University of Cape Town	Department of Physics
Computational Physics Honours Module	**PHY4000W-CP**

Tutorial 1

Due Wed, 22 March 2023, 12pm **T Dietel**

1. Implement the matrix operations listed below using arrays and loops in a programming language of your choice, and test your functions with the $n \times n$ matrix $\mathbf{M}$ and the n-dimensional vector $\mathbf{x}$, where

$$\begin{aligned} M_{ij} &= \big((37.1i + 91.7j^2) \mod 20.0\big) - 10.0 \\ x_i &= i \end{aligned}$$

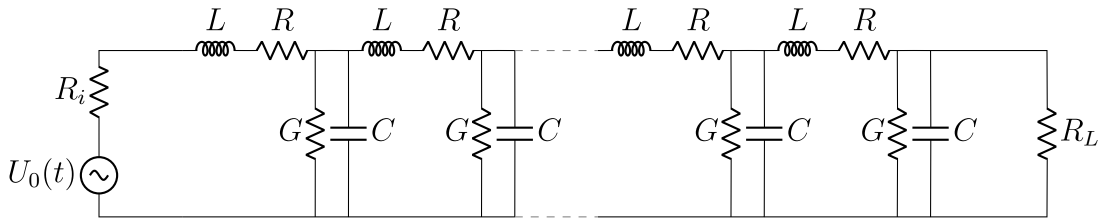
   Calculate and tabulate the values of $\mathbf{x}^T\mathbf{x}$ and $\mathbf{x}^T\mathbf{M}\mathbf{x}$ for $n = 5, 10, 20, 100, 1000$.

   For each of the following matrix operations, measure the execution time as a function of n, up to execution times of a few seconds. Compare the execution times of your implementation with a dedicated matrix library, e.g. the numpy package within python. Plot the execution times for all matrix operations and both implementations.

   - dot product of two vectors: $\mathbf{x}^T\mathbf{x}$
   - multiplication of a matrix with a vector: $\mathbf{M}\mathbf{x}$
   - multiplication of two vectors and a matrix: $\mathbf{x}^T\mathbf{M}\mathbf{x}$
   - multiplication of two matrices: $\mathbf{M}\mathbf{M}$

   What is the expected asymptotic behaviour of each operation? Compare with the observed behaviour. Try to estimate the number of floating point operations per second (FLOPS) of your system.

2. A transmission line (a.k.a. a cable) can be modelled with the Telegrapher's equations, as a circuit that includes the capacitance C and inductance L of the wire per unit length as well as the resistance R of the wires. Is also accounts for ohmic losses G due to the oscillating polarisation of the dielectric between the wires. We model the signal source as a combination of an ideal voltage source and an internal resistor $R_i = 50\,\text{Ohm}$.



We study a transmission line with a length of $5\,\text{m}$, which we segment into 500 cells for the calculation. Use $C = 110\,\text{pF}\,\text{m}^{-1}$, $L = 270\,\text{nH}\,\text{m}^{-1}$, $R = 30\,\mu\Omega\,\text{m}^{-1}$ and $G = 1\,\text{G}\Omega\,\text{m}^{-1}$ for your calculations.

(a) Using Kirchhoff's Laws, derive the set of equations that describes the transmission line. Solve the equations numerically for a sinusoidal input voltage with a frequency of $100\,\text{MHz}$, assuming an open-ended cable, a shorted cable and a cable with a termination resistor $R_L = 100\,\text{Ohm}$.

What is the voltage between the wires in the middle and at the end of the cable for these three cases?

(b) Plot the amplitude of the voltage along the cable in these three cases. Compare the three different wave forms.

(c) Find the value of the termination resistor R_L for which the difference between the minimum and maximum amplitude of the voltage along the transmission line is smallest.

A detector signal is transmitted over the cable. The signal is described by an exponential: $V(t) = V_0 e^{-(t-t_0)/\tau}\Theta(t - t_0)$, where t_0 is the start time of the signal and $\tau = 5\,\text{ns}$ is its decay time. Assume that the optimal termination resistor, as calculated in part (c), is used.

(d) Calculate the Fourier transform of the detector signal.

(e) The detector signal is measured via the voltage drop across the termination resistor, using e.g. an oscilloscope. Calculate and visualize the measured voltage across the termination resistor at the end of the transmission line.

(f) How fast does the signal travel in this transmission line?