

## Supplement for Lecture 7: Outliers and Influential Points

### Load Data from Textbook

```
data("SpeciesArea") # Load Data

species = SpeciesArea[,-c(4,5)] # Shorten Name and Remove Last 2 Columns

rm(SpeciesArea) #Removes Old Object from Environment

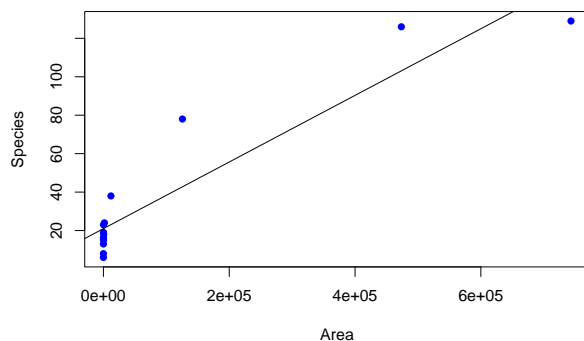
head(species)
```

```
##      Name   Area Species
## 1  Borneo 743244    129
## 2  Sumatra 473607    126
## 3   Java 125628     78
## 4  Bangka 11964     38
## 5 Bunguran 1594     24
## 6  Banggi   450     18
```

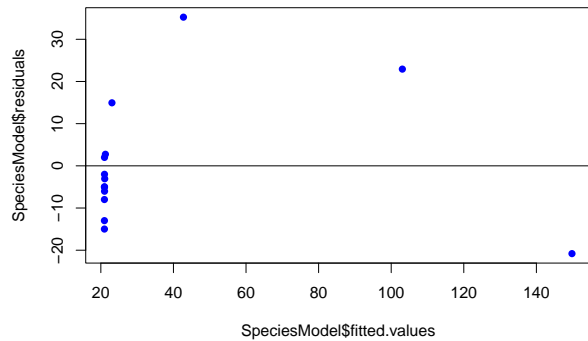
### Linear Model for Species vs Area

```
SpeciesModel=lm(Species~Area, data=species)

plot(Species ~ Area,data=species,pch=16,col="blue")
abline(SpeciesModel)
```

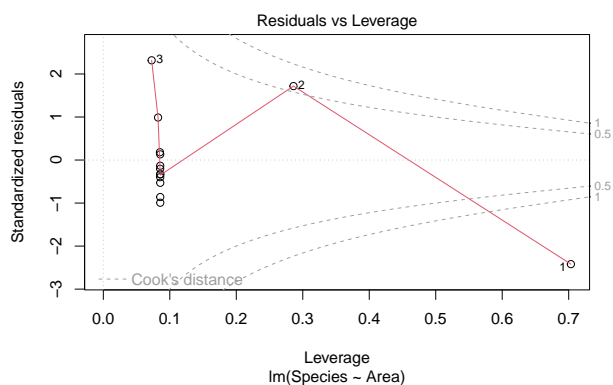
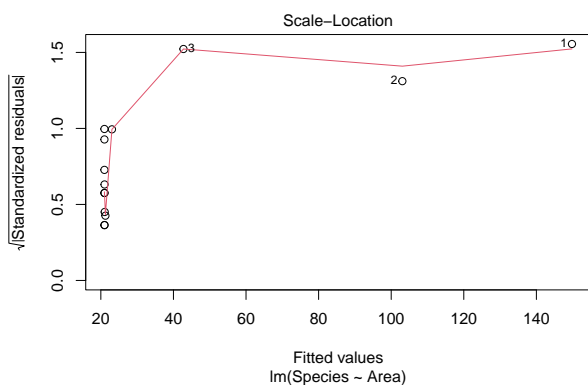
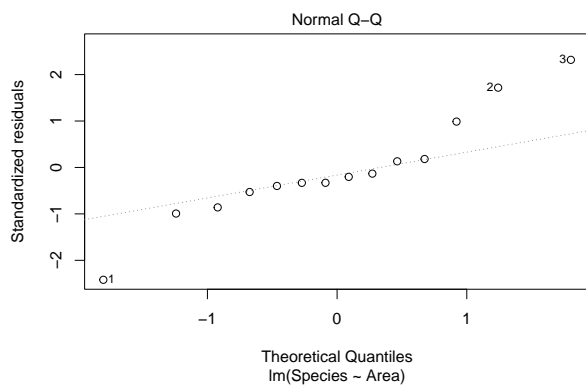
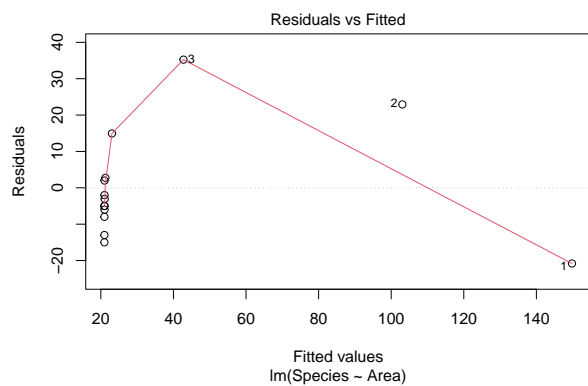


```
plot(SpeciesModel$residuals~SpeciesModel$fitted.values,pch=16,col="blue")
abline(0,0)
```



