ECE 581K Computer Project 4 (Due Oct 30th 11:59pm 2019)

The problems are from the reference book listed in the syllabus: "Intuitive Probability and Random Processes using MATLAB" by Steven Kay.

1. Problem 1

16.7 (⌣̈) (c,f) A *biased* random walk process is defined as $X[n] = \sum_{i=0}^{n} U[i]$, where $U[i]$ is a Bernoulli random process with

$$p_U[k] = \begin{cases} \frac{1}{4} & k = -1 \\ \frac{3}{4} & k = 1\,. \end{cases}$$

What is $E[X[n]]$ and $\mathrm{var}(X[n])$ as a function of n? Next, simulate on a computer a realization of this random process. What happens as $n \to \infty$ and why?

Please also try to tune the PMF of the Bernoulli random variable, P[K], and try to explain the results.

2. Problem 2

puter simulation. Specifically, generate $M = 10{,}000$ realizations of the AR random process $X[n] = 0.95X[n-1] + U[n]$ for $n = 0, 1, \ldots, 49$, where $U[n]$ is WGN with $\sigma_U^2 = 1$. Do so two ways: for the first set of realizations let $X[-1] = 0$ and for the second set of realizations let $X[-1] \sim \mathcal{N}(0, \sigma_U^2/(1-a^2))$, using a different random variable for each realization. Now estimate the variance for each sample time n, which is $r_X[0]$, by averaging $X^2[n]$ down the ensemble of realizations. Do you obtain the theoretical result of $r_X[0] = \sigma_U^2/(1-a^2)$?

3. Problem 3

18.5 (f,c) A discrete-time WSS random process $X[n]$ is defined by the difference equation $X[n] = aX[n-1] + U[n] - bU[n-1]$, where $U[n]$ is a discrete-time white noise random process with variance $\sigma_U^2 = 1$. Plot the PSD of $X[n]$ if $a = 0.9, b = 0.2$ and also if $a = 0.2, b = 0.9$ and explain your results.

4. Problem 4

18.13 (⌣̈) (f,c) A zero mean signal with PSD $P_S(f) = 2 - 2\cos(2\pi f)$ is embedded in white noise with variance $\sigma_W^2 = 1$. Plot the frequency response of the optimal Wiener smoother. Also, compute the minimum MSE. Hint: For the MSE use a "sum" approximation to the integral (see Problem 1.14).

5. Problem 5

18.14 (c) In this problem we simulate the Wiener smoother. First generate $N = 50$ samples of a signal $S[n]$, which is an AR random process (assumes that $U[n]$ is white Gaussian noise) with $a = 0.25$ and $\sigma_U^2 = 0.5$. Remember to set the initial condition $S[-1] \sim \mathcal{N}(0, \sigma_U^2/(1-a^2))$. Next add white Gaussian noise $W[n]$ with $\sigma_W^2 = 1$ to the AR random process realization. Finally, use the MATLAB code in the chapter to smooth the noise-corrupted signal. Plot the true signal and the smoothed signal. How well does the smoother perform?

6. Problem 6

20.30 $(\ddot{\smile})$ **(w)** It is desired to generate a realization of a WSS Gaussian random process by filtering WGN with an LSI filter. If the desired PSD is $P_X(f) = 2 - 2\cos(2\pi f)$, explain how to do this.

20.31 $(\ddot{\smile})$ **(c)** Using the results of Problem 20.30, generate a realization of $X[n]$. To verify that your data generation appears correct, estimate the ACS for $k = 0, 1, \ldots, 9$ and compare it to the theoretical ACS.

7. Problem 7

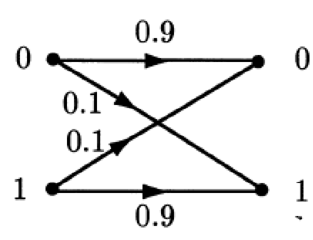
21.7 $(\ddot{\smile})$ **(f,c)** For a Poisson random process with an arrival rate of λ use a computer simulation to estimate the arrival rate if $\lambda = 2$ and also if $\lambda = 5$. To do so relate λ to the average number of arrivals in $[0, t]$. Hint: Use the MATLAB code in Section 21.7.

8. Problem 8

21.20 $(\ddot{\smile})$ **(c)** Use a computer simulation to generate multiple realizations of a Poisson random process with $\lambda = 1$. Then, use the simulation to estimate $P[T_2 \leq 1]$. Compare your result to the true value. Hint: Use the MATLAB code in Section 21.7.

9. Problem 9

22.9 $(\ddot{\smile})$ **(w,c)** A digital communication system transmits a 0 or a 1. After 10 miles of cable a repeater decodes the bit and declares it either a 0 or a 1. The probability of a decoding error is 0.1 as shown schematically in Figure 22.11. It is then retransmitted to the next repeater located 10 miles away. If the repeaters are all located 10 miles apart and the communication system is 50 miles in length, find the probability of an error if a 0 is initially transmitted. Hint: You will need a computer to work this problem.



10. Problem 10

22.15 (w,c) There are three lightbulbs that are always on in a room. At the beginning of each day the custodian checks to see if at least one lightbulb is working. If all three lightbulbs have failed, then he will replace them all. During the day each lightbulb will fail with a probability of $1/2$ and the failure is independent of the other lightbulbs failing. Letting the state be the number of working lightbulbs draw the state probability diagram and determine the transition probability matrix. Show that eventually all three bulbs must fail and the custodian will then have to replace them. Hint: You will need a computer to work this problem.