The NBA as an Evolving Multivariate System

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This article examines the NBA as an evolving system from the viewpoint of evolutionary biologist Stephen Jay Gould who wrote extensively on the disappearance of the .400 hitter in major league baseball, arguing that this is actually a sign of improvement in the quality of play. After reviewing his argument, we use multivariate performance measures of professional basketball players to see if similar characteristics of evolution are observed. It is found that basketball is beginning to exhibit some of these characteristics in recent years, albeit to a much lesser extent than that observed in baseball.

KEY WORDS: Generalized variance; Improvement; Multivariate time series; Sports performance; Trace.

1. INTRODUCTION

The enterprise of sports has been steadily rising in popularity all over the world. With ubiquitous marketing efforts and increasing participation that cuts across race, gender, and national boundaries, the economic and social importance of professional sports has increased dramatically. The revenues generated by major professional sports run into billions of dollars, from paid attendance at games as well as large television audiences. For example, the last two National Basketball Association (NBA) championship finals were commercially televised to more than 600 million households around the world. The popularity of sports has also stimulated various studies of sporting events and athletes participating in them. Scientific studies ranging from sports medicine to the characteristics of the human body in terms of physiology, morphology, or biochemistry are becoming common.

One of the more interesting windows on professional sports was opened by evolutionary biologist Stephen Jay Gould (1986, 1996), who is also an avid fan of baseball. Gould was intrigued by the disappearance of the .400 hitter in major-league baseball since Ted Williams of the Boston Red Sox achieved it for the last time in 1941. Previously, common explanations of this phenomenon had involved the nostalgic conclusion that contemporary hitters are simply not as good as great hitters of the past. In contrast, Gould argued that the phenomenon was really a sign of improvement in the level of play. The improvement manifests itself in a reduction of variation in batting averages, not in an upward trend in averages. In terms of relative distance from the league average, an individual batting average of .380 achieved today may be as good or better than .400 achieved several decades ago. By the same token, .400 hitting achieved today would be even more extraordinary than it was in 1941. Similarly, the breaking of a 37-year-old home run record by *two* players in 1998 is a very rare achievement that is likely to remain unsurpassed for a long time.

With this perspective, Chatterjee and Yilmaz (1991) examined the performance of teams in major-league baseball, rather than individual players. They found that the variability in the winning percentages of teams in both the National and American leagues have been declining over time. In a more recent article, Brooks, Chatterjee, and Yilmaz (1997) considered the question of who should be considered the greatest rebounder of all time in the NBA. Their analyses support the point that it is inappropriate to look at specific performance numbers such as rebounds per game without also considering the variability in those numbers.

In this article, we use the same perspective as a backdrop in looking at the NBA to see if observations similar to those in baseball can be made in the NBA as well. It will be noted, however, that baseball has two characteristics that make it suitable for a study of long-term behavior. First, it has been in existence for more than a century without many drastic changes in the basic rules of play. Second, hitting performance in baseball is easily measured because it involves one person against another, and the results can be expressed in terms of a single number such as the batting average. Measurement of individual performance in a team-oriented sport like basketball is more difficult unless one is only interested in a specific aspect of performance.

Unlike the related discussions mentioned previously, several measures of individual player performance were used in this study. The performance characteristics we have used pertain to scoring, rebounding, playmaking, ballhandling, and defensive quickness. We considered these measures as components of a multivariate performance vector. As far as we are aware, this is a first among the reported studies of sports performance. Multivariate measurement of performance means that variability must be studied using generalized notions of standard deviation or variance. We examine variability in this manner to see if it exhibits a discernible pattern over time.

In the next section, we briefly review Gould's theory as it pertains to competitive sports, followed by a brief history of the NBA. Description of the performance measures we used, our data, and basic analyses are given next, followed by the main findings. The article ends with some concluding remarks.

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2. A SUMMARY OF GOULD'S THEORY

The traditional view in evolutionary biology maintains that, through changes occurring over long periods of time, living things progress from primitive organisms toward more complex and advanced species. These changes occur through a process of natural selection whereby unfit members of population are eliminated, and only the fittest members survive. This process serves to increase the species' potential for survival in changing environments.

In contrast to this traditional view, Gould (1996) suggested that trends in evolution are characterized by *random* changes, not by a directed and purposeful progress toward higher complexity. Beginning with an abundance of single-cell organisms, more complex species are added in a random manner, but the vast majority remain close to minimal complexity. The resulting distribution of the complexity of species is right-skewed, with one tail fixed at the left wall (single cell), and the other tail slowly and randomly moving to the right.

