Statistical Inference I	**Prof. Chin-Tsang Chiang**
Lecture Notes 6	
October 24, 2015	**Scribe: Wei-Chang Lee, Chi-Ning Chou**

T.B.D

0.1 Density function

Theorem 1 *$P(X = x) = F(x) - F(x^-)$, where $F(x^-) = \lim_{y \uparrow x} F(y)$.*

Proof: Since $P(X = x) = F(X \leq x) - F(X < x)$ and notice that $y \downarrow x$ then $\{X \leq y\} \downarrow \{X \leq x\}$ and $y \uparrow x$ then $\{X \leq y\} \uparrow \{X < x\}$,we have

$$P(X = x) = F(x) - F(x^-)$$

■

When X is a discrete random variable, $F(x)$ is a step function which jumps at possible outcome.

Theorem 2 *If F(x) is differentiable, there exists a unique probability density function f(x) such that $F(c) = \int_{-\infty}^{c} f(x)dx \ \forall c \in \mathcal{R}$.*

Proof: The result is quite trivial, just apply the Fundamental Theorem of Calculus, we get

$$F'(x) = f(x)$$

■

The question arises when considering the physical meaning of $f(x)$, is $f(x)$ a probability measure? Go back to the definition of differentiation, we have

$$F'(x) = \lim_{\triangle \to 0} \frac{F(x + \frac{\triangle}{2}) - F(x - \frac{\triangle}{2})}{\triangle} = \frac{P((x - \frac{\triangle}{2}, x + \frac{\triangle}{2}))}{\triangle} = \frac{Probability}{Interval}$$

$f(x)$ is not usually probability measure, it is of intensity/density sense.

0.2 Quantative Description of Poisson Random Variable

Q: Please **quantitatively** describe the Poisson random variable.

- It's a **counting process**. That is, $N(t)$ that counts the number of appearances before time t.
- (**Boundary condition**) $N(0) = 0$

- (**Stationary**) $\forall t_1 < t_2$, $N(t_2) - N(t_1) \sim N(t_2 - t_1)$
- (**Independence**) $\forall t_1 < t_2 < t_3 < t_4$, $N(t_4) - N(t_3) \sim N(t_2) - N(t_1)$
- (**Fixed frequency**) $\lim_{\Delta \to 0^+} \frac{Pr[N(\Delta) - N(0) = 1]}{\Delta} = \lambda$, and $\lim_{\Delta \to 0^+} \frac{Pr[N(\Delta) - N(0) > 1]}{\Delta} = 0$
- (**Density function**) $f_\lambda(t, k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \mathbf{1}_{\{k = 0, 1, 2, \ldots\}}$