

Guided Tour of Machine Learning in Finance

Week 3: Unsupervised Learning

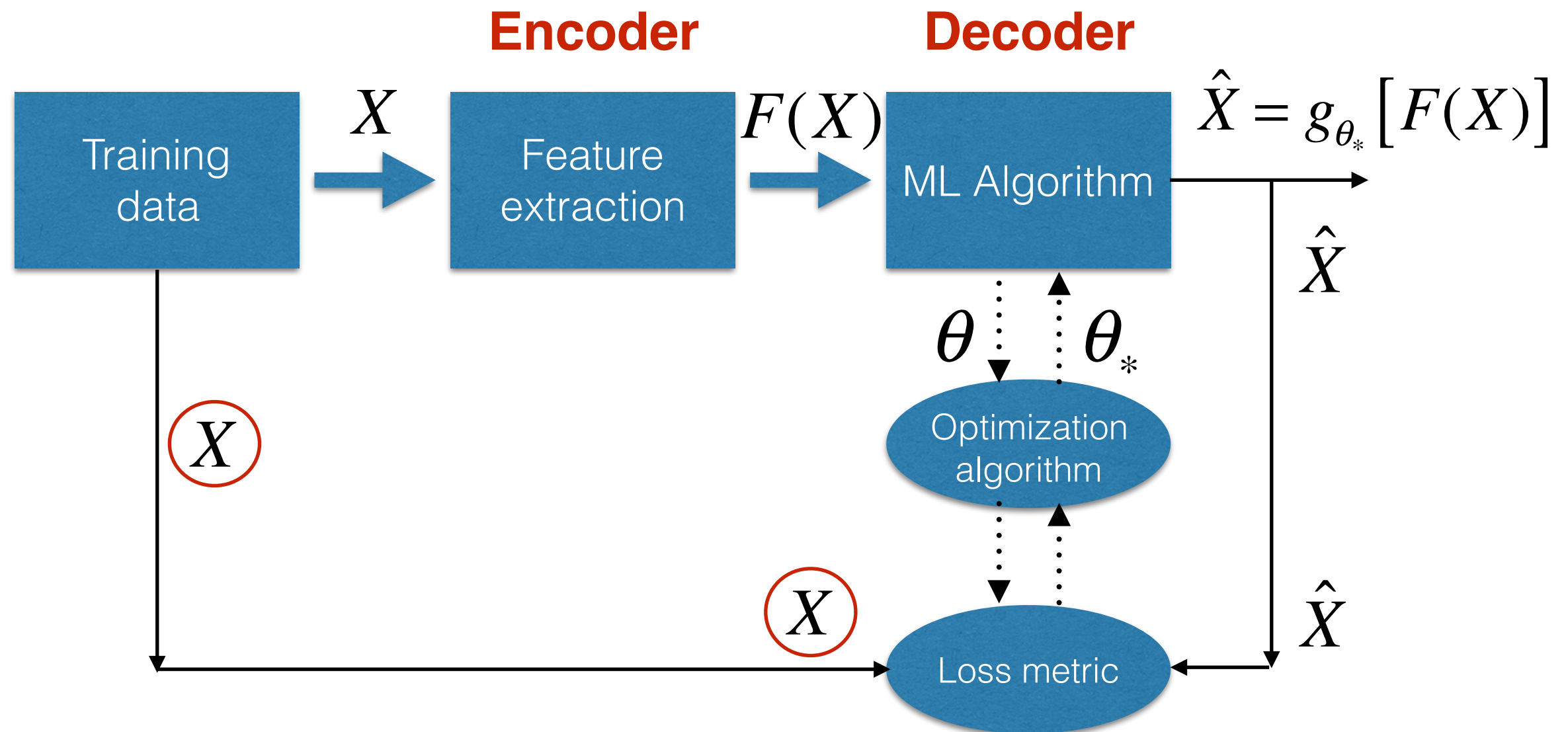
3-1-5-Autoencoder

Igor Halperin

NYU Tandon School of Engineering, 2017

Unsupervised learning diagram: Autoencoder

Autoencoder: replace Y with X !