## Default Plots from lm Object

```
plot(SpeciesModel) #Default Argument which=c(1,2,3,5)
```



```
#See ?plot.lm (6 Plots Available)
```

```
#Plot 1: Residuals vs Fitted (We Did This Already)
```

```
#Plot 2: Normal QQ (We Did This Already)
```

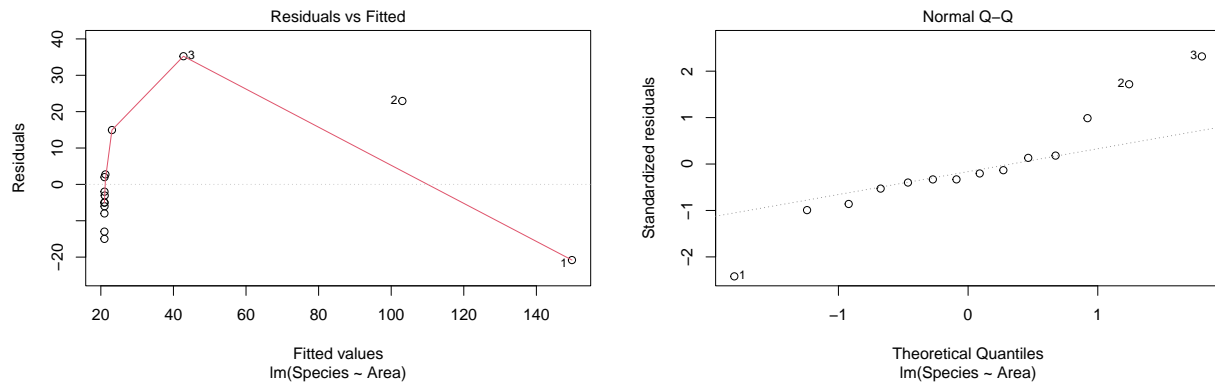
```
#Plot 3: Square Root of Absolute Standardized Residual vs Fitted (Check for Outliers)
```

```
#Plot 4: Not Part of Default
```

```
#Plot 5: Residuals vs Leverage (Check for Influence)
```

*#Plot 6: Not Part of Default*

```
plot(SpeciesModel, which=c(1,2)) #Only Want First Two
```



```
#plot(SpeciesModel, which=1:2)
```

```
#plot(SpeciesModel, 1:2)
```

## Identifying Outliers

*#Alternate to SpeciesModel\$residuals*

```
residuals(SpeciesModel)
```

```
##          1          2          3          4          5          6          7
## -20.807685  22.927548  35.241524  14.942506   2.739901  -3.061813  -6.017442
##          8          9         10         11         12         13         14
##  -2.006349   1.996424  -5.003402  -4.991789 -12.988150  -7.986070 -14.985203
```

```
max(residuals(SpeciesModel)) #Largest
```

```
## [1] 35.24152
```

```
min(residuals(SpeciesModel)) #Smallest
```

```
## [1] -20.80769
```

```
max(abs(residuals(SpeciesModel))) #Farthest Away from 0
```

```
## [1] 35.24152
```

```
which.max(abs(residuals(SpeciesModel))) #Returns Index of the Max
```

```
## 3
```

```
## 3
```

