# Midterm #1

This exam will consist of a combination of **multiple-choice** and **short answer**. Each **multiple-choice** question has only one correct answer. For all **multiple-choice** questions, identify your answer by circling exactly one of the options. No partial credit will be given for **multiple-choice** questions. **Short answer** questions should be supported by analysis or argument. On **short answer** questions, identify your answer by circling or risk losing points. For the majority of **short answer** questions, you have the ability to get partial credit if good work is shown. Read the instructions for each question because deviations exist.

You have only 50 minutes to complete this exam. The exam consists of 15 questions. You are not permitted to use your textbook, notes, calculator, computer, cell phone or any other resource on this exam. If you violate this policy, you violate the honor code of the university.

## 1 Information for Questions 1.1 to 1.2

Cook Out is the greatest restaurant for exotic American cuisine. Cook Out produces the only burger endorsed by Dr. Worldwide. Each week it costs \$300 to operate the business and an additional \$2.00 per burger they make. For convenience, we assume that Cook Out sells every burger they make.

### 1.1 If they sell each delicious burger for \$5.00, how many burgers do they need to sell each week to break-even? (3 Points)

**Solution:**

$$x^* = \frac{300}{5-2} = 100 \text{ burgers}$$

### 1.2 Out of the following statements involving the break-even point, circle the one true statement. (2 Points)

**a) If you decrease the price per burger, the break-even point will increase.**

**b)** If you decrease the cost per burger, the break-even point will increase.

**c)** If you decrease the fixed cost to operate the business, the break-even point will increase.

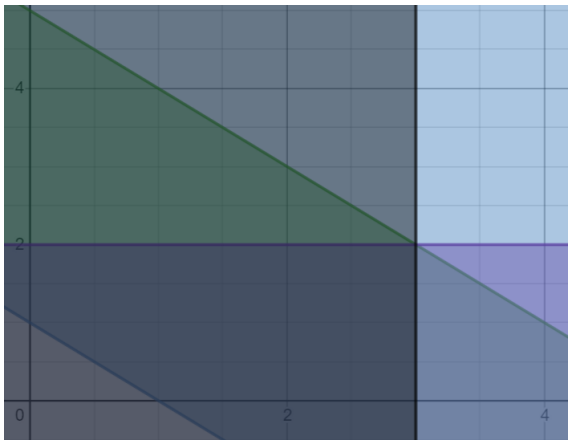
**d)** None of the previous statements are true.

## 2 Information for Questions 2.1 to 2.2

Consider the following set of linear constraints:

$$\begin{aligned} x + y &\leq 5 \\ x + y &\geq 1 \\ x &\leq 3 \\ y &\leq 2 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

**2.1 Shade the feasible region on the $(x, y)$-plane according to the given set of constraints. (6 Points)**




**2.2 How many extreme points does the feasible region have? Circle the correct number. (2 Points)**

**a)** 3

**b)** 4

**c)** **5**

**d)** 6

# 3 Information for Questions 3.1 and 3.2

I plan on using my new skills in business analytics to create a nutritional supplement company to steal money from people by making them think they need my supplements to survive. We plan on selling four different supplements. Each supplement is individually packaged and based on a mixture of 4 different vitamins/minerals. The first supplement *H2Orange* contains 60 mg of vitamin C, 500 mg of calcium, and 20 mg of potassium. The second supplement *Vitamin C-3PO* contains 60 mg of vitamin C, 200 mg of calcium, and 10 mg of iron. The third supplement *Ironic* contains 250 mg of calcium and 15 mg of iron. The last supplement *Vitamin G* contains 1,000 mg of potassium and 15 mg of iron. We would like to maximize the total profit. The profit per supplement is \$20, \$30, \$40, and \$70 for *H2Orange*, *Vitamin C-3PO*, *Ironic*, and *Vitamin G*, respectfully. Let $x_1$, $x_2$, $x_3$, $x_4$ represent the the number of supplements to produce. How many supplements of each type should we produce to maximize profit if we only have $7,000$ mg of each of the four different vitamins/minerals?

