Game Theory and Applications

南京大学

高 尉

Course Information and Textbooks

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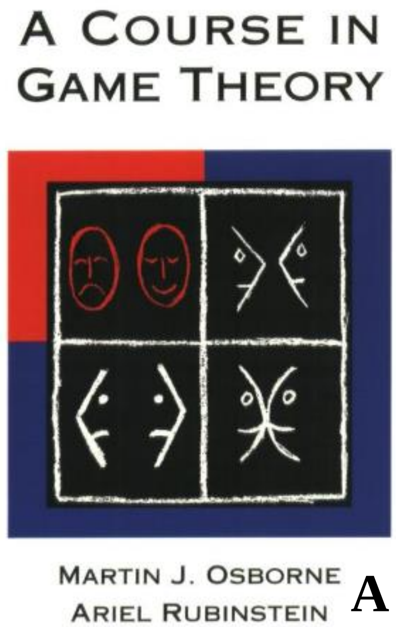
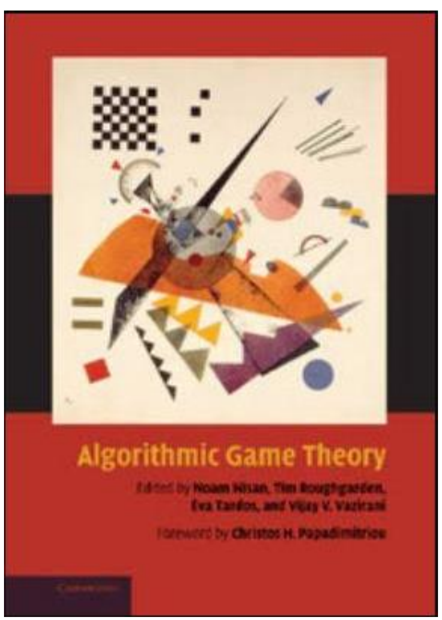


A Course in Game Theory

– Martin J. Osborne and Ariel Rubinstein

– MIT Press 1994

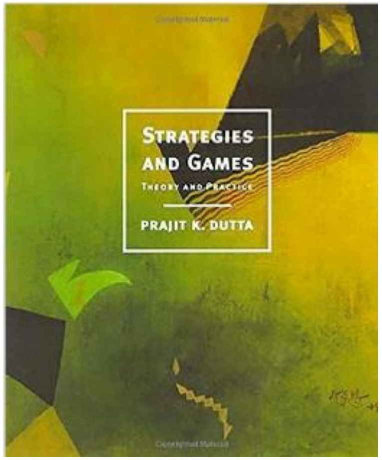


Algorithmic Game Theory

– Noam Nisan, Tim Roughgarden and Eva Tardos

– Cambridge University Press 2007

Textbooks

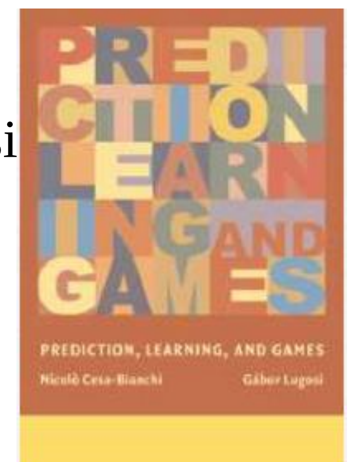


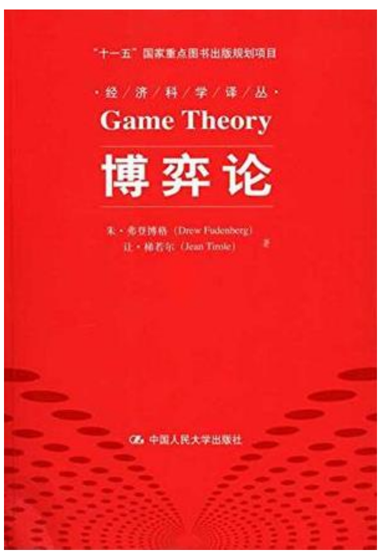
Strategies and Games: Theory and Applications

