Wir sehen $l_1 @ l_2 = \text{append } l_1\ l_2$ nach Definition, somit gilt Lemma 2 auch für äquivalente @-Aufrufe

1) I. Anf: $l = []$:

$\text{reverse}\ (\text{fold\_left}\ (\text{fun } a\ x \to f x :: a)\ [a]\ []) \overset{\text{Def. fold}}{=} \text{reverse}\ [a]$

$\overset{\text{Lemma 1}}{=} \text{reverse}\ [a] @ [] \overset{\text{Def. map}}{=} \text{reverse}\ [a] @ (\text{map } f\ [])$

I. Schritt: $l = xs \Rightarrow l = x::xs$

I. Schluss: $\text{reverse}(\text{fold\_left}\ (\text{fun } a x \to f x :: a)\ [a]\ (x::xs))$

$\overset{\text{Def. fold}}{=} \text{reverse}(\text{———} \quad '' \quad \text{———})\ [f x :: a]\ xs)$

$\overset{\text{I. Annahme}}{=} \text{reverse}\ [f x :: a] @ (\text{map } f\ xs)$

$\overset{\text{Def. reverse}}{=} (\text{reverse}\ [a] @ [f x]) @ (\text{map } f\ xs)$

$\overset{\text{Lemma 2}}{=} \text{reverse}\ [a] @ ([f x] @ (\text{map } f\ xs))$

$\overset{\text{Def. @}}{=} \text{reverse}\ [a] @ (f x :: ([] @ (\text{map } f\ xs)))$

$\overset{\text{Def @}}{=} \text{reverse}\ [a] @ (f x :: \text{map } f\ xs)$

$\overset{\text{Def. map}}{=} \text{reverse}\ [a] @ (\text{map } f\ (x::xs))$ □

2) $\text{reverse}\ (\text{fold\_left}\ (\text{fun } a x \to f x :: a)\ []\ l)$

$\overset{1)}{=} \text{reverse}\ [] @ \text{map } f\ l \overset{\text{Def. rev.}}{=} [] @ \text{map } f\ l \overset{\text{Def. @}}{=} \text{map } f\ l$ □

3) I. Anf: $l = []$:

$\text{fold\_left}\ (\text{fun } a x \to x+a)\ i\ [] \overset{\text{Def. fold}}{=} i \overset{\text{Def. sum}}{=} \text{sum } i\ []$

I. Schritt: $l = xs \Rightarrow l = x::xs$

I. Schluss: $\text{fold\_left}\ (\text{fun } a x \to x+a)\ i\ (x::xs)$

$\overset{\text{Def. fold}}{=} \text{fold\_left}\ (\text{fun } a x \to x+a)\ (x+i)\ xs$

Def. sum

$\overset{\text{I. Ann.}}{=} \text{sum } (x+i)\ xs = \text{sum } i\ (x::xs)$ □