PCA as a dimension reduction method

We had a linear transform (linear encoder) of the data $\mathbf{Z} = \mathbf{XV}$ parametrized by a $p \times k$ orthogonal matrix \mathbf{V} with $\mathbf{V}\mathbf{V}^T = \mathbf{I}$

$$\begin{matrix} \text{height } N & \text{width } k \\ \mathbf{Z} \end{matrix} = \begin{matrix} \text{height } N & \text{width } p \\ \mathbf{X} \end{matrix} \times \begin{matrix} \text{width } k & \text{height } p \\ \mathbf{V} \end{matrix}$$

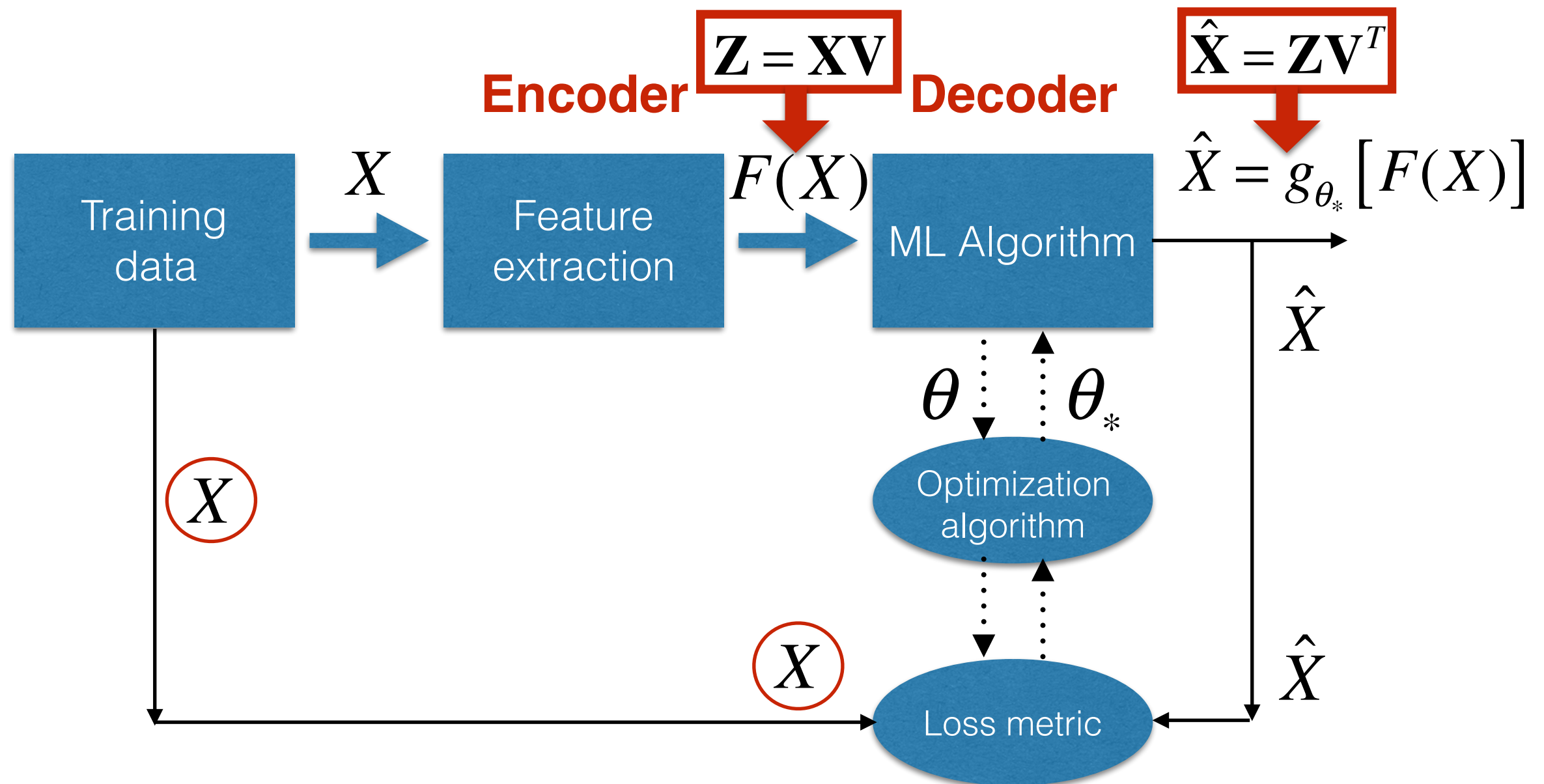
If $k = p$, $\mathbf{V} = \mathbf{U}$
this is just a rotation
of the eigenvalue
decomposition

