Multiple Comparisons and Fisher's LSD

READING:	5.7
EXERCISES:	CH 5. 45, 47, 62, 64
ASSIGNED:	HW 10
PRODUCER:	DR. MARIO

IMG CREDIT: ALEX RIEGERT-WATERS

Motivation

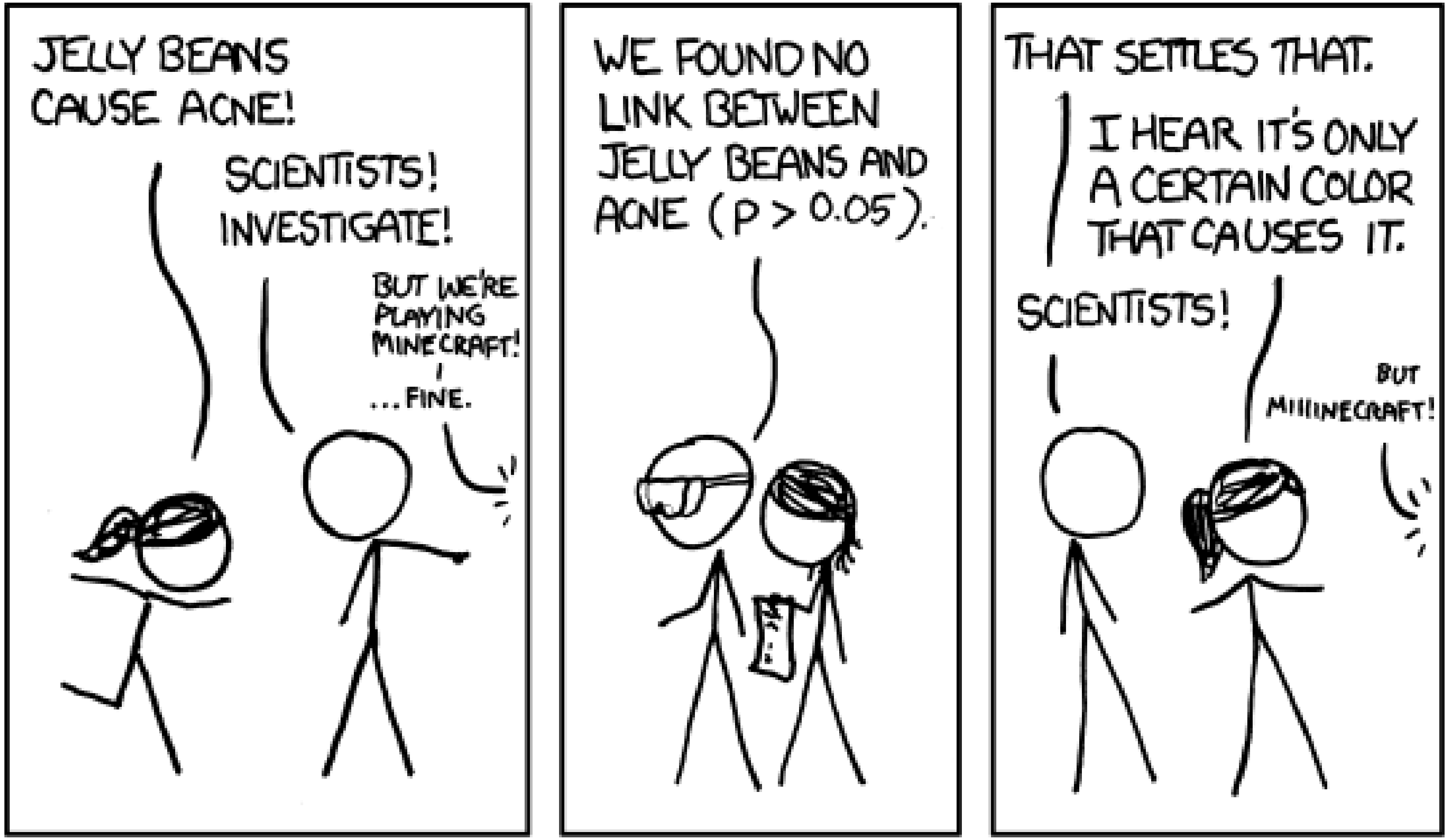
- Types of Errors

	$\boldsymbol{Fail\ to\ Reject\ H_0}$	$\boldsymbol{Reject\ H_0}$, Accept $\mathrm{H_a}$
