Confidence Intervals and Effect Sizes

READING:	5.5
EXERCISES:	NONE
ASSIGNED:	HW 10
PRODUCER:	DR. MARIO

Recall ANOVA F-test

- Question: *What do we learn if we achieve significance?*

$$H_a: \mu_i \neq \mu_j \text{ for some } i \neq j$$

- Follow-up Questions:
  - *How much uncertainty do we have regarding the group means?*
  - *Which group means are different?*
  - *Do the differences matter?*

Example: Grades on Different Exams

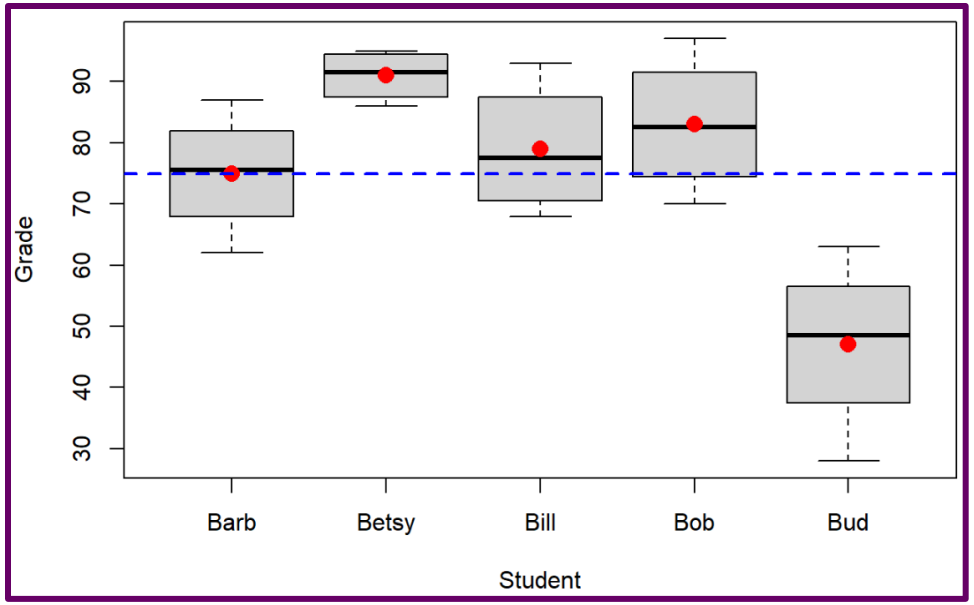
- There are Four Different Exams (1, 2, 3, 4)
- There are Five Students (Barb, Betsy, Bill, Bob, Bud)
- Each Student Takes All Four Exams
- Question: *Is there a significant difference in the average grade among the five individuals?*

Example: Grades on Different Exams

- Plot of Grades for all Four Students

```
Exams = read.csv("Exams4.csv")

boxplot(Grade~Student,data=Exams)
points(tapply(Exams$Grade, Exams$Student, mean),
       col="red",pch=16,cex=1.5)
abline(h=mean(Exams$Grade),lty=2,lwd=2,col="blue")
```



Example: Grades on Different Exams

- ANOVA Model and Output

```
amodS = aov(Grade~Student,data=Exams)
summary(amodS)
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)
## Student      4   4480  1120.0     9.6 0.000468 ***
## Residuals   15   1750   116.7
```

- We Have Evidence that At Least One Student Has an Average Grade that is Different Than the Average Grade of Another Student

Estimating Group Means

- Confidence Interval For Group Mean

$$\bar{y}_i \pm t_{0.975,n-K} \sqrt{\frac{MSE}{n_i}}$$

- Works When Different Groups Have Different Sample Sizes
- Groups With More Data -> Smaller Confidence Intervals
- *MSE* is Fixed For All Groups Because of Homoscedasticity Condition