*#Bad Idea: Remove Data Point with Largest Outlier*

```
species.without.3 = species[-which.max(abs(residuals(SpeciesModel))),]
species.without.3
```

```
##          Name    Area Species
## 1      Borneo 743244    129
## 2      Sumatra 473607    126
## 4      Bangka 11964     38
```

```
## 5      Bunguran    1594    24
## 6      Banggi      450     18
## 7      Jemaja      194     15
## 8      Karimata Besar 130     19
## 9      Tioman      114     23
## 10     Siantan     113     16
## 11     Sirhassan    46     16
## 12     Redang      25      8
## 13     Penebangan   13     13
## 14 Perhentian Besar 8       6
```

#### *#Standardizing Residuals*

```
SSE=sum(residuals(SpeciesModel)^2)
n=length(residuals(SpeciesModel))
std.error.regression = sqrt(SSE/(n-2))
std.error.regression #Expect to be Off By 15 Species On Average When Using Model to Predict
```

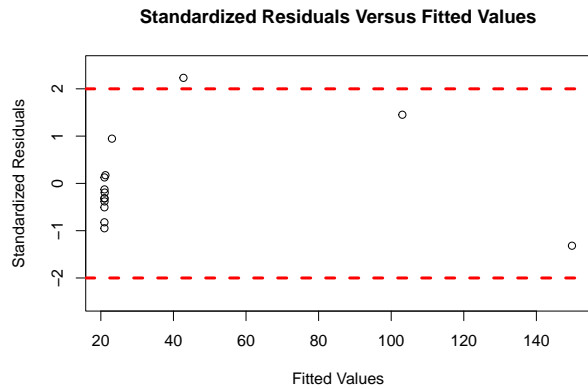
```
## [1] 15.79018
```

```
summary(SpeciesModel)
```

```
##
## Call:
## lm(formula = Species ~ Area, data = species)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -20.808  -7.494  -4.027   2.554  35.242
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.098e+01  4.621e+00  4.541 0.000677 ***
## Area         1.733e-04  1.942e-05  8.925 1.21e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 15.79 on 12 degrees of freedom
## Multiple R-squared:  0.8691, Adjusted R-squared:  0.8582
## F-statistic: 79.66 on 1 and 12 DF,  p-value: 1.206e-06
```

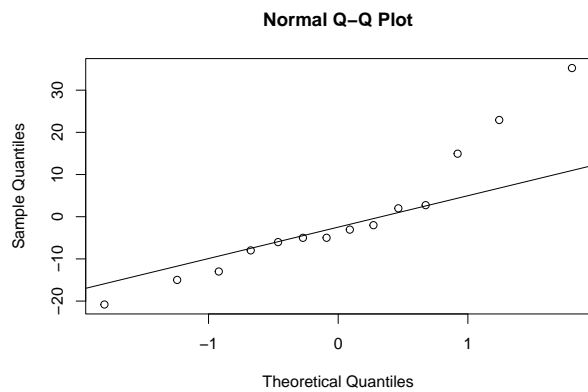
#### *#Plot of Standardized Residuals*

```
std.res=residuals(SpeciesModel)/std.error.regression
plot(y=std.res,x=fitted(SpeciesModel),
     main="Standardized Residuals Versus Fitted Values",
     xlab="Fitted Values", ylab="Standardized Residuals",
     ylim=c(-2.5,2.5))
abline(h=c(-2,2),
       col="red",
       lty=c(2,2),
       lwd=c(3,3))
```

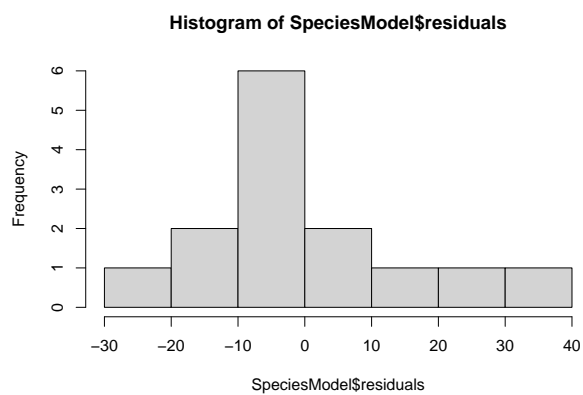


## Assess Conditions of Residuals

```
qqnorm(SpeciesModel$residuals)
qqline(SpeciesModel$residuals)
```



```
hist(SpeciesModel$residuals)
```



