Lecture 7

Produced by Dr. Worldwide

Welcome to the 305

Ex: Investment

- Investor Kathy Allen has $70,000 to divide across multiple investments
  - Municipal bonds with 8.5% return
  - Certificate of deposit with 5% return
  - Treasury bills with 6.5% return
  - Growth stock with 13% return

- Q: How much should Kathy invest to maximize return?

- Guidelines for diversification
  - No more than 20% of the total investment should be in municipal bonds
  - Amount invested in CDs shouldn't exceed amount invested in the rest
  - At least 30% of the investment should be in treasury bills and CDs
  - More invested in CDs & treasury bills than in the other two by a ratio of at least 1.2 to 1
  - Kathy wants to invest the entire $70,000

Ex: Investment

- Decision variables
  - $x_1 = Dollars\ invested\ in\ municipal\ bonds$
  - $x_2 = Dollars\ invested\ in\ CDs$
  - $x_3 = Dollars\ invested\ in\ Treasury\ Bills$
  - $x_4 = Dollars\ invested\ in\ Growth\ Stock$

- Linear program

Maximize $\quad 0.085x_1 + 0.05x_2 + 0.065x_3 + 0.13x_4$

Subject to

$$
\begin{aligned}
& x_1 + x_2 + x_3 + x_4 = 70000 \\
& x_1/(x_1 + x_2 + x_3 + x_4) \leq 0.2 \\
& x_2 \leq x_1 + x_3 + x_4 \\
& (x_2 + x_3)/(x_1 + x_2 + x_3 + x_4) \geq 0.3 \\
& (x_2 + x_3)/(x_1 + x_4) \geq 1.2 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

Ex: Investment

- Linear program in standard form

Maximize $\quad 0.085x_1 + 0.05x_2 + 0.065x_3 + 0.13x_4$

Subject to
$$
\begin{aligned}
&x_1 + x_2 + x_3 + x_4 = 70000 \\
&0.8x_1 - 0.2x_2 - 0.2x_3 - 0.2x_4 \le 0 \\
&-x_1 + x_2 - x_3 - x_4 \le 0 \\
&0.3x_1 - 0.7x_2 - 0.7x_3 + 0.3x_4 \le 0 \\
&1.2x_1 - x_2 - x_3 + 1.2x_4 \le 0 \\
&x_1, x_2, x_3, x_4 \ge 0
\end{aligned}
$$

- Download Investment-1.xlsx from course website from link Sheet 1

- Optimal solution $(x_1, x_2, x_3, x_4) = (0,0,38181,3181.18)$

Ex: Investment

- Sensitivity analysis

Variable Cells

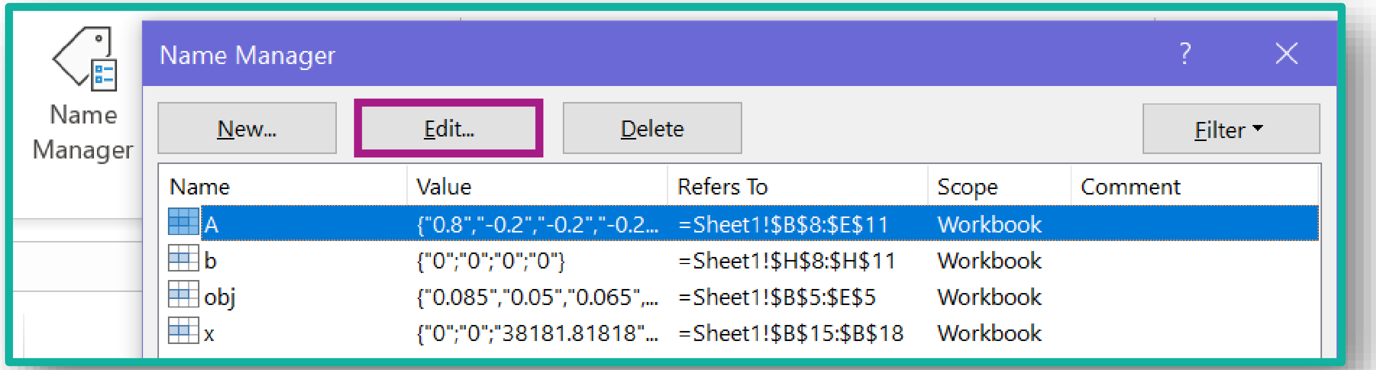
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$15	Municipal bonds = ($)	0	-0.045	0.085	0.045	1E+30
$B$16	CDs = ($)	0	-0.015	0.05	0.015	1E+30
$B$17	Treasury bills = ($)	38181.81818	0	0.065	0.065	0.015
$B$18	Growth stock = ($)	31818.18182	0	0.13	1E+30	0.045

