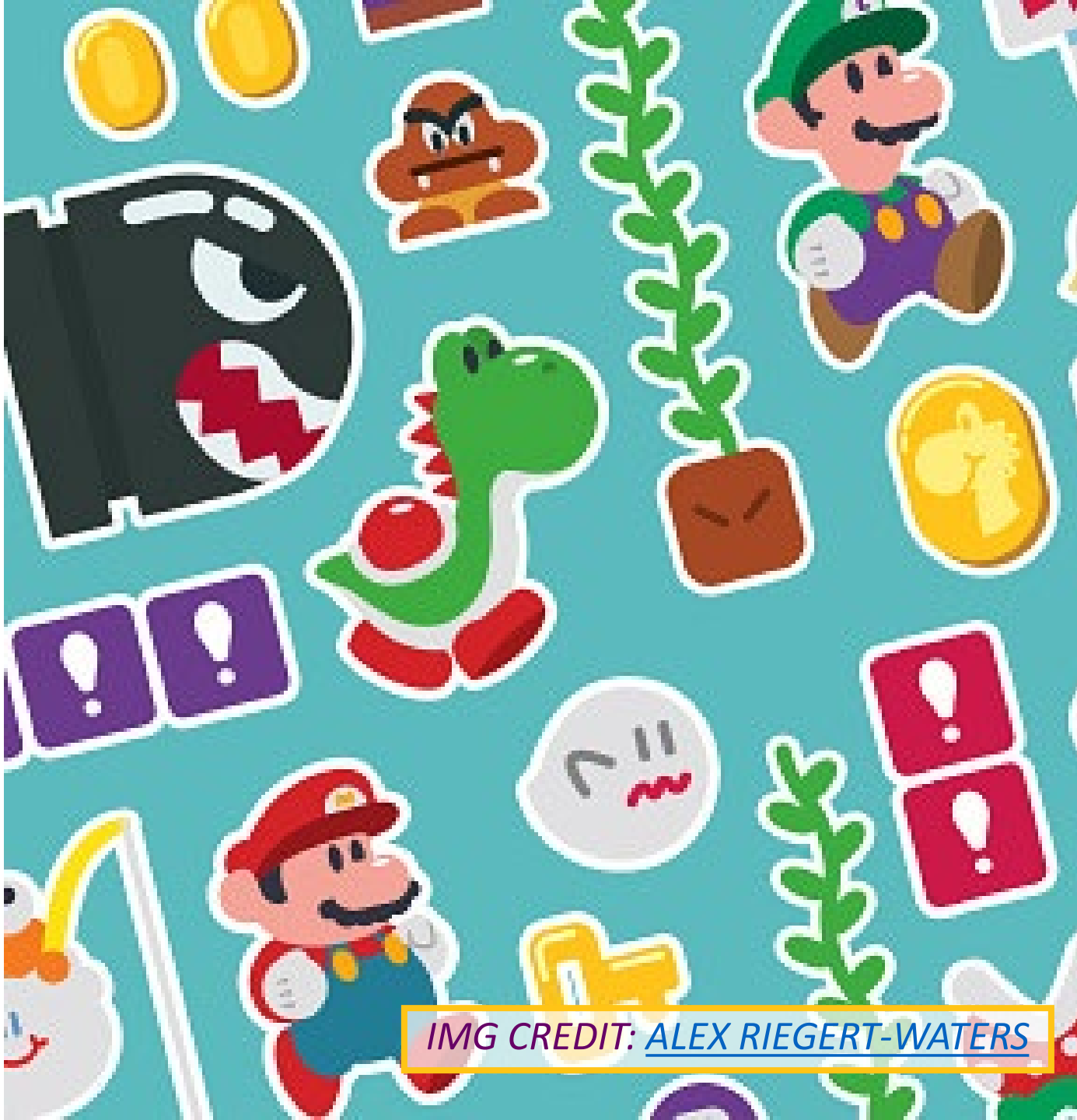
Two-Way ANOVA

READING:	6.3 – 6.5, 7.4 – 7.6, 8.7
EXERCISES:	CH 6. 38 CH 7. 34
ASSIGNED:	NONE
PRODUCER:	DR. MARIO



IMG CREDIT: ALEX RIEGERT-WATERS

Example: Grades on Different Exams

- Data

	Barb	Betsy	Bill	Bob	Bud	Mean
Exam #1:	62	94	68	86	50	72
Exam #2:	87	95	93	97	63	87
Exam #3:	74	86	82	70	28	68
Exam #4:	77	89	73	79	47	73
Mean	75	91	79	83	47	75

- Using One-Way ANOVA:
  - No Significant Differences Between Exams
  - Significant Differences Between Students

- Question: *Can we use **both** factors to explain variability in scores?*

Simple Block Design

- **Simple Block Design** has two factors with exactly one observation in each combination of factors

- Example: Examine Effect of Different Treatments at Different Severities
  - Factor A (Treatments) has *I* Levels
  - Factor B (Severity) has *J* Levels
  - Sample Size is $n = I * J$ in a Simple Block Design

- Question: *What is the problem of this design?*

Two-Way ANOVA

- Means Version

Treatment: $i \in \{1,2,\cdots,I\}$

Block: $j \in \{1,2,\cdots,J\}$

$$Y = \mu_{ij} + \epsilon$$

Mean of *Y* for Treatment *i* and Block *j*

- Effects Version (Additive)

$$Y = \mu + \alpha_i + \beta_j + \epsilon$$

Same Assumptions About Error Term

Grand Mean

Effect of Block *j*

Effect of Treatment *i*

Two-Way ANOVA

- Model

$$Y = \mu + \alpha_i + \beta_j + \epsilon$$

- Estimation of Model

Parameter	Estimate
μ	$\bar{y}$
α_i	$\bar{y}_i - \bar{y}$
β_j	$\bar{y}_j - \bar{y}$
σ_ϵ	$\sqrt{MSE}$

Factor A: $i \in \{1,2,\cdots,I\}$

Factor B: $j \in \{1,2,\cdots,J\}$

Group Means for A: $\bar{y}_i$

Group Means for B: $\bar{y}_j$

Two-Way ANOVA

- Partition of Variation

$$SST = \sum_i \sum_j (y_{ij} - \bar{y})^2 = (n-1)s^2$$

$$SSA = \sum_i \sum_j (\bar{y}_i - \bar{y})^2 = \sum_i J(\bar{y}_i - \bar{y})^2$$

$$SSB = \sum_i \sum_j (\bar{y}_j - \bar{y})^2 = \sum_j I(\bar{y}_j - \bar{y})^2$$

$$SSE = SST - SSA - SSB$$

Example: Diet Programs

- adf

Thank You

Make Reasonable Decisions