Multiple Tests

READING:	8.2
EXERCISES:	CH 8. 10, 13
ASSIGNED:	HW 10
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Pairwise Comparisons After ANOVA

- Want to Test Hypotheses for Each i and j at the 5% Significance Level

$$H_0: \mu_i - \mu_j = 0 \quad \text{vs} \quad H_a: \mu_i - \mu_j \neq 0$$

- Create Confidence Intervals for Each **Unique** $\mu_i - \mu_j$

$$\bar{y}_i - \bar{y}_j \pm \boxed{cv} \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

cv = "Critical Value"

Controls Width and Based Off Significance Level

- Goal: Explore Different Options for **Critical Values**

Pairwise Comparisons After ANOVA

- 95% Confidence Interval for $\mu_i - \mu_j$

- Interpretation: *Method results in a confidence interval that* ***contains*** *the true $\mu_i - \mu_j$ in 95% of cases if repeated infinite times.*

- Conclusion: *5% of the infinite cases will result in confidence intervals that* ***do not contain*** *the true $\mu_i - \mu_j$*

- More Confidence Intervals -> Chance of Getting an Interval that Doesn't Contain the True $\mu_i - \mu_j$ Increases (**Familywise Error Rate**)

Example: Grades on Different Exams

- Five Students (Barb, Betsy, Bill, Bob, Bud) Take Four Exams (1, 2, 3, 4)
- Question: *For each **pair** of students, is there a significant difference in the average grade for the **two** students?*
- There are 10 Questions Here Because There are 10 Unique Pairs
- Familywise Error Rate if We Create 10 Different 95% CI's

$$1 - 0.95^{10} = 40\%$$

Fisher’s LSD

- Liberal Approach
  - Small Critical Values -> Narrow Intervals
  - More Likely to Claim that a Difference Exists

- Method:
  - Step 1: Verify ANOVA F-test is Significant
  - Step 2: Compute All Intervals

$$\bar{y}_i - \bar{y}_j \pm t_{0.975,n-K}\sqrt{MSE\left(\frac{1}{n_i}+\frac{1}{n_j}\right)}$$

Example: Grades on Different Exams

- ANOVA Model and Output

```
Exams = read.csv("Exams4.csv")

amodS = aov(Grade~Student,data=Exams)
summary(amodS)
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)
## Student      4   4480  1120.0     9.6 0.000468
## Residuals   15   1750   116.7
```

Step 1 of Fisher's LSD
Achieved Significance!!

Example: Grades on Different Exams

- Margin of Error the Same for Every Pair

```
t_LSD = qt(1 - 0.05/2, amodS$df.residual)
MSE = summary(amodS)[[1]][2,3]
LSD = t_LSD * sqrt(MSE)*sqrt(1/4 + 1/4)
round(LSD,2)
```

```
## [1] 16.28
```

- Sample Means of 5 Students

```
tapply(Exams$Grade,Exams$Student,mean)
```

```
##   Barb Betsy  Bill   Bob   Bud
##     75    91    79    83    47
```

- Difference Between Bud and Each Other Student is **More Than** 16.28

```
abs(c(75,91,79,83)-47)
```

```
## [1] 28 44 32 36
```

Example: Grades on Different Exams

- All Pairwise t-Tests Under Fisher's LSD

```
pairwise.t.test(Exams$Grade, Exams$Student, p.adj = 'none')
```

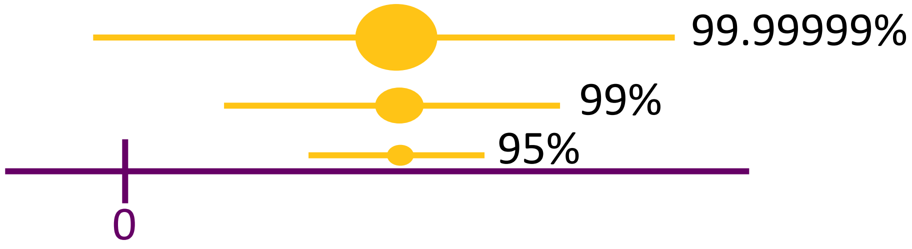
```
##
##  Pairwise comparisons using t tests with pooled SD
##
## data:  Exams$Grade and Exams$Student
##
##       Barb    Betsy    Bill    Bob
## Betsy 0.05357 -        -       -
## Bill  0.60812 0.13699  -       -
## Bob   0.31148 0.31148  0.60812 -
## Bud   0.00229 0.000038 0.00079 0.00028
##
## P value adjustment method: none
```

Notice:
$|75 - 91| = 16 < 16.28$

Agreement with Previous Slide

Bonferroni Adjustment

- Conservative Approach -> Decrease α for Each CI -> Wider Intervals



- Method:
  - Step 1: Choose a Familywise Error Rate α^*
  - Step 2: Decide How Many Intervals You are Computing. This is m.
  - Step 3: Individual Error Rate for Each CI is $\alpha = \frac{\alpha^*}{m}$
  - Step 4: Compute Intervals

$$\bar{y}_i - \bar{y}_j \pm t_{1-\frac{\alpha}{2}, n-K} \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Example: Grades on Different Exams

- Margin of Error the Same for Every Pair

```
t_Bon = qt(1 - (0.05/10)/2, amodS$df.residual)
Bon = t_Bon * sqrt(MSE)*sqrt(1/4 + 1/4)
round(Bon,2)
```

```
## [1] 25.1
```

- Sample Means of 5 Students

```
tapply(Exams$Grade,Exams$Student,mean)
```

```
##  Barb Betsy  Bill   Bob   Bud
##    75    91    79    83    47
```

