



Football VI

Produced by Dr. Mario | UNC STOR 390



Why Don't Teams Always Pass?



❖ Passing Plays Seem to Dominate Running Plays

❖ Simple Model of Play Selection

- ❖ Offensive Choice: Run or Pass
- ❖ Defensive Choice: Defend Run or Defend Pass
- ❖ Consider the Payoff Matrix

	Run Defense	Pass Defense
Offense Runs	-5	5
Offense Passes	10	0

- ❖ Offense Gains 1 Yard = Defense Loses 1 Yard
- ❖ Positive = Good For Offense





Why Don't Teams Always Pass?



- ❖ James von Neumann and Oskar Morgenstern
 - ❖ Foundation of Game Theory (1944)
 - ❖ Built From Mathematics and Economics
 - ❖ Two-Person Zero Sum Games = Games Where 2 Players are In Total Conflict
 - ❖ Assumption 1: Row Player Wants to Maximize the Payoff
 - ❖ Assumption 2: Column Player Wants to Minimize the Payoff
- ❖ Payoff Matrix Applied to Offense Decisions
 - ❖ Suppose the Offense Passes Every Time
 - ❖ Defense Could Defend Offense Every Time
 - ❖ Offense Would Gain 0 Yards
 - ❖ Is There an Optimal Mixed Strategy for the Offense?





Why Don't Teams Always Pass?



❖ Mixed Strategy

❖ Let p Represent the Probability Offense Runs

❖ Suppose Run Defense is Chosen

$$E[\text{Gain} | \text{Run Defense}] = p(-5) + (1 - p)10 = -15p + 10$$

❖ Suppose Pass Defense is Chosen

$$E[\text{Gain} | \text{Pass Defense}] = p(5) + (1 - p)0 = 5p$$

❖ Suppose Offense Runs 75% of the Time

$$E[\text{Gain} | \text{Run Defense}] = -1.25$$

$$E[\text{Gain} | \text{Pass Defense}] = 3.75$$



Always Run Defense

❖ Suppose Offense Runs 25% of the Time

$$E[\text{Gain} | \text{Run Defense}] = 6.25$$

$$E[\text{Gain} | \text{Pass Defense}] = 1.25$$



Always Pass Defense





Why Don't Teams Always Pass?



- ❖ Optimal Strategy for Offense

- ❖ Balancing the Expected Gain

$$E[\text{Gain}|\text{Run Defense}] = -15p + 10 = 5p = E[\text{Gain}|\text{Pass Defense}]$$

$$-20p = -10$$

$$p = 0.5$$

- ❖ Offense Runs 50% of the Time = Defense Has No Optimal Strategy
 - ❖ No Matter What Defense Selects Expected Gain is 2.5 Yards
 - ❖ Value to the Offense (Row Player) is 2.5 Yards

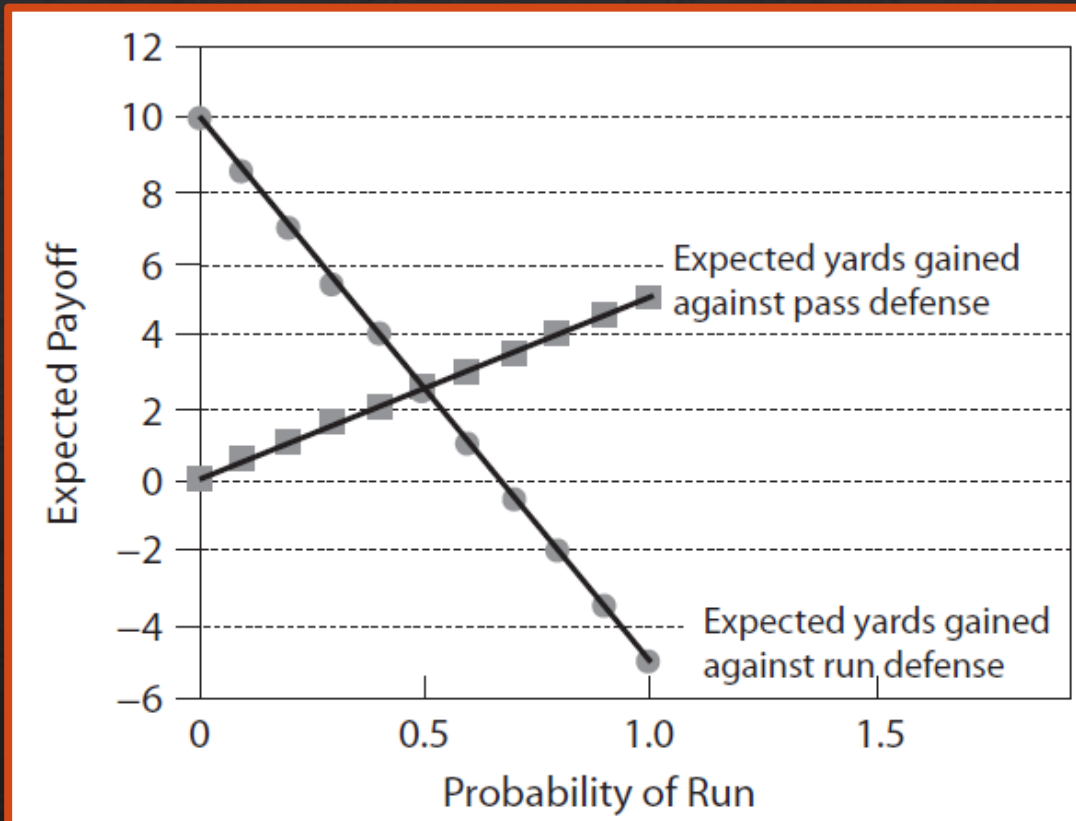




Why Don't Teams Always Pass?



