Statistical Inference I	**Prof. Chin-Tsang Chiang**
Lecture Notes 14	
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0.1 Continuous distribution

0.1.1 Uniform distribution

In continuous regime, we define a uniform random variable on a close interval $[a, b]$, where $a < b$, and denote it as Uni(a, b). If $X \sim$Uni(a, b), then

- $f_X(x|a, b) = \frac{1}{b-a}$
- $\mathbb{E}[X|a, b] = \frac{a+b}{2}$
- $Var[X|a, b] = \frac{(b-a)^2}{12}$

0.1.2 Exponential family

Here, we define three highly related continuous random variables: exponential, Weibull, and gamma. We first write down their distribution respectively, then introduce their relationship and properties.

Exponential: Exponential random variable captures a single interleaving time of a Poisson process with frequency $1/\beta$. If $X \sim$exponential(β)

$$f_X(x|\beta) = \frac{1}{\beta}e^{-x/\beta}\mathbf{1}_{(0,\infty)}(x)$$

Weibull: If $Y = X^{1/\gamma}$, where $X \sim$exponential(β) and $\gamma > 0$, we say Y has a Weibull(β, γ) distribution.

$$f_Y(y|\beta, \gamma) = \frac{\gamma}{\beta}y^{\gamma-1}e^{-y^{\gamma}/\beta}\mathbf{1}_{(0,\infty)}(y)$$

In other words, Weibull random variable is a power transformed version of exponential random variable. And as $\gamma = 1$, the Weibull degenerates to exponential.
Gamma: Intuitively, gamma distribution captures the total interleaving time up to more than one appearances. We use two parameters α, β to define a gamma random variable and denote it as Gamma(α, β). If $X \sim$Gamma(α, β), then

$$f_X(x|\alpha, \beta) = \frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^{\alpha}}\mathbf{1}_{(0,\infty)}$$

As we mentioned earlier, these three distributions are highly related to Poisson distribution in the sense that they describe the waiting time of a counting process given the number of desired observations while Poisson distribution captures the number of appearances given the amount of observing time. The two aspects are just two side of a coin, and we can use the following equation to relate them all together: Let $\{N(t)\}$ be a counting process and T be the corresponding waiting time for a single event to happen. We have

$$\{T > t\} = \{N(t) = 0\}$$

If we write down the probability of each side and do some computation, we can derive a relationship between exponential distribution and Poisson distribution.