**3.1 Write the linear programming model for this problem in standard form. (14 Points)**

$$\begin{aligned}
&\text{Maximize } Z = 20x_1 + 30x_2 + 40x_3 + 70x_4 \\
&\text{subject to} \\
&\qquad 60x_1 + 60x_2 \leq 7,000 \\
&\qquad 500x_1 + 200x_2 + 250x_3 \leq 7,000 \\
&\qquad 20x_1 + 1000x_4 \leq 7,000 \\
&\qquad 10x_2 + 15x_3 + 15x_4 \leq 7,000 \\
&\qquad x_1, x_2, x_3, x_4 \in \{0, 1, 2, 3, \cdots\}
\end{aligned}$$

**3.2 On Facebook, at least 50 people expressed interest to buy a single supplement of *Vitamin C-3PO*. Also based on the comments, we believe that the interest in *H2Orange* or *Vitamin C-3PO* is at least triple the interest in the other two supplements. Write two constraints in standard form to ensure that the expected demand from Facebook is at least met. (6 Points)**

$$\begin{aligned}
x_2 &\geq 50 \\
x_1 + x_2 - 3x_3 - 3x_4 &\geq 0
\end{aligned}$$

# 4 Information for Questions 4.1, 4.2, and 4.3

Consider the following Excel file that I created on my computer for your pleasure.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | | 3 | 2 | 1 | | 2 | |
| 3 | | 1 | 0 | 4 | | 1 | |
| 4 | | | | | | 3 | |
| 5 | | 1 | 1 | 1 | | | |
| 6 | | | | | | | |
| 7 | | | | | | | |
| 8 | | ***U*** | | | | | |
| 9 | | ***N*** | | | | | |
| 10 | | ***C*** | | | | | |
| 11 | | | | | | | |
| 12 | | | | | | | |
| 13 | | | | | | | |

**4.1 If I replace the $U$ with the formula "=MMULT(B5:D5,F2:F4)", what will the spreadsheet look like on my computer when the formula correctly evaluates? Write the changes in the spreadsheet below. (3 Points)**

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | 3 | 2 | 1 | | 2 |
| 3 | | 1 | 0 | 4 | | 1 |
| 4 | | | | | | 3 |
| 5 | | 1 | 1 | 1 | | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | ***6*** | | | | |
| 9 | | ***#VALUE!*** | | | | |
| 10 | | ***11*** | | | | |
| 11 | | **14** | | | | |

**4.2** If I replace the $N$ with the formula "=SUMPRODUCT(B5:D5,F2:F4)", what will the spreadsheet look like on my computer when the formula correctly evaluates? Write the changes in the spreadsheet below. (3 Points)

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | 3 | 2 | 1 | | 2 |
| 3 | | 1 | 0 | 4 | | 1 |
| 4 | | | | | | 3 |
| 5 | | 1 | 1 | 1 | | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | ***6*** | | | | |
| 9 | | ***#VALUE!*** | | | | |
| 10 | | ***11*** | | | | |
| 11 | | **14** | | | | |

**4.3** If I replace the $C$ with the formula "=MMULT(B2:D3,F2:F4)", what will the spreadsheet look like on my computer when the formula correctly evaluates? Write the changes in the spreadsheet below. (3 Points)

