Partitioning Variability ANOVA

READING:	2.2
EXERCISES:	CH 2. 15a, 17a
ASSIGNED:	MIDTERM 1
PRODUCER:	DR. MARIO

IMG CREDIT: ALEX RIEGERT-WATERS

Effectiveness of a Model

- Idea 1: *A simple linear regression model is* ***not effective*** *if we* ***cannot conclude*** *with confidence that the slope* $\boldsymbol{\beta_1 \neq 0}$

- Idea 2: *A simple linear regression model is* ***not effective*** *if we determine that the* ***predictions*** *from the fitted model only explain a* ***small*** *amount of the* ***total variability*** *of the response variable Y*

- Analysis of Variance (ANOVA) = Method for Implementing Idea 2

Partitioning Variability

- Textbook Explanation of Statistical Modeling

$$Data = Model + Error$$

- Partitioning Variability

$$\boxed{\text{TOTAL Variation in Response Variable Y}} = \boxed{\text{Variation Explained by MODEL}} + \boxed{\text{Unexplained Variation in RESIDUALS}}$$

- ANOVA Makes Inference Based Off This Partition

Partitioning Variability

- Mathematical Proof

$$y - \bar{y} = (\hat{y} - \bar{y}) + (y - \hat{y})$$

$$(y - \bar{y})^2 = (\hat{y} - \bar{y})^2 + 2(\hat{y} - \bar{y})(y - \hat{y}) + (y - \hat{y})^2$$

$$\begin{aligned}
\sum(y - \bar{y})^2 &= \sum(\hat{y} - \bar{y})^2 + \sum 2(\hat{y} - \bar{y})(y - \hat{y}) + \sum(y - \hat{y})^2 \\
&= \sum(\hat{y} - \bar{y})^2 + 2\sum(\hat{y}y - \hat{y}^2 - \bar{y}y + \bar{y}\hat{y}) + \sum(y - \hat{y})^2 \\
&= \sum(\hat{y} - \bar{y})^2 + 2\sum\hat{y}(y - \hat{y}) + 2\sum\bar{y}(\hat{y} - y) + \sum(y - \hat{y})^2 \\
&= \sum(\hat{y} - \bar{y})^2 + 2\boxed{\sum\hat{y}(y - \hat{y})} - 2\bar{y}\boxed{\sum(y - \hat{y})} + \sum(y - \hat{y})^2 \\
&= \sum(\hat{y} - \bar{y})^2 + \sum(y - \hat{y})^2
\end{aligned}$$