- Prajit K. Duta
- MIT Press 1999

Prediction, Learning and Games

- Nicolo Cesa-Bianchi and Gabor Lugosi
- MIT Press 1999



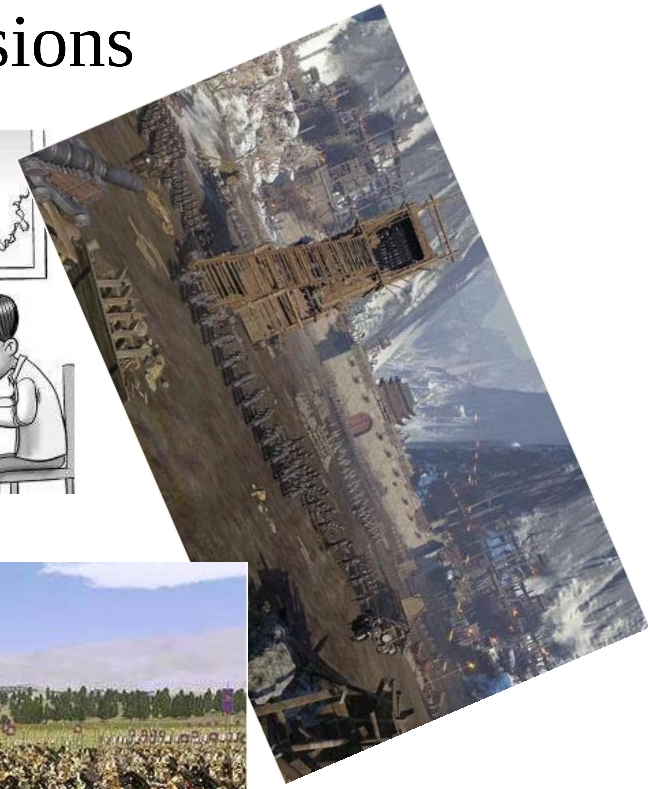


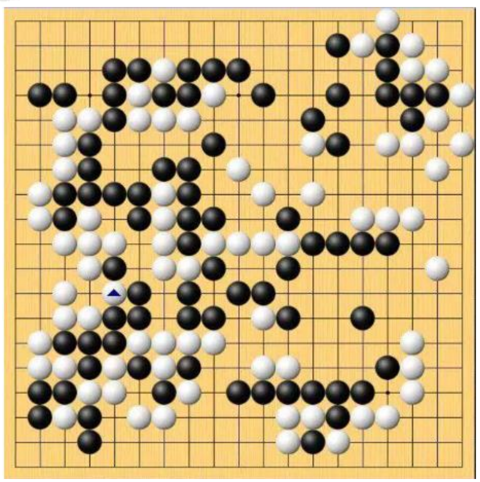
博弈论

- 人民大学出版社

What is Game

- A game: multi-person decisions/interacts, each outcome is affected by other and his own decisions





Key Elements for Game

- Players: Who is interacting (1, 2, multi-persons)
- Strategies/Decision: What are their options
- Payoff: What are their incentives
- Information: What do you know
- Rationality: How do you think

Two Players Strategy Game: Payoff Matrix

Player 2
(Column)

		Strategy C		Strategy D	
Player 1 (Row)	Strategy A	P11	P21	P12	P22
	Strategy B	P13	P23	P14	P24

Note

- The strategies A and B many be similar/different from C and D
- P1i and P2j may be different

An Example: Prisoners' Dilemma

Prisoner A		Prisoner B: Confess	Prisoner B: Don't confess
	Confess	-6 -6	0 -12
	Don't confess	-12 0	-1 -1

Prisoners' Dilemma： Prisoner A

		Prisoner A	
		Confess	Don't confess
Prisoner B	Confess	-6	-12
	Don't confess	0	-1

Prisoner A: choose 'confess'

Prisoners' Dilemma： Prisoner B

Prisoner B \ Prisoner A	Confess	Don't confess
Confess	-6	0
Don't confess	-12	-1

Prisoner B: choose 'confess'

Prisoners' Dilemma （cont.）

		Prisoner A	
		Confess	Don't confess
Prisoner B	Confess	$\boxed{-6 \quad -6}$	0 -12
	Don't confess	-12 0	-1 -1

Each single optimal decision is not global optimum

No-cooperative

Applications of Prisoners' Dilemma

- Lesson for military: consider the safety of two nations if they disarm (cooperate) or both heavily armed?
- Market Strategies: Two rival companies offer small discounts and retain a good market share, or offer huge discounts?
- Cooperation depend on morality, or the complicated dynamics of environment.

