

Cryptologie symétrique 1/2

Damien Vergnaud

Sorbonne Université

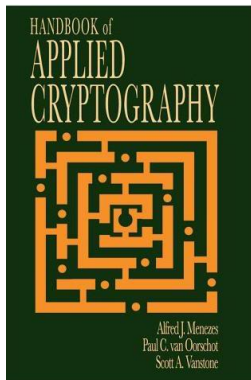
CRYPTO 1

References



Cryptographie : Théorie et pratique
Douglas Stinson

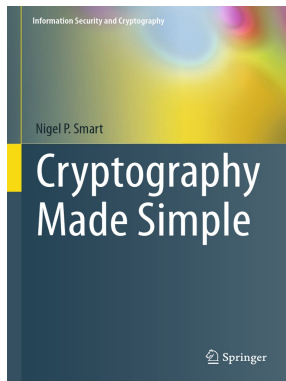
References



Handbook of Applied Cryptography

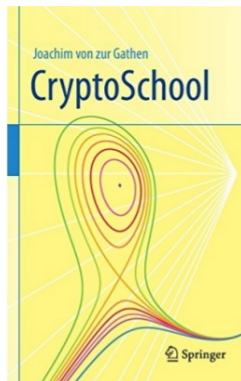
Alfred J. Menezes, Paul C. van Oorschot, Scott A. Vanstone

References

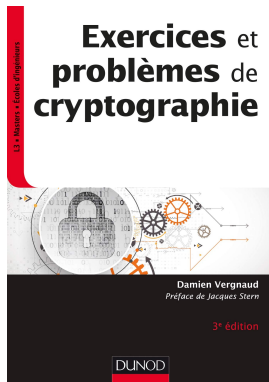


Cryptography Made Simple
Nigel P. Smart

References



CryptoSchool
Joachim von zur Gathen



Exercices et problèmes de cryptographie
Damien Vergnaud

Contents

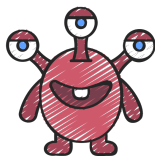
1 Introduction

- Security Objectives
- Terminology
- Kerckhoffs's principle
- One-Time Pad

2 Block Ciphers

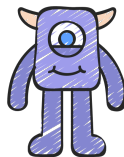
- Block ciphers - Definition
- Mode of operations
- Feistel Scheme
- Data Encryption Standard (DES)

Characters



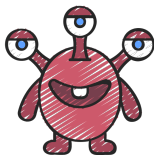
Alice

Characters



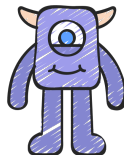
Bob

The basic goal: communication



Alice

internet, phone line, ...



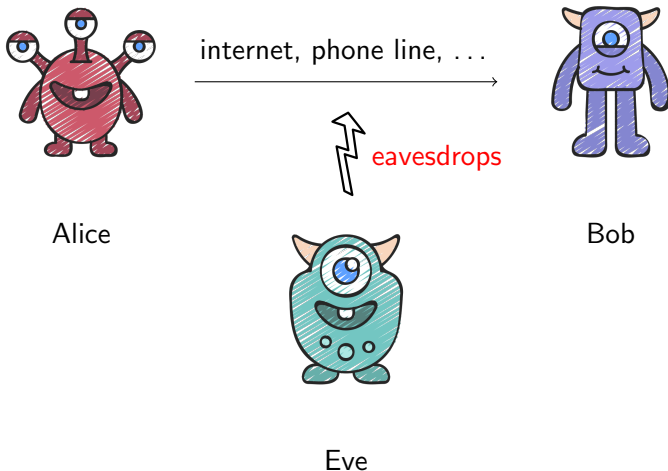
Bob

Another character



Eve

The basic goal: **secure** communication



Information security objectives

Cryptography = practice and study of **hiding information**

confidentiality	keeping information secret from all but those who are authorized to see it
integrity	ensuring information has not been altered by unauthorized or unknown means
message authentication	corroborating the source of information
signature	a means to bind information to an entity
receipt	acknowledgement that information has been received
anonymity	concealing the identity of an entity involved in some process
non-repudiation	preventing the denial of previous commitments or actions
<i>etc</i>	<i>etc</i>

Information security objectives

Cryptography = practice and study of **hiding information**

confidentiality	keeping information secret from all but those who are authorized to see it
integrity	ensuring information has not been altered by unauthorized or unknown means
message authentication	corroborating the source of information
signature	a means to bind information to an entity
receipt	acknowledgement that information has been received
anonymity	concealing the identity of an entity involved in some process
non-repudiation	preventing the denial of previous commitments or actions
<i>etc</i>	<i>etc</i>

Information security objectives

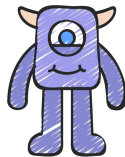
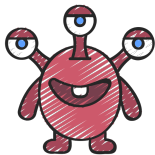
Cryptography = practice and study of **hiding information**

confidentiality	keeping information secret from all but those who are authorized to see it
integrity	ensuring information has not been altered by unauthorized or unknown means
message authentication	corroborating the source of information
signature	a means to bind information to an entity
receipt	acknowledgement that information has been received
anonymity	concealing the identity of an entity involved in some process
non-repudiation	preventing the denial of previous commitments or actions
<i>etc</i>	<i>etc</i>