Like the left wall in biological complexity, there is an invisible right wall signifying the biomechanical limits of human ability in sports. Through the centuries, athletes have been slowly approaching these limits. Since hitters in baseball, even average or below-average ones, are converging upon the right wall, the *variation* in batting averages has been declining through the decades. Improvement in hitting does not necessarily lead to an increase in batting averages, because pitching has also been improving. In an entire century of professional baseball, yearly hitting averages fluctuated around a stable level (the overall average in this century is around .262) while variation in the averages has been declining.

Declining variation combined with a stable average level implies that best hitting performances in the current era are closer to the average level than those in the past. Alternatively, a .400 batting average is farther above the average today; thus, it is much more difficult to attain. For illustration, Table 1 shows some hitting statistics in baseball in all complete decades since 1901. Observe that batting averages show moderate fluctuations but no distinct trend, while standard deviations exhibit a decreasing trend over time. The standardized Z-scores show that the best of the past and the best of the present are similar in their relative position to the overall average. George Brett's .390 in 1980 is quite comparable to Ty Cobb's .420 in 1911. Looking only at numeric batting averages without considering their variability, we would fail to appreciate the deeper significance of the underlying story. One thus comes to the conclusion that a system can be expected to exhibit a moderation of extremes in its behavior as it attains stability, and this is intrinsically connected to the concept of "improvement."

Table 1. Some Batting Statistics in Baseball in all Complete Decades Since 1901 (means and standard deviations are rounded to three decimal places)

Decade	Mean	Std. deviation	Highest average	Standardized score
1901–1910	.253	.040	.426 (Nap Lajoie, 1901)	4.32
1911–1920	.258	.038	.420 (Ty Cobb, 1911)	4.26
1921–1930	.286	.038	.424 (Rogers Hornsby, 1924)	3.63
1931–1940	.276	.033	.390 (Al Simmons, 1931)	3.46
1941–1950	.260	.033	.406 (Ted Williams, 1941)	4.42
1951–1960	.259	.032	.388 (Ted Williams, 1957)	4.03
1961–1970	.250	.032	.361 (Norm Cash, 1961)	3.47
1971–1980	.257	.032	.390 (George Brett, 1980)	4.16
1981–1990	.259	.032	.370 (Tony Gwynn, 1987)	3.47

3. BRIEF HISTORY OF THE NBA

The history of the National Basketball Association can be readily broken down into three periods: 1946–1950, the 1950s through middle 1960s, and the time from middle 1960s to the present. The earliest period was marked by a great deal of instability. A number of teams failed or moved from one city to another, sometimes representing more than one city. There were frequent changes in the game rules, rapid player turnover, and the quality of play was generally low.

From the 1950s through middle 1960s, the NBA became more established and stable. Teams settled permanently in major cities, several new teams were added, and failures of teams declined sharply as the era progressed. The last team failure occurred in 1955 (old Baltimore Bullets) after which there were 8 teams in the league (compared with 29 teams today). Numerous rule changes were instituted in order to increase fan interest (Jares 1971; Hill and Baron 1988; and Pluto 1992). The 24-second shot clock was implemented in 1955 to speed up the game and to prevent stalling tactics that had become common.

The modern era of the NBA began in mid-1960s. After this time, rule changes were fewer and farther between, and they had limited impact on the game. One exception is the three-point shot which was implemented at the beginning of the 1979–1980 season. By providing a bonus for scoring from a long distance (beyond 23 feet 9 inches), this change also helped open up the area under the basket often clogged by defending teams.

The modern era can also be viewed in terms of two rather distinct periods. Between the mid-1960s and early 1980s, the NBA did not have the spectacular marketing and media exposure it gained since that time. The start of the latest period of great economic success coincides with the entry of Larry Bird and Magic Johnson into the NBA in 1979, two players of extraordinary appeal and talent, and the implementation of the three-point shot in the same year. Johnson and Bird had played against each other in an exciting NCAA basketball final in 1979, and subsequently, college basketball also became an extremely popular spectacle. The spring sports season in this country is now dominated by the NCAA basketball tournament and the NBA playoffs. As a result, the pool of players that feeds the NBA has become much larger in size, and higher in the level of talent.

