Assignment #6 Solutions

due Friday, September 25th, 2020

1

Assume x_{ij} is the indicator of whether the edge (i, j) is chosen to be part of the route, i.e., $x_{ij} = 1$ if it is part of the route and $x_{ij} = 0$ otherwise.

To reduce the number of variables we can assume that items flow only from a lower node number to a higher node number, i.e., we ignore x_{ij} if $i \geq j$. At the same time, we assume the time taken from node i to note j is c_{ij}, $i < j$.. Then the model is as follows:

$$\min z = \sum_{i<j} c_{ij} x_{ij}$$
$$s.t.$$
$$x_{12} + x_{13} + x_{14} = 1$$
$$x_{12} - x_{23} - x_{25} = 0$$
$$x_{13} + x_{23} - x_{34} - x_{36} = 0$$
$$x_{14} + x_{34} - x_{46} = 0$$
$$x_{25} - x_{56} = 0$$
$$x_{56} + x_{36} + x_{46} = 1$$
$$0 \leq x_{ij} \leq 1,\ i < j,\ i = 1, 2, ..., 6,\ j = 1, 2, ..., 6 \textit{ and integer.}$$

Dijkstra algorithm

(a) To start, define the permanent set to be the origin, node 1.

(b) Next, find the shortest path from node 1 to any of its adjacent nodes: In this case, node 2 is the closest to 1, which we will add to the permanent set. Its distance to node 1 is 2.

(c) Next, we explore all the nodes adjacent to the nodes in the permanent set, i.e, $\{1, 2\}$. In this case, node 4 is the closest to node 1. Its distance to node 1 is 3.

(d) Continue with the same manner until all the nodes are in the permanent set, finally we will derive the following answer.

Then we can derive the shortest routes as follows.

Node 1 is adjacent to 2 and 4 ;
Node 2 is adjacent to 3 and 5;
Node 3 is adjacent to 6;

2

(a) Assume x_{ij} be the number of units transported through edge (i, j). And we assume the time taken from node i to note j is c_{ij}. Then the model is as follows:

$$
\begin{aligned}
&\min z = \sum_{i<j} c_{ij}(x_{ij} + x_{ji}) \\
&s.t. \\
&x_{12} + x_{15} + x_{16} + x_{17} - x_{21} - x_{51} - x_{61} - x_{71} = 9 \\
&x_{12} + x_{42} + x_{32} - x_{21} - x_{23} - x_{24} = 1 \\
&x_{23} + x_{43} - x_{32} - x_{34} = 1 \\
&x_{24} + x_{34} + x_{64} - x_{42} - x_{43} - x_{46} = 1 \\
&x_{15} + x_{75} + x_{85} + x_{10,5} - x_{51} - x_{57} - x_{58} - x_{510} = 1 \\
&x_{16} + x_{46} + x_{76} + x_{96} - x_{61} - x_{64} - x_{67} - x_{69} = 1 \\
&x_{17} + x_{57} + x_{67} + x_{87} + x_{97} - x_{71} - x_{75} - x_{76} - x_{78} - x_{79} = 1 \\
&x_{58} + x_{78} + x_{98} + x_{108} - x_{85} - x_{87} - x_{89} - x_{8,10} = 1 \\
&x_{69} + x_{79} + x_{89} - x_{96} - x_{97} - x_{98} = 1 \\
&x_{5,10} + x_{8,10} - x_{10,5} - x_{10,8} = 1 \\
&x_{ij} \geq 0, \;\; i = 1, 2, ..., 10, \; j = 1, 2, ..., 10 \; \textit{and integer.}
\end{aligned}
$$

Shortest route problem

One source to all other nodes (undirected)

Units shipped	Node	City	Node	City	Distance (minutes)
3	1	Inglewood	2	Westwood	25
2	1	Inglewood	5	Long Beach	48
1	1	Inglewood	6	Pasadena	50
3	1	Inglewood	7	Downey	32
1	2	Westwood	3	San Fermando Va	35
1	2	Westwood	4	Burbank	18
0	3	San Fermando Val	4	Burbank	28
0	4	Burbank	6	Pasadena	25
0	5	Long Beach	7	Downey	20
0	5	Long Beach	8	Anaaheim	27
1	5	Long Beach	10	Huntington Beach	24
0	6	Pasadena	7	Downey	45
0	6	Pasadena	9	Pomona	36
1	7	Downey	8	Anaheim	40
1	7	Downey	9	Pomona	29
0	8	Anaheim	9	Pomona	41
0	8	Anaheim	10	Huntington Beach	17