Terminology

- Cryptography
- Cryptanalysis (Cryptanalyst)
- Cryptology
- Cipher/Encryption Algorithm
- Encryption/Encipherment
- Decryption/Decipherment
- Plaintext
- Ciphertext

Secret key encryption



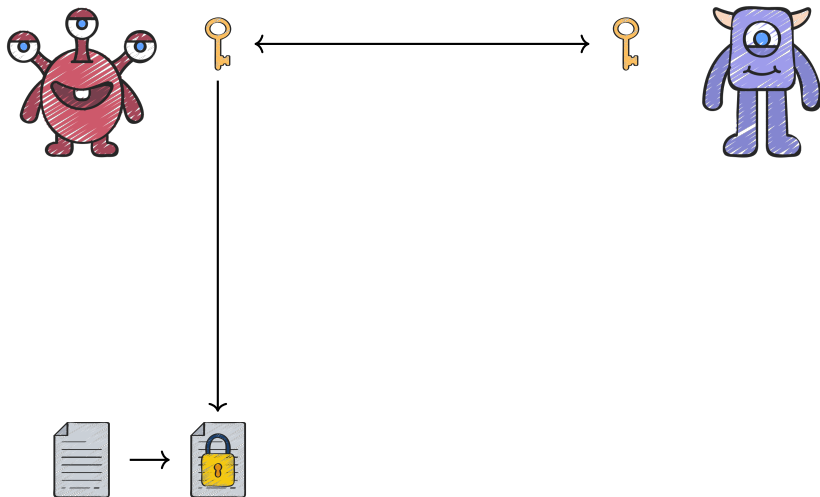
Secret key encryption



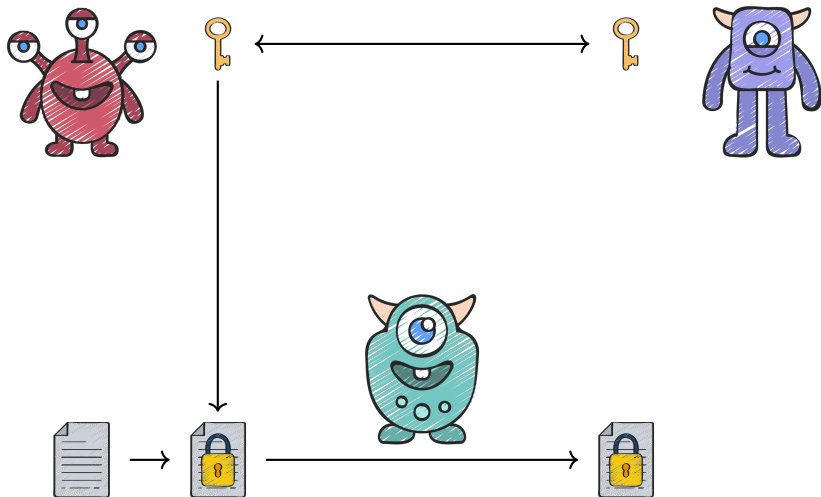
Secret key encryption



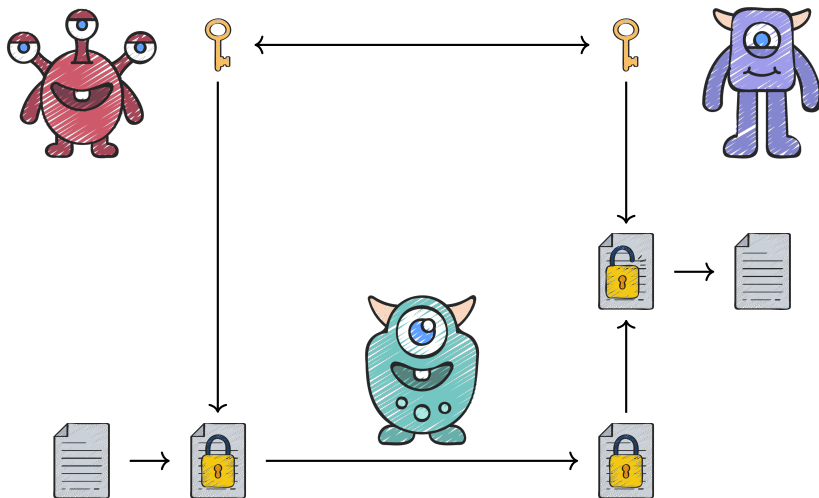
Secret key encryption



Secret key encryption



Secret key encryption



Kerckhoffs's principle

In 1883 Auguste Kerckhoffs wrote two journal articles on *La Cryptographie Militaire*:

