Triangular grids

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Half-rectangular grids

An $m \times n$ half-rectangular-grid is the set of non-negative integer points (x, y) such that $0 \leq x \leq n-1$ and $0 \leq y \leq m-1$ and $(m-1)x \leq (n-1)y$.

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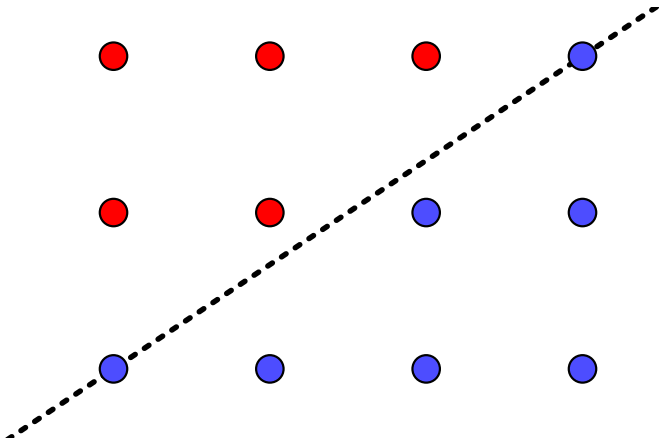

Covering while omitting a point

Theorem

Let G be an $m \times n$ half-rectangular grid, and let $P = (x_0, y_0) \in G$ be any point. The minimum number of lines required to cover $G \setminus P$ without covering P is $n - \lceil \frac{n-m}{m-1} y_0 \rceil - 1$.

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As a special case, in case of $m = n$, the minimum number of lines to cover $G \setminus P$ without covering P is $n - 1$, regardless of position of P.

Covering with multiplicity

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Theorem (Basit, Clifton, Horn (2023))

Number of lines required to cover an $n \times n$ half-rectangular grid with multiplicity k is greater than $\frac{2}{3}nk$

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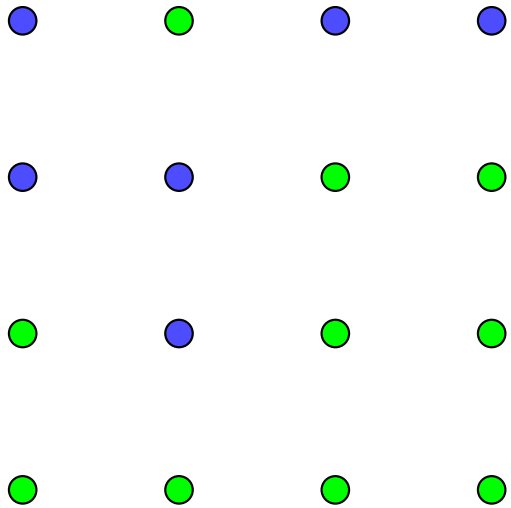

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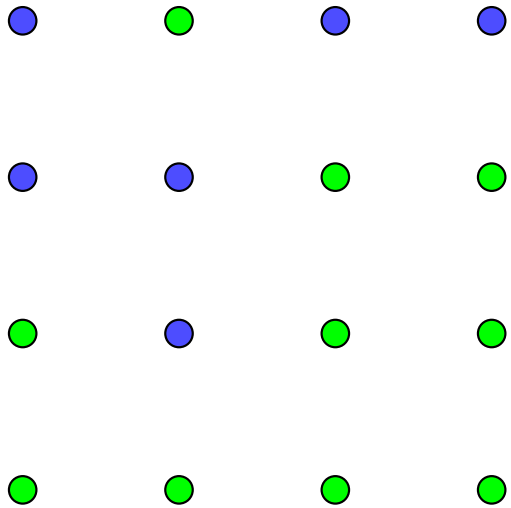

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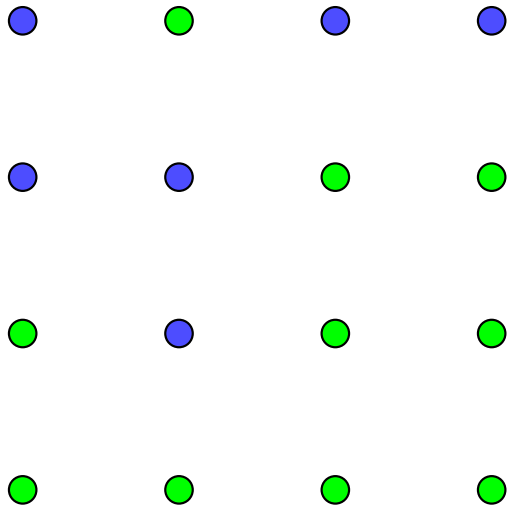

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Theorem

Number of lines required to cover any general triangular grid with multiplicity k is at least $nk\left(1 - e^{\frac{1}{2n}-1} - \frac{2}{n}\right) \geq nk\left(1 - \frac{1}{e} - O\left(\frac{1}{n}\right)\right)$.

Some Open Problems

1. Is the constant $1 - \frac{1}{e}$ in the previous theorem optimal?

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1. Is the constant $1 - \frac{1}{e}$ in the previous theorem optimal?
2. What is a good lower bound on covering $m \times n$ half-rectangular grids with multiplicity?