GAMBLING III

Produced by Dr. Mario

UNC STOR 538

MONEY MANAGEMENT

- Thought Experiment
  - Fact 1: UNC is Better Than Duke at Almost Everything
  - Fact 2: In All Circumstances, P(UNC > Duke) = 95%
  - Fact 3: You Have $10 Billion
  - Scenario: Competition of UNC vs Duke
  - Decision: How Much Should You Bet on UNC Winning?
    - $10
    - $100
    - $1,000
    - $100,000
    - All In and Ballin'
  - Goal: Choose an Amount For Long Term Gains

- Not "How Much?", But "What Percent?"

MONEY MANAGEMENT

- J.L. Kelly, Jr.
  - Scientist at Bell Labs
  - Formulated the Kelly Criterion
  - Methodology Used by Mathematician Ed Thorp to Make Money on Roulette and Blackjack
  - Ideas Used in Financial Investment

- Kelly Criterion = Optimal Percentage of Bankroll to Place on Bet in Order to Maximize Expected Growth of Wealth

- Simplify Ideas to $1

MONEY MANAGEMENT

- Situation: Start with $1.00
  - C = Starting Capital = 1
  - X = Bankroll After Bet
  - f = Fraction of Capital Placed on Bet (Unit)
  - p = Probability of Winning Bet
  - q = Probability of Losing Bet = 1-p
  - W = Profit Made Per $1 Bet When Won
  - L = Loss Per $1 Bet When Lost

- Maximize For Kelly Criterion

$$E[\log(X)]$$

MONEY MANAGEMENT

- Typically, for 50-50 Bet,
  - W = 100/110 = 0.91 = 91%
  - L =1
- Suppose Money Line is Bears +220
  - W = 220/100 = 2.2 = 220%
  - L = 1
- Suppose Money Line is Bears -220
  - W = 100/220 = 0.454 = 45.4%
  - L = 1

MONEY MANAGEMENT

- **Expected Value for Maximization**

$$E[\log(X)] = p \times \log(1 + fW) + q \times \log(1 - fL)$$

- **Taking the Derivative in Respect to f**

$$\frac{d}{df} E[\log(X)] = p \times \frac{W}{1 + fW} - q \times \frac{L}{1 - fL} \overset{\text{Set}}{=} 0$$

- **Solve Equation for f**

$$f = \frac{\mathrm{pW} - \mathrm{qL}}{W \times L}$$ → Edge Favoring Bettor

MONEY MANAGEMENT

- **Restructuring**

$$f = \frac{p}{L} - \frac{q}{W} = \frac{p}{L} - \frac{1-p}{W} = \frac{p}{L} - \frac{1}{W} + \frac{p}{W} = -\frac{1}{W} + \left(\frac{1}{L} + \frac{1}{W}\right) \times p$$

Linear Model: y=a+bX

- **Example 1: Bet on a Spread With 60% Chance of Winning**

$$f = \frac{pW - qL}{W \times L} = \frac{0.6 \times 0.909 - 0.4 \times 1}{0.909 \times 1} = 0.16 = 16\%$$