4. PERFORMANCE MEASURES

Because a lot of points are scored in a typical NBA game, it is not surprising that offensive prowess is an important factor in determining an NBA player's performance. The most commonly reported measure of offensive performance is the *average number of points scored per game* (PPG), even though this can be a misleading number since it does not account for playing time. A more informative measure is *points per minute played* (PPM), obtained by dividing the total number of points scored by a player by the number of minutes played. Data on this measure is only available since 1951 because teams did not keep track of minutes played before this time.

Scoring is complemented by several other dimensions of performance. A player's offensive playmaking ability and creativity is commonly measured by *assists per minute* (APM). Other important attributes of performance include rebounding, defense, and ballhandling. *Rebounds per minute* (RPM) is used to represent a player's agility and strength in gaining possession of loose balls. A common measure of defensive quickness is *steals per minute* (SPM), and a measure of general ballhandling skill is *turnovers per minute* (TPM). The use of performance measures in terms of per-minute rates rather than per-game averages or totals is also in agreement with that recommended by Naik and Khattree (1996).

Although there are numerous other less-common measures of performance, we conducted our analyses using these five measures. All of the data come from *Microsoft Complete NBA* CD-ROM (1994) and Sachare (1991). In these sources, data on the first three measures (PPM, APM, RPM) spanned the years 1951–1994, whereas the data on the last two measures (SPM, TPM) were available only for the years 1973–1994.

5. DATA ANALYSIS

In analyzing the data, our main concern was to look at the NBA as an evolving system over time. In this investigation, we analyzed data related to individual player performance (analysis of team performance will be the subject of another paper). The specific questions we asked included the following:

1. Is there a discernible pattern of change in scoring, the most important indicator of offensive performance?

2. Is there a pattern of declining variation in performance as indicated by multivariate measures of variability? In other words, does the phenomenon of shrinking tails and moderating extremes have a multivariate analogy?

3. Is there a pattern in the best overall individual performance as measured by a multivariate standardized score? Relative to the league average, are the best performances of today comparable to the best of the past?

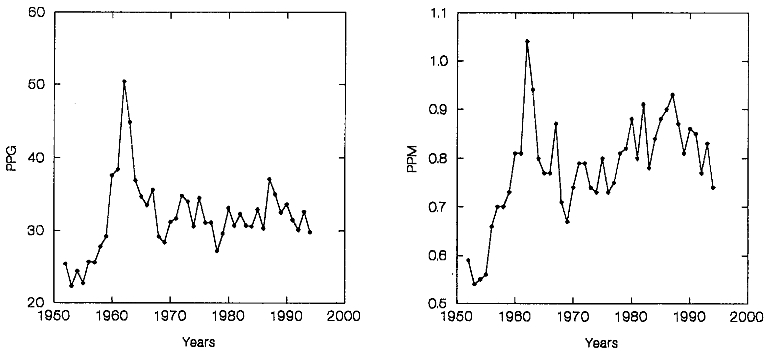
For the period between 1951 and 1994, time series plots of the league-leading PPG and PPM in each year are shown in Figures 1(a, b), respectively. Both graphs show an increase in points scored up to the middle 1960s. An extreme observation in each graph belongs to Wilt Chamberlain in the 1961–1962 season (PPG=50.4 and PPM=1.04, including a record 100 points in one game). After the mid-1960s, PPG remains relatively steady around an average of about 32 points, but a declining pattern since mid-1980s is discernible. PPM also exhibits these features, but a more exaggerated rise in early 1980s followed by the declining pattern can be seen.

It is worthwhile to emphasize that this decline should not be interpreted as a deterioration in offensive performance or quality of the players. This period includes most of the playing career of Michael Jordan, believed by many to be the greatest offensive player ever. The situation here may indeed be similar to the disappearance of the .400 hitter in baseball. The apparent decline can be a sign of improvement because the average scores may be lower because of a rise in the quality of competition (larger player population base, higher financial incentives, more players involved in scoring, improved defensive play, etc.). To get a better idea of this possibility, it is desirable to look at the variability in the performance of the players over time.

To examine variability, data was represented as an $n \times p$ matrix $\mathbf{X}_t$ whose n rows consist of the data for n players participating in year $t, t = 1, 2, \ldots, T$, and its p columns correspond to the selected performance measures. For the period 1951–1994, we used $p = 3$ measures consisting of points scored (PPM), rebounds (RPM), and assists (APM). For the period 1973–1994, we used $p = 5$ measures by adding steals (SPM) and turnovers (TPM) to the data vectors. For year t, we computed the variance-covariance matrix Σ_t, along with two common measures of multivariate dispersion. One of these is the *generalized variance*, defined as the determinant of $|\Sigma_t|$ of Σ_t. The second measure is the *trace* of Σ_t, the sum of diagonal elements which are the variances of performance measures.

