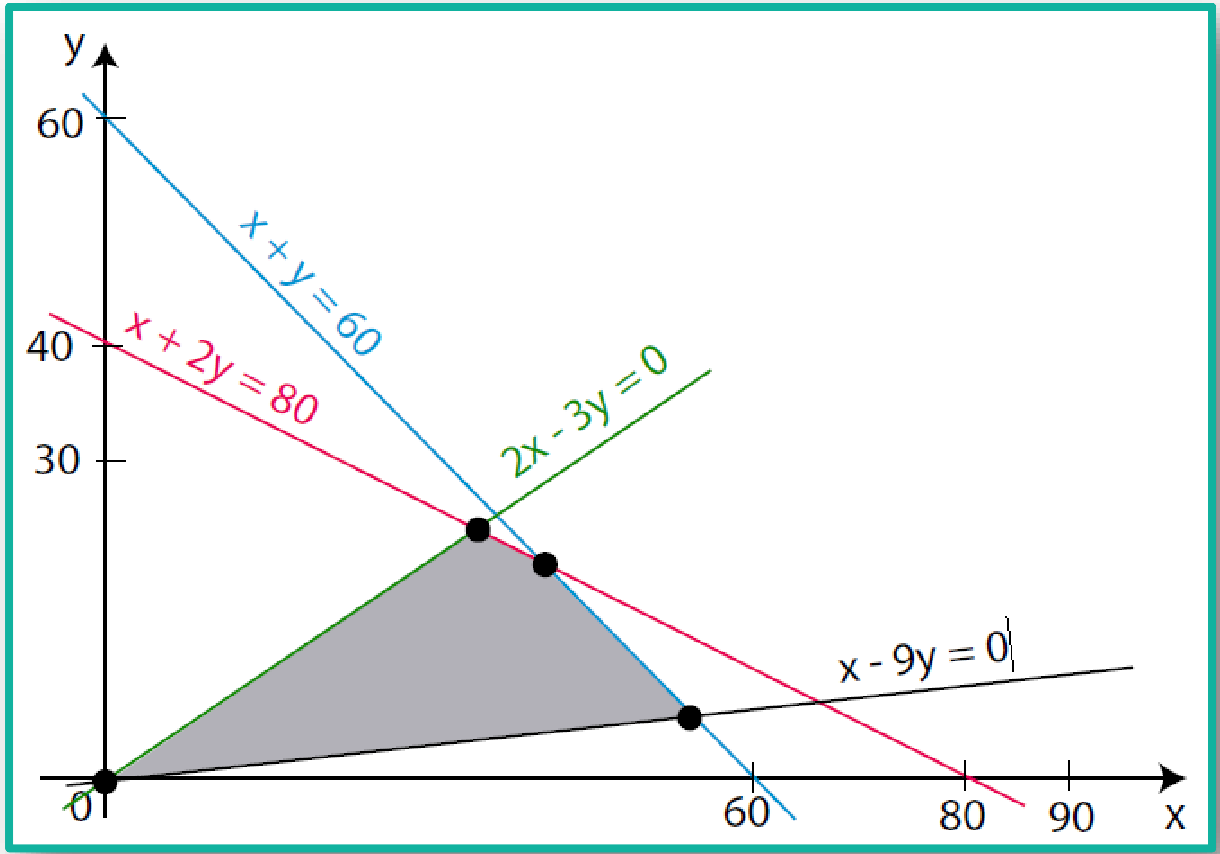
- Complete linear program

$$
\begin{array}{ll}
\text{Maximize} & 12x + 16y \\
\text{Subject to} & x + y \leq 60 \\
& x + 2y \leq 80 \\
& 2x - 3y \geq 0 \\
& x - 9y \geq 0 \\
& x \geq 0 \\
& y \geq 0
\end{array}
$$

- Since there are 2 decision variables, we can solve it graphically

Ex: *The Possibility* Restaurant

- Graph of feasible region (use origin to determine which side to shade)

