



- University bookstore is considering several expansion projects
- Some projects require 2-years and some projects require 3 years

	NPV return	Projec	ct costs	/year (\$1,000s)
Project	(\$1,000s)	1	2	3
1. Website	\$120	\$55	\$40	\$25
2. Warehouse	85	45	35	20
<ol><li>Clothing department</li></ol>	105	60	25	-
4. Computer department	140	50	35	30
5. ATMs	70	30	30	-
Available funds per year		\$150	\$110	\$60

• Not enough space available for computer and clothing department





- Q: Which projects should the director select to maximize returns?
- Binary decision variables (indicator variables)

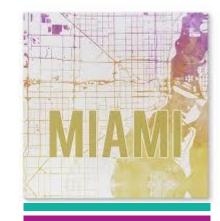
• 
$$x_1 = \begin{cases} 1 & if website selected \\ 0 & otherwise \end{cases}$$

• 
$$x_2 = \begin{cases} 1 & if warehouse selected \\ 0 & otherwise \end{cases}$$

• 
$$x_3 = \begin{cases} 1 & if \ clothing \ department \ selected \\ 0 & otherwise \end{cases}$$

• 
$$x_4 = \begin{cases} 1 & if \ computer \ department \ selected \\ 0 & otherwise \end{cases}$$

• 
$$x_5 = \begin{cases} 1 & if \ ATM \ selected \\ 0 & otherwise \end{cases}$$

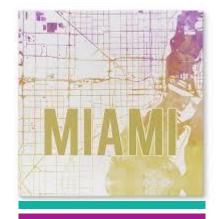




Linear program in standard form

Maximize 
$$120x_1 + 85x_2 + 105x_3 + 140x_4 + 70x_5$$
 Subject to 
$$55x_1 + 45x_2 + 60x_3 + 50x_4 + 30x_5 \le 150$$
 
$$40x_1 + 35x_2 + 25x_3 + 35x_4 + 30x_5 \le 110$$
 
$$25x_1 + 20x_2 + 30x_4 \le 60$$
 
$$x_3 + x_4 \le 1$$
 
$$x_1, x_2, x_3, x_4 \in \{0,1\}$$

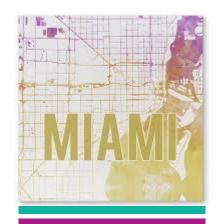
- Decision variables are binary making this a 0-1 Integer Model
- Download MachineShop.xlsx from course website from link Sheet 1







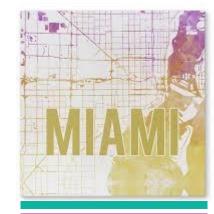
	Α	В	С	D	Е	F	G	Н	1
1	Capital budgeting								
2									
3	Projects to pursue	Website	Warehouse	Clothing Dept.	Computer Dept.	ATMs			
4									
5	NPV return (\$1000s):	120	85	105	140	70			
6	Resources:						Spent	Constraint	Available
7	Budget year 1	55	45	60	50	30	135	<=	150
8	Budget year 2	40	35	25	35	30	105	<=	110
9	Budget year 3	25	20	0	30	0	55	<=	60
10	Space constraint	0	0	1	1	0	1	<=	1
11									
12	Indicators of selected p	rojects:			Subject to the	e Constrai	nts:		
13	Website	1							
14	Warehouse	0			\$B\$13:\$B\$17	7 = binary			
15	Clothing Dept.	0			\$G\$10 <= \$I\$10				
16	Computer Dept.	1		\$G\$7 <= \$I\$7					
17	ATMs	1							
18	NPV return =	330		\$G\$8 <= \$I\$8					
					\$G\$9 <= \$I\$	59			





- American Parcel Service (APS) has determined it needs to add several new package distribution hubs to service cities east of the Mississippi River
- APS desires to construct the minimum set of new hubs in the following 12 cities so that there is a hub within 300 miles of each city

City	Cities within 300 miles
1. Atlanta	Atlanta, Charlote, Nashville
2. Boston	Boston, New York
3. Charlotte	Atlanta, Charlotte, Richmond
4. Cincinnati	Cincinnati, Detroit, Indianapolis, Nashville, Pittsburgh
5. Detroit	Cincinnati, Detroit, Indianapolis, Milwaukee, Pittsburgh
6. Indianapolis	Cincinnati, Detroit, Indianapolis, Milwaukee, Nashville, St. Louis
7. Milwaukee	Detroit, Indianapolis, Milwaukee
8. Nashville	Atlanta, Cincinnati, Indianapolis, Nashville, St. Louis
9. New York	Boston, New York, Richmond
10. Pittsburgh	Cincinatti, Detroit, Pittsburgh, Richmond
11. Richmond	Charlotte, New York, Pittsburgh, Richmond
12. St. Louis	Indianapolis, Nashville, St. Louis