Constraints

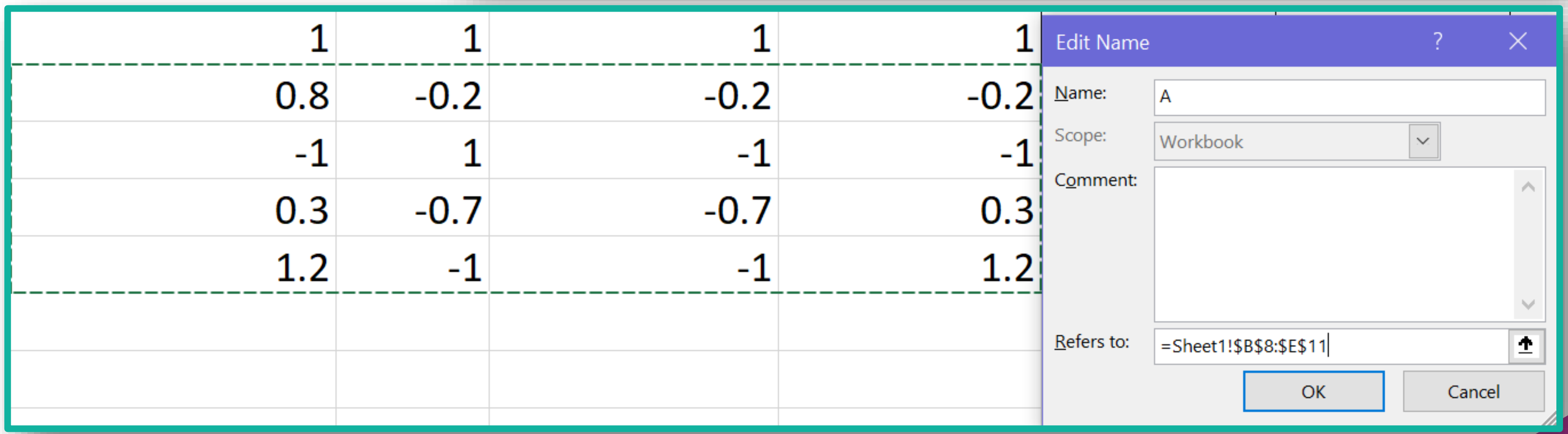
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$7	Total investment Usage	70000	0.094545455	70000	1E+30	70000
$F$8	Constraint 1 Usage	-14000	0	0	1E+30	14000
$F$9	Constraint 2 Usage	-70000	0	0	1E+30	70000
$F$10	Constraint 3 Usage	-17181.81818	0	0	1E+30	17181.81818
$F$11	Constraint 4 Usage	6.54836E-11	0.029545455	0	37800	70000

