4190.407 Algorithms HW 1-2

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In this assignment, I implemented 3 different multiplication algorithms: traditional grade school, Karatsuba and Toom-Cook (Toom-3). The time complexity with respect to the integer size N, is:

- $O(N^2)$ for the grade school algorithm.
- $O\left(N^{\log 3}\right) \simeq O(N^{1.58})$ for the Karatsuba algorithm.
- $O\left(N^{\frac{\log 5}{\log 2}}\right) \simeq O(N^{1.46})$ for the Toom-3 algorithm.

Implementation

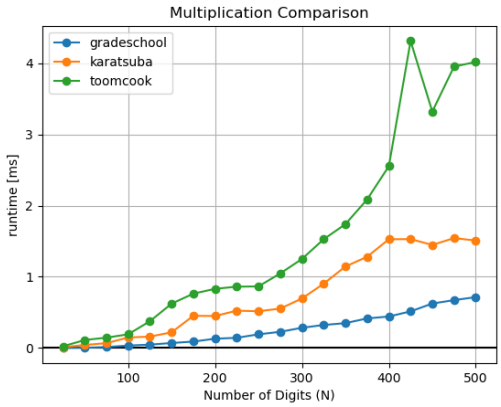
I used C++ to implement the integer class (the `Bigint` class) and the three multiplication algorithms. I used Python for test case generation, algorithm testing and plotting. For detailed description about the code structure, please refer to README.md inside the directory.

How to build and run

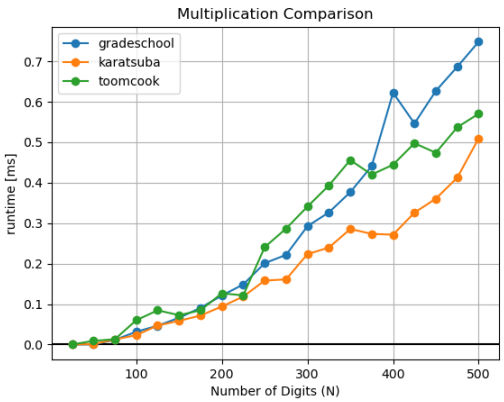
- The C++ program is located in ./build/ directory. You can compile the source code by running **`$./build.sh`** (in Linux environment).
- You can run automatic testing by running **`$ python3 ./test.py`**. This includes random testcase generation, testing and comparing runtime for 3 algorithms.
- You can also test for specific testcases by manually invoking the C++ program:
  `$ ./build/run_algo [gradeschool/karatsuba/toomcook] [INPUT FILE PATH] [OUTPUT FILE PATH] [TIME MEASUREMENT FILE PATH]`

Test Results

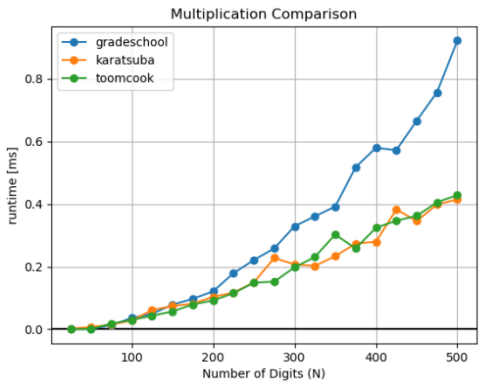
N	Sample Input	Output
25	4791403878675630055387839 1179428639403940381365630	5651118957461160988722566349069706405578114573570
100	5068182314676529060433912922911326765600131335789577169451983026156890832228946064028472175305849231 8976973205401268376773733200352147797585561595003706868298309344033776473020381254002077210190615777	2398498778186053908890875268501688152752610550908936201901825833848188424982533397204866169263917724008214221933946122935348447048265328124478035800262517419111347636448873271840268470190699182971962127611598949406248200101228320507454455021717988308971642521776132278582411658944707811401284065207



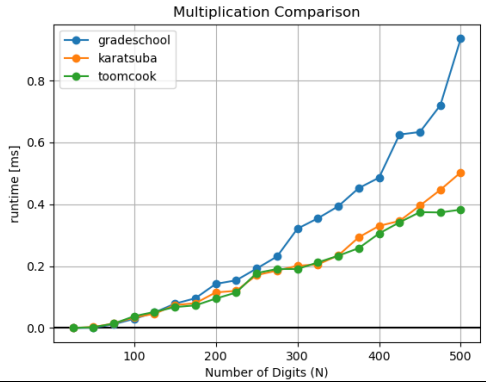
(a) BASECASE_N = 5



(b) BASECASE_N = 30



(c) BASECASE_N = 70



(d) BASECASE_N = 150

While the time complexity of Karatsuba and Toom-Cook algorithms are superior to that of grade school algorithms, the large number of leaf nodes in recursive call makes the coefficient large enough to offset this advantage. So, in practice, we need to treat all Ns smaller than threshold value as base cases and use grade school algorithm to handle these cases.

Let's call the threshold value BASECASE_N. If BASECASE_N is small (in the case of (a)), grade school algorithm outperforms the other two algorithms. However, as BASECASE_N grows, The Karatsuba and Toom-Cook algorithm operate significantly faster (in the cases of (b) and (c)). Especially, we can observe that Toom-Cook is slightly faster than Karatsuba at $N = 150$ (in the case of (d)).