Speed up Dijkstra

Shortest Path Problems:

- Single source, any / all destination problem
- Single source, single target
- All pairs of shortest path

☐ Recall Dijkstra

Initialize( ) ← $d[s]=0$, $d[u \neq s] = \infty$

$Q \leftarrow V[G]$

while $Q \neq \phi$

do $u \leftarrow$ EXTRACT_MIN(Q)

for each vertex $v \in Adj[u]$

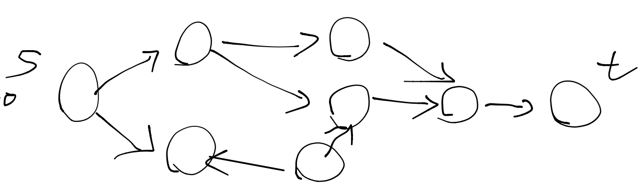
do RELAX(u, v, w)

$\pi[v] \leftarrow u$, $d[v] \downarrow$

- Single target: stop if $u = t$

☐ Bi-directional Search

$d_f[s]=0$



$d_b[t] = 0$

Alternate forward search from s

backward search from t (following edges backwards)

π_f : normal π_b : follow edge back

$d_f[u]$: distances of forward search
$d_b[u]$: .... backward search
priority Queue: Q_f : forward
Q_b : backward

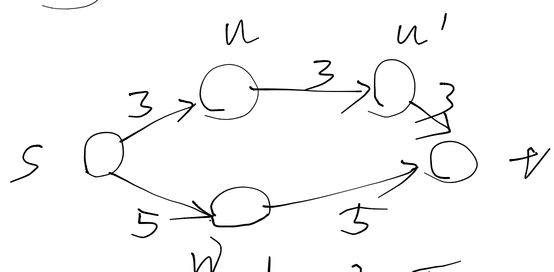
Question: What is the termination condition?
Answer: Some vertex u has been processed both in the forward search & backward search (deleted from both Q_f, Q_b)

Question: how do we find the shortest path $s \to t$?
Claim: If w was processed first from both Q_f, Q_t
① find S.P. using π_f from s to w
② find S.P. using π_b from t to w (backwards)

① Apply $\pi_f[w]$, $\pi_f \pi_f[w]$,
② Apply $\pi_b[w]$, $\pi_b \pi_b[w]$

w may not on the Shorst Path (S.P.)

Backward:



$d_f[w] = 5$

$d_b[u'] = 3 \qquad d_b[w] = 5 \qquad d_b[t] = 0$

find an x (possibly diff. from w) has minimum value of $d_f[x] + d_b[x]$

□ Goal-Directed Search

Modify edge weights with potential function

$\bar{w}(u, v) = w(u, v) - \lambda(u) + \lambda(v)$

Correctness

$$\bar{w}(p) = w(p) - \lambda_t(s) + \lambda_t(t)$$

landmark $l \in V$

Precompute: $\delta(u, l)$

$\lambda_t(u) = \delta(u, l) - \delta(t, l)$