CMPT 383 Comparative Programming Languages

Homework 6 Solution

This homework is due by 11:59pm PT on Wednesday Mar 23, 2022. No late submission is accepted. Please save your answers in a single file called `h6_firstname_lastname.pdf` and submit it to Canvas. You may also write on paper and scan it (or take a picture) into a PDF. Please make sure the text is readable.

Requirements:

- Please include an environment in the judgments even if it is not used.

1. (20 points) Consider the FUN language that we learned, provide a big-step operational semantics to expression e_1`<=`e_2. The expression evaluates to `true` if $e_1 \leq e_2$. Otherwise, it evaluates to `false`.

Solution:

$$\frac{E \vdash e_1 : c_1 \quad E \vdash e_2 : c_2 \quad c_1 \leq c_2}{E \vdash e_1\texttt{<=}e_2 : \texttt{true}} \qquad \frac{E \vdash e_1 : c_1 \quad E \vdash e_2 : c_2 \quad c_1 > c_2}{E \vdash e_1\texttt{<=}e_2 : \texttt{false}}$$

2. (20 points) Consider the FUN language, prove the following expression evaluates to 3 with respect to the big-step operational semantics that can handle recursion.

$$\texttt{let } x = 2 \texttt{ in } x + 1$$

Solution:

$$\frac{E \vdash 2 : 2 \qquad \dfrac{E[x \triangleleft 2] \vdash 1 : 1 \qquad \dfrac{\texttt{Ident } x \quad E[x \triangleleft 2](x) = 2}{E[x \triangleleft 2] \vdash x : 2}}{E[x \triangleleft 2] \vdash x + 1 : 3}}{E \vdash \texttt{let } x = 2 \texttt{ in } x + 1 : 3}$$

Note that the top line corresponds to the rule for identifiers. The middle line corresponds to the rule for the + operator. The bottom line corresponds to the rule for let bindings.

3. (30 points) Suppose we add a program construct called `testSign` to the FUN language with the following syntax

$$\begin{array}{rcl} e & ::= & \ldots \quad \text{(all existing productions in FUN)} \\ & | & \texttt{'testSign'}\ e\ e\ e\ e \end{array}$$

The evaluation result of `testSign` e_1 e_2 e_3 e_4 is

- the result of e_2, if e_1 evaluates to a negative number
- the result of e_3, if e_1 evaluates to zero
- the result of e_4, if e_1 evaluates to a positive number

Provide a big-step operational semantics for `testSign`.

Solution:

$$\frac{E \vdash e_1 : c \quad c < 0 \qquad E \vdash e_2 : v}{E \vdash \texttt{testSign}\ e_1\ e_2\ e_3\ e_4 : v} \qquad \frac{E \vdash e_1 : 0 \qquad E \vdash e_3 : v}{E \vdash \texttt{testSign}\ e_1\ e_2\ e_3\ e_4 : v} \qquad \frac{E \vdash e_1 : c \quad c > 0 \qquad E \vdash e_4 : v}{E \vdash \texttt{testSign}\ e_1\ e_2\ e_3\ e_4 : v}$$

4. (30 points) Consider the `testSign` in Quesiton 3, provide a small-step operational semantics for `testSign`. Note that for expression `testSign` $e_1\ e_2\ e_3\ e_4$, the expression e_1 should be evaluated first. You can assume the small-step operational semantics for other FUN constructs already exists. Hint: here is the small-step operational semantics for `if-then-else`.

$$\frac{\langle e_1, E\rangle \to \langle e_1', E'\rangle}{\langle \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3, E\rangle \to \langle \texttt{if}\ e_1'\ \texttt{then}\ e_2\ \texttt{else}\ e_3, E'\rangle}$$

$$\frac{}{\langle \texttt{if true then}\ e_2\ \texttt{else}\ e_3, E\rangle \to \langle e_2, E\rangle}$$

$$\frac{}{\langle \texttt{if false then}\ e_2\ \texttt{else}\ e_3, E\rangle \to \langle e_3, E\rangle}$$

Solution:

$$\frac{\langle e_1, E\rangle \to \langle e_1', E'\rangle}{\langle \texttt{testSign}\ e_1\ e_2\ e_3\ e_4, E\rangle \to \langle \texttt{testSign}\ e_1'\ e_2\ e_3\ e_4, E'\rangle}$$

$$\frac{c < 0}{\langle \texttt{testSign}\ c\ e_2\ e_3\ e_4, E\rangle \to \langle e_2, E\rangle}$$

$$\frac{}{\langle \texttt{testSign}\ 0\ e_2\ e_3\ e_4, E\rangle \to \langle e_3, E\rangle}$$

$$\frac{c > 0}{\langle \texttt{testSign}\ c\ e_2\ e_3\ e_4, E\rangle \to \langle e_4, E\rangle}$$