Identifying Unusual Points in Regression

READING:	4.4
EXERCISES:	CH 4. 11abcd
ASSIGNED:	HW 4
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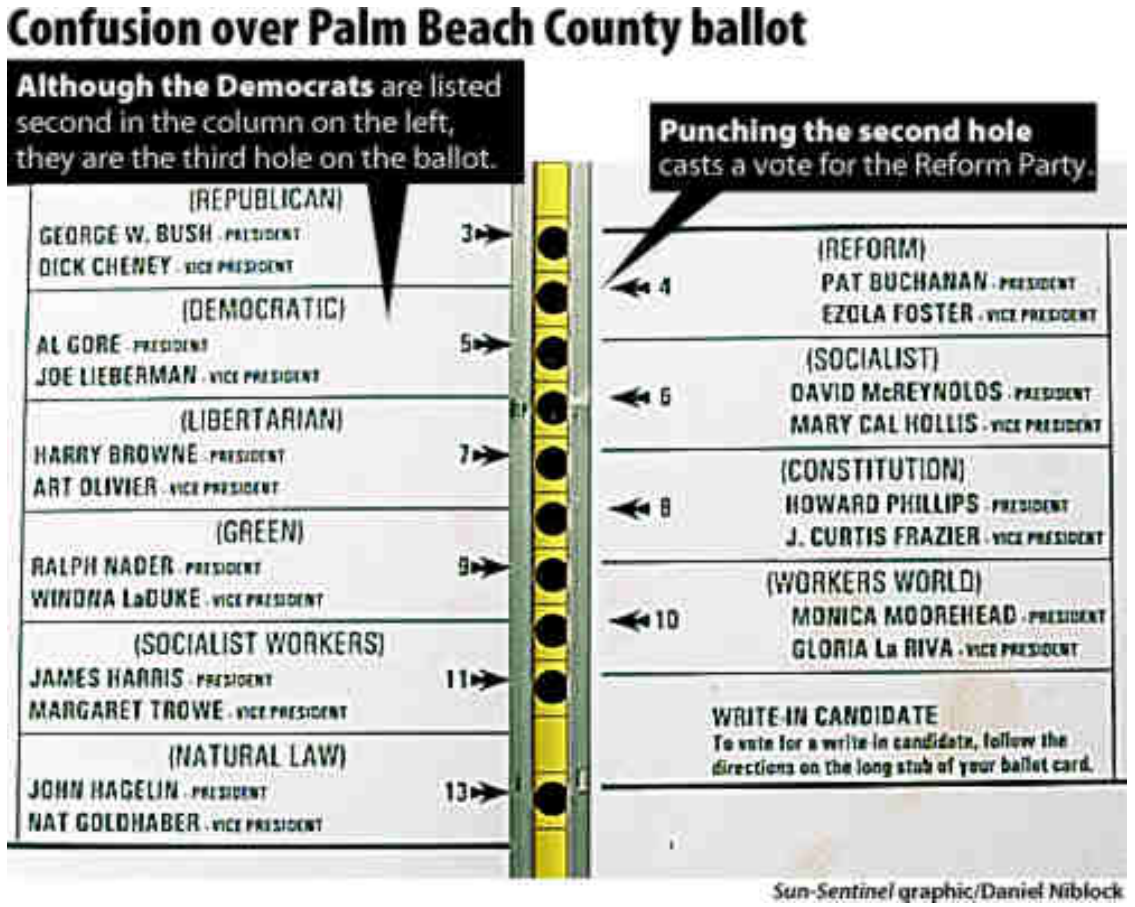
Leverage

- "Potential" Impact on Linear Regression Fit
- Formula: $h_i = \frac{1}{n} + \frac{(x_i - \bar{x})^2}{\sum(x_i - \bar{x})^2}$
- Sum of Leverages: $\sum h_i = \sum \frac{1}{n} + \frac{\sum(x_i - \bar{x})^2}{\sum(x_i - \bar{x})^2} = 1 + 1 = 2$
- Average Leverage or Typical Leverage: $\bar{h} = \frac{\sum h_i}{n} = \frac{2}{n}$
- Less Data or Farther from $\bar{x}$ = More Leverage

Example: Butterfly Ballots

- 2000 Election
  - Bush (Republican)
  - Gore (Democrat)
  - Buchanan (Third Party)
- Unusual Results in Palm Beach
  - Nonstandard “Butterfly Ballot”
  - Accidental Voting?