$\boldsymbol{H_0\ is\ True}$	Correct	Type 1 Error (α)
$\boldsymbol{H_a\ is\ True}$	Type 2 Error ($\boldsymbol{\beta}$)	Correct

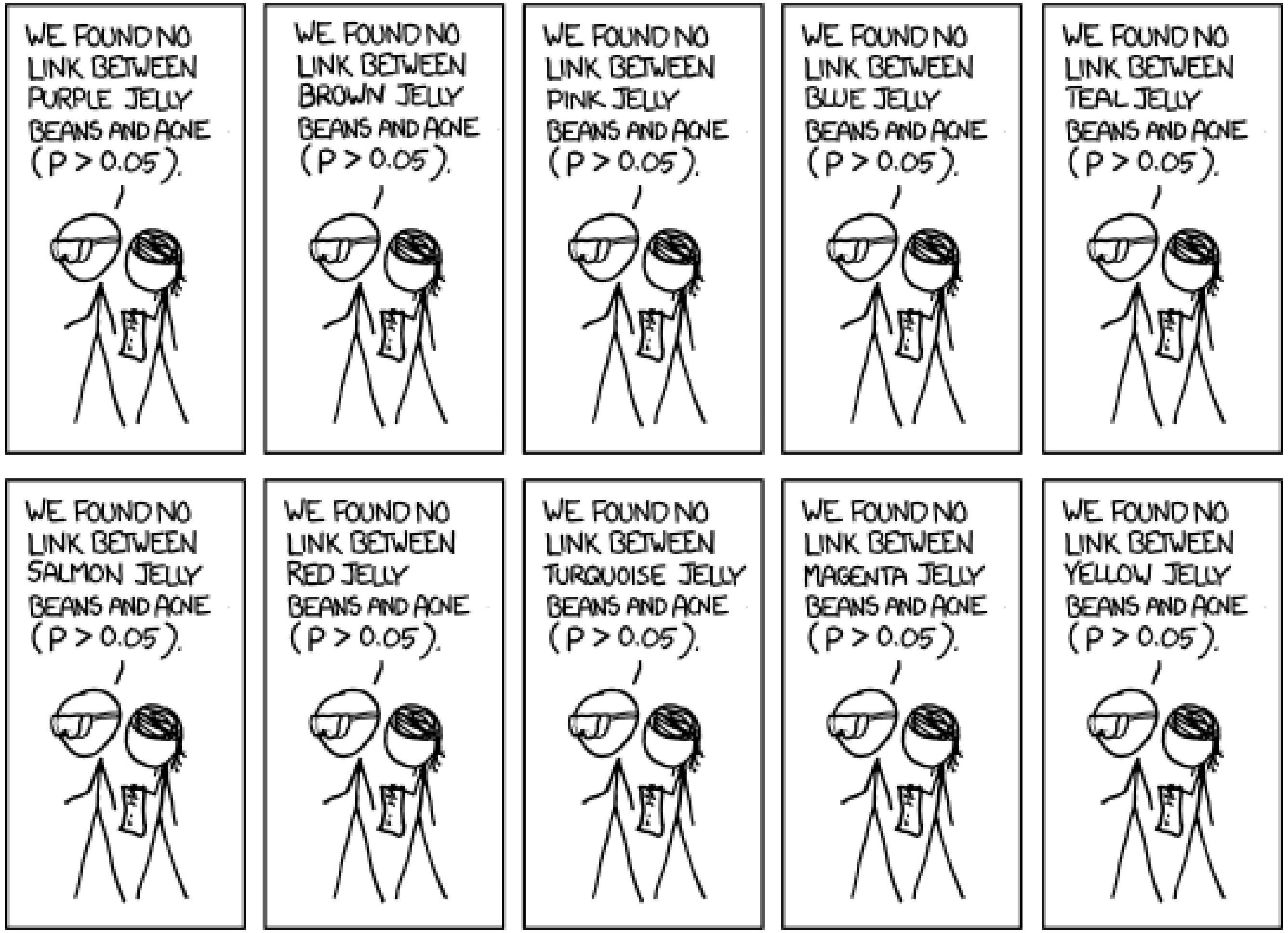
- When Doing Hypothesis Tests or Confidence Intervals, We Typically Control $\boldsymbol{\alpha}$ Which is the Probability of a Type 1 Error (i.e. $\boldsymbol{\alpha} = 0.05$)