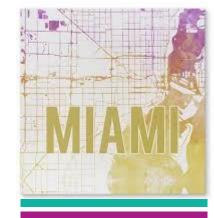




- Binary decision variables (indicator variables)
  - $x_i = \begin{cases} 1 & \text{if city } i \text{ is selected to be a hub} \\ 0 & \text{otherwise} \end{cases}$
  - $i \in \{1, 2, \dots, 12\}$
- Q: How can we select the minimum number of hubs that cover all the cities?
- Objective function

$$Z = x_1 + x_2 + \dots + x_{12} = \sum_{i=1}^{12} x_i$$

• We need to specify individual constraints for all 12 cities because we need to cover all 12 cities







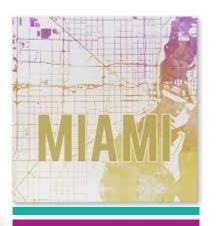
City	Cities within 300 miles
<ol> <li>Atlanta</li> </ol>	Atlanta, Charlote, Nashville
2. Boston	Boston, New York
3. Charlotte	Atlanta, Charlotte, Richmond

$x_1 + x_3 + x_8 \ge 1$	(To Cover Atlanta)
$x_2 + x_9 \ge 1$	(To Cover Boston)
$x_1 + x_3 + x_{11} \ge 1$	(To Cover Charlotte)

- Download SetCovering.xlsx from course website from link Sheet 2
- Run Excel Solver to find the solution



- 1. Atlanta
- 2. Boston
- 3. Charlotte
- 4. Cincinnati
- 5. Detroit
- 6. Indianapolis
- 7. Milwaukee
- 8. Nashville
- 9. New York
- 10. Pittsburgh
- 11. Richmond
- 12. St. Louis

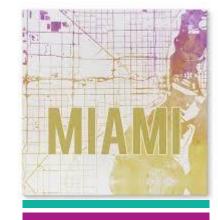




- Minimum number of distribution hubs needed is 4
  - Optimal solution given below with duplicate cities underlined

1. Atlanta	Atlanta, <u>Charlote</u> , <u>Nashville</u>
2. Boston	Boston, New York
6. Indianapolis	Cincinnati, Detroit, Indianapolis, Milwaukee, Nashville, St. Louis
11. Richmond	Charlotte, New York, Pittsburgh, Richmond

Other optimum solutions exist i.e. {Boston, Charlotte, Detroit, St. Louis}





- General transportation problem
  - There are sources and destinations
  - Sources have supply and destinations have demand
  - A cost is associated to transport units along each route
- Wheat is harvested in the Midwest and stored in grain elevators in 3 different cities
  - Kansas City
  - Omaha
  - Des Moines
- These grain elevators supply flour mills in 3 different cities
  - Chicago
  - St. Louis
  - Cincinnati







Grain Elevator	Supply	
<ol> <li>Kansas City</li> </ol>	150	
2. Omaha	175	
3. Des Moines	275	
Total	600 tons	

Mill	Demand
A. Chicago	200
B. St. Louis	100
C. Cincinnati	300
Total	600 tons

- Grain is shipped in railroad cars, each capable of holding 1 ton of wheat
- Transportation cost per ton (railroad car) of wheat

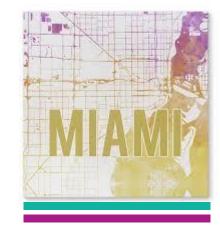
		Mill	
Grain elevator	A. Chicago	B. St. Louis	C. Cincinnati
1. Kansas City	\$6	\$8	\$10
2. Omaha	7	11	11
3. Des Moines	4	5	12





- Q: How many tons of wheat should be shipped on each route to minimize cost?
- Decision variables
  - $x_{ij}$  = number of tons of grain to ship from i to j
  - $i \in \{1,2,3\}$
  - $j \in \{A, B, C\}$
- Objective function

$$Z = 6x_{1A} + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} + 4x_{3A} + 5x_{3B} + 12x_{3C}$$





