
# Football V

Produced by Dr. Mario | UNC STOR 538

# Why Don't Teams Always Pass?

- Passing Plays Seem to Dominate Running Plays

- Simple Model of Play Selection
  - Offensive Choice: Run or Pass
  - Defensive Choice: Defend Run or Defend Pass
  - Consider the Payoff Matrix


| | Run Defense | Pass Defense |
|---|---|---|
| Offense Runs | -5 | 5 |
| Offense Passes | 10 | 0 |

  - Offense Gains 1 Yard = Defense Loses 1 Yard
  - Positive = Good For Offense

# Why Don't Teams Always Pass?

- James von Neumann and Oskar Morgenstern
  - Foundation of Game Theory (1944)
  - Built From Mathematics and Economics
  - Two-Person Zero Sum Games = Games Where 2 Players are In Total Conflict
  - Assumption 1: Row Player Wants to Maximize the Payoff
  - Assumption 2: Column Player Wants to Minimize the Payoff


- Payoff Matrix Applied to Offense Decisions
  - Suppose the Offense Passes Every Time
  - Defense Could Defend Passing Every Time
  - Offense Would Gain 0 Yards
  - Is There an Optimal Mixed Strategy for the Offense?

# Why Don't Teams Always Pass?

- Mixed Strategy
  - Let $p$ Represent the Probability Offense Runs
  - Suppose Run Defense is Chosen

$$E[Gain|Run\ Defense] = p(-5) + (1-p)10 = -15p + 10$$

  - Suppose Pass Defense is Chosen

$$E[Gain|Pass\ Defense] = p(5) + (1-p)0 = 5p$$

  - Suppose Offense Runs 75% of the Time

$$E[Gain|Run\ Defense] = -1.25$$
$$E[Gain|Pass\ Defense] = 3.75$$

← Always Run Defense

  - Suppose Offense Runs 25% of the Time

$$E[Gain|Run\ Defense] = 6.25$$
$$E[Gain|Pass\ Defense] = 1.25$$

← Always Pass Defense

# Why Don't Teams Always Pass?

- Optimal Strategy for Offense
  - Balancing the Expected Gain

$$E[Gain|Run\ Defense] = -15p + 10 = 5p = \mathrm{E[Gain|Pass\ Defense]}$$

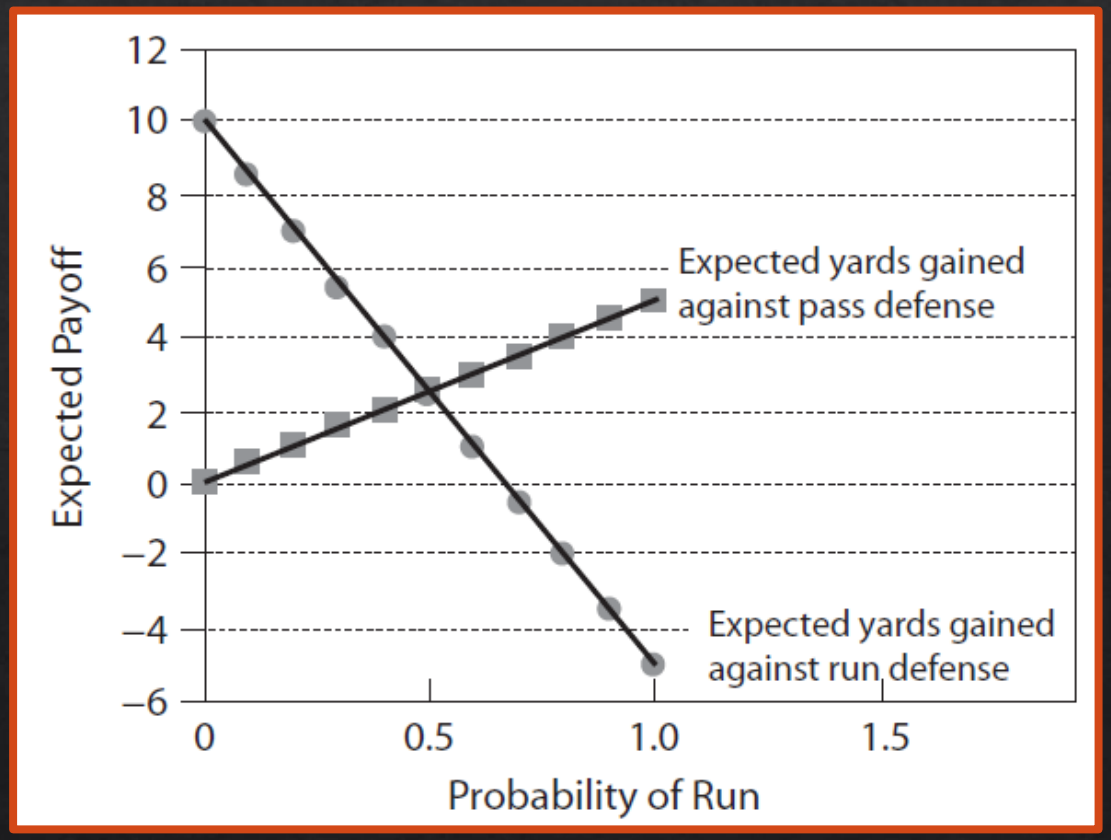
$$-20p = -10$$

$$p = 0.5$$

- Offense Runs 50% of the Time = Defense Has No Optimal Strategy
- No Matter What Defense Selects Expected Gain is 2.5 Yards
- Value to the Offense (Row Player) is 2.5 Yards

# Why Don't Teams Always Pass?

- Optimal Strategy for Offense
  - Visualization



# Why Don't Teams Always Pass?

- Interesting Conclusion
  - Recall the Payoff Matrix

| | Run Defense | Pass Defense |
|---|---|---|
| Offense Runs | -5 | 5 |
| Offense Passes | 10 | 0 |


- Defense Guesses Correctly = Passing is 5 Yards Better
- Defense Guesses Incorrectly = Passing is 5 Yards Better
- Passing Seems to Dominate
- Optimal Strategy is to Mix Up Offense and Defense
- Problems or Ideas?