- Typically, Researchers are Hoping to Achieve Significance

Motivation

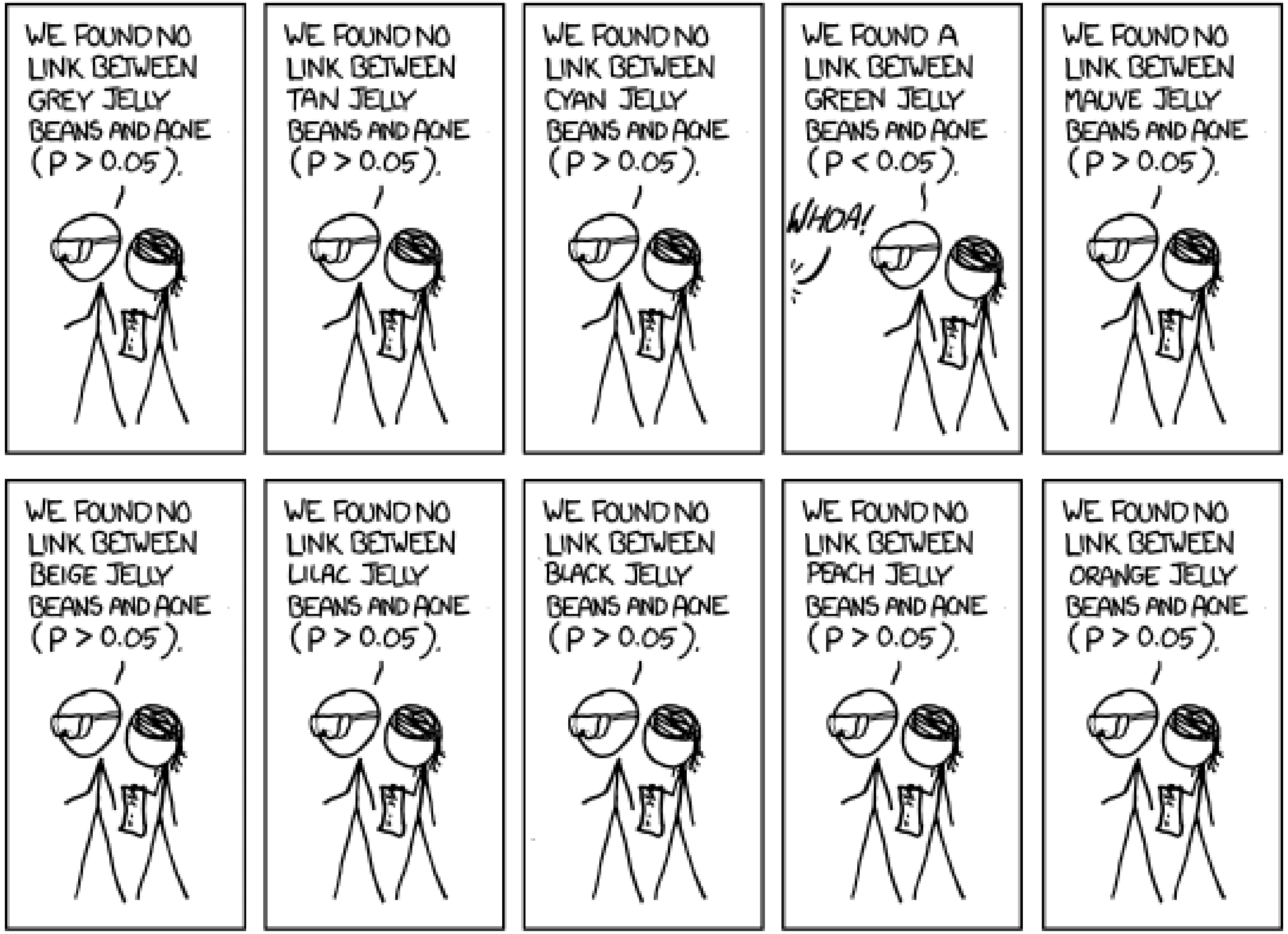

JELLY BEANS CAUSE ACNE!
SCIENTISTS! INVESTIGATE!
BUT WE'RE PLAYING MINECRAFT!
...FINE.
WE FOUND NO LINK BETWEEN JELLY BEANS AND ACNE (P > 0.05).
THAT SETTLES THAT.
I HEAR IT'S ONLY A CERTAIN COLOR THAT CAUSES IT.
SCIENTISTS!
BUT MIIIINECRAFT!

Motivation

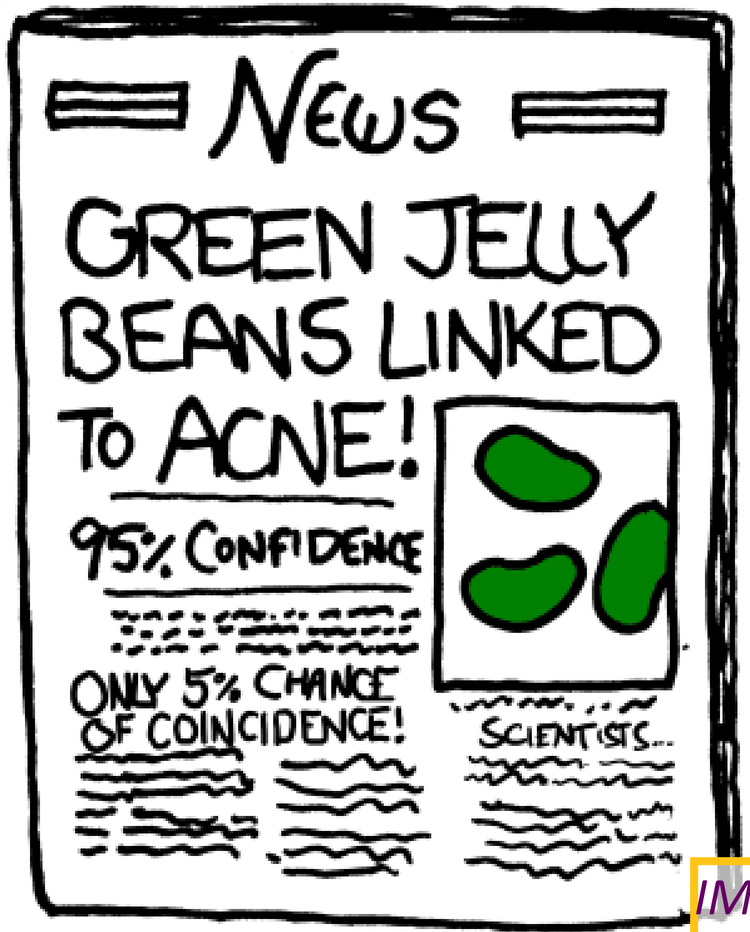


WE FOUND NO LINK BETWEEN PURPLE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BROWN JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN PINK JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BLUE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN TEAL JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN SALMON JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN RED JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN TURQUOISE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN MAGENTA JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN YELLOW JELLY BEANS AND ACNE (P > 0.05).