A **decoded signal** (inverse transform) is obtained as $\hat{\mathbf{X}} = \mathbf{ZV}^T$

$$\begin{matrix} \text{height } N & \text{width } p \\ \hat{\mathbf{X}} \end{matrix} = \begin{matrix} \text{height } N & \text{width } k \\ \mathbf{Z} \end{matrix} \times \begin{matrix} \text{width } p & \text{height } k \\ \mathbf{V}^T \end{matrix}$$

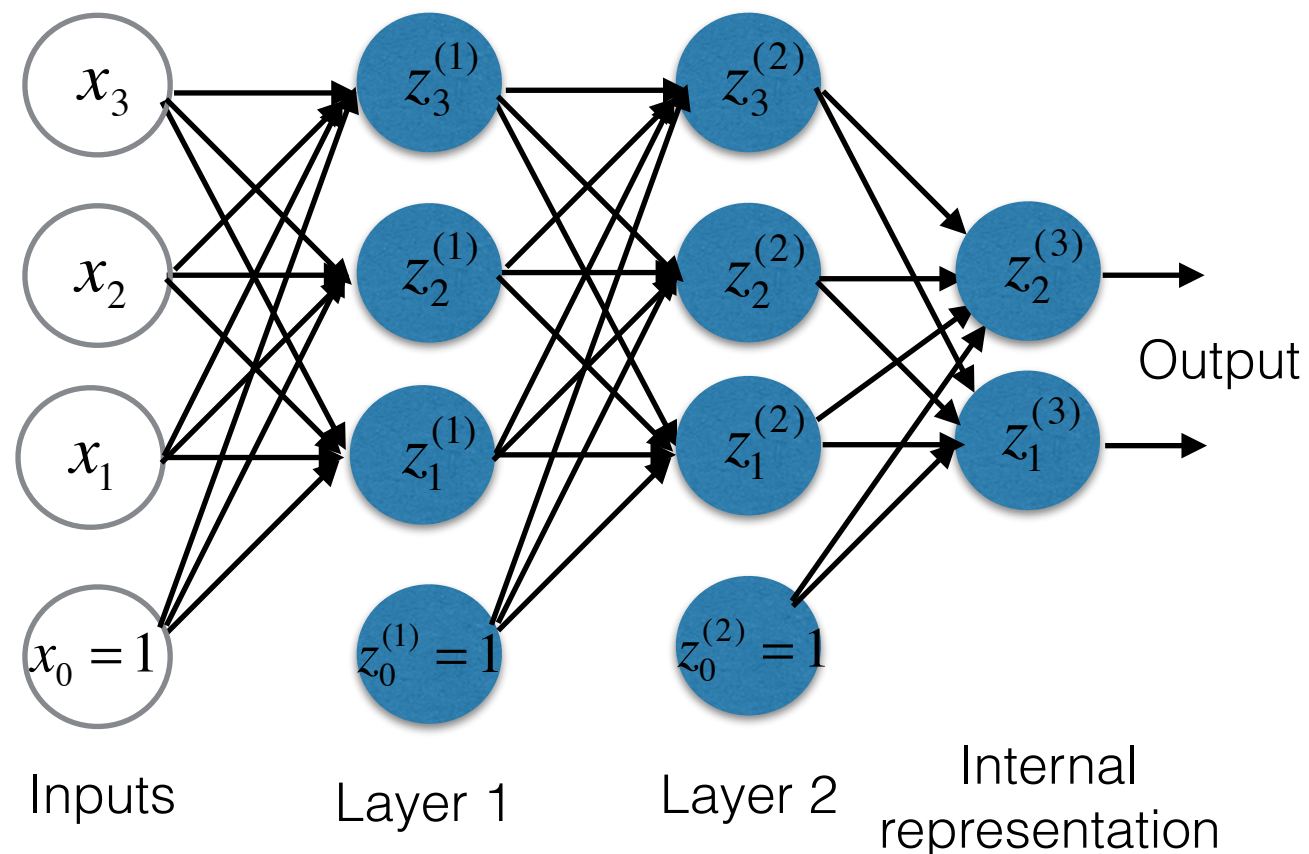
Unsupervised learning diagram: Autoencoder