Units shippe	Node	City	Node	City	Distance (minutes)
0	2	Westwood	1	Inglewood	25
0	5	Long Beach	1	Inglewood	48
0	6	Pasadena	1	Inglewood	50
0	7	Downey	1	Inglewood	32
0	3	San Fermando	2	Westwood	35
0	4	Burbank	2	Westwood	18
0	4	Burbank	3	San Fermand	28
0	6	Pasadena	4	Burbank	25
0	7	Downey	5	Long Beach	20
0	8	Anaaheim	5	Long Beach	27
0	10	Huntington Be	5	Long Beach	24
0	7	Downey	6	Pasadena	45
0	9	Pomona	6	Pasadena	36
0	8	Anaheim	7	Downey	40
0	9	Pomona	7	Downey	29
0	9	Pomona	8	Anaheim	41
0	10	Huntington Be	8	Anaheim	17
				Total	463

Flow constraints:

Node	Network Flo	Constraint	Value
1	9	=	9
2	1	=	1
3	1	=	1
4	1	=	1
5	1	=	1
6	1	=	1
7	1	=	1
8	1	=	1
9	1	=	1
10	1	=	1

Figure 1: One source to all other nodes (undirected graph)

The shortest route are the following:

$1 \to 2 \to 3$
$1 \to 2 \to 4$
$1 \to 5 \to 10$
$1 \to 6$
$1 \to 7 \to 8$
$1 \to 7 \to 9$

(b) Assume x_{ij} is the indicator of whether the edge (i, j) is chosen to be part of the route, i.e., $x_{ij} = 1$ if it is part of the route and $x_{ij} = 0$ otherwise. And we assume the time taken from node i to note j is c_{ij}. Then the model is as follows:

$$
\begin{aligned}
&\min z = \sum_{i<j} c_{ij}x_{ij} \\
&s.t. \\
&x_{12} + x_{15} + x_{16} + x_{17} = 9 \\
&x_{12} - x_{23} - x_{24} = 1 \\
&x_{23} - x_{34} = 1 \\
&x_{24} + x_{34} - x_{46} = 1 \\
&x_{15} - x_{57} - x_{58} - x_{5,10} = 1 \\
&x_{16} + x_{46} - x_{67} - x_{69} = 1 \\
&x_{17} + x_{57} + x_{67} - x_{78} - x_{79} = 1 \\
&x_{58} + x_{78} - x_{89} - x_{8,10} = 1 \\
&x_{69} + x_{79} + x_{89} = 1 \\
&x_{5,10} + x_{8,10} = 1 \\
&0 \le x_{ij} \le 1,\ i<j,\ i = 1,2,...,10,\ j = 1,2,...,10 \ \mathit{and\ integer}.
\end{aligned}
$$

Shortest route problem

One source to all destination (directed)

Units shipped	Node	City	Node	City	Distance (minutes)
3	1	Inglewood	2	Westwood	25
2	1	Inglewood	5	Long Beach	48
1	1	Inglewood	6	Pasadena	50
3	1	Inglewood	7	Downey	32
1	2	Westwood	3	San Fermando Val	35
1	2	Westwood	4	Burbank	18
0	3	San Fermando Vall	4	Burbank	28
0	4	Burbank	6	Pasadena	25
0	5	Long Beach	7	Downey	20
0	5	Long Beach	8	Anaaheim	27
1	5	Long Beach	10	Huntington Beach	24
0	6	Pasadena	7	Downey	45
0	6	Pasadena	9	Pomona	36
1	7	Downey	8	Anaheim	40
1	7	Downey	9	Pomona	29
0	8	Anaheim	9	Pomona	41
0	8	Anaheim	10	Huntington Beach	17
				Total	463

Flow constraints:

Node	Network Flow	Constraint	Value
1	9	=	9
2	1	=	1
3	1	=	1
4	1	=	1
5	1	=	1
6	1	=	1
7	1	=	1
8	1	=	1
9	1	=	1
10	1	=	1

Figure 2: One source to all other nodes (directed graph)

The shortest route are the following:

$1 \to 2 \to 3$
$1 \to 2 \to 4$
$1 \to 5 \to 10$
$1 \to 6$
$1 \to 7 \to 8$
$1 \to 7 \to 9$

(c) Yes, it does matters because the shortest paths from 1 to every other node may change.