# Transformation

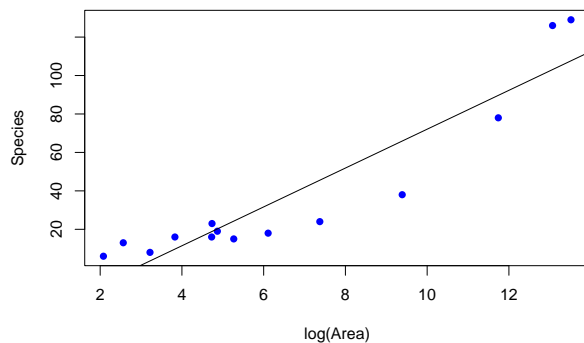
Three Possible Options Using Natural Logarithm Transformation Only

- $y = \beta_0 + \beta_1 \log(x) + \epsilon$
- $\log(y) = \beta_0 + \beta_1 x + \epsilon$
- $\log(y) = \beta_0 + \beta_1 \log(x) + \epsilon$

*# Run the Code and Try All Three To Choose Best*

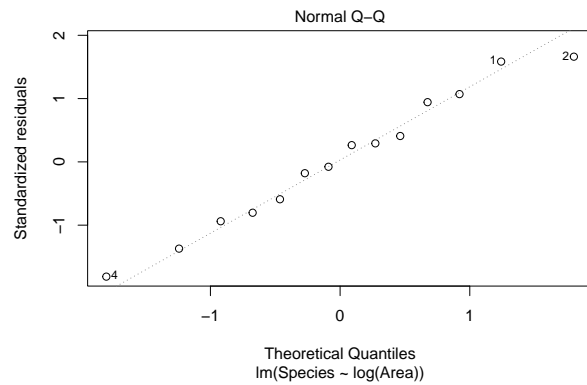
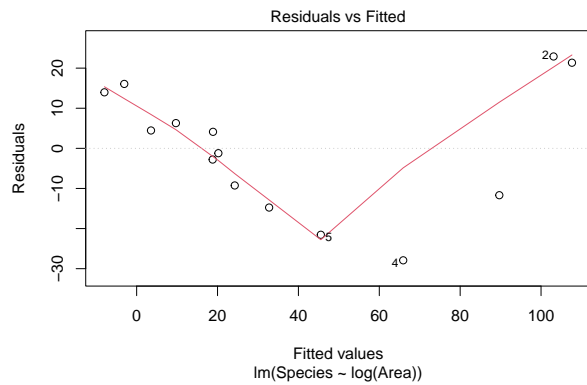
```
TransMod=lm(Species~log(Area), data=species)
summary(TransMod)
```

```
##
## Call:
## lm(formula = Species ~ log(Area), data = species)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -27.916 -11.075   1.455  12.052  22.905
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -28.989      8.923   -3.249  0.00697 **
## log(Area)     10.107      1.178    8.580 1.82e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 16.34 on 12 degrees of freedom
## Multiple R-squared:  0.8599, Adjusted R-squared:  0.8482
## F-statistic: 73.62 on 1 and 12 DF,  p-value: 1.822e-06
plot(Species ~ log(Area),data=species,pch=16,col="blue")
abline(TransMod)
```



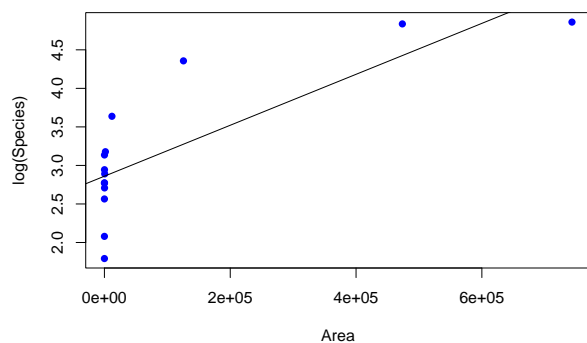
*#abline(v=c(5,12),col=c("red","red"),lwd=c(3,3),lty=c(3,3)) #Run for First Option*

```
plot(TransMod,c(1,2))
```