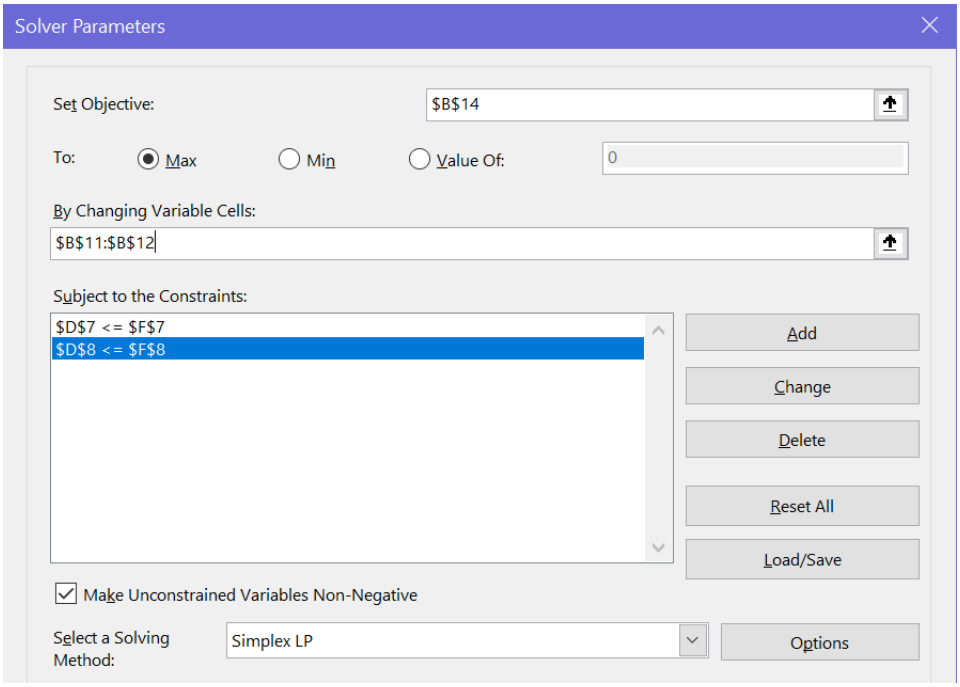
| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | 3 | 2 | 1 | | 2 |
| 3 | | 1 | 0 | 4 | | 1 |
| 4 | | | | | | 3 |
| 5 | | 1 | 1 | 1 | | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | ***6*** | | | | |
| 9 | | ***#VALUE!*** | | | | |
| 10 | | ***11*** | | | | |
| 11 | | **14** | | | | |

# 5 Information for Questions 5.1, 5.2, and 5.3

When Dr. Mario is not teaching, he is an entertainer for kid's birthday parties. He entertains as either a magician or a juggler. Each week Dr. Mario has exactly 40 units of mental energy and 40 units of physical energy to spend on this nonsense. Each hour being a magician requires 3 units of mental energy and 2.5 units of physical energy. Each hour being a juggler requires 5.2 units of mental energy and 7.2 units of physical energy. Each hour that Dr. Mario does magic results in a profit of \$50, and each hour that Dr. Mario juggles results in a profit of \$100. How many hours should Dr. Mario spend each week being a magician and a juggler to maximize profit?

The Excel spreadsheet used to organize this linear programming problem and the Excel Solver setup used to solve this linear programming problem are seen below:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **Entertainment** | | | | | |
| 2 | | | | | | |
| 3 | *Performance Type* | *Magician* | *Juggler* | | | |
| 4 | *Profit Per Hour* | 50 | 100 | | | |
| 5 | | | | | | |
| 6 | *Resources* | *Magician* | *Juggler* | *Usage* | *Constraint* | *Availability* |
| 7 | Mental Energy | 3 | 5.2 | 0 | <= | 40 |
| 8 | Physical Energy | 2.5 | 7.2 | 0 | <= | 40 |
| 9 | | | | | | |
| 10 | *Hours Performing as:* | | | | | |
| 11 | *Magician* | 0 | | | | |
| 12 | *Juggler* | 0 | | | | |
| 13 | | | | | | |
| 14 | *Profit* | 0 | | | | |



The sensitivity analysis from Excel is provided below:

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$11 | Magician | 9.302325581 | 0 | 50 | 7.692307692 | 15.27777778 |
| $B$12 | Juggler | 2.325581395 | 0 | 100 | 44 | 13.33333333 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $D$7 | Mental Energy Usage | 40 | 12.79069767 | 40 | 8 | 11.11111111 |
| $D$8 | Physical Energy Usage | 40 | 4.651162791 | 40 | 15.38461538 | 6.666666667 |

### 5.1 What type of linear programming model is most appropriate? (2 Points)

**a)** Total Integer Model

**b)** 0-1 Integer Model

**c)** Mixed Integer Model

**d) None of the above**

### 5.2 What is the correct formula used in cell B14 for the objective function? (2 Points)

**a)** =MMULT(B11:B12,B4:C4)

**b)** =MMULT(B11:B12,B4:C4)

**c)** =SUMPRODUCT(B4:C4,B11:B12)

**d) =MMULT(B4:C4,B11:B12)**

**e)** None of the above will work

### 5.3 Based on the sensitivity analysis, which statement is true? (2 Points)

**a) If Dr. Mario decides to increase his profit as a magician to $57 per hour, the optimal solution will stay the same.**

**b)** If Dr. Mario can increase either his mental energy capacity or his physical energy capacity for free, he should increase his physical energy capacity because it is cheaper.