3

Assume x_{ij} is the indicator of whether the edge (i, j) is chosen to be part of the route, i.e., $x_{ij} = 1$ if it is part of the route and $x_{ij} = 0$ otherwise. To reduce the number of variables we can assume that items flow only from a lower node number to a higher node number, i.e., we ignore x_{ij} if $i \geq j$. At the same time, we assume the time taken from node i to note j is c_{ij}, $i < j$. Then the model is as follows:

$$\min z = \sum_{i<j} c_{ij} x_{ij}$$

$s.t.$

$$x_{12} + x_{13} + x_{14} + x_{15} - x_{21} + x_{31} + x_{41} + x_{51} = 1$$
$$x_{12} + x_{32} + x_{62} + x_{92} - x_{21} - x_{23} - x_{26} - x_{29} = 0$$
$$x_{13} + x_{23} + x_{43} + x_{63} + x_{73} + x_{83} - x_{31} - x_{32} - x_{34} - x_{36} - x_{37} - x_{38} = 0$$
$$x_{14} + x_{34} + x_{54} + x_{74} - x_{41} + x_{43} - x_{45} - x_{47} = 0$$
$$x_{15} + x_{45} + x_{75} + x_{14,5} - x_{51} - x_{54} - x_{57} - x_{5,14} = 0$$
$$x_{26} + x_{36} + x_{86} + x_{96} - x_{62} - x_{63} - x_{68} - x_{69} = 0$$
$$x_{37} + x_{47} + x_{57} + x_{87} + x_{10,7} - x_{73} - x_{74} - x_{75} - x_{78} - x_{7,10} = 0$$
$$x_{38} + x_{68} + x_{78} + x_{11,8} + x_{12,8} - x_{83} - x_{86} - x_{87} - x_{8,11} - x_{8,12} = 0$$
$$x_{29} + x_{69} + x_{11,9} + x_{13,9} - x_{92} - x_{96} - x_{9,11} - x_{9,13} = 0$$
$$x_{7,10} + x_{12,10} + x_{14,10} - x_{10,7} - x_{10,12} - x_{10,14} = 0$$
$$x_{8,11} + x_{9,11} + x_{12,11} + x_{13,11} - x_{11,8} - x_{11,9} - x_{11,12} - x_{11,13} = 0$$
$$x_{8,12} + x_{10,12} + x_{11,12} + x_{15,12} + x_{16,12} - x_{12,8} - x_{12,10} - x_{12,11} - x_{12,15} - x_{12,16} = 0$$
$$x_{9,13} + x_{15,13} - x_{13,9} - x_{13,15} = 0$$
$$x_{5,14} + x_{10,14} + x_{16,14} - x_{14,5} + x_{14,10} - x_{14,16} = 0$$
$$x_{11,15} + x_{12,15} + x_{13,15} + x_{17,15} - x_{15,11} + x_{15,12} + x_{15,13} - x_{15,17} = 0$$
$$x_{12,16} + x_{14,16} + x_{17,16} - x_{16,12} - x_{16,14} - x_{16,17} = 0$$
$$x_{15,17} + x_{16,17} - x_{17,15} - x_{17,16} = 1$$
$$0 \leq x_{ij} \leq 1,\ i < j\ i = 1, 2, ..., 17,\ j = 1, 2, ..., 17\ \textit{and integer}.$$

Shortest route problem

Select branch	Node	Node	Distance (days)	Select branc	Node	Node	Distance (days)
0	1	2	3	0	2	1	3
0	1	3	4	0	3	1	4
1	1	4	2	0	4	1	2
0	1	5	5	0	5	1	5
0	2	3	4	0	3	2	4
0	2	6	2	0	6	2	2
0	2	9	6	0	9	2	6
0	3	4	1	0	4	3	1
0	3	6	3	0	6	3	3
0	3	7	5	0	7	3	5
0	3	8	4	0	8	3	4
0	4	5	2	0	5	4	2
1	4	7	4	0	7	4	4
0	5	7	3	0	7	5	3
0	5	14	6	0	14	5	6
0	6	8	3	0	8	6	3
0	6	9	3	0	9	6	3
0	7	8	3	0	8	7	3
1	7	10	2	0	10	7	2
0	8	11	2	0	11	8	2
0	8	12	4	0	12	8	4
0	9	11	2	0	11	9	2
0	9	13	4	0	13	9	4
1	10	12	2	0	12	10	2
0	10	14	3	0	14	10	3
0	11	12	5	0	12	11	5
0	11	13	1	0	13	11	1
0	11	15	2	0	15	11	2
0	12	15	3	0	15	12	3
1	12	16	1	0	16	12	1
0	13	15	3	0	15	13	3
0	14	16	7	0	16	14	7
0	15	17	5	0	17	15	5
1	16	17	3	0	17	16	3
						Total	14

Flow constraints:

Node	Network Flo	Constraint	Value
1	1	=	1
2	0	=	0
3	0	=	0
4	0	=	0
5	0	=	0
6	0	=	0
7	0	=	0
8	0	=	0
9	0	=	0
10	0	=	0
11	0	=	0
12	0	=	0
13	0	=	0
14	0	=	0
15	0	=	0
16	0	=	0
17	1	=	1

Figure 3: Shortest path from George's camp to coast

The shortest route are the following:

$1 \rightarrow 4 \rightarrow 7 \rightarrow 10 \rightarrow 12 \rightarrow 16 \rightarrow 17$

The total time from 1 to 17 is 14 days.