- Le système doit être matériellement, sinon mathématiquement, indéchiffrable
 - **The system must be practically, if not mathematically, indecipherable**
- Il faut qu'il n'exige pas le secret, et qu'il puisse sans inconvénient tomber entre les mains de l'ennemi
 - It must not be required to be secret, and it must be able to fall into the hands of the enemy without inconvenience
- La clef doit pouvoir en être communiquée et retenue sans le secours de notes écrites, et être changée ou modifiée au gré des correspondants
 - Its key must be communicable and retainable without the help of written notes, and changeable or modifiable at the will of the correspondents

Kerckhoffs's principle

In 1883 Auguste Kerckhoffs wrote two journal articles on *La Cryptographie Militaire*:

- Le système doit être matériellement, sinon mathématiquement, indéchiffrable
 - **The system must be practically, if not mathematically, indecipherable**
- Il faut qu'il n'exige pas le secret, et qu'il puisse sans inconvénient tomber entre les mains de l'ennemi
 - **It must not be required to be secret, and it must be able to fall into the hands of the enemy without inconvenience**
- La clef doit pouvoir en être communiquée et retenue sans le secours de notes écrites, et être changée ou modifiée au gré des correspondants
 - Its key must be communicable and retainable without the help of written notes, and changeable or modifiable at the will of the correspondents

Kerckhoffs's principle

In 1883 Auguste Kerckhoffs wrote two journal articles on *La Cryptographie Militaire*:

- Le système doit être matériellement, sinon mathématiquement, indéchiffrable
 - **The system must be practically, if not mathematically, indecipherable**
- Il faut qu'il n'exige pas le secret, et qu'il puisse sans inconvénient tomber entre les mains de l'ennemi
 - **It must not be required to be secret, and it must be able to fall into the hands of the enemy without inconvenience**
- La clef doit pouvoir en être communiquée et retenue sans le secours de notes écrites, et être changée ou modifiée au gré des correspondants
 - **Its key must be communicable and retainable without the help of written notes, and changeable or modifiable at the will of the correspondents**

Kerckhoffs's principle

- Il faut qu'il soit applicable à la correspondance télégraphique
 - **It must be applicable to telegraphic correspondence**
- Il faut qu'il soit portable, et que son maniement ou son fonctionnement n'exige pas le concours de plusieurs personnes
 - **It must be portable, and its usage and function must not require the concurrence of several people**
- Enfin, il est nécessaire, vu les circonstances qui en commandent l'application, que le système soit d'un usage facile, ne demandant ni tension d'esprit, ni la connaissance d'une longue série de règles à observer
 - **Finally, it is necessary, given the circumstances that command its application, that the system be easy to use, requiring neither mental strain nor the knowledge of a long series of rules to observe**

Kerckhoffs's principle

- Il faut qu'il soit applicable à la correspondance télégraphique
 - **It must be applicable to telegraphic correspondence**
- Il faut qu'il soit portable, et que son maniement ou son fonctionnement n'exige pas le concours de plusieurs personnes
 - **It must be portable, and its usage and function must not require the concurrence of several people**
- Enfin, il est nécessaire, vu les circonstances qui en commandent l'application, que le système soit d'un usage facile, ne demandant ni tension d'esprit, ni la connaissance d'une longue série de règles à observer
 - Finally, it is necessary, given the circumstances that command its application, that the system be easy to use, requiring neither mental strain nor the knowledge of a long series of rules to observe

Kerckhoffs's principle

- Il faut qu'il soit applicable à la correspondance télégraphique
 - **It must be applicable to telegraphic correspondence**
- Il faut qu'il soit portable, et que son maniement ou son fonctionnement n'exige pas le concours de plusieurs personnes
 - **It must be portable, and its usage and function must not require the concurrence of several people**
- Enfin, il est nécessaire, vu les circonstances qui en commandent l'application, que le système soit d'un usage facile, ne demandant ni tension d'esprit, ni la connaissance d'une longue série de règles à observer
 - **Finally, it is necessary, given the circumstances that command its application, that the system be easy to use, requiring neither mental strain nor the knowledge of a long series of rules to observe**

Encryption: Security Notions

Encryption is supposed to provide confidentiality of the data.

But what exactly does this mean?

Security goal	But ...
Recovery of secret key is infeasible	True if data is sent in the clear
Obtaining plaintext from ciphertext is infeasible	Might be able to obtain half the plaintext
<i>etc</i>	<i>etc</i>

So what is a **secure** encryption scheme ?

Not an easy question to answer ...

Attackers should not be able to compute any information about m .

Encryption: Security Notions

Encryption is supposed to provide confidentiality of the data.

But what exactly does this mean?

Security goal	But ...
Recovery of secret key is infeasible	True if data is sent in the clear
Obtaining plaintext from ciphertext is infeasible	Might be able to obtain half the plaintext
<i>etc</i>	<i>etc</i>

So what is a **secure** encryption scheme ?

Not an easy question to answer ...

Attackers should not be able to compute any information about m .

How to formalize it ?

Attackers should not be able to compute any information about m .