**c)** It costs approximately $12.80 and $4.65 for each unit of mental energy and physical energy, respectfully.

**d)** None of the above statements are true.

# 6 Information for Questions 6.1, 6.2, and 6.3

Becky and Andreas are hosting a party for 6 precious children. At the end of the party, Andreas and Becky must team up to hug all these precious children so none of them go home crying. The table below shows the distance (in feet) between the adults (Andreas and Becky) and the 6 precious children (1,2,3,4,5,6):

| **Matrix of Distance** | | | | | | |
|---|---|---|---|---|---|---|
| | Precious Children | | | | | |
| Adult | *1* | *2* | *3* | *4* | *5* | *6* |
| *Andreas* | 30 | 10 | 24 | 17 | 25 | 6 |
| *Becky* | 20 | 25 | 13 | 18 | 13 | 6 |

Andreas and Becky want to know how they can make sure each child gets a hug but also minimize the total distance of their combined travel. The Excel spreadsheet below was created to solve this optimization problem.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | **Hugs** | | | | | | | |
| 2 | | | | | | | | |
| 3 | **Matrix of Distance** | | | | | | | |
| 4 | | Precious Children | | | | | | |
| 5 | Adult | *1* | *2* | *3* | *4* | *5* | *6* | |
| 6 | *Andreas* | 30 | 10 | 24 | 17 | 25 | 6 | |
| 7 | *Becky* | 20 | 25 | 13 | 18 | 13 | 6 | |
| 8 | | | | | | | | |
| 9 | **Decision Variables** | | | | | | | |
| 10 | | Precious Children | | | | | | |
| 11 | Adult | *1* | *2* | *3* | *4* | *5* | *6* | *Row Sum* |
| 12 | *Andreas* | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | *Becky* | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | *Column Sum* | 0 | 0 | 0 | 0 | 0 | 0 | |
| 15 | | | | | | | | |
| 16 | **Objective Function** | | | | | | | |
| 17 | *Total Distance* | 0 | | | | | | |

For consistency, define the decision variables $x_{ij} = 1$ if person $i$ hugs child $j$, and $x_{ij} = 0$ if person $i$ does not hug child $j$. In this representation, $i \in \{A, B\}$ and $j \in \{1, 2, 3, 4, 5, 6\}$. For this problem, there are twelve different decision variables. Use this notation when referencing the decision variables.

**6.1 Write out the formula that can be used in cell "B17" to create the objective function. (4 Points)**

$$= SUMPRODUCT(B6 : G7, B12 : G13)$$

**6.2 What are all the constraints for this unbalanced assignment problem? Write them out using the mathematical notation for the decision variables in standard form, and write them out in the way they would appear in Excel Solver. (18 Points)**

$$\begin{aligned} x_{A1} + x_{B1} &= 1 \\ x_{A2} + x_{B2} &= 1 \\ x_{A3} + x_{B3} &= 1 \\ x_{A4} + x_{B4} &= 1 \\ x_{A5} + x_{B5} &= 1 \\ x_{A6} + x_{B6} &= 1 \\ x_{i,j} &\in \{0, 1\} \end{aligned}$$

$$\begin{aligned} \$B\$14 : \$G\$14 &= 1 \\ \$B\$12 : \$G\$13 &= binary \end{aligned}$$

**6.3 Are there multiple optimal solutions, no optimal solutions, or exactly 1 optimal solution for this problem? Give a reason for your answer in complete sentences to get full credit. (4 Points)**

We can find a solution by assigning either Andreas or Becky to each child based on who is closer. There are multiple optimal solutions because the 6th child is equidistant to Andreas and Becky. If we switch who hugs child 6, then we will have the same total distance but a different solution. There are exactly 2 optimal solutions for this problem.