Assignment #7 Solutions

due Friday, October 11th, 2019

1 Assume x_{ij} is the amount of flow (thousands of cars) going through edge (i, j). Let C_{ij} be the flow capacity of edge (i, j). Then the model is as follows:

$$
\begin{aligned}
&\max z = x_{12} + x_{13} + x_{14} \\
&s.t. \\
&x_{12} + x_{32} + x_{52} - x_{23} - x_{25} = 0 \\
&x_{13} + x_{23} + x_{53} + x_{63} - x_{32} - x_{35} - x_{36} = 0 \\
&x_{14} + x_{64} + x_{74} - x_{46} - x_{47} = 0 \\
&x_{25} + x_{35} - x_{52} - x_{53} - x_{58} = 0 \\
&x_{36} + x_{46} + x_{76} - x_{63} - x_{64} - x_{67} - x_{68} = 0 \\
&x_{47} + x_{67} - x_{74} - x_{76} - x_{78} = 0 \\
&x_{ij} \leq C_{ij} \\
&x_{ij} \geq 0, \quad i = 1, 2, ..., 8, \; j = 1, 2, ..., 8 \; \textit{and integer.}
\end{aligned}
$$

We can solve the model using excel, shown in Fig **??**. From Fig **??**, we observe that maximum traffic flow the streets can accommodate is 21,000 cars. The amount of traffic along each street are shown in excel table A5:A24. The street would be able to handle the expected flow after a game.

Maximum flow problem

Select branc	Node	Node	Capacity
6	1	2	10
7	1	3	7
8	1	4	8
0	2	3	3
6	2	5	6
0	3	2	5
2	3	5	6
5	3	6	5
4	4	6	4
4	4	7	5
0	5	2	0
0	5	3	2
8	5	8	8
0	6	3	0
0	6	4	2
5	6	7	6
4	6	8	4
0	7	4	0
0	7	6	2
9	7	8	9
		Total	21

Flow constraints:

Node	Network Flo	Constraint	Value
2	0	=	0
3	0	=	0
4	0	=	0
5	0	=	0
6	0	=	0
7	0	=	0

Figure 1: Excel in Question 1

2

Assume x_{ij} is the amount of flow going through edge (i, j), $i < j$. Then the model is as follows:

$$
\begin{aligned}
&\max z = x_{12} + x_{13} + x_{14} + x_{15} \\
&s.t. \\
&x_{12} - x_{26} - x_{29} = 0 \\
&x_{13} - x_{36} - x_{37} = 0 \\
&x_{14} - x_{47} - x_{48} = 0 \\
&x_{15} - x_{58} - x_{5,11} = 0 \\
&x_{26} + x_{36} - x_{6,10} - x_{6,12} = 0 \\
&x_{37} + x_{47} + x_{67} - x_{78} - x_{7,10} = 0 \\
&x_{48} + x_{58} + x_{78} - x_{8,11} - x_{8,13} - x_{8,14} = 0 \\
&x_{29} - x_{9,12} = 0 \\
&x_{6,10} + x_{7,10} - x_{10,12} - x_{10,13} = 0 \\
&x_{5,11} + x_{8,11} - x_{11,14} = 0 \\
&x_{6,12} + x_{9,12} + x_{10,12} - x_{12,15} = 0 \\
&x_{10,13} + x_{8,13} + x_{12,13} - x_{13,15} = 0 \\
&x_{8,14} + x_{11,14} - x_{14,15} = 0 \\
&x_{ij} \geq 0,\ i < j,\ i = 1, 2, ..., 15,\ j = 1, 2, ..., 15 \ \textit{and integer}.
\end{aligned}
$$

We can solve the model using excel, shown in Fig **??**. We can see the maximum flow is 250. The number of units processed at each work center are shown in Excel A5:A31.

1	**Maximum flow problem**								
2									
3									
4	Select branch	Node	Node	Capacity					
5	70	1	2	90		*Flow constraints:*			
6	50	1	3	120		Node	Network Flo	Constraint	Value
7	60	1	4	75		2	0	=	0
8	70	1	5	85		3	0	=	0
9	25	2	6	25		4	0	=	0
10	45	2	9	45		5	0	=	0
11	30	3	6	30		6	0	=	0
12	20	3	7	20		7	0	=	0
13	35	4	7	35		8	0	=	0
14	25	4	8	25		9	0	=	0
15	40	5	8	40		10	0	=	0
16	30	5	11	30		11	0	=	0
17	55	6	10	65		12	0	=	0
18	0	6	12	75		13	0	=	0
19	15	7	8	50		14	0	=	0
20	40	7	10	40					
21	0	8	11	35					
22	0	8	13	60					
23	65	8	14	70					
24	45	9	12	50					
25	30	10	12	55					
26	65	10	13	65					
27	30	11	14	60					
28	60	12	13	60					
29	15	12	15	90					
30	125	13	15	190					
31	95	14	15	110					
32				250					

Figure 2: Excel in Question 2

3

Assume x_{ij} is the amount of flow going through edge (i, j). Let c_{ij} be the capacity of edge (i, j). Then the model is as follows:

$$
\begin{aligned}
&\min z = x_{84} + x_{86} + x_{87} \\
&s.t. \\
&x_{12} + x_{42} + x_{62} - x_{21} - x_{24} - x_{26} = 0 \\
&x_{43} + x_{53} - x_{31} - x_{34} - x_{35} = 0 \\
&x_{14} + x_{24} + x_{34} + x_{64} + x_{74} + x_{84} - x_{43} - x_{42} - x_{41} - x_{46} - x_{47} - x_{48} = 0 \\
&x_{35} + x_{75} - x_{53} - x_{57} = 0 \\
&x_{86} + x_{26} + x_{46} - x_{62} - x_{64} = 0 \\
&x_{87} + x_{47} + x_{57} - x_{74} - x_{75} = 0 \\
&0 \leq x_{ij} \leq c_{ij}, \quad i = 1, 2, ..., 8, \ j = 1, 2, ..., 8 \textit{ and integer.}
\end{aligned}
$$

The maximum number of flights the airline can schedule per day from Chicago to Los Angeles is 16. The number of flights along each route is shown in A5:A24.

Maximum flow problem

	Select branch	Node	Node	Capacity
5	7	2	1	10
6	0	2	4	7
7	0	2	6	8
8	1	3	1	2
9	0	3	4	4
10	0	3	5	6
11	8	4	1	9
12	4	4	2	4
13	0	4	3	0
14	0	4	6	5
15	0	4	7	7
16	1	5	3	1
17	0	5	7	5
18	3	6	2	8
19	0	6	4	2
20	3	7	4	3
21	1	7	5	1
22	9	8	4	9
23	3	8	6	3
24	4	8	7	5
25				16

Flow constraints:

Node	Network Flo	Constraint	Value
2	0	=	0
3	0	=	0
4	0	=	0
5	0	=	0
6	0	=	0
7	0	=	0

Figure 3: Excel in Question 3