The Relationship Between Empirical Process and Gaussian Process: An Example in Kolmogrov-Smirov Test
Stochostic Process: Final Project

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Abstract

Kolmogrov-Smirov test is a famous non-parametric goodness of fitting test. The Kolmogrov statistics: $D_n = \sup_{x \in \mathcal{R}} |\hat{F}_n(x) - F(x)|$ is the central idea in this statistical test. D_n is a *distribution-free* statistics. The convergence of D_n provides us a way to see that whether a source is sampled from the guessing distribution. Moreover, since the probability distribution of D_n will converge to that of a Brownian Bridge, the confidence interval can be calculated.

A distribution-free statistics, the Kolmogrov statistics, of empirical distribution converging to the Brownian Bridge is so amazing that we further dig into the relationship between empirical process and Gaussian process. Looking forward to find some interesting behaviour among them.

Keywords: Kolmogrov-Smirov test, Empirical Process, Brownian Bridge, Gaussian Process

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References

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2. *The Hunger Games*, Suzanne Collins. 2008. Scholastic. U.S. https://sites.google.com/site/the74thhungergamesbyced/download-the-hunger-games-trilogy-e-book-txt-file

Appendix

The code of this project can be found on Github: https://github.com/jerrychou82/MCMC_Break_St
It's welcome to discuss the code with me!