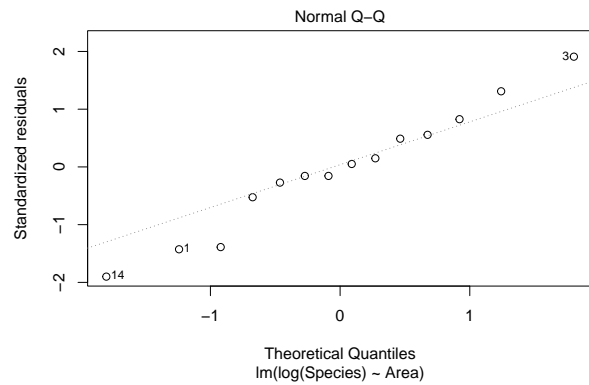
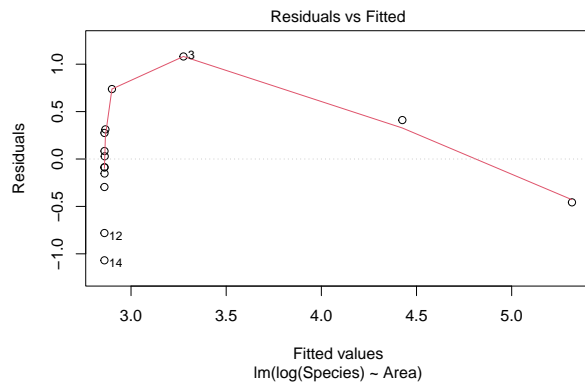


```
TransMod=lm(log(Species)~Area, data=species)
summary(TransMod)
```

```
##
## Call:
## lm(formula = log(Species) ~ Area, data = species)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.06844 -0.25964 -0.02951  0.30320  1.08130
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.860e+00  1.721e-01  16.62  1.2e-09 ***
## Area          3.305e-06  7.232e-07   4.57 0.000643 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.588 on 12 degrees of freedom
## Multiple R-squared:  0.6351, Adjusted R-squared:  0.6047
## F-statistic: 20.89 on 1 and 12 DF,  p-value: 0.0006433
plot(log(Species) ~ Area,data=species,pch=16,col="blue")
abline(TransMod)
```