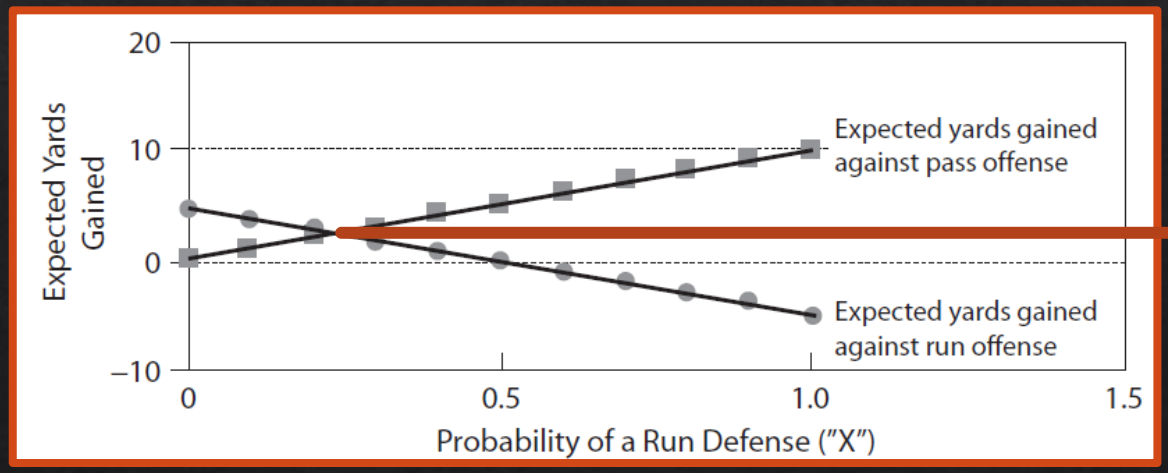
# Why Don't Teams Always Pass?

- Optimal Strategy for Defense
  - Let $q$ Represent the Probability Defense Calls Run Defense
  - Suppose Offense Decides to Run

$$E[Gain|Offense\ Runs] = q(-5) + (1-q)5 = -10q + 5$$

  - Suppose Offense Decides to Pass

$$E[Gain|Offense\ Passes] = q(10) + (1-q)0 = 10q$$




25% Run Defense

# Why Don't Teams Always Pass?

- Optimal Strategy for Defense
  - Defense Defends Runs 25% of the Time = Offense Has No Optimal Strategy
  - No Matter What Offense Selects Expected Gain is 2.5 Yards
  - Value to the Defense (Column Player) is 2.5 Yards
  - Important Conclusion = Value of Offense and Defense are Equal


- Generalization of Payoff Matrix
  - Let $r$ = Average Yards Gained on Run
  - Let $t$ = Average Yards Gained on Pass
  - Let $k$ = Impact of Defense Decision on Running Plays
  - Let $m$ = Multiplier of Impact of Defense Decision on Passing Plays

# Why Don't Teams Always Pass?

- Generalization of Payoff Matrix
  - Visualization

| | Run Defense | Pass Defense |
|---|---|---|
| Offense Runs | r-k | r+k |
| Offense Passes | t+mk | t-mk |

- Optimal Offensive Mix

$$p = \frac{m}{m+1}$$

- Optimal Defensive Mix

$$q = 0.5 + \frac{r-t}{2k(m+1)}$$

→ Defend Better Play More Than 50% of the Time

# Why Don’t Teams Always Pass?

- Interesting Insights From Game Theory
  - Acquire Better Quarterback
    - New QB Gains 3 More Yards on Average
    - Should We Pass More Often?
    - Replace $t$ with $t + 3$
    - Has No Impact on Optimal Run-Pass Mix
  - Acquire Better Running Back
    - New RB Gains 5 More Yards on Average Against Pass Defense
    - New RB Gains 0 More Yards on Average Against Run Defense

# Why Don't Teams Always Pass?

- Interesting Insights From Game Theory
  - Consider Payoff Matrix

| | Run Defense | Pass Defense |
|---|---|---|
| Offense Runs | -5 | 5 |
| Offense Passes | 5 | -5 |


  - Modified Payoff Matrix with Better Running Back

| | Run Defense | Pass Defense |
|---|---|---|
| Offense Runs | -5 | 10 |
| Offense Passes | 5 | -5 |

# Why Don't Teams Always Pass?

- Interesting Insights From Game Theory
  - Finding Values
    - $r = 2.5$
    - $\mathrm{k} = 7.5$
    - $\mathrm{t} = 0$
    - $5 = 7.5\mathrm{m} \longrightarrow \mathrm{m} = 2/3$
  - Optimal Run-Pass Mixture
    - Run 2/5 of Time
    - Pass 3/5 of Time
    - Expected Yards Gained Has Improved Overall from 0 Yards to 1 Yard
  - Running Back Has Improved But Offense Run Less
  - Optimal Defense Run-Pass Mixture
    - Defend Run 3/5 of Time
    - Defend Pass 2/5 of Time
  - What Did We Learn?

# Final Inspiration

I’m just here so I can pay my fines.

- Mahatma Mario