Motivation


WE FOUND NO LINK BETWEEN GREY JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN TAN JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN CYAN JELLY BEANS AND ACNE (P > 0.05).
WE FOUND A LINK BETWEEN GREEN JELLY BEANS AND ACNE (P < 0.05).
WHOA!
WE FOUND NO LINK BETWEEN MAUVE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BEIGE JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN LILAC JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN BLACK JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN PEACH JELLY BEANS AND ACNE (P > 0.05).
WE FOUND NO LINK BETWEEN ORANGE JELLY BEANS AND ACNE (P > 0.05).

Motivation



"If you torture your data long enough, they will confess."

- I. J. Good (Jack Good)

IMG CREDIT: [HTTPS://XKCD.COM/882/](https://xkcd.com/882/)

Motivation

- ANOVA Model $Y = \mu + \alpha_i + \epsilon$ where $i \in \{1,2,..,50\}$
  - Null Hypothesis is a Strict Statement (i.e. $H_0: \alpha_8 = 0$)
  - Alternative Hypothesis is the Opposite (i.e. $H_a: \alpha_8 \neq 0$)

- **Truth:** Null Hypothesis is Highly Unlikely to Actually Be True Therefore Just Keep Increasing Your Sample Size

- This is NOT the Focus of the Textbook

Motivation

- ANOVA Model $Y = \mu + \alpha_i + \epsilon$ where $i \in \{1,2,..,50\}$
    - Suppose $H_{0i}: \alpha_i = 0$ is True for all $i \in \{1,2,..,50\}$
    - $P(Type\ 1\ Error\ for\ i) = 0.05$
    - $P(At\ Least\ One\ Type\ 1\ Error) = 1 - (0.95)^{50} = 0.923$

- **Individual False Alarm Rate** is the Chance of a Type 1 Error for a Single Test (i.e. 0.05)

- **Familywise False Alarm Rate** is the Chance of at least one Type 1 Error Among a Family of Multiple Tests

Motivation

- Conclusion: *The probability of making a Type 1 Error increases as we increase the number of hypothesis tests we do or the number of confidence intervals we create.*

- **Ethical** Options to Manage this Problem
  - Do Only a Few Hypothesis Tests or Confidence Intervals that were Selected Prior to Acquiring Data/Performing Analysis
  - Adjust the Significance Level for Each Test So the Familywise False Alarm Rate is Small

Estimating Differences in Group Means

- Assume ANOVA F-Test Resulted in Rejecting Null Hypothesis

$$\mu_i \neq \mu_j \; for \; some \; i \neq j$$

- Confidence Interval For Differences Between Group Means

$$(\bar{y}_i - \bar{y}_j) \pm t_{0.975,n-K} \sqrt{\frac{MSE}{n_i} + \frac{MSE}{n_j}}$$

- We Want to Use the Confidence Interval to Test Hypotheses
  - $H_0: \mu_i = \mu_j$ (Assume to Be True)
  - $H_a: \mu_i \neq \mu_j$ (Accept if 0 is not in Interval)

Example: Grades on Different Exams

- ANOVA Model and Output

```
amodS = aov(Grade~Student,data=Exams)
summary(amodS)
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)
## Student      4   4480  1120.0     9.6 0.000468 ***
## Residuals   15   1750   116.7
```

- We Have Evidence that At Least One Student Has an Average Grade that is Different Than the Average Grade of Another Student

Example: Grades on Different Exams

- Question: *How many differences could we estimate?*

Barb vs. Betsy	Barb vs. Bill	Barb vs. Bob
Barb vs. Bud	Betsy vs. Bill	Betsy vs. Bob
Betsy vs. Bud	Bill vs. Bob	Bill vs. Bud
Bob vs. Bud		

- Formula: "5 Choose 2" $= \binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5\times4\times3!}{2!3!} = \frac{20}{2} = 10$

- Require 10 Confidence Intervals to Test All Pairwise Comparisons

Thank You

Make Reasonable Decisions