University of Cape Town	Department of Physics

Computational Physics
Honours Module

PHY4000W-CP

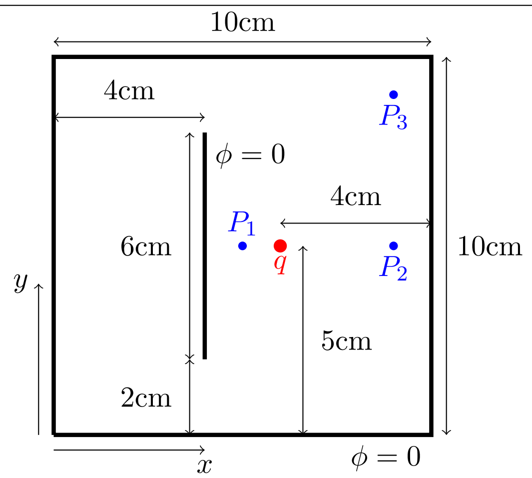
Tutorial 4

Due Wed, 17 May 2023, 12pm **T Dietel**

1. The Poisson equation describes the electric potential of a charge distribution.

$$\nabla^2 \phi(x,y) = -\rho(x,y)/\epsilon_0 \tag{1}$$

We are given a point charge $q = 1\mathrm{C}$ near an electrode inside a box at a potential $\phi = 0$ as depicted on the right.
All calculations should be done on a lattice with a spacing of $1\mathrm{mm}\times 1\mathrm{mm}$.



(a) Solve the Poisson equation and calculate the potential in the centre of the box (P_1), 1 cm from the right side of the box (P_2) and in the upper right corner (P_3), 1 cm from either side. Draw a map of the potential ϕ.

(b) Calculate and visualize the electric field. What is the electric field (magnitude and angle ϕ with respect to the x axis) at the points P_1, P_2 and P_3?

(c) Determine the charge distribution $\rho(y)$ along the electrode inside the box at $x = 4\mathrm{cm}$. Plot the charge distribution ρ vs. the coordinate along the electrode y.
What is the total charge on the electrode?

2. The oscillations of a massive string are described by the partial differential equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{T}{\rho(x)} \frac{\partial^2 u(x,t)}{\partial x^2},$$

where $u(x,t)$ is the displacement of the string at location x and time t, $\rho(x)$ is the mass density of the string and T the string tension. Both ends of the string are fixed, resulting in the boundary conditions $u(x=0,t) = u(x=L,t) = 0$, where L is the length of the string.

The problem can be solved by separation of variables, taking $u(x,t) = X(x)\Theta(t)$:

$$\frac{\Theta''(t)}{\Theta(t)} = \frac{T}{\rho(x)} \frac{X''(x)}{X(x)} = -\omega^2.$$

For the temporal part, the solutions are oscillations with frequency ω: $\Theta(t) = Ae^{\pm i\omega t}$, while the spatial part must satisfy the ordinary differential equation:

$$\frac{d^2X(x)}{dx^2} + \frac{\omega^2\rho(x)}{T} X(x) = 0.$$

Solve this differential equation by searching for the eigenvalues and eigenfunctions of $X(x)$. Determine the first 10 eigenvalues ω_n for an inhomogeneous string with mass density $\rho(x) = 1.3 - \frac{1}{2}\sin\left(\frac{\pi x}{L}\right)$ and a homogeneous string with mass density $\rho = 1$. Compare the eigenvalues for the angular frequency ω_n for the homogeneous and the inhomogeneous string. Draw the corresponding wave forms $X_n(x)$ for $n = 1, 5, 9$.