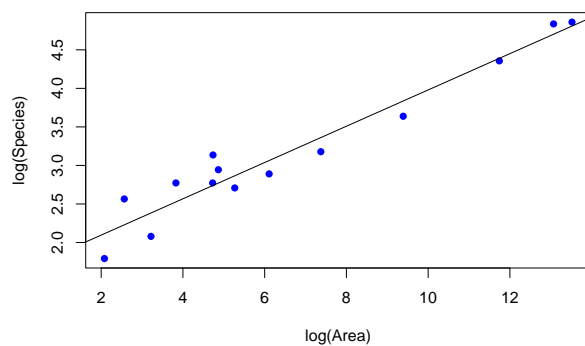


```
plot(TransMod,c(1,2))
```



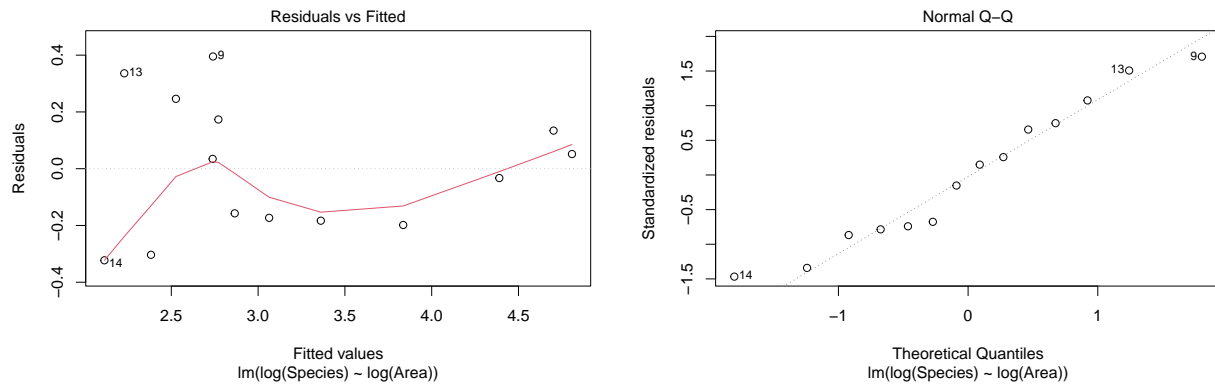
```
TransMod=lm(log(Species)~log(Area), data=species)
summary(TransMod)
```

```
##
## Call:
## lm(formula = log(Species) ~ log(Area), data = species)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.32280 -0.18071  0.00079  0.16356  0.39534
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1.6249     0.1326   12.26 3.81e-08 ***
## log(Area)      0.2355     0.0175   13.46 1.34e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2427 on 12 degrees of freedom
## Multiple R-squared:  0.9379, Adjusted R-squared:  0.9327
## F-statistic: 181.1 on 1 and 12 DF,  p-value: 1.335e-08
plot(log(Species) ~ log(Area),data=species,pch=16,col="blue")
abline(TransMod)
```



```
plot(TransMod,c(1,2))
```





## Getting Fitted Values, Residuals, and Predictions

Using the model we fit to make predictions:

$$\widehat{\log(y)} = \hat{\beta}_0 + \hat{\beta}_1 \log(x)$$

Transforming predictions back to the units of  $Y = \text{Number of Species}$

$$\hat{y} = e^{\widehat{\log(y)}} = e^{\hat{\beta}_0 + \hat{\beta}_1 \log(x)} = e^{\hat{\beta}_0} x^{\hat{\beta}_1}$$

If we run `coef(TransMod)`, we find our estimates

$$\hat{y} = e^{1.6249 + 0.2355 \log(x)} = e^{1.6249} x^{0.2355}$$

*#Recall Property of Logs*

```
10^log10(5)
```

```
## [1] 5
```

```
exp(log(7))
```

```
## [1] 7
```

*#Extract Coefficients*

```
as.numeric(coef(TransMod))
```

```
## [1] 1.6249050 0.2354745
```

```
beta0=as.numeric(coef(TransMod))[1]
```

```
beta1=as.numeric(coef(TransMod))[2]
```

*#Raw Predictions of Log(Species)*

```
species$logfit = fitted(TransMod)
```

*#3 Methods for Predictions of Actual Species*

```
species$fit1 = exp(species$logfit)
```

```
species$fit2= exp(beta0+beta1*log(species$Area))
```

```
species$fit3 = exp(beta0)*species$Area^beta1
```

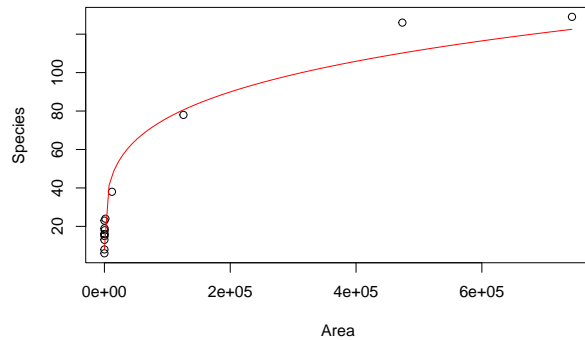
*#Obtain Residuals in Original Units of Species Variable to Calculate SE of Regression*

```
species$res = species$Species - species$fit1
sqrt(sum(species$res^2)/(length(species$res)-2)) #Standard Error of Regression
```

```
## [1] 6.551732
```

```
#Plot Fitted Model on Raw Untransformed Data
```

```
plot(Species~Area, data=species)
curve(exp(beta0+beta1*log(x)), add=TRUE, col="red")
```



```
#Extrapolate for the Island Australia (7,700,000 km^2)
```

```
exp(beta0+beta1*log(7700000))
```

```
## [1] 212.4615
```

```
plot(Species~Area, data=species, xlim=c(0,7700000), ylim=c(0,400))
curve(exp(beta0+beta1*log(x)), add=TRUE, col="red")
points(7700000, 379, col="blue", pch=16, cex=3) #I Wikipedia Like a Boss
```

