Assignment #12 Solutions

due Tuesday, November 17th, 2020

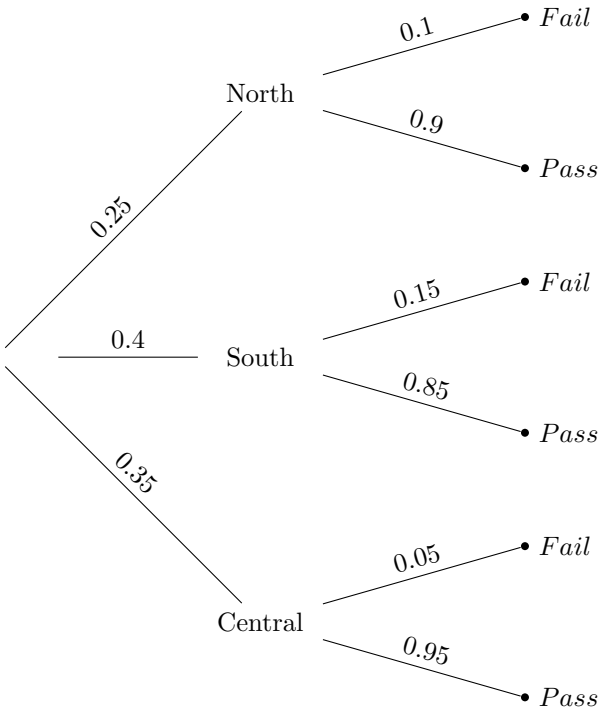
Problem 1 (20 points)

The probability a given patient is a smoker if the patient has a serious illness is

$$P = \frac{0.23 * 0.18}{0.23 * 0.18 + 0.77 * 0.06} = 0.47.$$

Problem 2

(a) (10 points) Below is the probability tree



(10 points)

$$P(Fail) = 0.25 * 0.1 + 0.4 * 0.15 + 0.35 * 0.05 = 0.1025$$

Problem 3

(a) (4 points) Below is the excel table for 3(a).

1	Emergency Call					
2						
3	Time btw Emergency calls (hr.)	Prbability	Cumulative	Range		
4	1	0.05	0.00	[0,0.05]	Expected Value	3.65
5	2	0.10	0.05	(0.05,0.15]	Average	3.9444444
6	3	0.30	0.15	(0.15,0.45]		
7	4	0.30	0.45	(0.45,0.75]		
8	5	0.20	0.75	(0.75,0.95]		
9	6	0.05	0.95	(0.95,1]		
10						
11	Day	Interarrivals	Service Time	Cumulative Time		
12	0	0.908240386	5	5		
13	0	0.994306853	6	11		
14	0	0.7595289	5	16		
15	0	0.108162916	2	18		
16	0	0.272107379	3	21		
17	1	0.380666699	3	24		
18	1	0.047170953	1	25		
19	1	0.912748126	5	30		
20	1	0.328741449	3	33		
21	1	0.574886413	4	37		
22	1	0.511095194	4	41		
23	1	0.604722732	4	45		
24	2	0.489741246	4	49		
25	2	0.156203357	3	52		
26	2	0.400499892	3	55		
27	2	0.80381764	5	60		
28	2	0.913955543	5	65		
29	2	0.988293226	6	71		
30	3	0.100908854	2	73		

(b) (8 points) The expected value of time between calls is 3.65 hr, while the average time between calls is 3.94 hr. In this simulation, the average time between calls is slightly higher than the expected value of time.

The expected value of time between call is a theoretical value. As the number of simulations increases, the sample mean of average time between calls would become close to the expected value.

(c) (8 points) 18 calls were made during the 3-day period. One cannot assume this is an average umber of calls per 3-day period. Run simulation multiple times. In each simulation, record the total number of calls made during the 3-day period. Then calculate the average of total number of calls made during the 3-day period.

Problem 4 (20 points)

Figure 1 shows the simulation results for problem 4. Note that 2 out of 10 times, the robber is within 2 blocks of the store. (Can use either Taxi cab metric or Euclidean norm).

