Statistical Inference I **Prof. Chin-Tsang Chiang**

Lecture Notes 7

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1 Quantative Description of Poisson Random Variable

Q: Please **quantitatively** describe the Poisson random variable.

- It's a **counting process**. That is, $N(t)$ that counts the number of appearances before time t.
- (**Boundary condition**) $N(0) = 0$
- (**Stationary**) $\forall t_1 < t_2$, $N(t_2) - N(t_1) \sim N(t_2 - t_1)$
- (**Independence**) $\forall t_1 < t_2 < t_3 < t_4$, $N(t_4) - N(t_3) \sim N(t_2) - N(t_1)$
- (**Fixed frequency**) $\lim_{\Delta \to 0^+} \frac{Pr[N(\Delta) - N(0) = 1]}{\Delta} = \lambda$, and $\lim_{\Delta \to 0^+} \frac{Pr[N(\Delta) - N(0) > 1]}{\Delta} = 0$
- (**Density function**) $f_\lambda(t, k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \mathbf{1}_{\{k=0,1,2,\ldots\}}$