Geometrically, generalized variance is a volume measure which is proportional to the squared volume generated by the vectors of deviations from attribute means, and trace is a distance measure which is proportional to the sum of the squared lengths of the deviation vectors. It is noted that, with multivariate data, both measures of variability have some shortcomings that cannot be avoided. For example, two different covariance matrices can yield the same value of generalized variance or trace. Additionally, trace also ignores all covariance terms. It is hoped that use of both measures gives a more complete picture of variability in multivariate data, especially if they exhibit similar patterns over time. In the present case, this is what we found.

We should also note that there is some discussion in the literature concerning the choice of the variance-covariance matrix as opposed to the correlation matrix in computing generalized variance (see, e.g., Naik and Khattree 1996). Since the correlation matrix forces all variables to have equal variances, it does not allow the identification of those variables that contribute more significantly to total variability. We opted for the covariance matrix since it retains this information by expressing variability in performance measures in their commonly used scales. Also, we use the population dispersion matrix Σ_t (as opposed to the sample matrix $\mathbf{S}_t$) since our data come from the entire population



(a) (b)

Figure 1. Plots of a league-leading PPG (a), and league-leading PPM (b) for the years 1951–1994.

of participating NBA players who actually played in each year.

6. RESULTS

Figures 2(a, b) show plots of generalized variance and trace, respectively, for the period 1951–1994 using $p = 3$ performance measures. The pattern in Figure 2(a) is somewhat similar to that observed in Figure 1(b) for PPM, showing a distinct rise in early 1960s and mid-1980s, followed by a declining trend over the last decade of data. In Figure 2(b), trace shows a pattern somewhat similar to that in Figure 1(a) for PPG. This figure also exhibits a decline in trace since the mid-1980s.

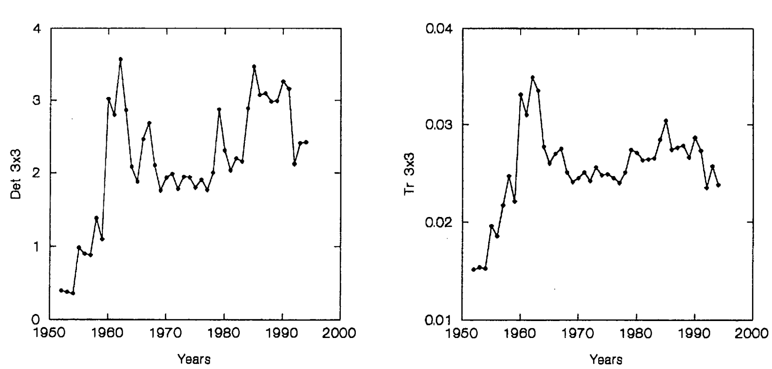
Figures 3(a, b) show plots of generalized variance and trace, respectively, for the period 1973–1994 using $p = 5$ attributes. The patterns in these three figures are very similar to the patterns seen in the right halves of the plots in Figures 2(a, b). Thus, it appears that the addition of two new attributes does not significantly alter the patterns of variability observed with three attributes.

To summarize these observations, evolution of individual performance in the NBA shows significant fluctuations in the first two decades. The pattern of improvement, in the form of a steady decline in variability, appears only in the last decade of data. If this pattern holds, we should see a continuation of shrinking tails of performance statistics, and further moderation of extreme values. This is a big if, however, since the NBA continues to change at a rather rapid pace. For example, the distance of the three-point shot was lowered (to 22 feet) prior to the 1994–1995 season. The following season, two new teams representing Toronto and Vancouver were added, officially rendering the "National" in the league's name a misnomer. Such changes can undoubtedly have confounding effects on the stability of the system.

We next looked at the data for the best overall players in the league in each year, in terms of their distance from the mean. For the player who was voted the most valuable player (MVP) in the league, we computed the squared generalized distance D^2 of his data vector from the vector of league averages. For the ith player in year t, this is defined as

$$D^2_{it} = (\mathbf{x}_{it} - \mu_{it})'\Sigma_t^{-1}(\mathbf{x}_{it} - \mu_{it}),$$

where $\mathbf{x}_{it}$ is the column vector of data for the ith player in year t, and μ_{it} is column vector of means for the p attributes. The square root of D^2, which is known as the Mahalanobis distance, is a multivariate generalization of the univariate standardized Z-score. It is noted that, while Z can be positive or negative in sign, D is always positive by convention. Also, the maximum observed value in a set of univariate data has the largest Z-value, but the observation with the maximum D value may not have the maximum value of any variable in the multivariate case. In fact, an observation that has the *minimum* value of one or more performance measures can have the largest Mahalanobis distance from the mean. Consequently, maximization of D (or D^2) is not a reasonable criterion in determining the best players in the league. On the other hand, since MVPs typically perform well on multiple attributes, comparison of the values for the



(a) (b)

Figure 2. Plots of generalized variance denoted as Det 3 × 3 (a), and trace denoted as Tr 3 × 3 with p = 3 variables (b).