Ex: Investment

- Created variables



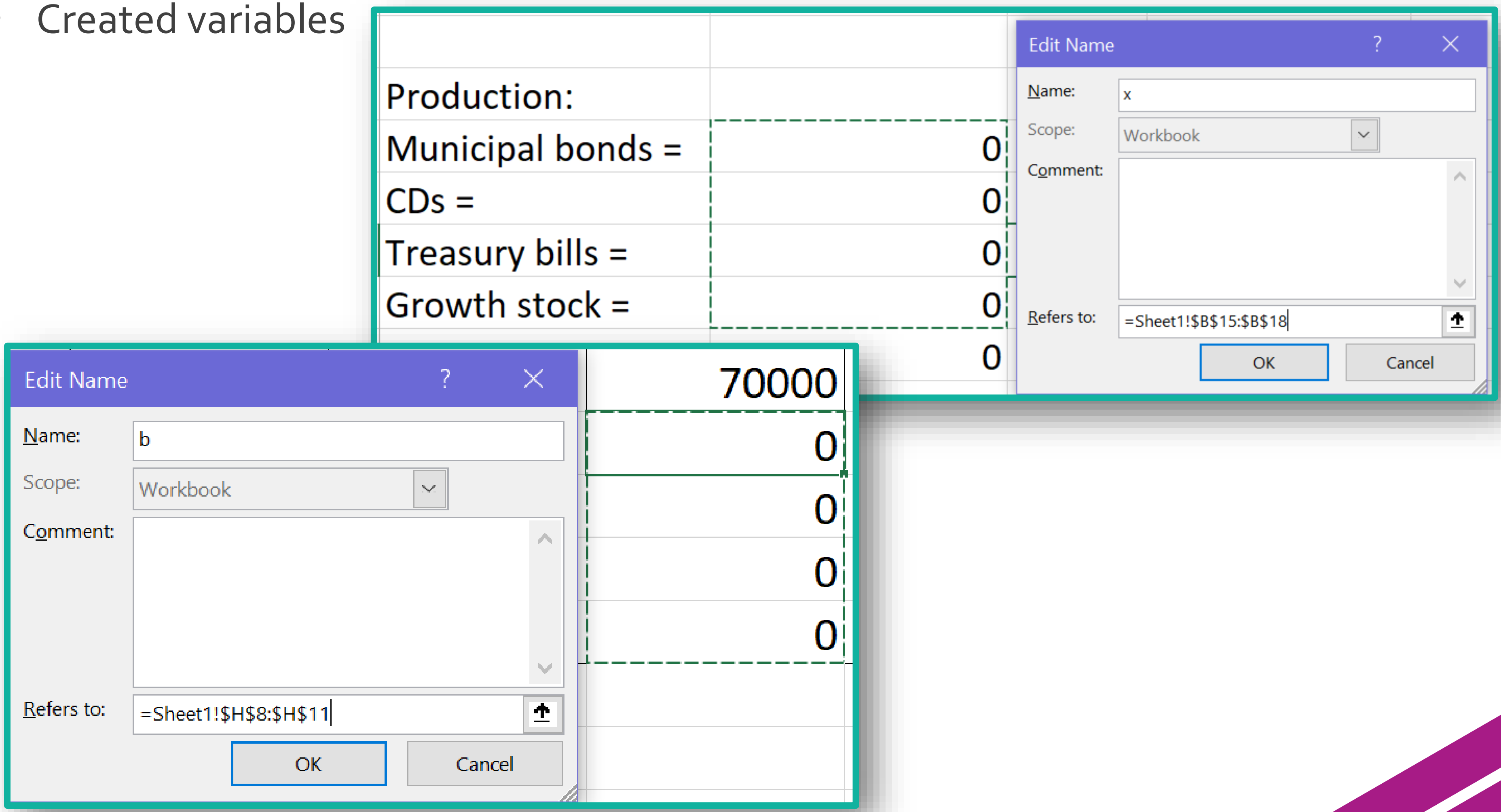
Name	Value	Refers To	Scope	Comment
A	{"0.8","-0.2","-0.2","-0.2...	=Sheet1!$B$8:$E$11	Workbook	
b	{"0";"0";"0";"0"}	=Sheet1!$H$8:$H$11	Workbook	
obj	{"0.085","0.05","0.065",...	=Sheet1!$B$5:$E$5	Workbook	
x	{"0";"0";"38181.81818"...	=Sheet1!$B$15:$B$18	Workbook	



