Assignment #8 Solutions

due Friday, October 9th, 2020

Problem 1 (20 points)

We can apply the minimal spanning tree algorithm to derive the minimal spanning tree as follows:

Node 1 is adjacent to 4

Node 2 is adjacent to 4

Node 3 is adjacent to 6

Node 4 is adjacent to 6

Node 5 is adjacent to 7

Node 6 is adjacent to 7

Node 7 is adjacent to 8

The total length of the path way is 1160 yards.

Problem 2 (20 points)

We can apply the minimal spanning tree algorithm to derive the minimal spanning tree as follows:

Node 1 is adjacent to 2 and 4

Node 3 is adjacent to 6

Node 4 is adjacent to 6

Node 5 is adjacent to 8

Node 6 is adjacent to 9

Node 7 is adjacent to 10

Node 9 is adjacent to 10 and 11

Node 10 is adjacent to 12 and 13

Node 12 is adjacent to 14

The total number of refurbished sidewalks is 1086 feet.

Problem 3 (20 points)

The model is as follows:

$$
\begin{aligned}
\min z = & \; P_1 d_1^-, P_2 d_2^-, P_3 d_1^+, P_4 d_3^+ \\
s.t. & \\
& 5x_1 + 2x_2 + 4x_3 + d_1^- - d_1^+ = 240 \\
& 3x_1 + 5x_2 + 2x_3 + d_2^- - d_2^+ = 500 \\
& 4x_1 + 6x_2 + 3x_3 + d_3^- - d_3^+ = 400 \\
& x_i, d_i^-, d_i^+ \geq 0, i = 1, 2, 3.
\end{aligned}
$$

Problem 4

(a) (20 points) Let $x_i \in \{0, 1\}$, for $i = 1, ..., 8$. When x_i is 1, it means that we select project i, otherwise, we should not select project i.

Data:

b_i = development budget for project i ($1,000,000), i = 1,...,8.
p_i = number of research personnel for project i, i = 1,...,8.
s_i = expected annual sales for project i ($1,000,000), i = 1,...,8.

The model is as follows:

$$\min z = P_1 d_1^+, P_2 d_2^+, P_3 d_3^-, P_4 (d_4^- + d_5^-), P_5 d_6^+, P_6 d_7^-$$

$$s.t.$$

$$\sum_{i=1}^{8} b_i x_i + d_1^- - d_1^+ = 5$$

$$\sum_{i=1}^{8} p_i x_i + d_2^- - d_2^+ = 27$$

$$\sum_{i=1}^{8} s_i x_i + d_3^- - d_3^+ = 6.5$$

$$x_1 + x_3 + x_4 + x_6 + d_4^- - d_4^+ = 2$$

$$x_2 + x_5 + x_7 + x_8 + d_5^- - d_5^+ = 2$$

$$x_2 + x_3 + x_5 + x_6 + x_7 + d_6^- - d_6^+ = 3$$

$$x_5 + x_6 + d_7^- - d_7^+ = 2$$

$$x_i \in \{0, 1\}, i = 1, ..., 8, d_j^-, d_j^+ \geq 0, j = 1, ..., 7.$$

(b) (20 points)

	Goal programming for Problem 5																								
Priority 1																									
Minimize	d1^+																								
Objective function	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0			
Variables	x1	x2	x3	x4	x5	x6	x7	x8	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	d4^-	d4^+	d5^-	d5^+	d6^-	d6^+	d7^-	d7^+	Used	Constraint	Value
Budget ($1,000,000)	0.675	1.05	0.725	0.43	1.24	0.89	1.62	1.2	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	5	=	5
Research Personnel	6	5	7	8	10	6	7	6	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	27	=	27
Expected Annual Sale	0.82	1.75	1.6	1.9	0.93	1.7	1.3	1.8	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	6.5	=	6.5
Offensive Projects	1	0	1	1	0	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	2	=	2
Defensive Projects	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	2	=	2
Risky Projects	0	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	3	=	3
Required Projects	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	2	=	2

x1	0
x2	0
x3	0
x4	1
x5	0
x6	1
x7	1
x8	0
d1^-	2.06
d1^+	0
d2^-	6
d2^+	0
d3^-	1.6
d3^+	0
d4^-	0
d4^+	1.11E-16
d5^-	1
d5^+	0
d6^-	1
d6^+	0
d7^-	1
d7^+	0
Objective function	0