Table 1. Squared Mahalanobis Distances for NBA's Most Valuable Players (p = 3)

Year	MVP	D^2	MVP's D^2 maximum?	Team won division
1956	Bob Pettit	9.16	N	N
1957	Bob Cousy	16.77	Y	Y
1958	Bill Russell	15.94	N	Y
1959	Bob Pettit	10.74	N	Y
1960	Wilt Chamberlain	14.13	N	N
1961	Bill Russell	11.95	N	Y
1962	Bill Russell	13.33	N	Y
1963	Bill Russell	17.73	Y	Y
1964	Oscar Robertson	22.10	Y	N
1965	Bill Russell	16.01	N	Y
1966	Wilt Chamberlain	15.68	N	Y
1967	Wilt Chamberlain	20.20	N	Y
1968	Wilt Chamberlain	23.36	Y	Y
1969	Wes Unseld	8.94	N	Y
1970	Willis Reed	3.53	N	Y
1971	Kareem Abdul-Jabbar	14.69	Y	Y
1972	Kareem Abdul-Jabbar	17.95	Y	Y
1973	Dave Cowens	5.46	N	N
1974	Kareem Abdul-Jabbar	7.39	N	Y
1975	Bob McAdoo	14.80	N	N
1976	Kareem Abdul-Jabbar	15.80	Y	Y
1977	Kareem Abdul-Jabbar	13.19	Y	Y
1978	Bill Walton	15.38	N	Y
1979	Moses Malone	9.65	N	N
1980	Kareem Abdul-Jabbar	6.19	N	Y
1981	Julius Erving	7.07	N	N
1982	Moses Malone	10.42	Y	N
1983	Moses Malone	12.96	N	Y
1984	Larry Bird	9.79	N	Y
1985	Larry Bird	11.09	N	Y
1986	Larry Bird	10.16	N	Y
1987	Magic Johnson	22.85	Y	Y
1988	Michael Jordan	12.77	N	N
1989	Magic Johnson	24.62	Y	Y
1990	Magic Johnson	17.88	N	Y
1991	Michael Jordan	12.76	N	Y
1992	Michael Jordan	11.98	N	Y
1993	Charles Barkley	14.44	N	Y
1994	Hakeem Olajuwon	8.32	N	Y

MVPs or other top players can be informative in terms of their relative distance from the league mean.

For the period 1955–1994, Table 2 shows the MVP in each season, his D^2 value with $p = 3$ performance measures, whether or not the MVP had the highest D^2 among top players, and whether or not MVPs team won its division. As was the case for variability, the addition of two measures for 1973–1994 did not make much difference in the results. It is seen that MVPs almost always played on teams that won their division (and always on teams which at least reached the playoffs). In 28 out of 39 seasons, there were other top players whose D^2 values were larger than the MVPs. While some of these players were well-rounded performers and helped their teams into the playoffs, others played on nonplayoff teams or they were specialists who excelled in one or two dimensions. For example, in 1973–

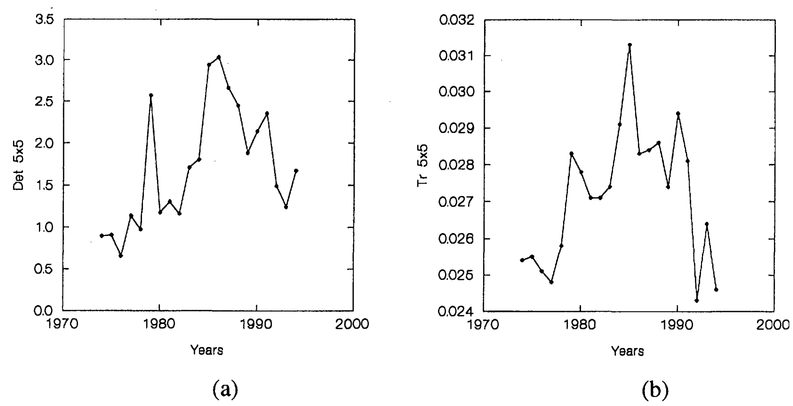


Figure 3. Plots of generalized variance denoted as Det 5 × 5 (a), and trace denoted by Tr 5 × 5 with p = 5 variables (b).