1	1	1	1
0.8	-0.2	-0.2	-0.2
-1	1	-1	-1
0.3	-0.7	-0.7	0.3
1.2	-1	-1	1.2

Ex: Investment

- Created variables



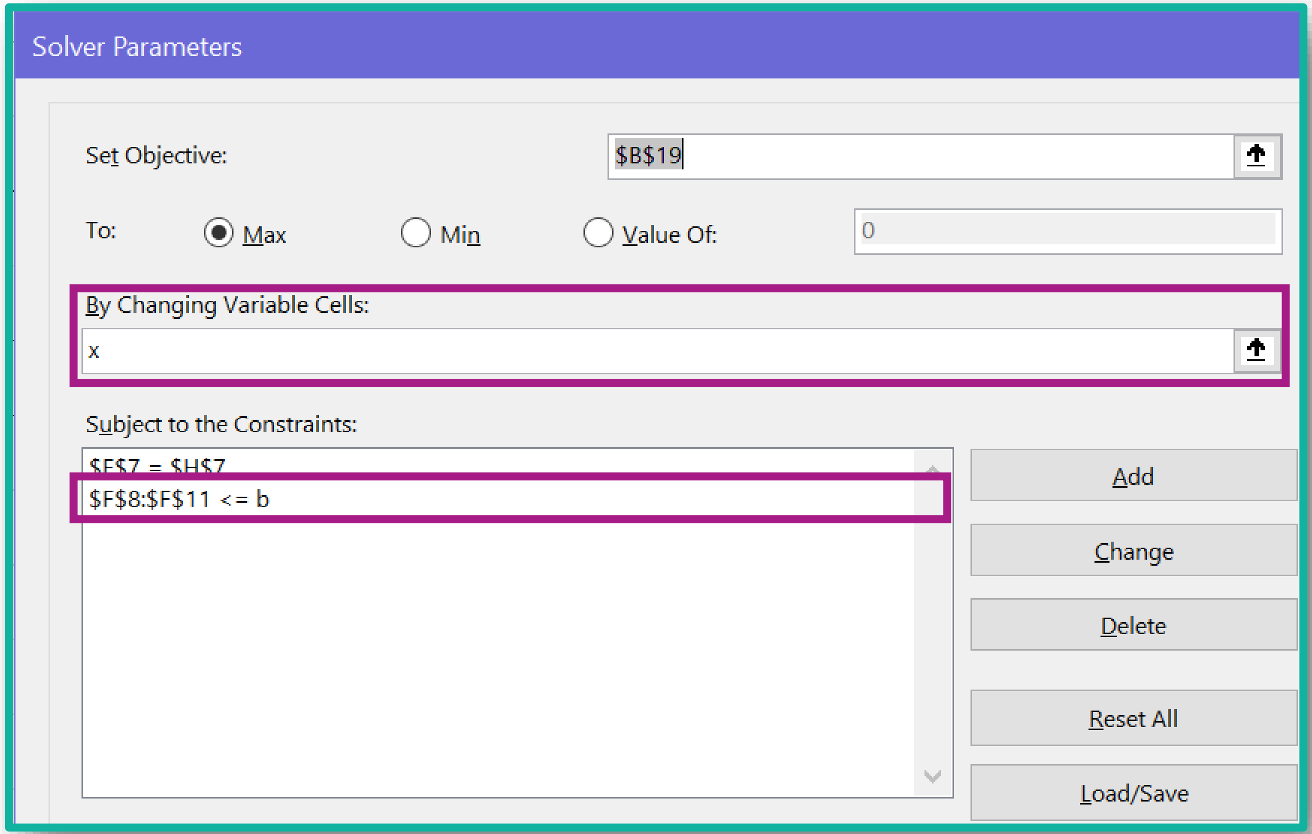
Ex: Investment

- Usage of variables

Products:	Municipal bonds	CDs	Treasury bills	Growth stock			
	($)	($)	($)	($)			
Return:	0.085	0.05	0.065	0.13			
Constraints:					Usage	Constraint	R.H.S.
Total investment	1	1	1	1	0	=	70000
Constraint 1	0.8	-0.2	-0.2	-0.2	=MMULT(A,x)		0
Constraint 2	-1	1	-1	-1	0	<=	0
Constraint 3	0.3	-0.7	-0.7	0.3	0	<=	0
Constraint 4	1.2	-1	-1	1.2	0	<=	0
Production:							
Municipal bonds =	0						
CDs =	0						
Treasury bills =	0						
Growth stock =	0						
Return =	0						

Ex: Investment

- Usage of variables



