# CMPT 383 Comparative Programming Languages

Homework 7 Solution

This homework is due by 11:59pm PT on Wednesday Apr 6, 2022. No late submission is accepted. Please save your answers in a single file called `h7_firstname_lastname.pdf` and submit it to Canvas. You may also write on paper and scan it (or take a picture) into a PDF. Please make sure the text is readable.

1. (20 points) Consider the FUN language with type annotations, prove the type of following expression is $Int \to Int$. In other words, show the derivation process using the T-XXX rules.

$$\texttt{lambda}\ x : \texttt{Int}.\ 1 + x$$

Solution:

$$\dfrac{\dfrac{\text{Int } 1}{\Gamma[x \triangleleft Int] \vdash 1 : Int}\ (\text{T-Int}) \qquad \dfrac{\text{Ident } x \quad \Gamma[x \triangleleft Int](x) = Int}{\Gamma[x \triangleleft Int] \vdash x : Int}\ (\text{T-Ident})}{\dfrac{\Gamma[x \triangleleft Int] \vdash 1 + x : Int}{\Gamma \vdash \texttt{lambda}\ x : \texttt{Int}.\ 1 + x : Int \to Int}\ (\text{T-Abs})}\ (\text{T-Plus})$$

2. (30 points) Consider the FUN language with type annotations, prove the type of following expression is $Int$ using T-XXX rules.

$$\texttt{let}\ f : \texttt{Int->Int} = \texttt{lambda}\ x : \texttt{Int}.\ x\ \texttt{in}\ \texttt{app}\ f\ 1$$

Solution:

$$\dfrac{\dfrac{\dfrac{\text{Ident } x \quad \Gamma[x \triangleleft Int](x) = Int}{\Gamma[x \triangleleft Int] \vdash x : Int}\ (\text{T-Ident})}{\Gamma \vdash \texttt{lambda}\ x : \texttt{Int}.\ x : Int \to Int}\ (\text{T-Abs}) \qquad \dfrac{\dfrac{\text{Ident } f \quad \Gamma[f \triangleleft Int \to Int](f) = Int \to Int}{\Gamma[f \triangleleft Int \to Int] \vdash f : Int \to Int}\ (\text{T-Ident}) \qquad \dfrac{\text{Int } 1}{\Gamma[f \triangleleft Int \to Int] \vdash 1 : Int}\ (\text{T-Int})}{\Gamma[f \triangleleft Int \to Int] \vdash \texttt{app}\ f\ 1 : Int}\ (\text{T-App})}{\Gamma \vdash \texttt{let}\ f : \texttt{Int->Int} = \texttt{lambda}\ x : \texttt{Int}.\ x\ \texttt{in}\ \texttt{app}\ f\ 1 : Int}\ (\text{T-Let})$$

3. (10 points) Find a most general unifier of the following constraints. You do not need to show the steps.

$$\{X_1 = X_2 \to X_3,\ X_2 = X_3 \to X_4,\ X_3 = Int\}$$

Solution:

$$[X_1 \mapsto (Int \to X_4) \to Int,\ X_2 \mapsto Int \to X_4,\ X_3 \mapsto Int]$$

4. (30 points) Consider the FUN language without type annotations, perform constraint-based type checking of the following expression using CT-XXX rules (use CT-Ident1 and CT-Ident2 instead of CT-Ident). You need to show the derivation steps.

$$\texttt{let}\ f = \texttt{lambda}\ x.\ x\ \texttt{in}\ \texttt{app}\ f\ 1$$

Solution:

$$
\dfrac{\text{fresh } X_1 \qquad \dfrac{\text{fresh } X_2 \qquad \dfrac{\text{Ident } x \quad x \in \text{dom}(\Gamma[f \triangleleft X_1][x \triangleleft X_2]) \quad \Gamma[f \triangleleft X_1][x \triangleleft X_2](x) = X_2}{\Gamma[f \triangleleft X_1][x \triangleleft X_2] \vdash x : X_2|\{\}}\,\text{(CT-Ident1)}}{\Gamma[f \triangleleft X_1] \vdash \texttt{lambda}\ x.\ x : X_2 \to X_2|\{\}}\,\text{(CT-Abs)} \qquad \dfrac{\text{fresh } X_3, X_4 \qquad \dfrac{\text{Ident } f \quad f \in \text{dom}(\Gamma[f \triangleleft X_1]) \quad \Gamma[f \triangleleft X_1](f) = X_1}{\Gamma[f \triangleleft X_1] \vdash f : X_1|\{\}}\,\text{(CT-Ident1)} \qquad \dfrac{\text{Int } 1}{\Gamma[f \triangleleft X_1] \vdash 1 : Int|\{\}}\,\text{(CT-Int)}}{\Gamma[f \triangleleft X_1] \vdash \texttt{app}\ f\ 1 : X_4|\{X_1 = X_3 \to X_4,\ Int = X_3\}}\,\text{(CT-App)}}{\Gamma \vdash \texttt{let}\ f = \texttt{lambda}\ x.\ x\ \texttt{in}\ \texttt{app}\ f\ 1 : X_4|\{X_1 = X_3 \to X_4,\ Int = X_3,\ X_1 = X_2 \to X_2\}}\,\text{(CT-Let)}
$$

5. (10 points) Consider Question 4 again, find a most general unifier of the final constraints. You do not need to show the steps.

Solution:

$$[X_1 \mapsto Int \to Int,\ X_2 \mapsto Int,\ X_3 \mapsto Int,\ X_4 \mapsto Int]$$