Assignment #5 Solutions

due Friday, September 27th, 2019

1

Let x_{ij} = the amount of steels (tons) supplied from City i to city j every week, where i = A,B,C and j = 1,2,3,4.
Data:

Let c_{ij} = shipping cost per ton from city i to city j,where i = A,B,C and j = 1,2,3,4.
S_j = weekly production in city i, where i = A, B, C.
D_i = weekly demand in city j, where j = 1,2,3. Then the model is as follows:

$$\min z = \sum_{i=A}^{C} \sum_{j=1}^{4} c_{ij} x_{ij}$$

$$s.t. \quad \sum_{i=A}^{C} x_{ij} \leq S_j, \ \forall j$$

$$\sum_{j=1}^{4} x_{ij} \geq D_i, \ \forall i$$

$$x_{B3} = 0$$

$$x_{ij} \geq 0, \forall i, j \ and \ integer.$$

The minimum cost is \$8260 with optimal solution shown below.

1	Variables	Destinations						
2	Sources	1. Detroit	2. St. Louis	3. Chicago	4. Norfolk	Slack demand	Steel shipped	*Supply*
3	A. Bethlehem	0	0	0	150	0	150	150
4	B. Birmingham	120	0	0	90	0	210	210
5	C. Gary	10	70	180	0	60	320	320
6	Steel shipped	130	70	180	240	60		
7	*Demand*	130	70	180	240	60		
8								
9								
10			Costs	Destinations				
11			Sources	1. Detroit	2. St. Louis	3. Chicago	4. Norfolk	
12			A. Bethlehem	14	9	16	18	
13			B. Birmingham	11	8	7	16	
14			C. Gary	16	12	10	22	
15								
16								
17								
18			Total cost =	8260				

2

Let i = A (Charlotte), B(Memphis), C (Louisville) and j = 1 (St. Louis), 2 (Atlanta), 3 (New York).
x_{ij} = number of trucks from warehouse i to terminal j every week, where i = A,B,C and j = 1,2,3.

Data:

Let p_{ij} = profit per truckload shipment from warehouse i to terminal j, where i = A,B,C and j = 1,2,3.

C_j = additional trucks capacity at terminal j, where j = 1,2,3. Then the model is as follows:

$$\max z = \sum_{i=A}^{C} \sum_{j=1}^{3} p_{ij} x_{ij}$$

$$s.t. \quad \sum_{j=1}^{3} x_{ij} = 30, \ \forall i$$

$$\sum_{i=A}^{C} x_{ij} \le C_j, \ \forall j$$

$$x_{ij} \ge 0, \ \forall i, j \ and \ integer.$$

From the table, we can see that the maximum profit is \$159,000.

1	**Transporting steel to plants**										
2											
3	Variables	Terminal						Profit	Terminal		
4	Warehouses	1. St. Louis	2. Atlanta	3. New York	Steel shipped	*Trucks*		Warehouses	1. St. Louis	2. Atlanta	3. New York
5	A. Charlotte	0	30	0	30	30		A. Charlotte	1800	2100	1600
6	B. Memphis	30	0	0	30	30		B. Memphis	1000	700	900
7	C. Louisville	0	0	30	30	30		C. Louisville	1400	800	2200
8	Steel shipped	30	30	30							
9	*Extra truck space*	40	60	50							
10											
11	Total profit =	159000									

3

(a) Let i = 1 (Math), 2 (History), 3 (English), 4 (Biology), (Spanish), 5 (Psychology)
j = 1(8M), 2(9M), 3(11M), 4(12M), 5(14M), 6(8T), 7(9T), 8(11T), 9(12T), 10(14T)

$$x_{ij} = \begin{cases} 1 & \text{if enroll in course i section j, } i = 1, ..., 5, j = 1, ..., 10 \\ 0 & \text{otherwise} \end{cases}$$

Data:

$$y_{ij} = \begin{cases} 1 & \text{if course i section j is available, } i = 1, ..., 5, j = 1, ..., 10 \\ 0 & \text{otherwise} \end{cases}$$

p_{ij} = level of preference for course i section j, $i = 1, ..., 5, j = 1, ..., 10$ and $p_{ij} \in \{1, .., 8\}$, where 1 is the most preferred, and 8 is the least preferred.

$$\min z = \sum_{i=1}^{5} \sum_{j=1}^{10} p_{ij} x_{ij}$$

$$s.t. \quad \sum_{j=1}^{10} x_{ij} = 1, \ \forall i$$

$$x_{ij} \le y_{ij}, \ \forall i, j$$

$$0 \le x_{ij} \le 1, \ \forall i, j \ and \ integer$$

The minimum preference is 10 with optimal solution shown below.

1		Time													
2	Indicator	1. 8M	2. 8T	3. 9M	4. 9T	5. 11M	6. 11T	7. 12M	8. 12T	9. 14M	10. 14T	# sessions		only one session		min preference
3	Course															10
4	A. Math	0	0	0	0	0	0	0	1	0	0	1	=	1		
5	B. History	0	0	0	0	0	1	0	0	0	0	1	=	1		
6	C. English	0	0	0	1	0	0	0	0	0	0	1	=	1		
7	D. Biology	0	0	0	0	1	0	0	0	0	0	1	=	1		
8	E. Spanish	0	0	0	0	0	0	1	0	0	0	1	=	1		
9	F. Psychology	0	0	0	0	0	0	0	0	0	1	1	=	1		
10	time slot	0	0	0	1	1	1	1	1	0	1					
11		<=	<=	<=	<=	<=	<=	<=	<=	<=	<=					
12		1	1	1	1	1	1	1	1	1	1					
13																
14		Time														
15	Preference	1. 8M	2. 8T	3. 9M	4. 9T	5. 11M	6. 11T	7. 12M	8. 12T	9. 14M	10. 14T					
16	Course															
17	A. Math	8	7	6	3	4	1	5	2	10000	10000					
18	B. History	6	5	10000	10000	2	1	10000	10000	4	3					
19	C. English	10000	10000	8	1	4	2	7	5	6	3					
20	D. Biology	7	6	5	10000	2	10000	3	10000	4	1					
21	E. Spanish	10000	4	10000	1	2	10000	3	10000	10000	10000					
22	F. Psychology	6	999	999	4	999	2	999	3	5	1					