- Q: What other variable was created and how is it being used?

Ex: Transportation

- Best Boy retail chain ships televisions from 3 of its distribution warehouses to three of its retail stores monthly
- Each warehouse has a fixed supply per month and fixed demand per month
- Q: How many TVs should be shipped from each warehouse to each store to minimize the total cost of transportation?
- Supply (700 TVs) and Demand (600 TVs)

Warehouse	Supply (TVs)
1. Cincinnati	300
2. Atlanta	200
3. Pittsburgh	200

Store	Demand (TVs)
A. New York	150
B. Dallas	250
C. Detroit	200

Ex: Transportation

- Shipping cost per TV for each route

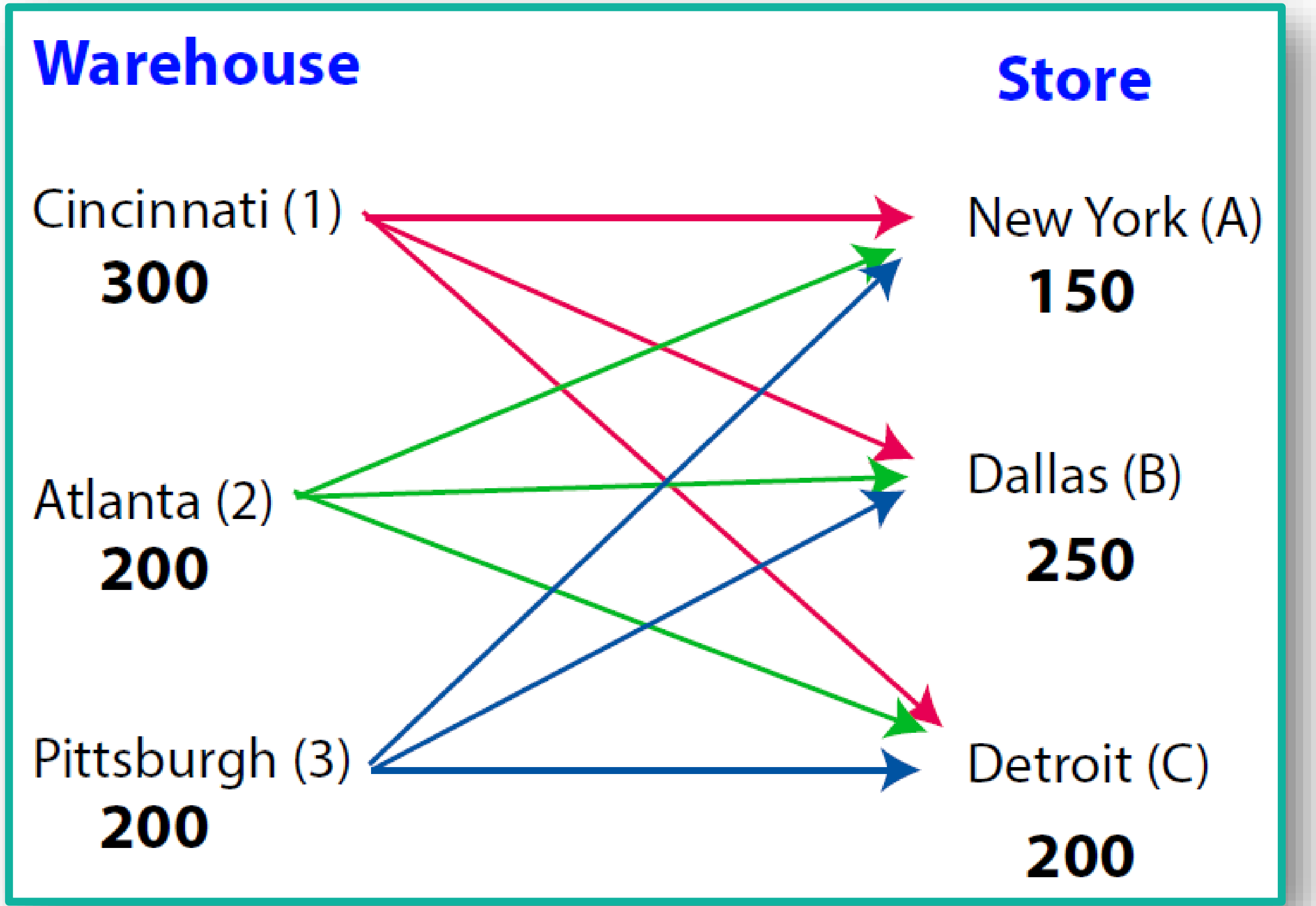
Warehouse
1. Cincinnati
2. Atlanta
3. Pittsburgh