Figure 1: Excel in Question 5, Priority 1

	Goal programming for Problem 5																								
Priority 3																									
Minimize	d3^-																								
Objective function	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0			
Variables	x1	x2	x3	x4	x5	x6	x7	x8	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	d4^-	d4^+	d5^-	d5^+	d6^-	d6^+	d7^-	d7^+	Used	Constraint	Value
Budget ($1,000,000)	0.675	1.050	0.725	0.430	1.240	0.890	1.620	1.200	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	5	=	5
Research Personnel	6	5	7	8	10	6	7	6	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	27	=	27
Expected Annual Sale	0.82	1.75	1.60	1.90	0.93	1.70	1.30	1.80	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	6.5	=	6.5
Offensive Projects	1	0	1	1	0	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	2	=	2
Defensive Projects	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	2	=	2
Risky Projects	0	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	3	=	3
Required Projects	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	2	=	2
Goal 1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	=	0
Goal 2	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	=	0

x1	0
x2	0
x3	0
x4	1
x5	0
x6	1
x7	1
x8	1
d1^-	0.86
d1^+	0
d2^-	0
d2^+	0
d3^-	0
d3^+	0.2
d4^-	0
d4^+	0
d5^-	0
d5^+	0
d6^-	1
d6^+	0
d7^-	1
d7^+	0
Objective function	0

Figure 2: Excel in Question 5, Priority 3

	Goal programming for Problem 5																								
Priority 6																									
Minimize	d7^-																								
Objective function	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0			
Variables	x1	x2	x3	x4	x5	x6	x7	x8	d1^-	d1^+	d2^-	d2^+	d3^-	d3^+	d4^-	d4^+	d5^-	d5^+	d6^-	d6^+	d7^-	d7^+	Used	Constraint	Value
Budget ($1,000,000)	0.675	1.050	0.725	0.430	1.240	0.890	1.620	1.200	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	5	=	5
Research Personnel	6	5	7	8	10	6	7	6	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	27	=	27
Expected Annual Sale	0.82	1.75	1.60	1.90	0.93	1.70	1.30	1.80	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	6.5	=	6.5
Offensive Projects	1	0	1	1	0	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	2	=	2
Defensive Projects	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	2	=	2
Risky Projects	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	3	=	3
Required Projects	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	2	=	2
Goal 1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	=	0
Goal 2	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	=	0
Goal 3	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	=	0
Goal 4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1.11E-16	=	0
Goal 5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	=	0

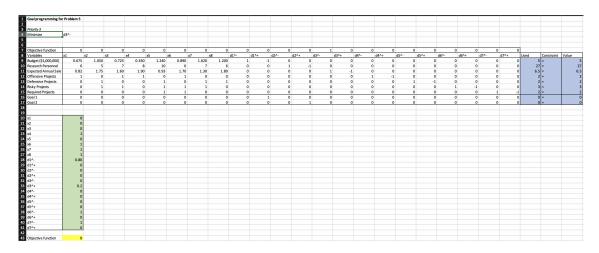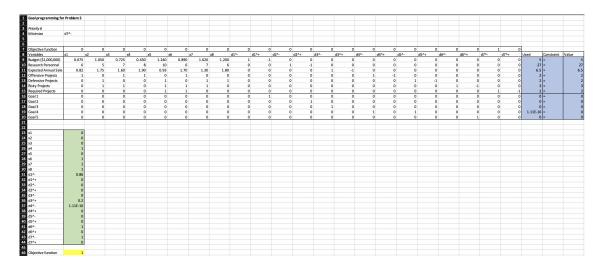|	
x1	0
x2	0
x3	0
x4	1
x5	0
x6	1
x7	1
x8	1
d1^-	0.86
d1^+	0
d2^-	0
d2^+	0
d3^-	0
d3^+	0.2
d4^-	1.11E-16
d4^+	0
d5^-	0
d5^+	0
d6^-	1
d6^+	0
d7^-	1
d7^+	0
Objective function	1

Figure 3: Excel in Question 5, Priority 6

We can solve the model using excel, shown in Fig 1, 2, 3. To best achieve its goals, project 4 6 7 8 are taken. (or project 2 4 6 7).