Inference for the Regression Slope

READING:	2.1
EXERCISES:	CH 2. 15b, 17bcd, 19bc
ASSIGNED:	HW 5
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Motivation

- Goal: Estimate a Linear Model for the Relationship Y versus X

$$Y = \beta_0 + \beta_1 X + \epsilon$$

- Strategy: Get Sample of 100 Observations and Fit Line

$$\hat{Y} = \widehat{\beta_0} + \widehat{\beta_1} X = 3 - 5X$$

- Question: *What would happen if we got a sample of a different 100 observations, and refit the line?*

Uncertainty in Estimated Slope

- Standard Error of the Slope

$$SE_{\widehat{\beta_1}} = \sqrt{\frac{1}{n-2} * \frac{\sum(y_i - \hat{y})^2}{\sum(x_i - \bar{x})^2}} = \frac{\hat{\sigma}_\epsilon}{\sqrt{\sum(x_i - \bar{x})^2}}$$

- Measures How Much We Expect the Estimated Slope to Vary from One Sample to the Next

- Measures Our Uncertainty About $\hat{\beta}_1$ as an Estimate of β_1

Confidence Interval for Slope

- Goal: Give Range of Possible Values for β_1
- Formula for 95% Confidence Interval:

$$\hat{\beta}_1 \pm \mathrm{t}_{0.025,n-2} * SE_{\widehat{\beta_1}}$$

- Critical Value $\mathrm{t}_{0.025,n-2}$ is the 97.5 percentile on the t-Distribution with $n-2$ Degrees of Freedom
- This Positive and Negative Versions of the Critical Value Divide the **Appropriate** t-Distribution into Middle 95% and Outside 5%

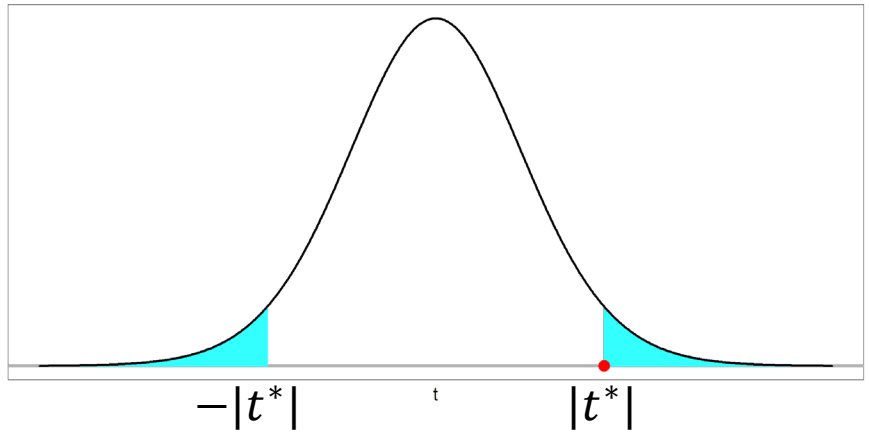
t-Test for Slope

- Step 1: Choose a Significance Level α
  - Assume 0.05 if Not Specified
- Step 2: State Hypotheses (Null and Alternative)
  - $\mathrm{H}_0: \beta_1 = 0$
  - $\mathrm{H}_\mathrm{a}: \beta_1 \neq 0$
- Step 3: Acquire Data and Perform Simple Linear Regression
  - $\hat{\beta}_1 = Estimated\ Slope$
  - $\hat{\sigma}_\epsilon = Standard\ Error\ of\ Regression$

t-Test for Slope

- Step 4: Test Statistic
  - $t^* = {\widehat{\beta_1}}/{SE_{\widehat{\beta_1}}}$

- Step 5: P-value
  - Use t-Distribution with $n - 2$ Degrees of Freedom
  - Two-Sided or Non-Directional
  - P-value = Shaded Area



t-Test for Slope

- Step 6: Decision
  - P-value < 0.05, then Reject Null and Accept Alternative
  - P-value > 0.05, then Fail to Reject the Null

- Step 7: Interpret Results to Audience
  - Interpret for People with Background in Basic Math
  - Should Use Words that are Based off the Context of the Data
  - What Does “Statistically Significant” Mean to the Audience

Inference for the Slope in R

- The **summary()** function in R prints out the test statistic and p-value
- The **confint()** function in R prints out a 95% CI for the slope

```
mod = lm(adj_fatal~yd,data=fatal)
summary(mod)
```

```
##
## Call:
## lm(formula = adj_fatal ~ yd, data = fatal)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.119634 -0.040335 -0.007417  0.034376  0.205392
##
## Coefficients:
##             Estimate Std. Error t value   Pr(>|t|)
## (Intercept) 0.104129   0.022873   4.552 0.00000744 ***
## yd          0.005374   0.001219   4.407 0.00001414 ***
```

```
confint(mod)
```

```
##                  2.5 %      97.5 %
## (Intercept) 0.059135422 0.149122822
## yd          0.002975191 0.007772437
```

Conclusion

- Learning What Is Classically Meant When a Person States a Predictor Variable to be “Statistically Significant”
- We Can Use the Confidence Interval to Conduct a Hypothesis Test
- A Confidence Interval is Superior to a Hypothesis Test
- **Always** Supplement Your Conclusion from a Hypothesis Test with a Confidence Interval

Thank You

Make Reasonable Decisions