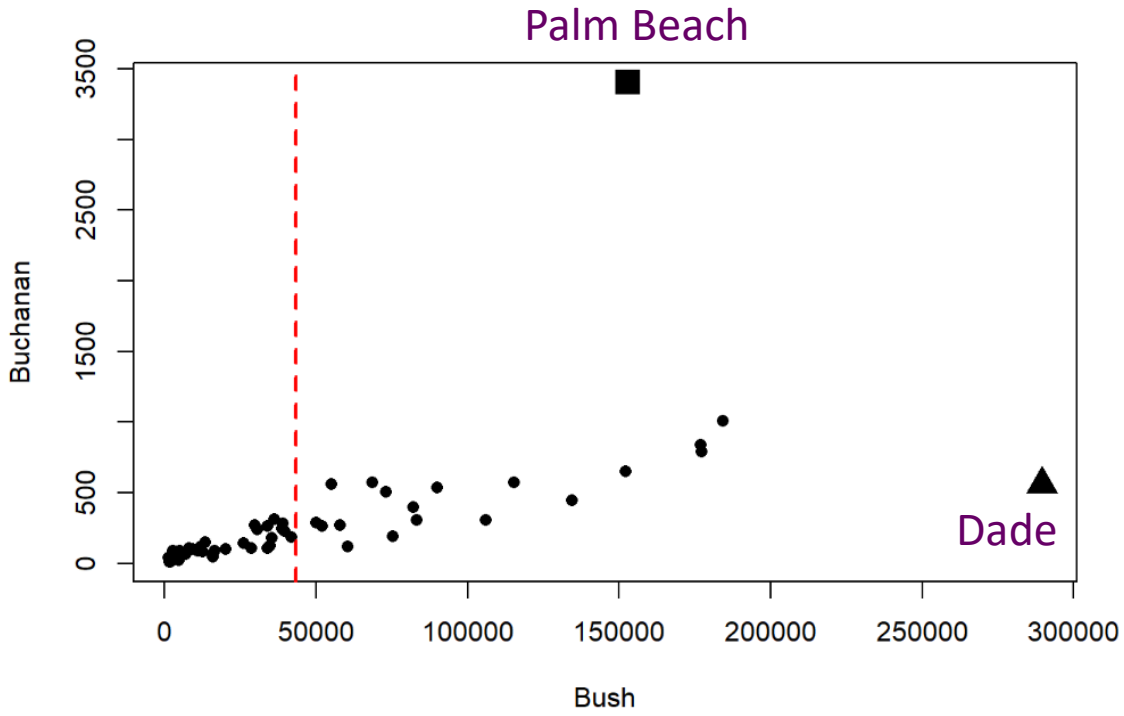


Example: Butterfly Ballots

```
library(Stat2Data)
library(mosaic)

data("PalmBeach")

plot(Buchanan~Bush,data= PalmBeach,pch=16)
abline(v=mean(PalmBeach$Bush),col="red",lty=2,lwd=2)
points(x=PalmBeach$Bush[which(PalmBeach$County=="PALM BEACH")],
       y=PalmBeach$Buchanan[which(PalmBeach$County=="PALM BEACH")],
       cex=2,pch=15)
points(x=PalmBeach$Bush[which(PalmBeach$County=="DADE")],
       y=PalmBeach$Buchanan[which(PalmBeach$County=="DADE")],
       cex=2,pch=17)
```



Example: Butterfly Ballots

```
#Calculate Average Number of Votes for Bush
avg.x = mean(PalmBeach$Bush)
devsquared= (PalmBeach$Bush - avg.x)^2
#Calculate Leverage
Leverage = 1/nrow(PalmBeach) + devsquared/sum(devsquared)
#Save Leverage Into Data
PalmBeach$Leverage= Leverage
#Print Data in Order of Leverage
head(PalmBeach[order(PalmBeach$Leverage,decreasing=TRUE),],5)
```

```
##           County Buchanan   Bush   Leverage
## 13          DADE      561 289456 0.29747301
## 52     PINELLAS     1010 184312 0.10761608
## 6       BROWARD      789 177279 0.09859725
## 29 HILLSBOROUGH      836 176967 0.09820784
## 50   PALM BEACH     3407 152846 0.07085197
```

Very Unusual Leverage

"Triple the Typical Leverage"

$$3 * \frac{2}{n} = 3 * \frac{2}{67} = 0.09$$

Standardized Residual

- Old Formula:

$$stdres_i = \frac{y_i - \hat{y}_i}{\widehat{\sigma_\epsilon}}$$

- New Formula:

$$stdres_i = \frac{y_i - \hat{y}_i}{\widehat{\sigma_\epsilon}\sqrt{1 - h_i}}$$

- Further Adjust for Leverage Since Some Points Impact More
- Points with More Leverage Tend to Higher Standardized Residuals

Studentized Residual

- Old Formula:

$$studres_i = \frac{y_i - \hat{y}_i}{\widehat{\sigma_{(i)}}}$$

- New Formula:

$$studres_i = \frac{y_i - \hat{y}_i}{\widehat{\sigma_{(i)}}\sqrt{1 - h_i}}$$

- We Calculate Standard Error of Regression After Removing Point
- Line Doesn’t Change, Then Standardized Equals Studentized

Cook's Distance

- Formula:

$$D_i = \frac{stdres_i^2}{k+1}\left(\frac{h_i}{1-h_i}\right) \qquad \text{for SLR} \rightarrow k = 1$$