田忌赛马

齐威王 \ 田忌	上	中	下
上	1　-1	1　-1	1　-1
中	-1　1	1　-1	1　-1
下	-1　1	-1　1	1　-1

1：齐威王（上）vs 田忌（下）
2：齐威王（中）vs 田忌（上）
3：齐威王（下）vs 田忌（中）

Imperfect information
Random Strategies

What is Game Theory (博弈论)

- Game theory = Multi-person decision theory

- Game theory: **study of mathematical models of conflict and cooperation between intelligent rational decision-makers** (Wikipedia)

  - Game theory is highly mathematical
  - Game theory assumes all human interactions can be understood and navigated by presumptions
  - Abstraction of real complex situation
  - Finding acceptable, if not optimal, strategies in conflict situations

The Importance of Game Theory

- All intelligent beings make decisions all the time.
- AI needs to perform these tasks as a result.
- Help to analyze situations more rationally, and formulate an acceptable alternative with respect to circumstance.

Key Elements of Games Theory

- Player
- Strategy/Decision
- Payoff
- Information
- Rationality

Players

- A player is a decision maker and can be anything from individuals to entire nations.
- Players have the ability to choose among a set of possible actions.
- Games are often characterized by the fixed number of players.

Strategies

- A strategy is a set of actions available to a player.
- Strategies may be simple or complex.
- In non-cooperative games each player is uncertain about what the other will do since players can not reach agreements among themselves.

Payoffs

- Payoffs are the final returns to the players at the conclusion of the game.
- Payoffs are usually measure in utility although sometimes measure monetarily.
- In general, players are able to rank the payoffs from most preferred to least preferred.
- Players seek the highest payoff available.

Information

- Various rules in game
- The set of strategies for each players
- The payoff matrix
- All information about the game

Rationality

Assumptions:

- humans are rational beings
- humans always seek the best alternative in a set of possible strategies
-

Why assume rationality?

- narrow down the range of possibilities
- predictability

History of Game Theory: Milestone I

John Von Neumann (mathematician)

Oskar Morgenstern (economist)

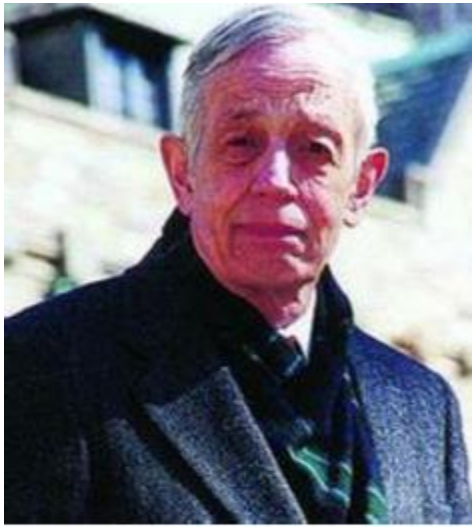
"Theory of game and economic Behavior" Princeton University Press 1944

① Mathematical method to analyze games

② A new scientific approach to the study of economics

History of Game Theory: Milestone II

John Forbes Nash (1928-2015)



Main contribution: Nash Equilibrium

1) In non-cooperative games,

Neither player has an incentive to change strategy, given the other player's choice

2) Proof of the existence of Nash Equilibrium

Nash Equilibrium of Prisoners' Dilemma

		Prisoner A	
		Confess	Don't confess
Prisoner B	Confess	$\boxed{-6 \quad -6}$	0 -12
	Don't confess	-12 0	-1 -1

History of Game Theory: Prosperity

- Wide applications (after 1950s): economics, computer science, artificial intelligence…

- Nobel Prize in Economics
  - 1994, Nash, Selten and Harsanyi
  - 2005, Thomas Schelling and Robert Aumann
  - 2007, Leonid Hurwicz, Eric Maskin and Roger Myerson
  - 2012, Alvin E. Roth and Lloyd S. Shapley
  - 2014, Jean Tirole

Types of Games

- # of players:
  - 1, 2, multi-persons games
- Orders of players, time and repeat
  - Simultaneous and sequential
- Payoff
  - Zero sum and non-zero sum

Types of Games (cont.)