Store
A. New York
B. Dallas
C. Detroit

	To Store		
From Warehouse	**A**	**B**	**C**
1	\$16	\$18	\$11
2	14	12	13
3	13	15	17

Ex: Transportation

- Visual of all routes (supply > demand)



Ex: Transportation

- Decision variables
  - Need to have one for each of the 9 routes
  - x_{ij} = number of televisions from warehouse i to store j
  - $i = 1,2,3 \quad \& \quad j = A, B, C$

- Linear program in standard form

Minimize $\quad 16x_{1A} + 18x_{1B} + 11x_{1C} + 14x_{2A} + 12x_{2B} + 13x_{2C} + 13x_{3A} + 15x_{3B} + 17x_{3C}$

Subject to

$x_{1A} + x_{1B} + x_{1C} \leq 300$ (Cincinnati supply)

$x_{2A} + x_{2B} + x_{2C} \leq 200$ (Atlanta supply)

$x_{3A} + x_{3B} + x_{3C} \leq 200$ (Pittsburgh supply)

$x_{1A} + x_{2A} + x_{3A} \geq 150$ (New York demand)

$x_{1B} + x_{2B} + x_{3B} \geq 250$ (Dallas demand)

$x_{1C} + x_{2C} + x_{3C} \geq 200$ (Detroit demand)

Ex: Transportation

- Download Transportation-1.xlsx from course website from link Sheet 2
- Sheet called Standard contains the standard linear program format and the sheet called Alternative contains a more compact form of the same linear program
- Focus on Alternative sheet

	A	B	C	D	E	F	G
4	***Warehouse***	New York	Dallas	Detroit	***TV sets shipped***	***Constraint***	***Supply***
5	Cincinatti	0	0	200	200	<=	300
6	Altanta	0	200	0	200	<=	200
7	Pittsburgh	150	50	0	200	<=	200
8	***TV sets shipped***	150	250	200			
9	***Constraint***	>=	>=	>=			
10	***Demand***	150	250	200			
11	***Cost ($)***	7300					
12							
13							
14	***Warehouse***	New York	Dallas	Detroit			
15	Cincinatti	16	18	11			
16	Altanta	14	12	13			
17	Pittsburgh	13	15	17			

Ex: Transportation

- Use of SUMPRODUCT in creation of objective function

	A	B	C	D
4	***Warehouse***	New York	Dallas	Detroit
5	Cincinatti	0	0	200
6	Altanta	0	200	0
7	Pittsburgh	150	50	0
8	***TV sets shipped***	150	250	200
9	***Constraint***	>=	>=	>=
10	***Demand***	150	250	200
11	***Cost ($)***	7300		
12		=SUMPRODUCT(B5:D7, B15,D17)		
13				
14	***Warehouse***	New York	Dallas	Detroit
15	Cincinatti	16	18	11
16	Altanta	14	12	13
17	Pittsburgh	13	15	17

