Partitioning Variability ANOVA

READING:	2.2
EXERCISES:	CH 2. 15a, 17a
ASSIGNED:	MIDTERM 1
PRODUCER:	DR. MARIO

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Effectiveness of a Model

- Idea 1: *A simple linear regression model is **not effective** if we **cannot conclude** with confidence that the slope* $\boldsymbol{\beta_1 \neq 0}$
- Idea 2: *A simple linear regression model is **not effective** if we determine that the **predictions** from the fitted model only explain a **small** amount of the **total variability** of the response variable Y*
- Analysis of Variance (ANOVA) = Method for Implementing Idea 2

Partitioning Variability

- Textbook Explanation of Statistical Modeling

$$Data = Model + Error$$

- Partitioning Variability

TOTAL Variation in Response Variable Y	=	Variation Explained by MODEL	+	Unexplained Variation in RESIDUALS

- ANOVA Makes Inference Based Off This Partition

Partitioning Variability

- Mathematical Proof

$$y - \bar{y} = (\hat{y} - \bar{y}) + (y - \hat{y})$$

$$(y - \bar{y})^2 = (\hat{y} - \bar{y})^2 + 2(\hat{y} - \bar{y})(y - \hat{y}) + (y - \hat{y})^2$$

$$\begin{aligned}
\sum(y - \bar{y})^2 &= \sum(\hat{y} - \bar{y})^2 + \sum 2(\hat{y} - \bar{y})(y - \hat{y}) + \sum(y - \hat{y})^2 \\
&= \sum(\hat{y} - \bar{y})^2 + 2\sum(\hat{y}y - \hat{y}^2 - \bar{y}y + \bar{y}\hat{y}) + \sum(y - \hat{y})^2 \\
&= \sum(\hat{y} - \bar{y})^2 + 2\sum\hat{y}(y - \hat{y}) + 2\sum\bar{y}(\hat{y} - y) + \sum(y - \hat{y})^2 \\
&= \sum(\hat{y} - \bar{y})^2 + 2\boxed{\sum\hat{y}(y - \hat{y})} - 2\bar{y}\boxed{\sum(y - \hat{y})} + \sum(y - \hat{y})^2 \\
&= \sum(\hat{y} - \bar{y})^2 + \sum(y - \hat{y})^2
\end{aligned}$$

Partitioning Variability

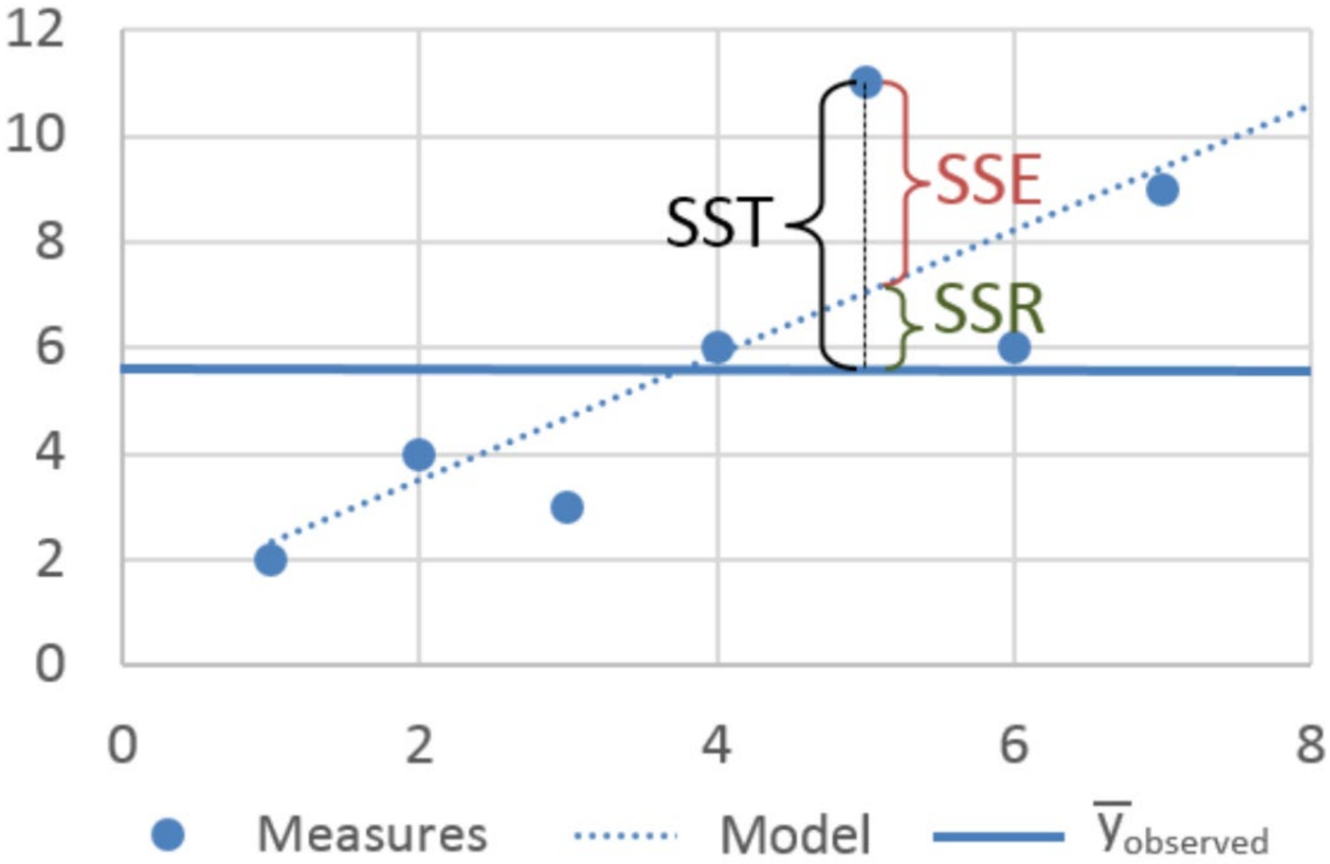
- Conclusion

$$\sum(y-\bar{y})^2 = \sum(\hat{y}-\bar{y})^2 + \sum(y-\hat{y})^2$$

$$SSTotal \;=\; SSModel \;+\; SSE$$

- In Many Textbooks, $SSModel$ is sometimes called SSR for Sum of Squares Regression

Partitioning Variability



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ANOVA Table

- Classic Table

Source	d.f.	Sum of Squares	Mean Square	F	P-value
Model	1	$SSModel$	$SSModel/1$	$\frac{MSModel}{MSE}$	$F_{1,n-2}$
Residual	$n-2$	SSE	$SSE/(n-2)$		
Total	$n-1$	$SSTotal$			

- Degrees of Freedom and Sum of Squares for Total are Both a Summation of Values for Model and Residual

Thank You

Make Reasonable Decisions