#### Constraints

$$x_{1A} + x_{1B} + x_{1C} = 150$$
  
 $x_{2A} + x_{2B} + x_{2C} = 175$   
 $x_{3A} + x_{3B} + x_{3C} = 275$ 

$$x_{1A} + x_{2A} + x_{3A} = 200$$
  
 $x_{1B} + x_{2B} + x_{3B} = 100$   
 $x_{1C} + x_{2C} + x_{3C} = 300$ 

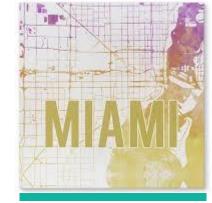
$$x_{ij} \in \{0,1,2,\cdots\}$$

#### Optimal solution

(Kansas City Supply)(Omaha Supply)(Des Moines Supply)

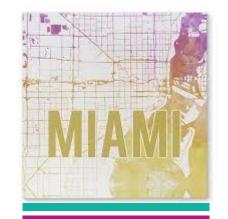
(Chicago Demand)(St. Louis Demand)(Cincinnati Demand)

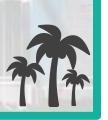
Variables	Destinations	5			
Sources	A. Chicago	B. St. Louis	C. Cincinnati	Grain shippe	Supply
1. Kansas City	25	0	125	150	150
2. Omaha	0	0	175	175	175
3. Des Moines	175	100	0	275	275
Grain shipped	200	100	300		
Demand	200	100	300		
Total cost =	4525				





- Balanced versus unbalanced
  - When total supply equals total demand, the problem is balanced
  - When total supply doesn't equal total demand, the problem is unbalanced
  - Current grain transportation problem is balanced (600 Supply = 600 Demand)
- Modifications for unbalanced transportation problems
  - If total supply is smaller than total demand, we replace the equalities in the demand constraints to be  $\leq$
  - If total supply is bigger than total demand, we replace the equalities in the supply constraints to be  $\leq$
  - Alternative approach is to create slack variables to absorb excess
  - When total supply is smaller than total demand, the slack variables act as fictitious sources
  - When total supply is bigger than total demand, the slack variables act as fictitious destinations

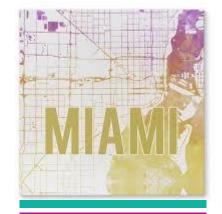




- Suppose we change Cinicinnati's demand from 300 to 350
- New constraints

$$\begin{array}{lll} x_{1A} + x_{1B} + x_{1C} = 150 & \text{(Kansas City Supply)} \\ x_{2A} + x_{2B} + x_{2C} = 175 & \text{(Omaha Supply)} \\ x_{3A} + x_{3B} + x_{3C} = 275 & \text{(Des Moines Supply)} \\ x_{1A} + x_{2A} + x_{3A} \leq 200 & \text{(Chicago Demand)} \\ x_{1B} + x_{2B} + x_{3B} \leq 100 & \text{(St. Louis Demand)} \\ x_{1C} + x_{2C} + x_{3C} \leq 350 & \text{(Cincinnati Demand)} \\ \end{array}$$

• Total supply (600) is smaller than total demand (650)





- Include fictitious source "Slack (S)"
- Modified constraints with slack variable

$$x_{1A} + x_{1B} + x_{1C} = 150$$

$$x_{2A} + x_{2B} + x_{2C} = 175$$

$$x_{3A} + x_{3B} + x_{3C} = 275$$

$$x_{SA} + x_{SB} + x_{SC} = 50$$

$$x_{1A} + x_{2A} + x_{3A} + x_{SA} = 200$$

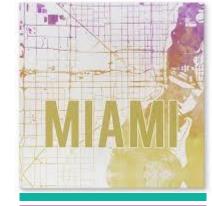
$$x_{1B} + x_{2B} + x_{3B} + x_{SB} = 100$$

$$x_{1C} + x_{2C} + x_{3C} + x_{SC} = 350$$

 $x_{ij} \in \{0,1,2,\cdots\}$ 

(Kansas City Supply)(Omaha Supply)(Des Moines Supply)(Fictitious "Slack" Supply)

(Chicago Demand)(St. Louis Demand)(Cincinnati Demand)

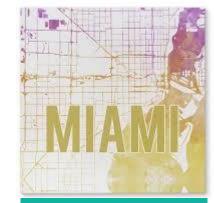






Variables	Destinations	•			
Sources	A. Chicago	B. St. Louis	C. Cincinnati	Grain shipped	Supply
1. Kansas City	0	0	150	150	150
2. Omaha	25	0	150	175	175
3. Des Moines	175	100	0	275	275
4. Slack source	0	0	50	50	50
Grain shipped	200	100	350		
Demand	200	100	350		
Total cost =	4525				

• Q: Why is all the grain from slack going to Cincinnati?









### The End