- Amount of Influence Depends On:
  - Leverage
  - Distance from Line of "Best" Fit

- Large Cook's D Indicates Point Strongly Influences Regression

Rules of Thumb

- We Can Calculate All Four Measures On Each Observation

- Table Showing Accepted Definitions of
  - "Moderately Unusual"
  - "Very Unusual"

Statistic	Moderately Unusual	Very Unusual
Leverage	Above $2(^2/_n)$	Above $3(^2/_n)$
Standardized Residual	Beyond ± 2	Beyond ± 3
Studentized Residual	Beyond ± 2	Beyond ± 3
Cook's D	Above 0.5	Above 1

Example: Butterfly Ballots

```
#Fit Model
vote.mod = lm(Buchanan~Bush, data=PalmBeach)
#Extract Standard Error of Regression
sigma.reg = summary(vote.mod)$sigma
#Save Actual Residuals
PalmBeach$Residual = vote.mod$residuals
#Calculate and Save Standardized Residual
PalmBeach$StdRes = PalmBeach$Residual/(sigma.reg*sqrt(1-PalmBeach$Leverage))
#Calculate and Save Cook's Distance
PalmBeach$CookD=cooks.distance(vote.mod)
#Find Very Unusual Cook's Distance
PalmBeach[which(PalmBeach$CookD>1),]
```

```
##         County Buchanan   Bush   Leverage  Residual    StdRes    CookD
## 13        DADE      561 289456 0.29747301 -907.4953 -3.059180 1.981366
## 50 PALM BEACH     3407 152846 0.07085197 2610.1926  7.651072 2.231935
```

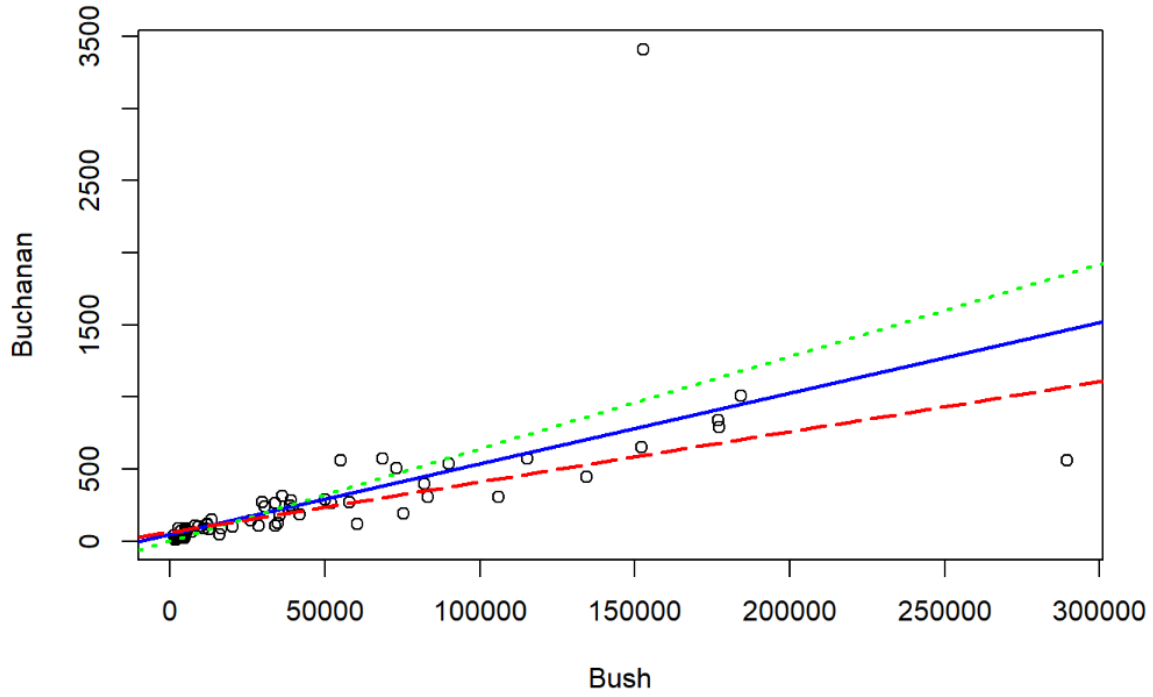
Example: Butterfly Ballots

```
vote.mod = lm(Buchanan~Bush, data=PalmBeach)

noDade = PalmBeach[which(PalmBeach$County!="DADE"),]
vote.mod.noDade = lm(Buchanan~Bush, data=noDade)

noPB = PalmBeach[which(PalmBeach$County!="PALM BEACH"),]
vote.mod.noPB = lm(Buchanan~Bush, data=noPB)


plot(Buchanan~Bush,data=PalmBeach)
abline(vote.mod, col="blue",lwd=2)
abline(vote.mod.noDade, col="green",lwd=2,lty=3)
abline(vote.mod.noPB, col="red",lwd=2,lty=5)
```



Thank You

Make Reasonable Decisions