- Information
  - Perfect information and imperfect information
- Rationality
  - Cooperative or non-cooperative
- Strategies/Decision
  - Finite and infinite strategies
- …

1-Person Game

	T. Cost	wicked w.	p. of w. w.
land	1000	200	5%
waterway	600	1200	20%

How to choose?

Expected expense of land = 1000+200*5% =1010
Expected expense of waterway = 600+1200*20% =840

How about only one time?

2-Persons Game: Simple Nim

- Rule
  - Two players carry coins in turn
  - A player remove exactly 1 or 2 coins/turn
  - The winner is the one taking the last coin.

Lemma: Suppose that player A and B are playing the simple Nim game, where at each round, a player can remove between 1 and k coins, then a player has a winning strategy if he can take coins so as to leave i(k+1) coins.

Proof by induction I: For i=1, A leaves k+1 coins, then B selects x coins ($1\leq x\leq k$). A takes the leaves and wins.

2-Persons Game: Simple Nim

Proof by induction II. Assume the statement is true for $i=n$ i.e., if A leaves $n(k+1)$ coins, then A wins.

Suppose A leaves $(n+1)(k+1)$ coins. If B select x: $1 \leq x \leq k$, then A selects $k+1-x$, and leaves $n(k+1)$. By induction, A wins.

This lemma holds by induction for all i

Multiple-Persons Game: Pirate Game

Five Pirates A > B > C > D > E have 1000 gold coins, and decide how to distribute them

Pirate Rules

- The most senior pirate first proposes a plan of distribution. All pirates vote on whether to accept this distribution
- If the majority (including tie vote) accepts the plan, the coins are dispersed and the game ends
- If the majority rejects the plan, the proposer is thrown overboard from the pirate ship and dies, and the next most senior pirate makes a new proposal to begin the system again
- The process repeats until a plan is accepted or if one pirate lefts

Multiple-Persons Game: Pirate Game (cont.)

Five Pirates A > B > C > D > E have 1000 gold coins, and decide how to distribute them

Four decision factors

- Each pirate wants to survive
- Each pirate tries to maximize the number of gold coins if survival
- Each pirate would prefer to throw another overboard, if all other results would be equal
- The pirates do not trust each other, no cooperation

How to play?

-Average distributed

Multiple-Persons Game: Pirate Game (cont.)

- For D and E

  decisions: D:1000 E:0

- For C, D and E

  decisions: C: 999 D:0 E:1

- For B, C, D and E

  decisions: B:999 C:0 D:1 E:0

- For A, B, C, D and E

  decisions: A:998 B:0 C:1 D:0 E:1

Cooperative vs Non-Cooperative Game

Cooperation often leads to higher payoffs

- Prisoners' Dilemma

		Prisoner A	
		Confess	Don't confess
Prisoner B	Confess	-6 -6	0 -12
	Don't confess	-12 0	-1 -1

- More examples
  - Countries cooperation on trade
  - Cartel: formation of monopoly by multiple organizations

Zero vs Non-Zero Sum Game

- Zero-Sum game: the total payoff among players is zero, i.e., neither create nor destroy in playing game

Rock-Paper-Scissors

		Player 2 Rock	Paper	Scissors
Player 1	Rock	0 0	-1 1	1 -1
	Paper	1 -1	0 0	-1 1
	Scissors	-1 1	1 -1	0 0

Many zero-sum games in our daily lives

- War, resources, trade …

Zero vs Non-Zero Sum Game

- Non-Zero-Sum game: the total payoff among players is not zero, may increase or decrease in playing game

Battle of sexes
性别战

		Man: Boxing		Man: Ballet	
Woman	Boxing	2	3	0	0
Woman	Ballet	1	1	3	2

Most real-life games are non-zero-sum:

- China-vs-American trade
- Create an organization/company
- …

Simultaneous and Sequential Game

- Simultaneous Game: make actions simultaneously

  Rock-Paper-Scissors

  Battle of sexes

- Sequential/Dynamic Game: make actions one by one

Simultaneous and Sequential Game (cont.)