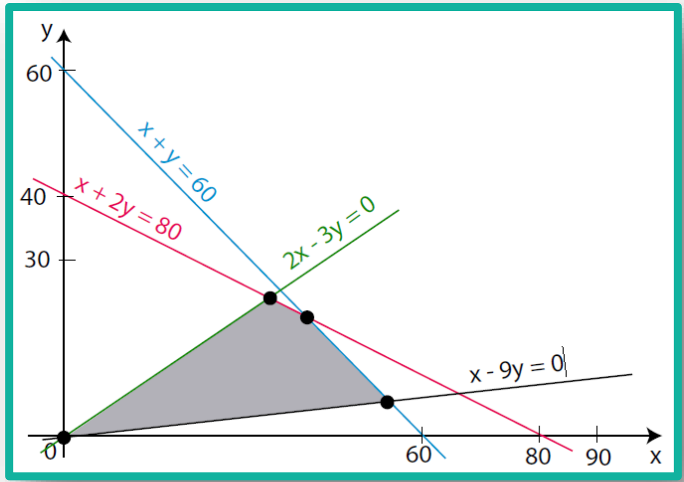


Ex: *The Possibility* Restaurant

- Find the corners of the feasible region
  - Origin: (0,0)
  - Intersection of Green and Red: (34.3,22.8)

$$2(80-2y)-3y=0$$
$$160-4y-3y=0$$
$$160-7y=0$$
$$y=\frac{160}{7}=22.8$$
$$x=80-2y=80-2(22.8)=34.3$$

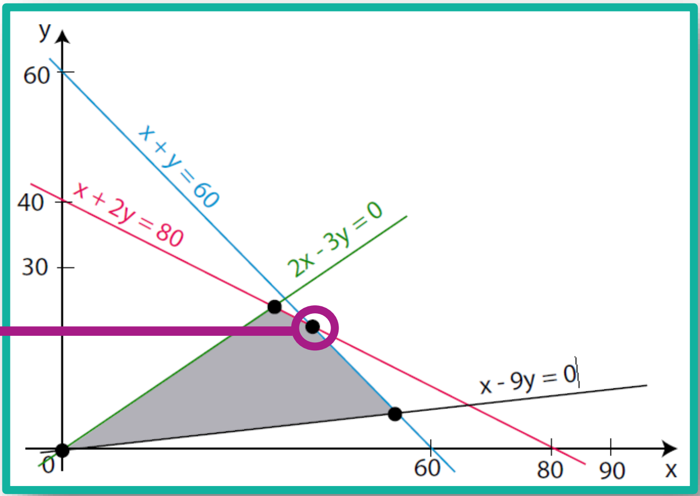
  - Intersection of Blue and Red: (40,20)
  - Intersection of Blue and Black: (54,6)



Ex: *The Possibility* Restaurant

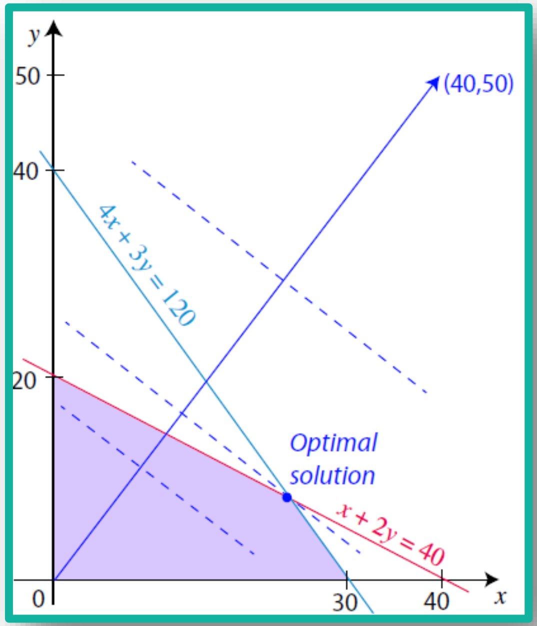
- Evaluate objective function at extreme points

x	y	12x+16y
0	0	0
34.3	22.8	776.4
40	20	800
54	6	744



Ex: *The Possibility* Restaurant

- Alternative approach: use growth vector and level curves (contours)
  - Computing all extreme points can be time-consuming
  - For objective function in form $Z = ax + by$ the growth vector is the vector starting at the origin and in the direction of (a, b)
  - The last perpendicular line along the growth curve that intersects the feasible region will intersect at the optimal solution



The End

Dale