Example: Grades on Different Exams

- Confidence Intervals for Average Grade for Each Student

```
out = summary(amodS)

ybars = tapply(Exams$Grade,Exams$Student,mean)
t.value = qt(0.975,amodS$df.residual)
MSE = out[[1]][2,3]
counts = tapply(Exams$Grade,Exams$Student,length)

Lower = ybars-t.value*sqrt(MSE/counts)
Upper = ybars+t.value*sqrt(MSE/counts)

data.frame(Lower,Upper)
```

	Lower	Upper
Barb	63.48886	86.51114
Betsy	79.48886	102.51114
Bill	67.48886	90.51114
Bob	71.48886	94.51114
Bud	35.48886	58.51114

Estimating Differences in Group Means

- Confidence Interval For Differences Between Group Means

$$(\bar{y}_1 - \bar{y}_2) \pm t_{0.975,n-K} \sqrt{\frac{MSE}{n_1} + \frac{MSE}{n_2}}$$

- Works for Comparing the Two Groups
- Use this Interval to Estimate $\mu_1 - \mu_2$
- If 0 is in the Interval -> No Evidence that $\mu_1 - \mu_2 \neq 0$

Example: Grades on Different Exams

- Question: *How many differences could we estimate?*

Barb vs. Betsy	Barb vs. Bill	Barb vs. Bob
Barb vs. Bud	Betsy vs. Bill	Betsy vs. Bob
Betsy vs. Bud	Bill vs. Bob	Bill vs. Bud
Bob vs. Bud		

- Formula: "5 Choose 2" $= \binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5\times4\times3!}{2!3!} = \frac{20}{2} = 10$

Example: Grades on Different Exams

- Barb vs Betsy

```
diff = (ybars[1]-ybars[2])
lower = round(diff-t.value*sqrt(MSE/counts[1]+MSE/counts[2]),2)
upper = round(diff+t.value*sqrt(MSE/counts[1]+MSE/counts[2]),2)

paste("(",lower,",",upper,")",sep="")
```

```
## [1] "(-32.28,0.28)"
```

- Barb vs Bud

```
diff = (ybars[1]-ybars[5])
lower = round(diff-t.value*sqrt(MSE/counts[1]+MSE/counts[5]),2)
upper = round(diff+t.value*sqrt(MSE/counts[1]+MSE/counts[5]),2)

paste("(",lower,",",upper,")",sep="")
```

```
## [1] "(11.72,44.28)"
```

Effect Size

- Based on the ANOVA (Effects) Model

$$Y = \mu + \alpha_i + \epsilon$$

- "Statistically Significant" is Often Misleading

- Example:
  - Hiring a Tutor to Help You Increase Your Grade in the Class
  - Question: *Would You Hire That Tutor if You Believe that Tutor Can Increase Your Grade in the Class by 1%?*

Effect Size

- Example:
  - You Are Taking STOR 320 and STOR 455
    - STOR 320: $\mu_{Grade} = 75, \sigma_{Grade} = 1$
    - STOR 455: $\mu_{Grade} = 75, \sigma_{Grade} = 10$
  - Tutor Can Help You Increase Your Grade by 5 Percentage Points
  - Question: *Would you hire that tutor to help you in STOR 320 or STOR 455? Why?*

Effect Size

- Effect Size is Based on a Ratio of the Estimated Effect and *RMSE*
- Formula for Single Group

$$D_i = \frac{\hat{\alpha}_i}{RMSE} = \frac{\bar{y}_i - \bar{y}}{RMSE}$$

- Formula for Difference Between Two Groups

$$D_{ij} = \frac{\bar{y}_i - \bar{y}_j}{RMSE}$$

Example: Grades on Different Exams

- *RMSE* of Model

```
MSE = out[[1]][2,3]
RMSE = sqrt(MSE)
RMSE
```

```
## [1] 10.80123
```

"Typical" Difference Between Points Scored and Expected Points Scored

- Effect Size of Bud

```
(ybars[5]-mean(Exams$Grade))
```

```
## Bud
## -28
```

```
(ybars[5]-mean(Exams$Grade))/RMSE
```

```
##        Bud
## -2.592296
```

The Average Grade Drops by About 2.59 Standard Deviations if You Are Bud

Thank You

Make Reasonable Decisions