Autoencoder:



Neural Encoder

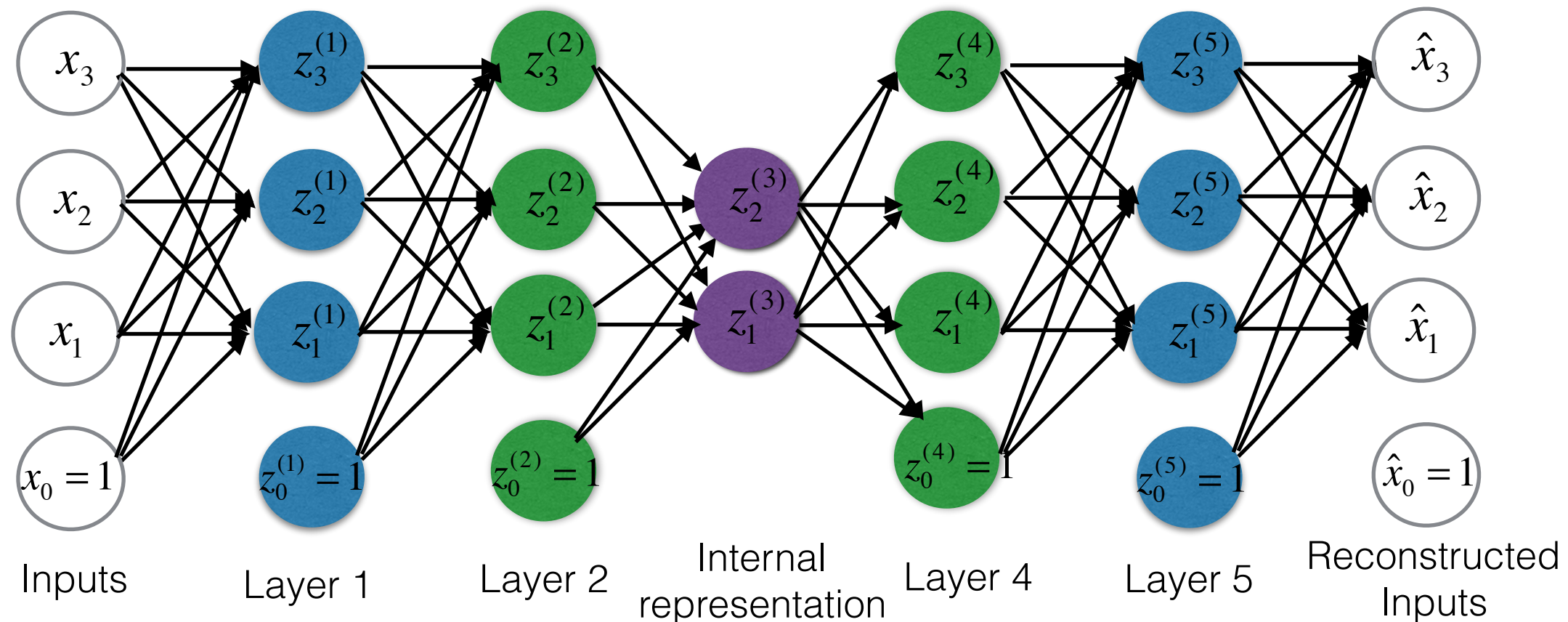
- **A Neural Encoder** is composed of layers of artificial neurons (perceptrons), with a final layer providing a low-dimensional representation of data



- For linear activation functions in all layers, this is equivalent to a single linear layer, which is equivalent to the Linear Encoder of the PCA
- If we use non-linear activations, then the depth of the network matters. For complex data, many layers might be needed to build a good internal representation.