Probabilistic approach

- M some random variable that takes values from \mathcal{M}
- K random variable distributed uniformly over \mathcal{K}
- $C = \mathcal{E}_K(M)$

Definition

An encryption scheme is perfectly secret if for every random variable M , every $m \in \mathcal{M}$ and every $c \in \mathcal{C}$ with $\Pr(C = c) > 0$:

$$\Pr(M = m) = \Pr(M = m | C = c)$$

$\leadsto C$ and M are independent

A perfectly secure scheme: **one-time pad**

Description

- $\ell \in \mathbb{N}$ a parameter. $\mathcal{M} = \mathcal{K} = \{0, 1\}^\ell$.
- Let \oplus denote component-wise XOR.
- **Vernam's cipher:** $\text{Enc}(K, m) = m \oplus K$ and $\text{Dec}(K, c) = c \oplus K$.



- One-time pad is **perfectly secret!**

$$\begin{aligned}\Pr(C = c | M = m) &= \Pr(K \oplus M = c | M = m) \\ &= \Pr(K = m \oplus c | M = m) = 2^{-\ell}\end{aligned}$$



- Each key cannot be used **more than once!**

$$\text{Enc}(K, m_0) \oplus \text{Enc}(K, m_1) = (m_0 \oplus K) \oplus (m_1 \oplus K) = m_0 \oplus m_1$$

- **optimal** in the class of perfectly secret schemes

Does encryption guarantee message integrity?

- **Idea:**

- Alice encrypts m and sends $c = \text{Enc}(K, m)$ to Bob.
- Bob computes $\text{Dec}(K, m)$, and if it “makes sense” accepts it.

- **Intuition:** only Alice knows K , so nobody else can produce a valid ciphertext.

It does not work!

Example

one-time pad.

Need a way to ensure that data arrives at destination in its original form (as sent by the sender and it is coming from an authenticated source)

Does encryption guarantee message integrity?

- **Idea:**

- Alice encrypts m and sends $c = \text{Enc}(K, m)$ to Bob.
- Bob computes $\text{Dec}(K, m)$, and if it “makes sense” accepts it.

- **Intuition:** only Alice knows K , so nobody else can produce a valid ciphertext.

It does not work!

Example

one-time pad.

Need a way to ensure that data arrives at destination in its original form (as sent by the sender and it is coming from an authenticated source)

Outline

1 Introduction

- Security Objectives
- Terminology
- Kerckhoffs's principle
- One-Time Pad

2 Block Ciphers

- Block ciphers - Definition
- Mode of operations
- Feistel Scheme
- Data Encryption Standard (DES)

Block ciphers

- **Block cipher**

- deterministic algorithm
- operates on fixed-length groups of bits, called **blocks**,
- an unvarying transformation specified by a **symmetric key**.

- **Design of block ciphers**

- based on the concept of an **iterated product cipher**
- suggested and analyzed by Claude Shannon
- by combining simple operations such as substitutions and permutations

- **Two main techniques**

- **Feistel network** or Feistel scheme
- **substitution-permutation networks** (SPN network)

Block ciphers

- **Block cipher**

- deterministic algorithm
- operates on fixed-length groups of bits, called **blocks**,
- an unvarying transformation specified by a **symmetric key**.

- **Design of block ciphers**

- based on the concept of an **iterated product cipher**
- suggested and analyzed by Claude Shannon
- by combining simple operations such as substitutions and permutations

- **Two main techniques**

- **Feistel network** or Feistel scheme
- **substitution-permutation networks** (SPN network)

Block ciphers

- **Block cipher**

- deterministic algorithm
- operates on fixed-length groups of bits, called **blocks**,
- an unvarying transformation specified by a **symmetric key**.

- **Design of block ciphers**

- based on the concept of an **iterated product cipher**
- suggested and analyzed by Claude Shannon
- by combining simple operations such as substitutions and permutations

- **Two main techniques**

- **Feistel network** or Feistel scheme
- **substitution-permutation networks** (SPN network)

Block ciphers - Definition

Problem: the plaintexts may be extremely long

~> hard to analyse security of the cipher.

Idea: Design ciphers that work on small blocks ...

two paired algorithms

- one for encryption \mathcal{E}
- one for decryption \mathcal{D}

that accept two inputs:

- an input block of size n bits
- a key of size k bits

$$\mathcal{E}_K(P) := \mathcal{E}(K, P) : \{0, 1\}^k \times \{0, 1\}^n \rightarrow \{0, 1\}^n$$

$$\mathcal{E}_K^{-1}(C) := \mathcal{D}_K(C) = \mathcal{D}(K, C) : \{0, 1\}^k \times \{0, 1\}^n \rightarrow \{0, 1\}^n$$

$$\boxed{\forall K, \forall P : \mathcal{D}_K(\mathcal{E}_K(P)) = P}$$

Block ciphers - Definition

Problem: the plaintexts may be extremely long

~> hard to analyse security of the cipher.

