University of Cape Town	Department of Physics

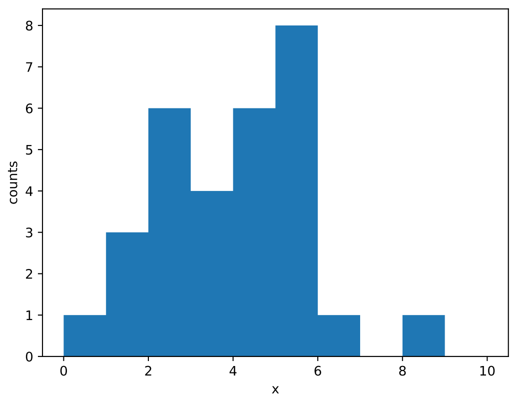
Computational Physics
Honours Module

PHY4000W-CP

Tutorial 2

Due Wed, 12 April 2023, 12pm **T Dietel**

1. We are analyzing a dataset that contains a normally distributed signal and a flat background, measured in 10 equidistant bins of x from 0 to 10 as shown in the figure. The normal distribiution has an (unknown) mean μ and standard deviation σ, and we define the signal strength S as the number of all measurements over the full range of x, i.e. not restricted to the range of the histogram. The background B shall be described by the average number of entries in a bin of width 1.



   (a) Implement a function to calculate the likelihood $\mathcal{L}$ for a parameter vector (S, μ, σ, B), given the measurements shown in the histogram. Calculate and report the likelihood for $S = 20$, $\mu = 5$, $\sigma = 2$ and $B = 1$?

   Note: What is the distribution of entries per bin? And how do you best deal with the finite bin width?

   (b) For which set of fit parameters is the likelihood maximal? What is the value of the likelihood $\mathcal{L}$ at this point?

   (c) Discuss the uncertainty in the neighborhood of the best fit, using appropriate plots of the likelihood.

2. Simulate the geometry of $^{208}\mathrm{Pb}+^{208}\mathrm{Pb}$ collisions in the Glauber Monte-Carlo model. In this picture, the Pb nuclei, which move in opposite directions along the z-axis with an impact parameter b, are seen as $A = 208$ independent nucleons that rest within the nucleus, i.e. have constant x and y.

   The nucleons are distributed in the nuclei according to the Woods-Saxon density distribution:

   $$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}},$$

   where $\rho(r)$ is the nucleon density in the nucleus, $R = A^{1/3} \times 1.2\mathrm{fm}$ is the radius of the nucleus and $a = 0.5\mathrm{fm}$ is a parameter for the "surface thickness". The factor ρ_0 can be determined from the normalization of the probability density function.

   Two nucleons interact if their distance in the x-y-plane is less than the radius of a disc with area σ: $d = \sqrt{\Delta x^2 + \Delta y^2} < \sqrt{\sigma/\pi}$, where $\sigma = 72\mathrm{mb} = 7.2\mathrm{fm}^2$ is the cross section for an inelastic nucleon-nucleon (N-N) interaction. Nucleons can interact multiple times, the number of N-N collisions N_{coll} can therefore be larger than the number of "participant" nucleons that suffer at least one interaction N_{part}. The Glauber Monte-Carlo method samples impact parameters and the distribution of the nucleons in the nuclei to determine the relation between b, N_{part} and N_{coll}.

   (a) What is the expected number of nucleons $N(r)$ at distance r from the center of the nucleus? Draw a large number of samples from $N(r)$ and plot the generated values of r.

   (b) Distribute the nucleons homogeneously over the sphere with radius r to determine x,y and z. Check isotropy and radial distribution of your results with suitable plots.

   (c) Sample impact parameters b, assuming that the impact parameter vectors $\vec{b} = (b_x, b_y)$ are uniformly distributed in the x-y-plane. How are the impact parameters b distributed? Check your results with suitable plots.

   (d) Determine N_{part} and N_{coll} for 10000 Pb-Pb collisions with random impact parameters b and at least one N-N collision. Plot N_{part} vs. b and N_{coll} vs. b and N_{coll} vs N_{part} . Histogram N_{part} and N_{coll} with logarithmic scale for the y axis.