Partitioning Variability

- Conclusion

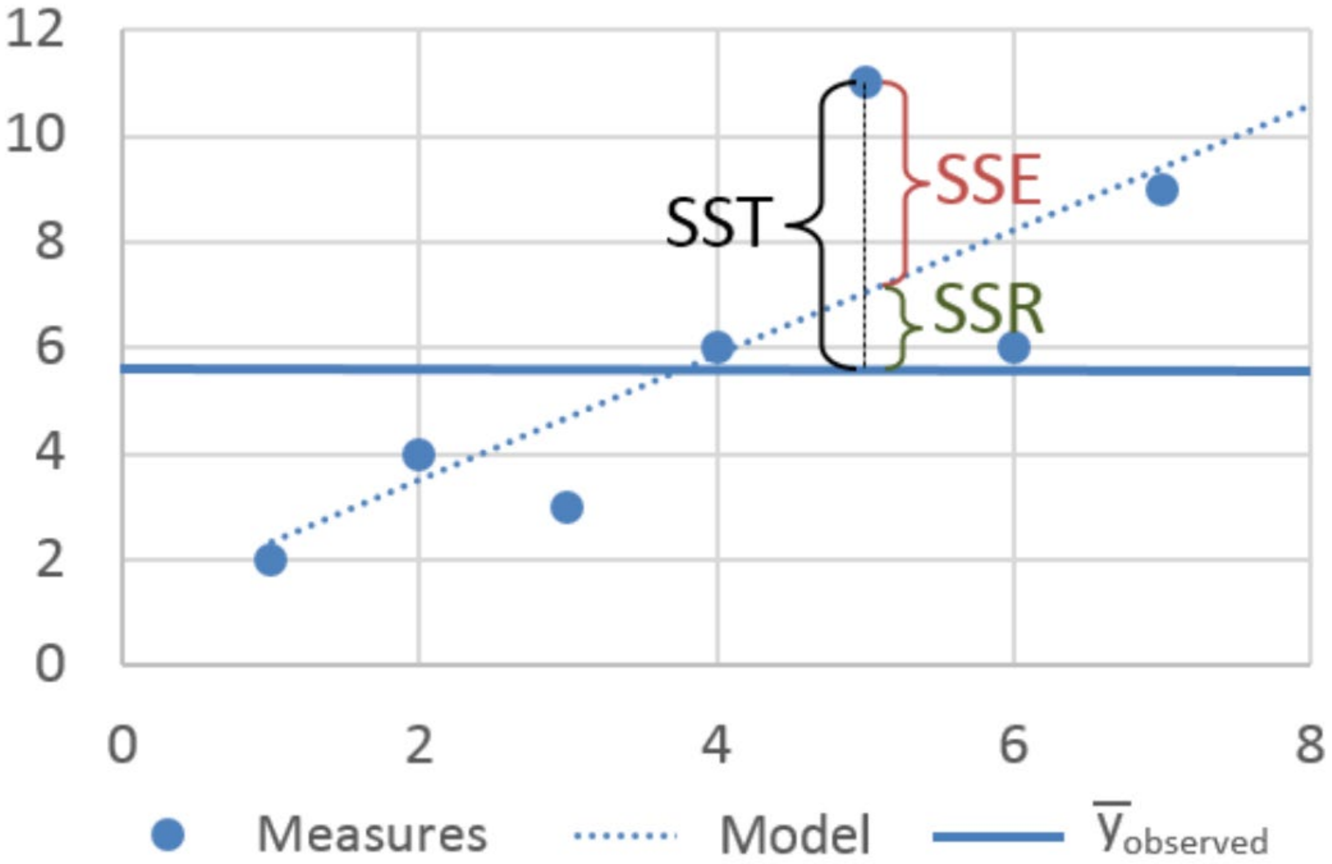
$$\sum(y-\bar{y})^2 = \sum(\hat{y}-\bar{y})^2 + \sum(y-\hat{y})^2$$

$$SSTotal \ = \ SSModel \ + \ SSE$$

- In General, We Want $SSModel$ to be **Larger Than** SSE

- In Many Textbooks, $SSModel$ is sometimes called SSR for Sum of Squares Regression

Partitioning Variability



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ANOVA Table

- Classic Table

Source	d.f.	Sum of Squares	Mean Square	F	P-value
Model	1	$SSModel$	$SSModel/1$	$\frac{MSModel}{MSE}$	$F_{1,n-2}$
Residual	$n-2$	SSE	$SSE/(n-2)$		
Total	$n-1$	$SSTotal$			

- Degrees of Freedom and Sum of Squares for Total are Both a Summation of Values for Model and Residual

ANOVA Test for Simple Linear Regression

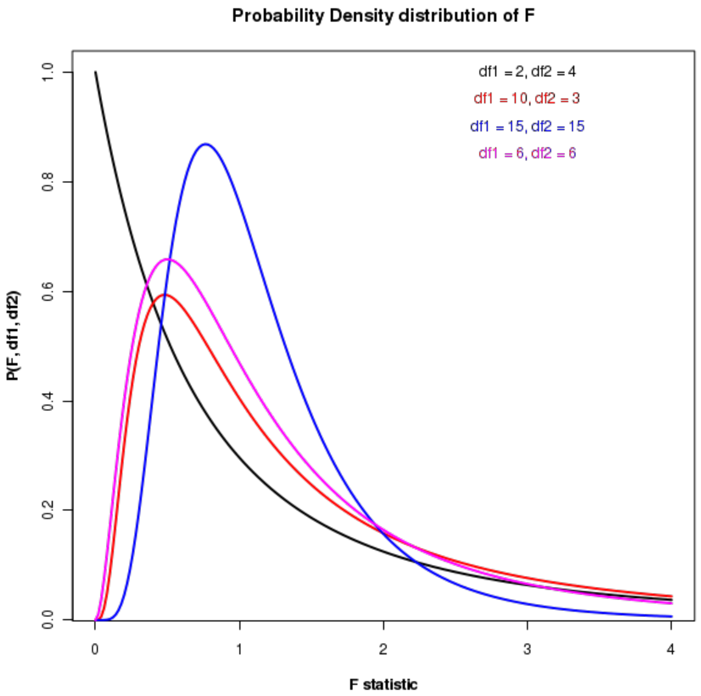
- Equivalent to the t-Test for the Slope
  - Identical Hypotheses (in Simple Linear Regression)
  - Perform Simple Linear Regression and Obtain Table
  - Different Test Statistic