- Simultaneous Game: Payoff matrices

Battle of sexes

		Man			
		Boxing		Ballet	
Woman	Boxing	2	3	0	0
	Ballet	0	0	3	2

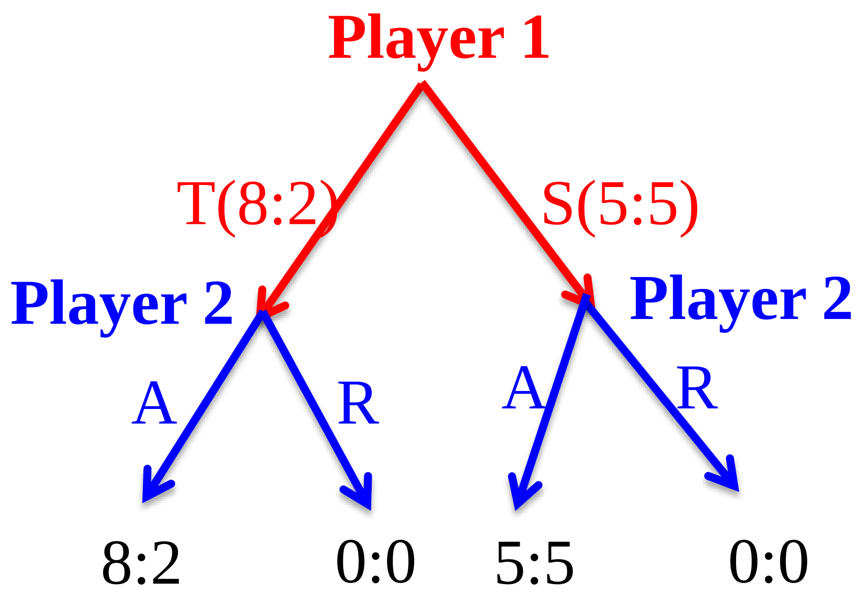
- Sequential/Dynamic game: tree

Perfect vs Imperfect Information

Sequential games

Perfect information game: all players know the actions previously made by all other players

Ultimatum Game
最后通牒博弈



Perfect vs Imperfect Information (cont.)

Sequential games

Imperfect information game: New players do not know some actions previously made by other players

Applications of Game Theory

- **Mathematics**
- **Computer Science**
- **Economics**
- **Biology**
- Political Science
- International Relations
- Philosophy
- Psychology
- Law
- **War**
- **Management**
- Sport
- **Game playing**

Limitations & Problems

- Assumes players always maximize their outcomes
- Some outcomes are difficult to provide a utility
- Not all of the payoffs can be quantified
- Not applicable to all problems

Contents

- Strategic game with perfect information
- Strategic game with imperfect information
- Extensive game with perfect information
- Extensive game with imperfect information
- Repeated game

Chapter

1. Introduction
2. Strategy Game and Nash Equilibrium
3. Mixed Strategy Game and Nash Equilibrium
4. Dominant Strategy Equilibrium and Rationality
5. Complexity and Computation of Finding Nash Equilibria
6. Applications I
7. Zero-Sum Game
8. Strategy Game with Incomplete Information
9. Extensive Game

Chapter

考核方式

- Home work: 20% (4-6次作业)
- Mid-Term exam: 20% (平时作业中两次最高分)
- Final exam: 60%

Preliminary Courses

- Calculus
- Linear algebra
- Probability

Exercises

- Let $\{a_n\}$ be a sequence of positive real number. Denote by $S_n = \sum_{i=1}^{n} a_i$. If $S_{n+1} \geq 2S_n$, then there exists a constant $c > 0$, such that $a_n \geq 2^n c$ for every positive n.

- Suppose that $(1,1,-1)$ is an eigenvector of matrix

$$\begin{bmatrix} 2 & -1 & b \\ 5 & a & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

  Solve $a, b,$ and the corresponding eigenvalue.

- For $\epsilon \in [0,1]$, prove that

$$\frac{1}{2}\left(1 + \sqrt{1 + 4\epsilon^2}\right) e^{1-\sqrt{1+4\epsilon^2}} \leq \mathrm{e}^{-(\epsilon^2-\epsilon^3)/2}$$