MONEY MANAGEMENT

- Relationship Between f and p

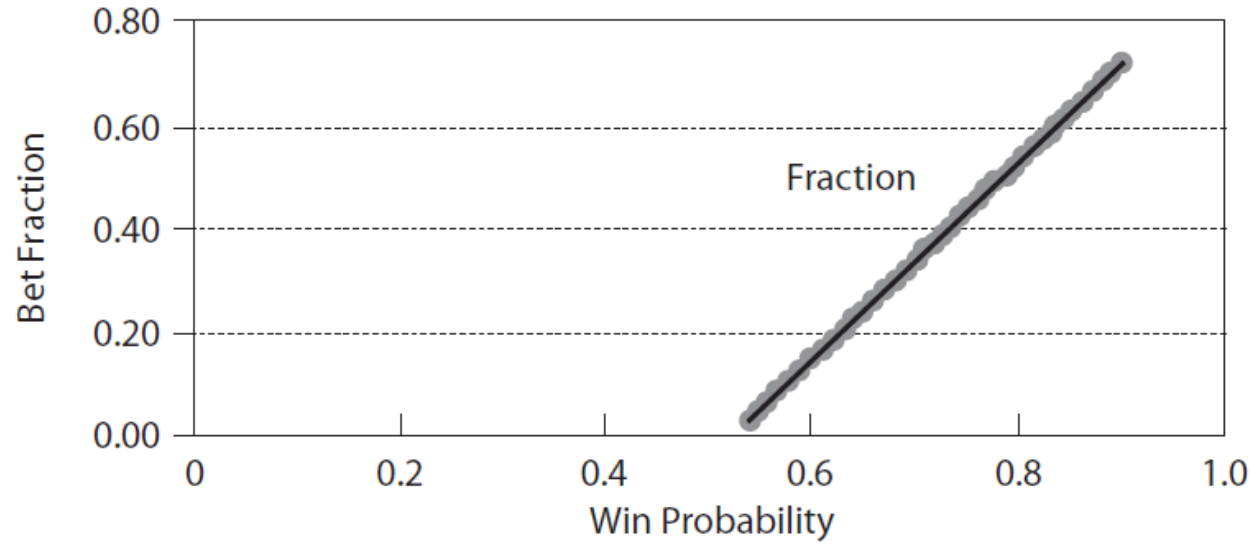

Figure 44.2. Optimal bet fraction as function of win probability.

MONEY MANAGEMENT

- **Relationship Between $E[\log(X)]$ and p**

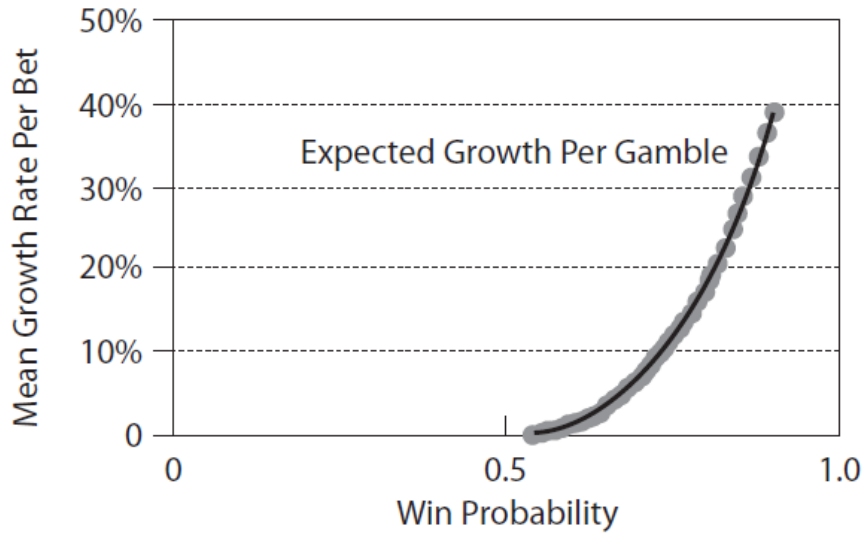

Figure 44.3. Average wealth growth per period as a function of win probability.

MONEY MANAGEMENT

- **Example 2: Bet on a Spread With 48% Chance of Winning**

$$f = \frac{pW - qL}{W \times L} = \frac{0.48 \times 0.909 - 0.52 \times 1}{0.909 \times 1} = -0.09 = -9\%$$

Don't Be an Idiot

- **Example 3: Bet on a Spread With 95% Chance of Winning**

$$f = \frac{pW - qL}{W \times L} = \frac{0.95 \times 0.909 - 0.05 \times 1}{0.909 \times 1} = 0.89 = 89\% \neq 100\%$$

Don't Be an Idiot

MONEY MANAGEMENT

- **Recall:** $E[\log(X)] = p \times \log(1 + fW) + q \times \log(1 - fL)$

- **Effect of Choosing Wrong** p

	K	L
2	**Fraction**	**Average growth rate**
3	0.05	0.006668091
4	0.1	0.010628669
5	0.15	0.011796783
6	0.2	0.010058642
7	0.25	0.005266501
8	0.3	–0.002768628
9	0.35	–0.014287401
10	0.4	–0.02959722
11	0.45	–0.049094515
12	0.5	–0.073298724

Figure 44.4. Average long-term growth rate versus fraction bet.

FINAL INSPIRATION

When I bet on horses,
I never lose. Why?
I bet on all the horses

-Tom Haverford