Idea: Design ciphers that work on small blocks ...

two paired algorithms

- one for **encryption** \mathcal{E}
- one for **decryption** \mathcal{D}

that accept two inputs:

- an input block of size n bits
- a key of size k bits

$$\begin{aligned}\mathcal{E}_K(P) &:= \mathcal{E}(K, P) : \{0, 1\}^k \times \{0, 1\}^n \rightarrow \{0, 1\}^n \\ \mathcal{E}_K^{-1}(C) &:= \mathcal{D}_K(C) = \mathcal{D}(K, C) : \{0, 1\}^k \times \{0, 1\}^n \rightarrow \{0, 1\}^n\end{aligned}$$

$$\boxed{\forall K, \forall P : \mathcal{D}_K(\mathcal{E}_K(P)) = P}$$

Block ciphers - Definition

Problem: the plaintexts may be extremely long

~> hard to analyse security of the cipher.

Idea: Design ciphers that work on small blocks ...

two paired algorithms

- one for **encryption** \mathcal{E}
- one for **decryption** \mathcal{D}

that accept two inputs:

- an input block of size n bits
- a key of size k bits

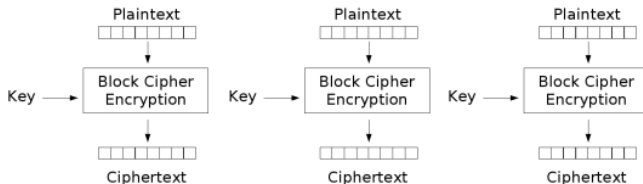
$$\begin{aligned}\mathcal{E}_K(P) &:= \mathcal{E}(K, P) : \{0, 1\}^k \times \{0, 1\}^n \rightarrow \{0, 1\}^n \\ \mathcal{E}_K^{-1}(C) &:= \mathcal{D}_K(C) = \mathcal{D}(K, C) : \{0, 1\}^k \times \{0, 1\}^n \rightarrow \{0, 1\}^n\end{aligned}$$

$$\boxed{\forall K, \forall P : \mathcal{D}_K(\mathcal{E}_K(P)) = P}$$

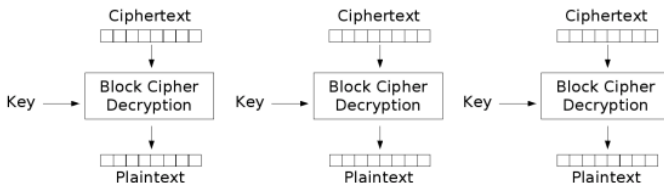
Mode of operations

- Block ciphers cannot be used **directly** for encryption.
- They are always used in some “modes of operation”
 - Electronic Codebook (ECB) mode
 - Cipher-Block Chaining (CBC) mode,
 - Output Feedback (OFB) mode,
 - Counter (CTR) mode,
 - ...

Electronic codebook (ECB)

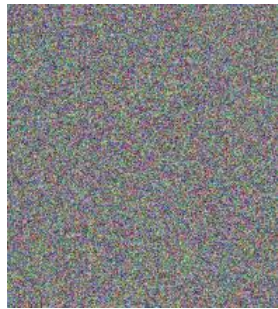
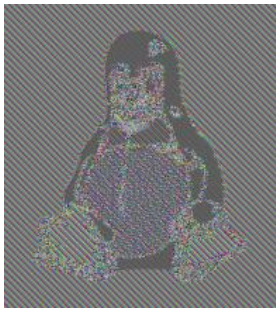


Electronic Codebook (ECB) mode encryption

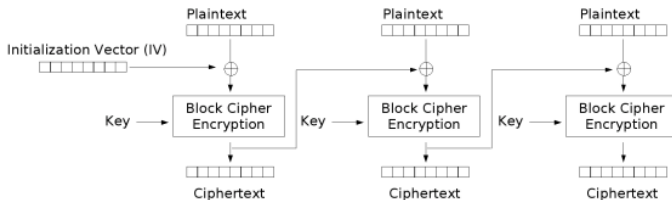


Electronic Codebook (ECB) mode decryption

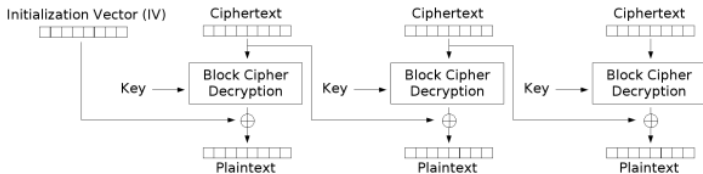
Electronic codebook (ECB) should not be used !



Cipher-block chaining (CBC)



Cipher Block Chaining (CBC) mode encryption



Cipher Block Chaining (CBC) mode decryption

Cipher-block chaining (CBC)

- **Error propagation?**



Error in block c_i affects only m_i and m_{i+1}
 \rightsquigarrow errors do not propagate

- **Can encryption be parallelized?**



No !

- **Can decryption be parallelized?**



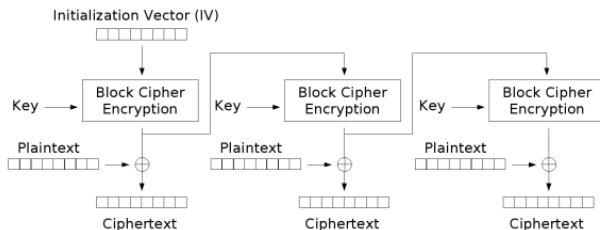
Yes !