Problem 5 (20 points)

(Sec 001) Figure 2 only shows one simulation for problem 5.

Step 1: Generate a sequence of random number $U_1, ..., U_{10} \sim U(0, 1)$

Step 2: Construct the distributional table for number of customers/day and use Vlookup to find X_i = the number of customers for day i, $i = 1, ..., 10$.

Step 3: Then for each day, there are a total of X_i customers. Generate random number $U_j \sim U(0, 1)$, $j = 1, ..., X_i$.

Step 4: Construct the distributional table for rental duration and use Vlookup to find the rental duration for every customer.

Note that students are required to explain how they simulated all the numbers in the table, including the Excel formulas they used.

(Sec 002)

Figure 2 only shows one simulation for problem 5.

Step 1: Generate a sequence of random number $U_1, ..., U_{10} \sim U(0, 1)$

Step 2: Construct the distributional table for number of customers/day and use Vlookup to find X_i = the number of customers for day i, $i = 1, ..., 10$.

Step 3: Then for each day, there are a total of X_i customers. Generate random number $U_j \sim U(0, 1)$, $j = 1, ..., X_i$.

Step 4: Construct the distributional table for rental duration and use Vlookup to find the rental duration for every customer.

Step 5: One can either use if... else... statement or manually compute to determine which car is available in each day. (This step is only required for Section 002)

For 2 out of 10 trials, the agency will not have a car on day 10. The probability that the agency will not have a car available on day 10 is approximately 0.2, which suggests that the agency should not expand its fleet. (The conclusion varies depending on the simulation results). To determine the optimal fleet size for the agency, one could try multiple trials with more cars available.

1	Robbery				
2					
3	Direction	Prbability	Cumulative	x coordinate	y coordinate
4	North	0.25	0.00	0	1
5	South	0.25	0.25	0	-1
6	East	0.25	0.50	1	0
7	West	0.25	0.75	-1	0
8					

	Trial 1			Trial 2			Trial 3			Trial 4			Trial 5		
10	x coordinate	y coordinate	Random number	x coordinate	y coordinate	Random number	x coordinate	y coordinate	Random number	x coordinate	y coordinate	Random number	x coordinate	y coordinate	Random number
11	1	0	0.566778054	-1	0	0.903714844	1	0	0.561505832	0	-1	0.444299134	0	1	0.078334161
12	0	-1	0.3573903	1	0	0.602396029	0	1	0.149115135	0	-1	0.257061734	-1	0	0.900899476
13	-1	0	0.890484896	0	-1	0.397069792	0	-1	0.303366015	0	1	0.046440918	1	0	0.707202489
14	0	1	0.201426324	0	-1	0.392105928	1	0	0.747004839	1	0	0.711208908	0	1	0.053712376
15	1	0	0.632075578	-1	0	0.792021135	1	0	0.602477872	0	1	0.217462213	1	0	0.663846803
16	1	0	0.743434255	0	1	0.188497868	0	1	0.14112495	1	0	0.588299392	0	-1	0.314186507
17	0	1	0.081767467	-1	0	0.773240597	0	1	0.11020338	0	-1	0.431246853	-1	0	0.850200533
18	0	-1	0.440262295	0	-1	0.350691056	0	1	0.034049599	0	1	0.133030168	0	1	0.094118602
19	0	-1	0.410505971	0	-1	0.386081955	-1	0	0.93404328	1	0	0.711383417	1	0	0.597642401
20	1	0	0.669066631	1	0	0.554497363	1	0	0.671711395	1	0	0.604190415	1	0	0.533046281
21	3	-1		-1	-3		3	3		4	0		2	2	
22															