(b)

$$\min z = \sum_{i=1}^{5}\sum_{j=1}^{10} p_{ij}x_{ij}$$

$$s.t. \quad \sum_{j=1}^{10} x_{ij} = 1, \ \forall i$$

$$x_{ij} \le y_{ij}, \ \forall i,j$$

$$\sum_{i=1}^{5} x_{i3} = 0$$

$$\sum_{i=1}^{5} x_{i8} = 0$$

$$x_{ij} \ge 0, \ \forall i,j \ and \ integer$$

The minimum preference is 15 with optimal solution shown below.

1		Time														
2	Indicator	1. 8M	2. 8T	3. 9M	4. 9T	5. 11M	6. 11T	7. 12M	8. 12T	9. 14M	10. 14T	# sessions		only one session		min preference
3	Course															15
4	A. Math	0	0	0	0	0	0	0	1	0	0	1	=	1		
5	B. History	0	0	0	0	0	0	0	0	1	0	1	=	1		
6	C. English	0	0	0	1	0	0	0	0	0	0	1	=	1		
7	D. Biology	0	0	0	0	0	0	1	0	0	0	1	=	1		
8	E. Spanish	0	1	0	0	0	0	0	0	0	0	1	=	1		
9	F. Psychology	0	0	0	0	0	0	0	0	0	1	1	=	1		
10	time slot	0	1	0	1	0	0	1	1	1	1					
11		<=	<=	<=	<=	<=	<=	<=	<=	<=	<=					
12		1	1	1	1	1	1	1	1	1	1					
13																
14		Time														
15	Preference	1. 8M	2. 8T	3. 9M	4. 9T	5. 11M	6. 11T	7. 12M	8. 12T	9. 14M	10. 14T					
16	Course															
17	A. Math	8	7	6	3	10000	10000	5	2	10000	10000					
18	B. History	6	5	10000	10000	10000	10000	10000	10000	4	3					
19	C. English	10000	10000	8	1	10000	10000	7	5	6	3					
20	D. Biology	7	6	5	10000	10000	10000	3	10000	4	1					
21	E. Spanish	10000	4	10000	1	10000	10000	3	10000	10000	10000					
22	F. Psychology	6	10000	10000	4	10000	10000	10000	3	5	1					

(c) Since there're only 5 class periods but we have 6 classes, it's impossible to take all classes on the same days.

4 Assume i = A (Adams), B (Baxter), C (Collins), D (Davis), E (Evans), F (Forrest), G (Gomez), H (Huang), I (Inchio), J (Jones), K (King), L (Lopez), and j = 1 (8am-4pm), 2 (4pm-midnight), 3(midnight-8am).

Let

$$x_{ij} = \begin{cases} 1 & \text{if nurse i is assigned to shift j, } i = 1, ..., 5, j = 1, ..., 10 \\ 0 & \text{otherwise} \end{cases}$$

Data: Let r_{ij} = rank assigned by nurse i to shift j, where i = A,...,L and j = 1,2,3. $r_{ij} \in \{1, 2, 3\}$
e_i = experience (in years) of nurse i, where i = A,...,L. Then the model is as follows:

Goal is to minimize the sum of $e_i * r_{ij} * x_{i,j}$. Note that such an objective function "penalizes" more the assignment of senior nurses to low-ranked shifts (higher valued ones).

$$\min z = \sum_{i=A}^{L} \sum_{j=1}^{3} e_i r_{ij} x_{ij}$$

$$s.t. \quad \sum_{j=1}^{3} x_{ij} = 1, \ \forall i$$

$$\sum_{i=A}^{L} x_{i1} = 5$$

$$\sum_{i=A}^{L} x_{i2} = 4$$

$$\sum_{i=A}^{L} x_{i3} = 3$$

$$0 \leq x_{ij} \geq 1, \ \forall i, j \ and \ integer$$

We can solve the model using excel shown below. The schedule is as follows:
Baxter, Collins, Evans, Forrest, King work on 8AM-4PM
Adams, Davis, Gomez, Jones work on 4PM-12AM
Huang, Inchio, Lopez work on 12AM-8am.

1		**Shifts**								**Shifts**		
2	**Variable**	8am - 4pm	4pm - midnight	midnight - 8am					**Preference***	8am - 4pm	4pm - midnight	midnight - 8am
3	**Nurse**				**shifts**		**1 shift per person**		**Nurse**			
4	A	0	1	0	1	=	1		A	2	4	6
5	B	1	0	0	1	=	1		B	5	15	10
6	C	1	0	0	1	=	1		C	7	14	21
7	D	0	1	0	1	=	1		D	3	1	2
8	E	1	0	0	1	=	1		E	3	9	6
9	F	1	0	0	1	=	1		F	4	8	12
10	G	0	1	0	1	=	1		G	2	1	3
11	H	0	0	1	1	=	1		H	3	2	1
12	I	0	0	1	1	=	1		I	2	6	4
13	J	0	1	0	1	=	1		J	6	3	9
14	K	1	0	0	1	=	1		K	5	15	10
15	L	0	0	1	1	=	1		L	4	6	2
16	People	5	4	3								
17		=	=	=								
18	Total demand	5	4	3								
19												
20	Total preference	40										