- **What if one bit of plaintext is changed (somewhere at the beginning)?**

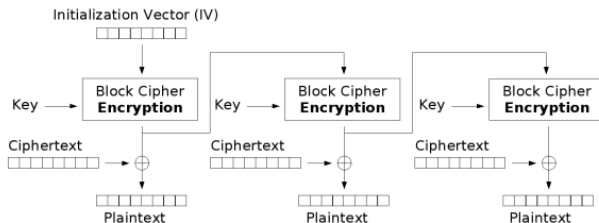


Everything needs to be recomputed
(not so good e.g. for disc encryption)

Output feedback (OFB)



Output Feedback (OFB) mode encryption



Output Feedback (OFB) mode decryption

Output feedback (OFB)

- **Error propagation?**



Error in block c_i affects only m_i and m_{i+1}
 \rightsquigarrow errors do not propagate

- **Can encryption/decryption be parallelized?**



No ! ...



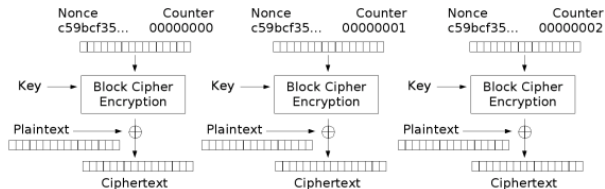
... but we can we can use **precomputation**

- **What if one bit of plaintext is changed (somewhere at the beginning)?**

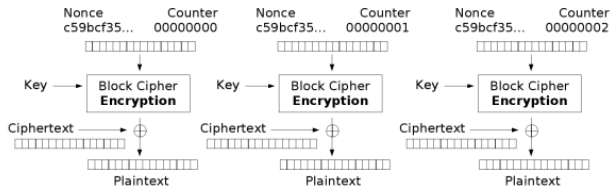


Only one block needs to be recomputed

Counter (CTR)



Counter (CTR) mode encryption



Counter (CTR) mode decryption

Counter (CTR)

- **Error propagation?**



Error in block c_i affects only c_i and c_{i+1}
 \rightsquigarrow errors do not propagate

- **Can encryption/decryption be parallelized?**



Yes ! ...



...and we can we can use **precomputation**



possible to decrypt one block without decrypting anything else

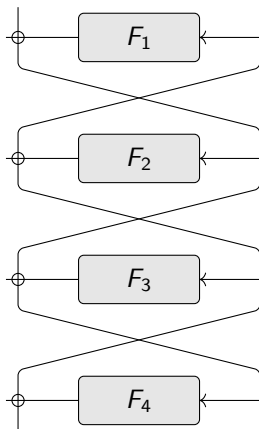
- **What if one bit of plaintext is changed (somewhere at the beginning)?**



Only one block needs to be recomputed

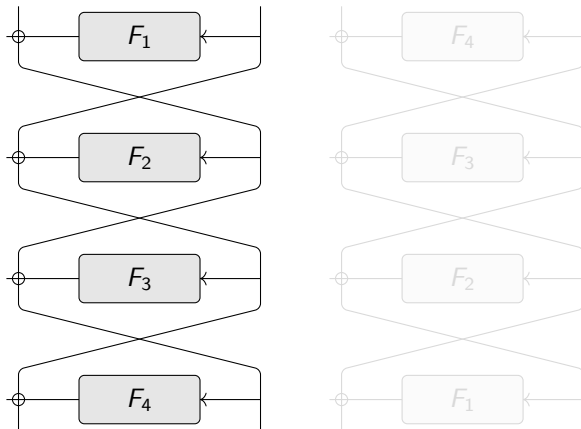
Feistel Scheme

- symmetric structure used in the construction of block ciphers
- invented by cryptographer Horst Feistel



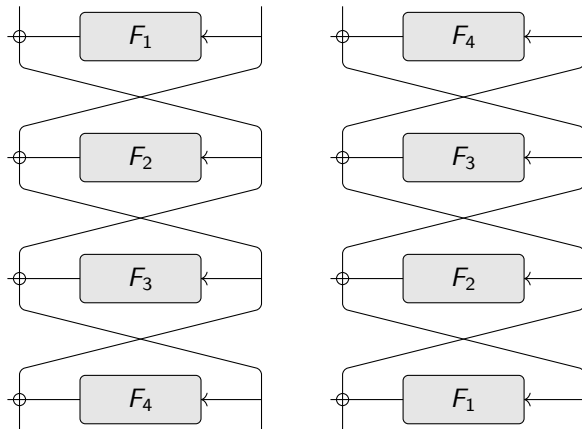
Feistel Scheme

- invertible !



Feistel Scheme

- invertible !



Data Encryption Standard (DES)

- First version designed by IBM in 1973-74, based on a **Lucifer cipher** (by Horst Feistel).
- **National Security Agency** (NSA) played some (unclear ...) role in the design of DES.
- made public in 1975.
- approved as a US federal standard in November 1976.