	Trial 6			Trial 7			Trial 8			Trial 9			Trial 10		
24	x coordinate	y coordinate	Random number	x coordinate	y coordinate	Random number	x coordinate	y coordinate	Random number	x coordinate	y coordinate	Random number	x coordinate	y coordinate	Random number
25	1	0	0.732614371	0	1	0.118982834	0	1	0.043196501	0	1	0.130888355	0	1	0.068386182
26	0	1	0.233653387	1	0	0.518350647	1	0	0.697017938	0	-1	0.342252825	0	-1	0.482374992
27	-1	0	0.85543986	-1	0	0.934199423	-1	0	0.971118488	1	0	0.685296538	-1	0	0.981192522
28	0	-1	0.274293137	1	0	0.604745533	1	0	0.700550117	1	0	0.593539707	0	-1	0.45700424
29	1	0	0.621725294	1	0	0.549919999	0	1	0.209279569	0	-1	0.264324486	0	1	0.246302244
30	0	-1	0.426414517	0	1	0.012596372	0	1	0.087644475	-1	0	0.772484821	0	-1	0.467892844
31	-1	0	0.894855939	0	1	0.086059611	0	1	0.208804101	0	1	0.003682592	-1	0	0.850968577
32	0	-1	0.261328402	-1	0	0.773341496	0	1	0.206307979	-1	0	0.76110943	0	1	0.183579371
33	-1	0	0.852708538	0	-1	0.396669988	0	-1	0.251131912	0	1	0.110272034	0	1	0.117134928
34	0	1	0.239713732	0	1	0.012187152	1	0	0.506085876	-1	0	0.797957205	-1	0	0.774982643
35	-1	-1		1	3		2	4		-1	1		-3	1	

Figure 1: Problem 4

1	Rental Car Agency											
2												
3	Number of Customers/Day	Prbability	Cumulative	Range		Rental Duration (days)	Prbability	Cumulative	Range			
4	0	0.20	0.00	[0,0.2]		1	0.10	0.00	[0,0.1]			
5	1	0.20	0.20	(0.2,0.4]		2	0.30	0.10	(0.1,0.4]			
6	2	0.50	0.40	(0.4,0.9]		3	0.40	0.40	(0.4,0.8]			
7	3	0.10	0.90	(0.9,1]		4	0.10	0.80	(0.8,0.9]			
8						5	0.10	0.90	(0.9,1]			
9												
10	Day	Number of Customers/Day	Random numbers	Rental Duration of customer 1	Random numbers	Rental Duration of customer 2	Random numbers	Rental Duration of customer 3	Random numbers			
11	1	2	0.845815578	3	0.455272708	1	0.039836255	3	0.506117658			
12	2	1	0.333857681	3	0.702263532	5	0.96365387	2	0.298177676			
13	3	2	0.843881777	2	0.373446879	2	0.122350003	3	0.44937429			
14	4	2	0.582818772	4	0.897535742	2	0.375369215	5	0.995761462			
15	5	2	0.669868301	2	0.15343813	1	0.000150164	5	0.969482262			
16	6	2	0.547334531	3	0.445121429	2	0.37484231	4	0.875176201			
17	7	2	0.541123489	3	0.609412224	3	0.503182606	3	0.765245535			
18	8	2	0.507133903	2	0.272417456	2	0.215330262	2	0.348925136			
19	9	2	0.629940343	3	0.560869903	3	0.400554161	4	0.836215319			
20	10	2	0.87245418	3	0.78975388	3	0.58111319	4	0.877946207			
21												
22	Avialability											
23	Day	Demand	Car 1	Car 2	Car 3	Car 4	Cars Available in the beginning	Customer loss		trail	at least 1 car available on Day 10	
24	1	2	3	1	0	0	4	0		1	1	
25	2	1	2	3	0	0	3	0		2	1	
26	3	2	1	2	2	2	2	0		3	0	
27	4	2	4	1	1	1	1	1		4	1	
28	5	2	3	2	1	0	3	0		5	1	
29	6	2	2	1	3	2	2	0		6	1	
30	7	2	1	3	2	1	1	1		7	1	
31	8	2	2	2	1	2	2	0		8	1	
32	9	2	1	1	3	1	1	1		9	0	
33	10	2	3	3	2	0	3	0		10	1	

Figure 2: Problem 5