- ❖ Optimal Strategy for Offense
- ❖ Visualization





Why Don't Teams Always Pass?



- ❖ Interesting Conclusion
- ❖ Recall the Payoff Matrix

	Run Defense	Pass Defense
Offense Runs	-5	5
Offense Passes	10	0

- ❖ Defense Guesses Correctly = Passing is 5 Yards Better
- ❖ Defense Guesses Incorrectly = Passing is 5 Yards Better
- ❖ Passing Seems to Dominate
- ❖ Optimal Strategy is to Mix Up Offense and Defense
- ❖ Problems or Ideas?





Why Don't Teams Always Pass?



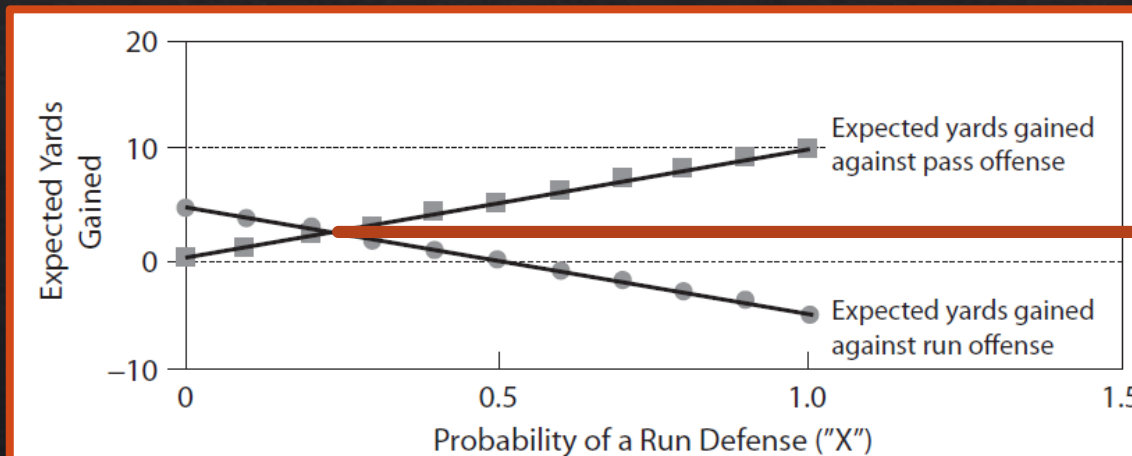
❖ Optimal Strategy for Defense

- ❖ Let q Represent the Probability Defense Calls Run Defense
- ❖ Suppose Offense Decides to Run

$$E[\text{Gain} | \text{Offense Runs}] = q(-5) + (1 - q)5 = -10q + 5$$

- ❖ Suppose Offense Decides to Pass

$$E[\text{Gain} | \text{Offense Passes}] = q(10) + (1 - q)0 = 10q$$



→ 25% Run Defense



Why Don't Teams Always Pass?



❖ Optimal Strategy for Defense

- ❖ Defense Defends Runs 25% of the Time = Offense Has No Optimal Strategy
- ❖ No Matter What Defense Selects Expected Gain is 2.5 Yards
- ❖ Value to the Defense (Column Player) is 2.5 Yards
- ❖ Important Conclusion = Value of Offense and Defense are Equal

❖ Generalization of Payoff Matrix

- ❖ Let r = Average Yards Gained on Run
- ❖ Let t = Average Yards Gained on Pass
- ❖ Let k = Impact of Defense Decision on Running Plays
- ❖ Let m = Multiplier of Impact of Defense Decision on Passing Plays





Why Don't Teams Always Pass?



❖ Generalization of Payoff Matrix

❖ Visualization

	Run Defense	Pass Defense
Offense Runs	$r-k$	$r+k$
Offense Passes	$t+mk$	$t-mk$

❖ Optimal Offensive Mix

$$p = \frac{m}{m+1}$$

❖ Optimal Defensive Mix

$$q = 0.5 + \frac{r-t}{2k(m+1)} \longrightarrow \text{Defend Better Play More Than 50\% of the Time}$$





Why Don't Teams Always Pass?



❖ Interesting Insights From Game Theory

❖ Acquire Better Quarterback

- ❖ New QB Gains 3 More Yards on Average
- ❖ Should We Pass More Often?
- ❖ Replace t with $t + 3$
- ❖ Has No Impact on Optimal Run-Pass Mix

❖ Acquire Better Running Back

- ❖ New RB Gains 5 More Yards on Average Against Pass Defense
- ❖ New RB Gains 0 More Yards on Average Against Run Defense





Why Don't Teams Always Pass?



❖ Interesting Insights From Game Theory

❖ Consider Payoff Matrix

	Run Defense	Pass Defense
Offense Runs	-5	5
Offense Passes	5	-5

❖ Modified Payoff Matrix with Better Running Back

	Run Defense	Pass Defense
Offense Runs	-5	10
Offense Passes	5	-5





Why Don't Teams Always Pass?



❖ Interesting Insights From Game Theory

❖ Finding Values

❖ $r = 2.5$

❖ $k = 7.5$

❖ $p = 0$

❖ $5 = 7.5m \longrightarrow m = 2/3$

❖ Optimal Run-Pass Mixture

❖ Run 2/5 of Time

❖ Pass 3/5 of Time

❖ Value of Offense Increased

❖ Running Back Has Improved But Offense Run Less

❖ Optimal Defense Run-Pass Mixture

❖ Defend Run 3/5 of Time

❖ Defend Pass 2/5 of Time

❖ What Did We Learn?





Final Inspiration

I'm just here so I can pay my fines.

- Mahatma Mario