Assignment #9 Solutions

due Friday, October 23th, 2020

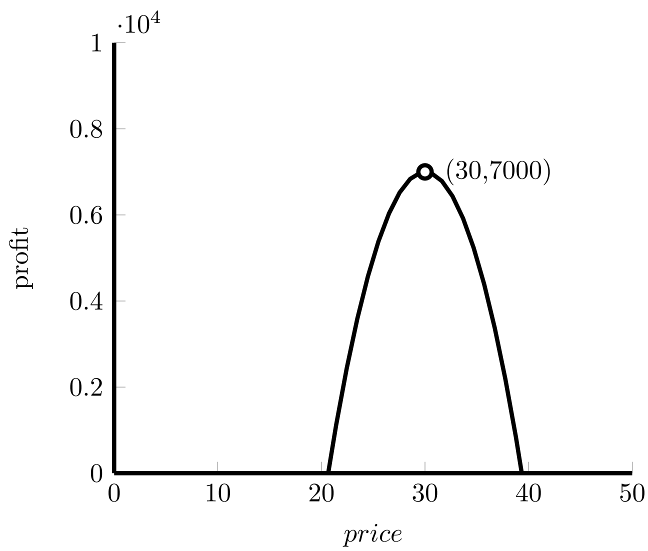
Problem 1 (20 points)

(a) The objective function $z = (4000 - 80p)(p - 10) - 25000 = -80p^2 + 4800p - 65000$, then we can take the derivative of z over p and set it to zero, namely,

$$\frac{\partial z}{\partial p} = -160p + 4800 = 0$$

Then we have $p^* = 30$, the optimal volume $v^* = 1,600$ and the optimal profit $z^* = 7000$.

(b)



Problem 2 (20 points)

(a) Using Excel solver, the optimal solution is $(x_1, x_2) = (15.45, 12.27)$ with profit $= 382.72$.

1	**Riverwood Paneling Company**					
2						
3	*Variables:*					
4	Colonial Paneling (x1)	15.454545				
5	Western Paneling (x2)	12.272727				
6						
7	Profit:	382.72727				
8						
9	*Constraints*	x1	x2	Used	Constraint	Allowed
10	Labor	1	2	40	=	40

(b) From the sensitivity report, we can see that the Lagrange Multiplier is 0.27. It reflects the approximate change in the objective function resulting from a unit change in the quantity (right-hand-side) value of the constraint equation. For this problem, if the quantity of labor hours is increased from 40 to 41 hours, the value of Z will increase by \$0.27—from \$382.73 to \$383.

1	Microsoft Excel 16.28 Sensitivity Report			
2	Worksheet: [hw10.xlsx]Q2 (a)			
3	Report Created: 11/5/19 1:20:37 AM			
4				
5				
6	Variable Cells			
7			Final	Reduced
8	Cell	Name	Value	Gradient
9	\$B\$4	Colonial Paneling (x1)	15.45454532	0
10	\$B\$5	Western Paneling (x2)	12.27272734	0
11				
12	Constraints			
13			Final	Lagrange
14	Cell	Name	Value	Multiplier
15	\$D\$10	Labor Used	40	0.272712457

Problem 3 (30 points)

Let (x, y) be the coordinate of the new distribution center.
Data: Let (x_i, y_i) be the coordinate of supplier i, i = A,B,C,D.
t_i be the annual number of truckloads from supplier i, i = A,B,C,D.

The distance between supplier i and new distribution center is $d_i = sqrt(x_i - x)^2 + (y_i - y)^2$. Hence, the objective function is

$$min \sum_{i \in \{A,B,C,D\}} d_i t_i$$

Using Excel solver, the location for the new distribution center is $(x, y) = (178.17, 483.18)$ with total transportation $= 68,171.95$ miles.

1	**Burger Doodle restaurant**					
2						
3	*Variables: new distribution center*					
4	x	178.17307				
5	y	483.18067				
6						
7	Total Distance	68171.951				
8						
9	*supplier*	x	y	distance	Annual Truckloads	total distance (supplier -> distribution center)
10	*A*	200	200	284.02061	65	18461.33979
11	*B*	100	500	79.961987	120	9595.438399
12	*C*	250	600	137.13447	90	12342.10217
13	D	500	300	370.30761	75	27773.07105

Problem 4 (30 points)

Let x_i be the proportion of money invest in stock i, i = 1,...,4.
Data: r_i be the annual return of stock i, i = 1,...,4.
ρ_{ij} is the correlation between stock i and stock j, where i = 1,...,4 and j = 1,..,4.
σ_i is the variance of stock i, i = 1,...,4

$$min \sum_{i=1}^{4} \sum_{j=1}^{4} \rho_i j \sigma_i \sigma_j x_i x_j$$

$$s.t. \sum_{i=1}^{4} r_i x_i \geq 0.12$$

$$\sum_{i=1}^{4} x_i = 1$$

$$x_i \geq 0, i = 1, ..., 4$$

Using Excel solver, the optimal solution is $(x_1, x_2, x_3, x_4) = (0.025, 0, 0.615, 0.359)$ with minimum portfolio variance $= 0.0361$, and total return $= 0.12$

1	**Investment portfolio**						
2							
3	*Variables*						
4	x1	0.02537561					
5	x2	0					
6	x3	0.61522538					
7	x4	0.359399					
8							
9	*Correlation matrix:*					*Return:*	*Variance:*
10	1	0.9	0.7	0.3		0.18	0.112
11	0.9	1	0.8	0.4		0.12	0.061
12	0.7	0.8	1	0.2		0.1	0.045
13	0.3	0.4	0.2	1		0.15	0.088
14							
15	Computing the covariance matrix (Sigma):						
16	0.112	0.07439032	0.04969507	0.02978322			
17	0.074390322	0.061	0.0419142	0.02930665			
18	0.04969507	0.0419142	0.045	0.01258571			
19	0.029783217	0.02930665	0.01258571	0.088			
20							
21	Computing the portfolio variance:						
22	x'*Sigma*x =	0.03613206					
23							
24	Portfolio variance:	0.03613206					
25	Portfolio return:	0.12	>=	0.12			
26	Sum of variables:	1	=	1			