- Difference Between Bud and Each Other Student is **More Than** 25.1

```
abs(c(75,91,79,83)-47)
```

```
## [1] 28 44 32 36
```

Example: Grades on Different Exams

- All Pairwise t-Tests Under Bonferroni Adjustment

```
pairwise.t.test(Exams$Grade, Exams$Student, p.adj = 'bonf')
```

```
##
##  Pairwise comparisons using t tests with pooled SD
##
## data:  Exams$Grade and Exams$Student
##
##       Barb    Betsy   Bill    Bob
## Betsy 0.53567 -       -       -
## Bill  1.00000 1.00000 -       -
## Bob   1.00000 1.00000 1.00000 -
## Bud   0.02293 0.00038 0.00789 0.00277
##
## P value adjustment method: bonferroni
```

Still All Less Than 0.05

Tukey's HSD

- Moderate Approach

- Method:
  - Step 1: Choose a Familywise Error Rate α^*
  - Step 2: Find Value of q from the *Studentized Range Distribution* Based on α^*, number of groups K, and $n - K$ d.f.
  - Step 4: Compute Intervals

$$\bar{y}_i - \bar{y}_j \pm \frac{q}{\sqrt{2}} \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

Example: Grades on Different Exams

- Margin of Error the Same for Every Pair

```
t_Tukey = qtukey(p= 1 - 0.05,
                 nmeans=5,
                 df=amodS$df.residual)
HSD = t_Tukey/sqrt(2) * sqrt(MSE)*sqrt(1/4 + 1/4)
round(HSD,2)
```

```
## [1] 23.58
```

- Notice:

$$Fisher's\ LSD < Tukey's\ HSD < Bonferroni$$

$$16.28 < 23.58 < 25.1$$

Example: Grades on Different Exams

- All Pairwise t-Tests Under TukeyHSD
- The Fourth Column Shows P-values
- Only Find Significance With Bud

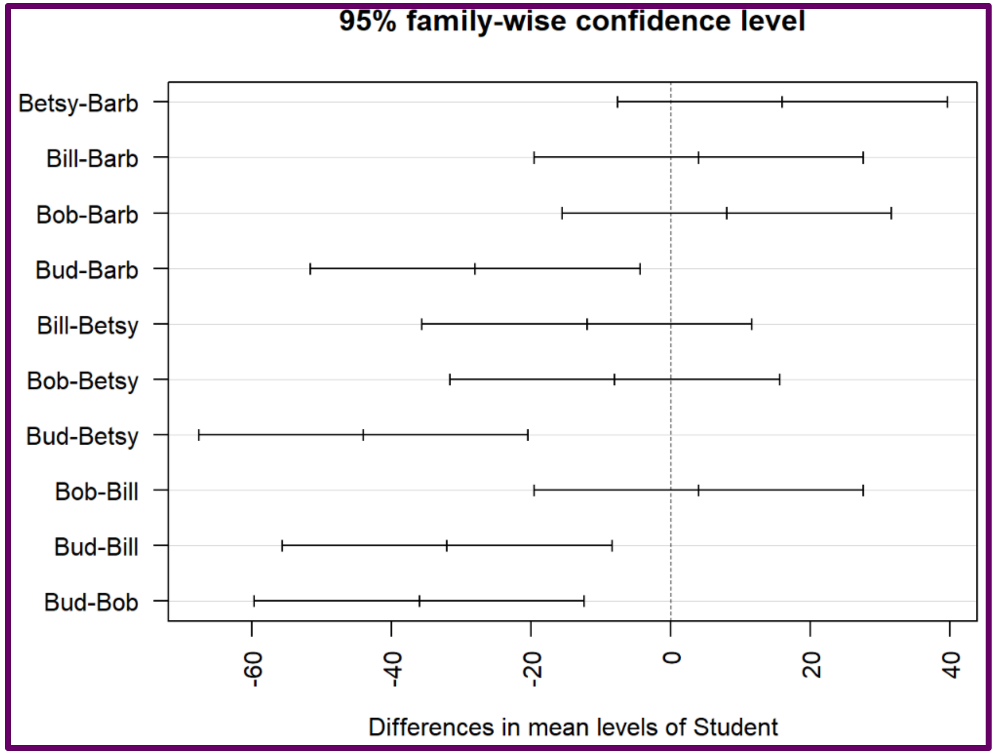
```
TukeyHSD(amodS)
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = Grade ~ Student, data = Exams)
##
## $Student
##             diff        lwr        upr     p adj
## Betsy-Barb    16  -7.584413  39.584413 0.2720310
## Bill-Barb      4 -19.584413  27.584413 0.9835150
## Bob-Barb       8 -15.584413  31.584413 0.8295529
## Bud-Barb     -28 -51.584413  -4.415587 0.0166293
## Bill-Betsy   -12 -35.584413  11.584413 0.5360462
## Bob-Betsy     -8 -31.584413  15.584413 0.8295529
## Bud-Betsy    -44 -67.584413 -20.415587 0.0003116
## Bob-Bill       4 -19.584413  27.584413 0.9835150
## Bud-Bill     -32 -55.584413  -8.415587 0.0060225
## Bud-Bob      -36 -59.584413 -12.415587 0.0021941
```

Example: Grades on Different Exams

- Plot of Confidence Intervals Using Tukey's HSD

```
par(mar=c(4,7,3,1))  #Adjusts Plot Margins
hsd = TukeyHSD(amodS) #Save Output from TukeyHSD()
plot(hsd, las=2) #Plot Output from TukeyHSD()
par(mar=c(5,4,4,2)) #Revert Back to Default Margins
```



- Calculation for Betsy vs Barb

$$(16 - 23.58, 16 + 23.58)$$
$$(-7.58, 39.58)$$

Thank You

Make Reasonable Decisions