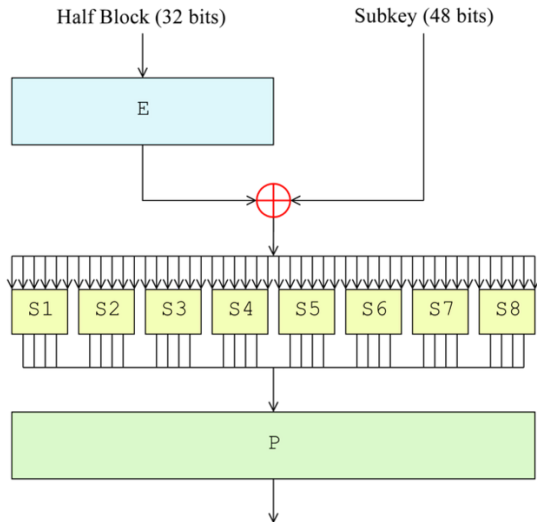
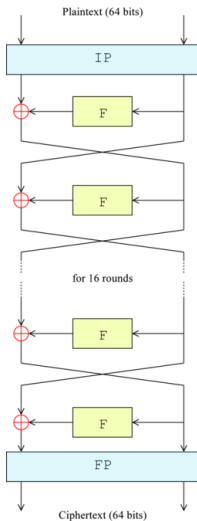
- Key length:
 - **effective**: 56 bits
 - **formally**: 64 bits (8 bits for checking parity).
- Block length: 64 bits

Data Encryption Standard (DES)

- First version designed by IBM in 1973-74, based on a **Lucifer cipher** (by Horst Feistel).
- **National Security Agency** (NSA) played some (unclear ...) role in the design of DES.
- made public in 1975.
- approved as a US federal standard in November 1976.

- Key length:
 - **effective:** 56 bits
 - **formally:** 64 bits (8 bits for checking parity).
- Block length: 64 bits

Data Encryption Standard (DES)

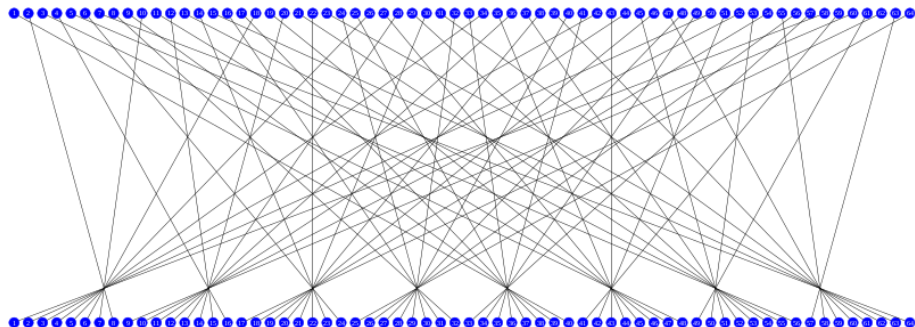


DES - Initial Permutation

58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

- specifies the input permutation on a 64-bit block.
- the first bit of the output is taken from the 58th bit of the input;
- the second bit from the 50th bit,
- ...
- the last bit is taken from the 7th bit of the input.

DES - Initial Permutation

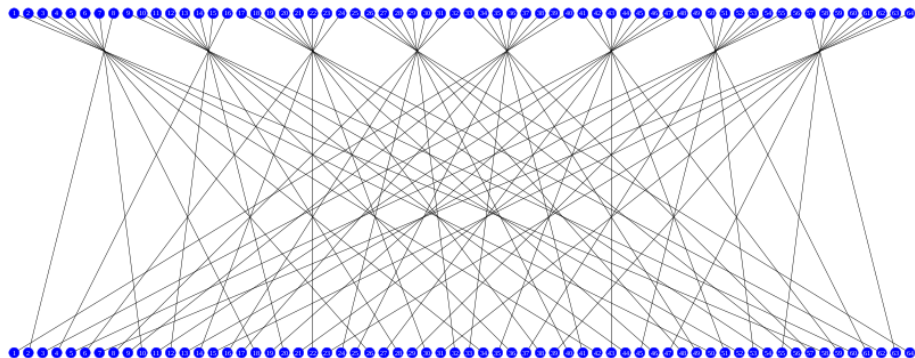


DES - Final Permutation

40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

- specifies the input permutation on a 64-bit block.
- the first bit of the output is taken from the 40th bit of the input;
- the second bit from the 8th bit,
- ...
- the last bit is taken from the 25th bit of the input.