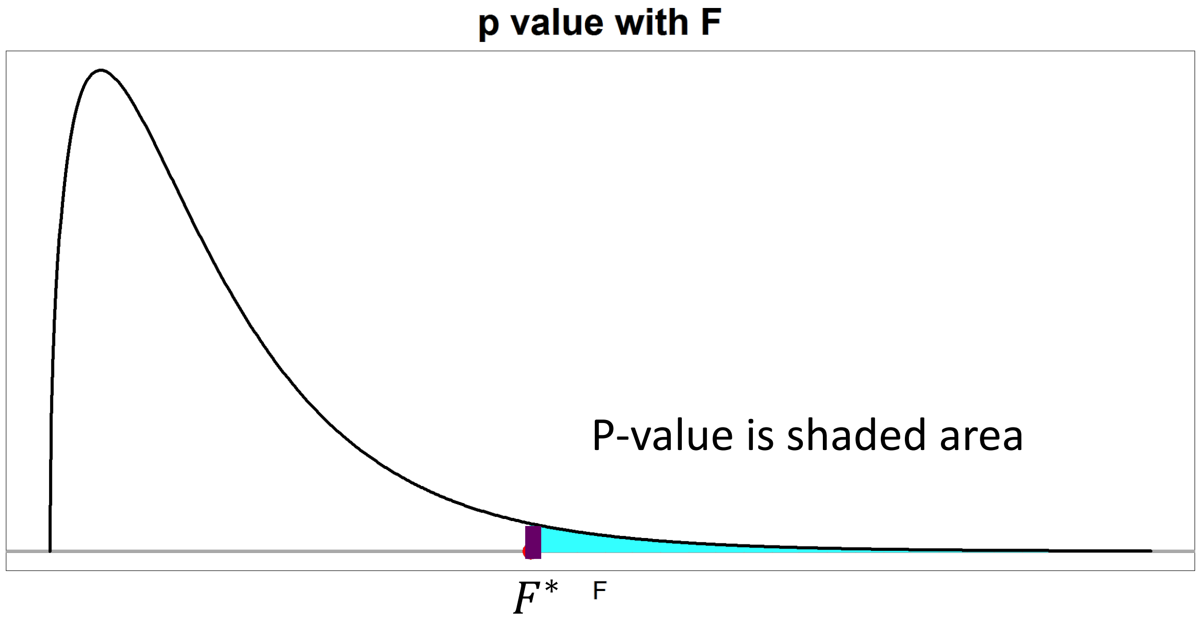
$$F^* = \frac{SSModel/1}{SSE/(n-2)} = \frac{MSModel}{MSE}$$

  - Compute p-value Based off F-Distribution and Not t-Distribution
    - F-distribution based off numerator degrees of freedom and denominator degrees of freedom
  - Decision and Interpretation is the Same (in SLR)

F-Distribution

- Typically, Right Skewed





- Used Typically When Test Statistic is a Ratio of Variances

Example: Fatalities

- Question: *Is there a linear relationship between the proportion of young drivers in a state and the number of vehicle fatalities in a state?*

- Data for States from 1982 Through 1988
  - From AER Package
  - *fatal* = Number of Vehicle Fatalities
  - *youngdrivers* = Percent of Drivers Aged 15 – 24 (inclusive)

- Question: *What is the problem with fitting a linear regression for the relationship **fatal** versus **youngdrivers**?*

Supplement for Lecture 10

- Conducting t-Test for Slope
- Confidence Interval for Parameters
- Getting ANOVA Table
- Conducting F-test and Comparing Results

Thank You

Make Reasonable Decisions