

极限和积分部分公式加强

Chunwei Yan

2012 年 10 月 17 日

1 极限

1.1 几个重要的等价无穷小

$x \rightarrow 0$

$\sin x \sim$	_____	$(1+x)^{\frac{1}{x}} =$	_____
$\sin x \sim$	_____	$\tan x \sim$	_____
$1 - \cos x \sim$	_____	$e^x - 1 \sim$	_____
$\ln 1 + x \sim$	_____	$(1+x)^a - 1 \sim$	_____
$\arcsin x \sim$	_____	$\arctan x \sim$	_____
$a^x - 1 \sim$	_____	$x^m + x^k (k > m > 0) \sim$	_____

几个极限

$\lim_{n \rightarrow \infty} \sqrt[n]{n} =$	_____	$\lim_{x \rightarrow 0^+} x^\delta (\ln x)^k =$	_____
$\lim_{x \rightarrow +\infty} x^k e^{-\delta x}$	_____	(常数 $k > 0$ $\delta > 0$)	

2 初等函数的导数公式

复习全书 P41 公式 2

$C' =$	_____	$\int 0dx =$	_____
$(x^a)' =$	_____	$\int ax^{a-1}dx =$	_____
$(a^x)' =$	_____	$\int a^x \ln a dx =$	_____
$(e^x)' =$	_____	$\int e^x dx =$	_____
$(\log ax)' =$	_____	$\int \frac{1}{x \ln a} dx =$	_____
$(\ln x)' =$	_____	$\int \frac{1}{x} dx =$	_____
$(\sin x)' =$	_____	$\int \cos x dx =$	_____
$(\cos x)' =$	_____	$\int -\sin x dx =$	_____
$(\tan x)' =$	_____	$\int \sec^2 x dx =$	_____
$\cot x' =$	_____	$-\int \csc^2 x dx =$	_____
$(\sec x)' =$	_____	$\int \sec x \tan x dx =$	_____
$(\csc x)' =$	_____	$-\int \csc x \cot x dx =$	_____
$(\arcsin x)' =$	_____	$\int \frac{1}{\sqrt{1-x^2}} dx =$	_____
$(\arccos x)' =$	_____	$-\int \frac{1}{\sqrt{1-x^2}} dx =$	_____
$(\arctan x)' =$	_____	$\int \frac{1}{1+x^2} dx =$	_____
$(\operatorname{arccot} x)' =$	_____	$-\int \frac{1}{1+x^2} dx =$	_____
$\int \frac{1}{\cos^2 x} dx =$	_____	$-\int \frac{1}{\sin^2 x} dx =$	_____

2.1 几个常见初等函数的 n 阶导数公式

复习全书 P42 (5)

$$\begin{aligned}
 (e^{ax})^{(n)} &= \underline{\hspace{2cm}} & \ln^{(n)}(1+x) &= \underline{\hspace{2cm}} \\
 (\sin ax)^{(n)} &= \underline{\hspace{2cm}} & ((1+x)^\alpha)^{(n)} &= \underline{\hspace{2cm}} \\
 (\cos ax)^{(n)} &= \underline{\hspace{2cm}}
 \end{aligned}$$

2.2 幂指数函数 $u(x)^{v(x)}$ 的求导法则与公式

复习全书 P42 (7)

$$u(x)^{v(x)} \underline{\hspace{2cm}}$$

2.3 反函数的一阶及二阶导数公式

复习全书 P42 (8)

设 $y = f(x)$ 可导, 且 $f'(x) \neq 0$, 则存在反函数 $x = \varphi(y)$, 且

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$

即

$$\varphi'(y) = \frac{1}{f'(x)}$$

若 $y = f(x)$ 存在二阶导数, 则

$$\varphi''(y) = \underline{\hspace{2cm}}$$

3 不定积分与定积分的计算

3.1 基本积分公式

$$\begin{array}{ll} \int x^\alpha dx = \underline{\hspace{2cm}} & \int \frac{1}{x} dx = \underline{\hspace{2cm}} \\ \int a^x dx = \underline{\hspace{2cm}} & \int e^x dx = \underline{\hspace{2cm}} \\ \int \sin x dx = \underline{\hspace{2cm}} & \int \cos x dx = \underline{\hspace{2cm}} \\ \int \tan x dx = \underline{\hspace{2cm}} & \int \cot x dx = \underline{\hspace{2cm}} \\ \int \sec x dx = \underline{\hspace{2cm}} & \int \csc x dx = \underline{\hspace{2cm}} \\ \int \sec^2 x dx = \underline{\hspace{2cm}} & \int \csc^2 x dx = \underline{\hspace{2cm}} \\ \int \frac{1}{a^2+x^2} dx = \underline{\hspace{2cm}} & \int \frac{1}{a^2-x^2} dx = \underline{\hspace{2cm}} \\ \int \frac{1}{\sqrt{a^2-x^2}} dx = \underline{\hspace{2cm}} & \int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \underline{\hspace{2cm}} \end{array}$$

4 基本积分方法

4.1 不定积分的凑微分求积分法

复习全书 P89 二

$$\begin{array}{ll} \int f(ax+b)dx = \underline{\hspace{2cm}} & \int f(\cos x) \sin x dx = \underline{\hspace{2cm}} \\ \int f(ax^2+bx+c)(2ax+b)dx = \underline{\hspace{2cm}} & \int f(\tan x) \sec^2 x dx = \underline{\hspace{2cm}} \\ \int f(\ln x) \frac{dx}{x} = \underline{\hspace{2cm}} & \int f(\arcsin x) \frac{dx}{\sqrt{1-x^2}} = \underline{\hspace{2cm}} \\ \int f(\sqrt{x}) \frac{dx}{\sqrt{x}} = \underline{\hspace{2cm}} & \int f(\arctan x) \frac{dx}{1+x^2} = \underline{\hspace{2cm}} \\ \int f(\sin x) \cos x dx = \underline{\hspace{2cm}} & \end{array}$$