http://courses.csail.mit.edu/6.006

Course Overview

- Efficient: procedures for solving scale problems
- Scalability
- classic data structures
- Real implementation in Python.

Content

- 8 modules

  Algorithmic thinking, peak finding,
  Sorting & trees: Event Simulation
  Hashing: Genome Comparisson
  Numerics: RSA encryption
  Graphs: Rubik's cube
  shortest paths: Caltech → MIT
  Dynamic programming: Image compression.
  Advanced topics:

Peak finder

- One dimentional version

a	b	c	d	e	f	g	h	i
1	2	3	4	5	6	7	8	9

a ~ i are numbers

Position 2 is a peak if and only if $b \geq a$ and $b \geq c$

Position 9 is a peak if $i \geq h$

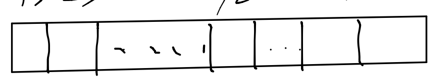
<u>Problem</u>: Find <u>a</u> peak if it exists.

<u>Argument</u>: Any array will have a peak

- straight forward algorithm

Start from left, go all the way to the right

$1, 2, \ldots \; n/2 \ldots n-1, \; n$

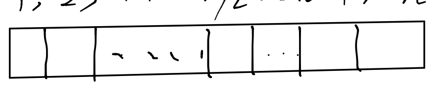


Worst-case Complexity : $\Theta(n)$ (on the order of n, a const. times n)

Asymptotic Complexity of this problem is linear

- Divide and Conquer

$1, 2, \ldots \; n/2 \ldots n-1, \; n$



If $a[n/2] < a[n/2 - 1]$ then only look at left half

$1, 2, \ldots \; n/2 - 1$ to look for a peak

Else if $a[n/2] < a[n/2 + 1]$ then look at $n/2 + 1, \ldots n$

to look for a peak

Else $a[n/2]$ is a peak

$$T(n) = T(n/2) + \Theta(1)$$

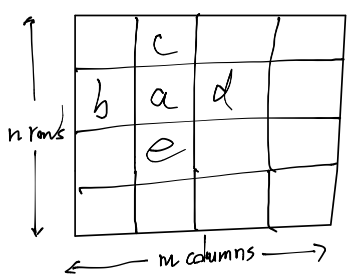
"Work" algorithm does on input of size n

Base case $T(1) = \Theta(1)$

$$T(n) = \Theta(1) + \Theta(1) + \cdots + \Theta(1) = \Theta(\log_2 n)$$

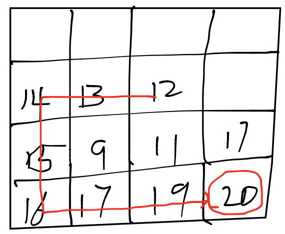
★ Can not do better for 1D version

□ 2D Version



a is a 2D peak if and only if $a \geq b$, $a \geq d$, $a \geq c$, $a \geq e$

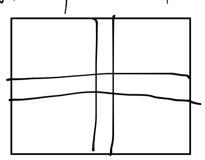
- Greedy ascent algorithm



Worst Complexity $\Theta(nm)$, $\Theta(n^2)$ if $m = n$

- Divide and Conquer

~~Attempt~~ #1



pick middle column $j = m/2$, find a 1D pick at (i, j)

use (i, j) as a start to find a 1D-peak on row i

$\boxed{\text{Incorrect}}$ 2D-peak may not exist on row i

~~Attempt~~ #2

- pick middle column $j = m/2$
- Find global maximum on column j at (i, j)

Compare $(i, j-1)$, (i, j), $(i, j+1)$

Pick left cols if $(i, j-1) > (i, j)$. similarly for right if $(i,j) \geq (i,j-1), (i,j+1)$, (i,j) is a 2D peak.

- Solve ~~the~~ new problem with half the number of columns
- when have a single column, find the global max $\Rightarrow$ done

$$T(n,m) = T(n, m/2) + \Theta(n)$$

$$T(n,1) = \Theta(n)$$

$$T(n,m) = \Theta(n) + \Theta(n) + \cdots \Theta(n) = \Theta(n \log_2 m)$$