1974, Kareem Abdul Jabbar was the MVP with $D^2 = 7.39$, but Slick Watts (on a team that did not do well) and Bob McAdoo (a pure scorer) had D^2 vales of 11.21 and 10.61.

To give a rough idea of how extreme D^2 values may be, we note that if the data came from a multivariate normal distribution, then D^2 values would follow a chi-square distribution with p degrees of freedom (see, for example, Johnson and Wichern, 1992). With $p = 3$, this means that D^2 would exceed 7.81 with probability .05, 11.34 with probability .01, and 12.84 with probability .005. Accordingly, most MVPs have distances that would fall approximately in the upper 5% tail of a chi-square distribution, and most have distances in the upper 1% tail.

Clearly, MVP selection involves more dimensions than the performance measures we used in this study. Unlike choosing a batting or scoring champion using an unambiguous, univariate measure, selection of the top player in the league (or more generally, a complete ranking of multivariate observations) is intrinsically difficult and ambiguous. In addition to numeric measures, reference is often made to "intangibles" in selecting an MVP, such as motivating other players on the team to play better. That the largest D^2 does not always belong to the MVP is not an accident.

As the third column in Table 2 indicates, squared Mahalanobis distances of the best players do not seem to exhibit a discernible trend, up or down. This is seen even more clearly in Figure 4 which shows a time series plot of D^2 values in Table 2. Thus, distances of top players from the league mean have apparently remained at about the same level through the years. Even though scoring or other statistics of league leaders may exhibit a decline over time, this apparently does not imply a decline of their performance relative to the overall mean for the league.

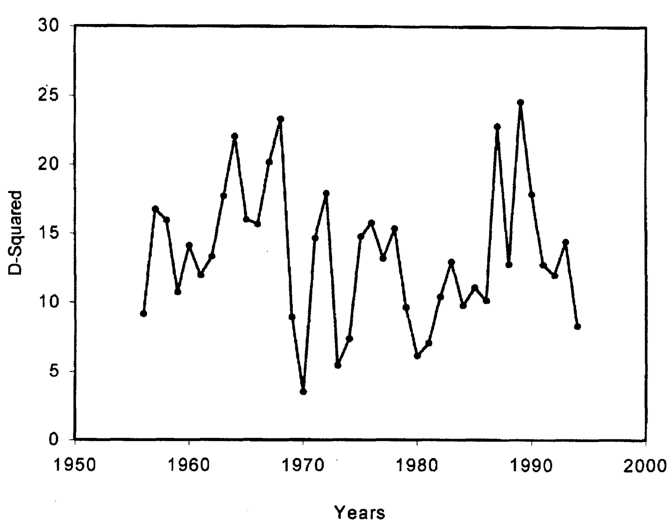


Figure 4. Plot of squared Mahalanobis distances of most valuable players in the NBA.

7. CONCLUSION

In the NBA, the steady decline in the variability of its performance statistics has not yet been observed to the same extent as it has in baseball, though there are indications that such a pattern is beginning to emerge. This may be due to the fact that baseball has been *the* national pastime for a very long time whereas basketball has come of age much more recently. From only 8 teams in 1955, the NBA has grown to 29 teams today, including two teams representing Canadian cities. Also, frequent changes in game rules have not subsided in the NBA, and brand new rules such as illegal defense are still being added. For example, the three-point shot was changed back to its original distance of 23 feet 9 inches in the 1997–1998 season, only three years after it had been reduced to 22 feet. By contrast, major league baseball has taken more than a century to grow to 28 teams, and with fewer rule changes along the way. Thus, the greater variability observed in the NBA is not entirely unexpected.

In the long run, economic and other external factors may have as much influence on professional sports as changes in game rules. Barring drastic new changes in the future, however, it is reasonable to expect an increase in the stability of the NBA in terms of performance statistics like those used in this study. This means that the fluctuations in the average levels should remain around a relatively stable level, and variability in performance measures should exhibit a slow decline in the future. It also means that, like .400 hitting in baseball, a 100 point game or a 50-point average, both achieved by Wilt Chamberlain in 1961, may indeed have disappeared forever.

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