Statistical Inference I

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Lecture Notes 7

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1 Quantative Description of Poisson Random Variable

Q: Please quantitatively describe the Poisson random variable.

- It's a **counting process**. That is, N(t) that counts the number of appearances before time t.
- (Boundary condition) N(0) = 0
- (Stationary) $\forall t_1 < t_2, N(t_2) N(t_1) \sim N(t_2 t_1)$
- (Independence) $\forall t_1 < t_2 < t_3 < t_4, \ N(t_4) N(t_3) \sim N(t_2) N(t_1)$
- $\bullet \ \ (\textbf{Fixed frequency}) \ \lim_{\Delta \to 0^+} \frac{Pr[N(\Delta) N(0) = 1]}{\Delta} = \lambda, \ \text{and} \ \lim_{\Delta \to 0^+} \frac{Pr[N(\Delta) N(0) > 1]}{\Delta} = 0$
- (Density function) $f_{\lambda}(t,k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \mathbf{1}_{\{k=0,1,2,\ldots\}}$