DES - Final Permutation

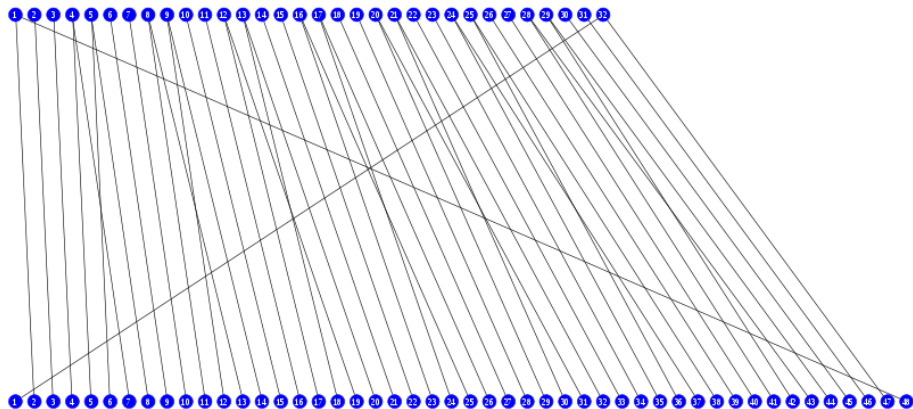


DES - Expansion Function

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

- some bits from the input are duplicated at the output
- the 32-bit half-block is expanded to 48 bits.

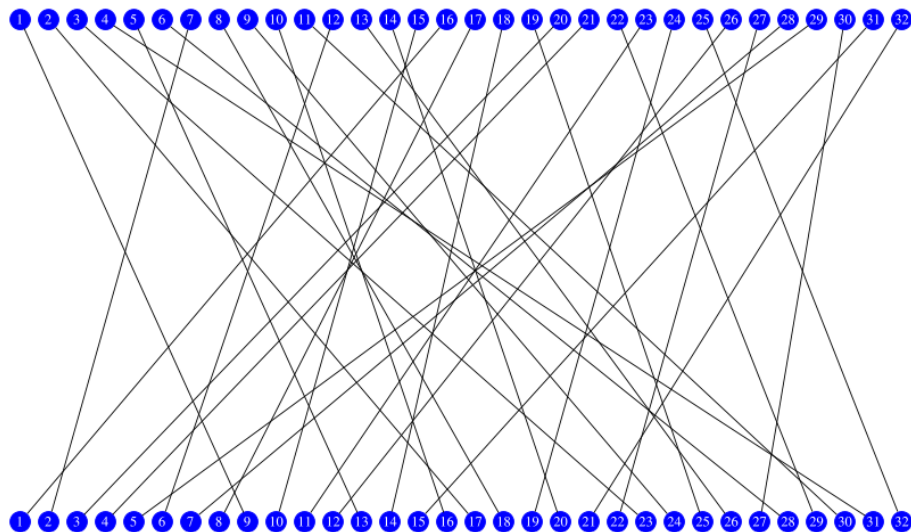
DES - Expansion Function



DES - Permutation P

16	7	20	21	29	12	28	17
1	15	23	26	5	18	31	10
2	8	24	14	32	27	3	9
19	13	30	6	22	11	4	25

DES - Permutation P



DES - S-box

- S-box: $\{0, 1\}^6 \rightarrow \{0, 1\}^4$

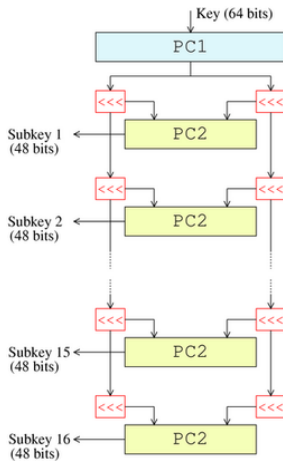
- Example of S_1 :

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	E	4	D	1	2	F	B	8	3	A	6	C	5	9	0	7
1	0	F	7	4	E	2	D	1	A	6	C	B	9	5	3	8
2	4	1	E	8	D	6	2	B	F	C	9	7	3	A	5	0
3	F	C	8	2	4	9	1	7	5	B	3	E	A	0	6	D

- $\alpha = \alpha_0\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5 \in \{0, 1\}^6$
 $S_1(\alpha) \rightsquigarrow$ entry indexed by row $\alpha_r = \alpha_0\alpha_5$ and column
 $\alpha_c = \alpha_1\alpha_2\alpha_3\alpha_4$.

Example : $S_1(011011) = 0101$ since $5 = 0101$ is at the intersection of row $1 = 01$ and column $D = 1101$.

DES Key Schedule



DES - Permuted Choice

- 56 bits of the key are selected by **Permuted Choice 1 (PC-1)**

57	49	41	33	25	17	9
1	58	50	42	34	26	18
10	2	59	51	43	35	27
19	11	3	60	52	44	36
63	55	47	39	31	23	15
7	62	54	46	38	30	22
14	6	61	53	45	37	29
21	13	5	28	20	12	4

- 48 subkey bits are selected by **Permuted Choice 2 (PC-2)**
(24 bits from the left half, and 24 from the right)

14	17	11	24	1	5	3	28
15	6	21	10	23	19	12	4
26	8	16	7	27	20	13	2
41	52	31	37	47	55	30	40
51	45	33	48	44	49	39	56
34	53	46	42	50	36	29	32

DES - Permuted Choice