Neural Encoder-Decoder

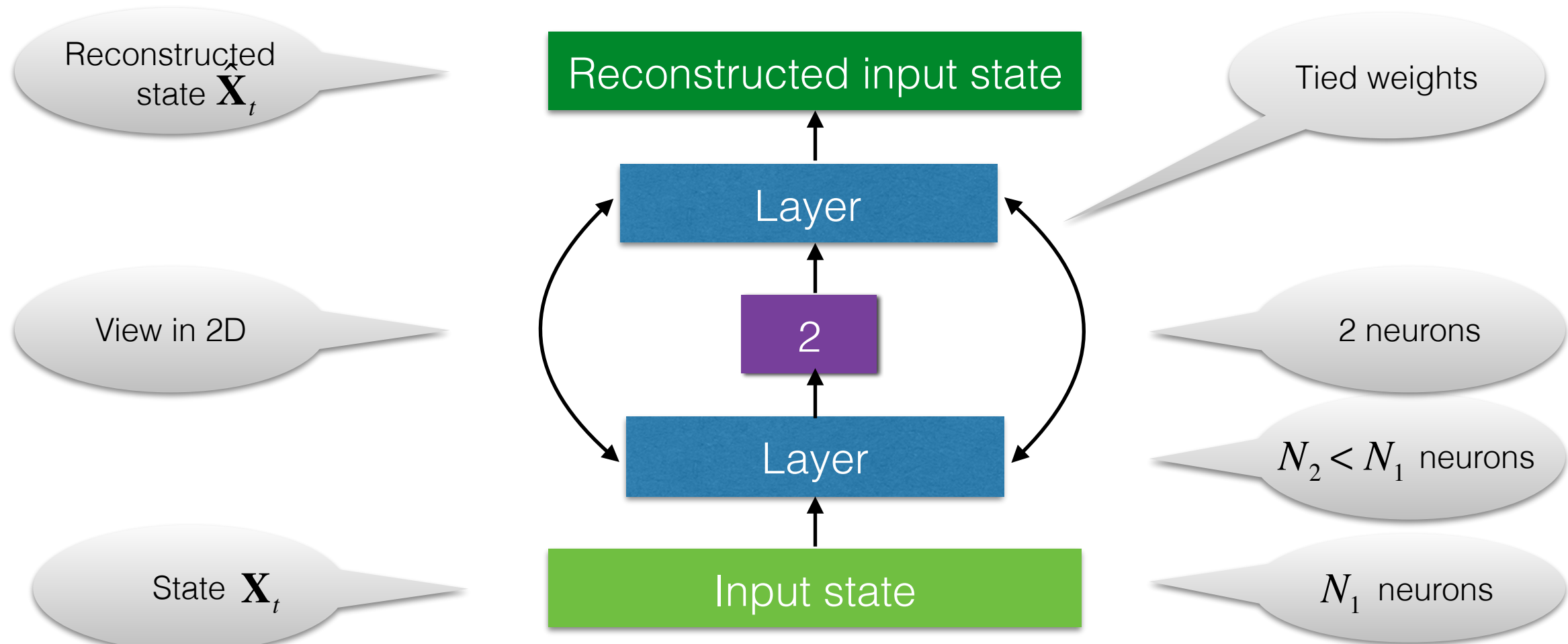
- A Neural Encoder/Decoder pair implements an Autoencoder:



- If the inner-most layer has only two neurons, their activation can be plotted for visualization of data with the autoencoder.
- A Stacked (multi-level) autoencoder may have layers with gradually decreasing number of neurons per layer
- Tied weights in the Encoder and Decoder are often used in practice

Visualizing data with autoencoder

Autoencoder provides a way to visualize a high-dimensional input vector by a non-linear projection onto a 2D space of neuron activations for the inner-most layer with 2 neurons:



Autoencoder is trained by minimizing the reconstruction error between input state \mathbf{X}_t and reconstructed state $\hat{\mathbf{X}}_t$

Control question

Select all correct answers

1. The PCA implements a Linear Encoder that converts correlated inputs \mathbf{X} into uncorrelated features \mathbf{Z} by a linear transform $\mathbf{Z} = \mathbf{XV}$
2. If the orthogonal matrix \mathbf{V} has dimension $p \times p$, i.e. it keeps all eigenvectors of correlation matrix of \mathbf{X} , then \mathbf{Z} preserves the total variation of \mathbf{X} .
3. When the data is noisy or non-stationary, \mathbf{Z} can have a higher variance than \mathbf{X} .
4. The PCA is a probabilistic method that enables simulating from data.

Correct answer: 1, 2