Ex: Transportation

- Searching for minimum of objective function

To: ○ Max ◉ Min ○ Value Of: 0

- Optimal solution
  $(x_{1A}, x_{1B}, x_{1C}, x_{2A}, x_{2B}, x_{2C}, x_{3A}, x_{3B}, x_{3C}) = (0,0,200,0,200,0,150,50,0)$

- Textbook uses equality for demand instead of greater than or equal to

Subject to the Constraints:

$B$8:$D$8 = $B$10:$D$10
$E$5:$E$7 <= $G$5:$G$7

	A	B	C	D
2				
3				
4	***Warehouse***	New York	Dallas	Detroit
5	Cincinatti	0	0	200
6	Altanta	0	200	0
7	Pittsburgh	150	50	0
8	***TV sets shipped***	150	250	200
9	***Constraint***	>=	>=	>=
10	***Demand***	150	250	200

Ex: Scheduling

- PM Computers assembles its own brand of laptops from component parts purchased overseas and domestically
- Most computers sold locally to the university, individuals, and businesses
- PM has production capacity to produce 160 computers per week with an additional 50 computers with overtime
- Cost per computer is $190 during regular time and $260 during overtime
- Additionally, it costs $10 per computer per week to hold a computer in inventory for future delivery
- PM wants to meet all customer orders with no shortages and quality service

Ex: Scheduling

- Order schedule for the next 6 weeks

Week	Computer Orders
1	105
2	170
3	230
4	180
5	150
6	250

- Q: How much regular time and overtime production is needed each week to meet its orders at the minimum total production cost?

Ex: Scheduling

- Each week, PM can produce computers either during regular time or during overtime.

- Each week, computers not used for an order are rolled over to the next week

- After the 6-week period, PM wants no inventory left over

- Decision variables
  - $r_j = regular\ production\ of\ computers\ per\ week\ j$
  - $o_j = overtime\ production\ of\ computers\ per\ week\ j$
  - $i_j = extra\ computers\ carried\ over\ as\ inventory\ in\ week\ j$
  - $j = 1{,}2{,}3{,}4{,}5{,}6$

Ex: Scheduling

- Linear program in standard form

Minimize

$$190(r_1 + r_2 + r_3 + r_4 + r_5 + r_6)$$
$$+260(o_1 + o_2 + o_3 + o_4 + o_5 + o_6)$$
$$+10(i_1 + i_2 + i_3 + i_4 + i_5)$$

Subject to

$$r_1 + o_1 - i_1 = 105$$
$$r_2 + o_2 + i_1 - i_2 = 170$$
$$r_3 + o_3 + i_2 - i_3 = 230$$
$$r_4 + o_4 + i_3 - i_4 = 180$$
$$r_5 + o_5 + i_4 - i_5 = 150$$
$$r_6 + o_6 + i_5 = 250$$

$$r_i \leq 160 \; for \; i \in \{1,2,3,4,5,6\}$$
$$o_i \leq 50 \; for \; i \in \{1,2,3,4,5,6\}$$

$$r_i, o_i, i_i \geq 0 \; for \; i \in \{1,2,3,4,5,6\}$$

Ex: Scheduling

- Download Multischedule-1.xlsx from course website from link Sheet 3
- Sheet called Standard contains the standard linear program format and the sheet called Alternative contains a more compact form of the same linear program
- From Standard, see the following solution from Excel Solver

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6
Regular	160	160	160	160	160	160
Overtime	0	0	25	20	30	50
Inventory	55	45	0	0	40	
Total	*160*	*215*	*230*	*180*	*190*	*250*
Required	*105*	*170*	*230*	*180*	*150*	*250*

- Try to acquire the same solution from the Alternative format

The End

Dale