A study note about classical mechanism

A predictable subtitle

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Contents

1 Two-body problem

All human things are subject to
decay, and when fate summons,
Monarchs must obey

Mac Flecknoe
John Dryden

1.1 An introduce of the problem

1.2 The lagrangian of two-body problem

Assume we have two particle with mass m_1 and m_2. At time $t = 0$, the initial positions of the particles are $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$. According to Newtons second law:

$$\boldsymbol{F}_1 \tag{1}$$

From gratational force we know

$$F = G\frac{m_1 m_2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|^2}$$

Hence the proentionel energy of the two-body system is

$$U(\boldsymbol{r}_1, \boldsymbol{r}_2) = -G\frac{m_1 m_2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|}$$

Here you can clearly see that $U(\boldsymbol{r}_1, \boldsymbol{r}_2)$ is depend on:

- the difference of $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$
- the magnitude of $U(\boldsymbol{r}_1, \boldsymbol{r}_2)$ or $U(\boldsymbol{r}_1, \boldsymbol{r}_2) = U(|\boldsymbol{r}_1 - \boldsymbol{r}_2|)$.

If we introduce a new variable, $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ where $\boldsymbol{r}$ is called *relative position*. The new variable maker U one more step easier to analyze. Which finally we have the potential energy for the tow body system is

$$U = U(r)$$

To conclude this section we write down the motion of two bodies with lagrangian formalism and we obtain

$$\mathcal{L} = \frac{1}{2}m_1\dot{r}^2 + \frac{1}{2}m_2\dot{r}_2^2 - U(r) \tag{2}$$

1.3 Reduced the problem to two single independent problems

$$\mathcal{L} = \frac{1}{2}M\dot{\boldsymbol{R}}^2 + (\frac{1}{2}\mu\dot{\boldsymbol{r}}^2 - U(r)) \tag{3}$$

where

$$\mathcal{L}_{cm} = \frac{1}{2} M \dot{\boldsymbol{R}}^2$$

$$\mathcal{L}_{rel} = \frac{1}{2} \mu \dot{\boldsymbol{r}}^2 - U(r)$$

1.4 The angulamentum

1.5 The euivalent one-dimentional problem