Replication of Tse (1998) and Tsui and Ho (2004)

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Abstract

Tse, in his 1998 paper explores the conditional volatility (CV) of the USD/YEN exchange rate during the time period of 1978-1994. He carries out various (G)ARCH model settings concluding that there any asymmetry in CV of the exchange rate does not exist, additionally arguing that there are no significant differences between stable and fractionally integrated models. Later this paper is replicated and tested by Tsui and Ho (2004) arguing, that even though most of the findings generally match Tse's (1998) results using USD/YEN exchange rate, exploring different currency pairs and using APARCH, FIAPARCH models indicates evidence of asymmetric CV. We find that our results are on par with Tse's results, but if the time period of our sample increases, then our estimates are higher than Tse's estimates. We also obtain asymmetry in the APARCH case for our sample, in line with Tsui and Ho's (2004) who made a replication study of Tse.

1. Introduction

Replication in economics is primarily done to see if a model or method is applicable for further use outside the specific data set. This paper aims to analyse Tse's (1998) manner of estimating CV of the exchange rate and verifying whether we arrive at the same inconclusive results of Tsui and Ho (2004) who argue that there exists a significant difference between fractionally integrated and stable models. They also augment Tse's (1998) research with additional currency pairs and by splitting the data set in multiple periods, finding that there exists asymmetric volatility in the exchange rate.

The remaining part of this paper consists of 3 sections. Section 2 gives a theoretical background to the models used in the analysis and a brief description of the data. This is followed by Section 3 that is divided in 3 subsections, each for 3 time periods used in the analysis. Finally, a section with conclusions ends this replication assignment with a discussion on our findings and a comparison with the previous literature.

2. Estimation Process

To continue the process of estimating CV, we firstly introduce (G)ARCH models in various settings, i.e, controlling for integration and asymmetric powers and indicate their differences.

An Overview on GARCH

After observing the existence of volatility clustering in time series, Robert Engle in 1982 introduced the ARCH (q) model account for this behaviour. However, as in practice, large q lags are required to capture time volatility, thus a more generalized approach was introduced to allow for the conditional variance to depend upon its own lags, i.e., Generalized ARCH (p,q), see eq. 1 - 3 (Westerlund, 2019).

$$y_t = \mu + \varepsilon_t \quad (1)$$

$$\varepsilon_t = \sqrt{h_t}\, v_t \quad (2)$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 h_{t-1} + \ldots + \beta_p h_{t-p} \quad (3)$$

Equation (1) denotes mean equation for the time-series of interest, (2) is variance equation for errors, whilst $v_t \sim iid\,(0,1)$, lastly (3) shows the full ARMA composition of h_t in the error term. Sum of both α *or* β coefficients should be less than 1 for stationarity.

An Overview on APARCH

The asymmetric-power ARCH (APARCH) model is a set of models introduced in 1993 by Engle, et al. The APARCH models, while combining other models in one framework, additionally captures asymmetry in return volatility. That is, volatility tends to increase more when returns are negative, as compared to positive returns of the same magnitude (Gentle, 2014). For this model, only variance equation differs as we can see in (4) (Tsui and Ho, 2004):

$$h_t^{\delta/2} = \eta + \alpha(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^{\delta} + h_{t-1}^{\delta/2} \quad (4)$$

When $\gamma > 0$, negative shocks give rise to higher volatility than positive shocks, thus if in upcoming regressions this coefficient will be statistically insignificant, it would support Tse's findings of having no asymmetric CV.

An overview on IGARCH

From (3), if we set $\alpha + \beta = 1$, the integrated GARCH or the IGARCH model is obtained. For this model volatility equation is expressed as follows in (5) (Westerlund, 2019):

$$\varepsilon_t^2 = \alpha_0 t + \varepsilon_0^2 + \sum_{k=0}^{t-1}(-\beta_1\eta_k + \eta_{k+1}) \quad (5)$$

Despite given restriction, IGARCH model can be strongly stationary even though it is not weakly stationary. IGARCH model implies infinite persistence of the conditional variance to a shock in squared returns. However, in most empirical situations the volatility process is found to be mean reverting (Tayefi, et al., 2012). Due to IGARCH models theoretically having the variance of the errors tending to infinity, the IGARCH models can be misspecified and have a tendency to overfit the data. A way to correct for this is to use the FIGARCH models in order to correct for the misspecification.

An Overview on FIGARCH

The FIGARCH model, while building on the GARCH model, aims to capture the effect of a long-term persistent shock on the conditional volatility. In scenario when $\alpha + \beta = 1$ is close to 1, we might observe unit root. To allow for alpha and beta to be close to each other and produce long lasting effect, one may use fractionally integrated GARCH (FIGARCH) model, see (6) (Tsui and Ho, 2004):

$$h_t = \frac{\eta}{1-\beta} + \lambda(L)\varepsilon_t^2 \quad (6)$$

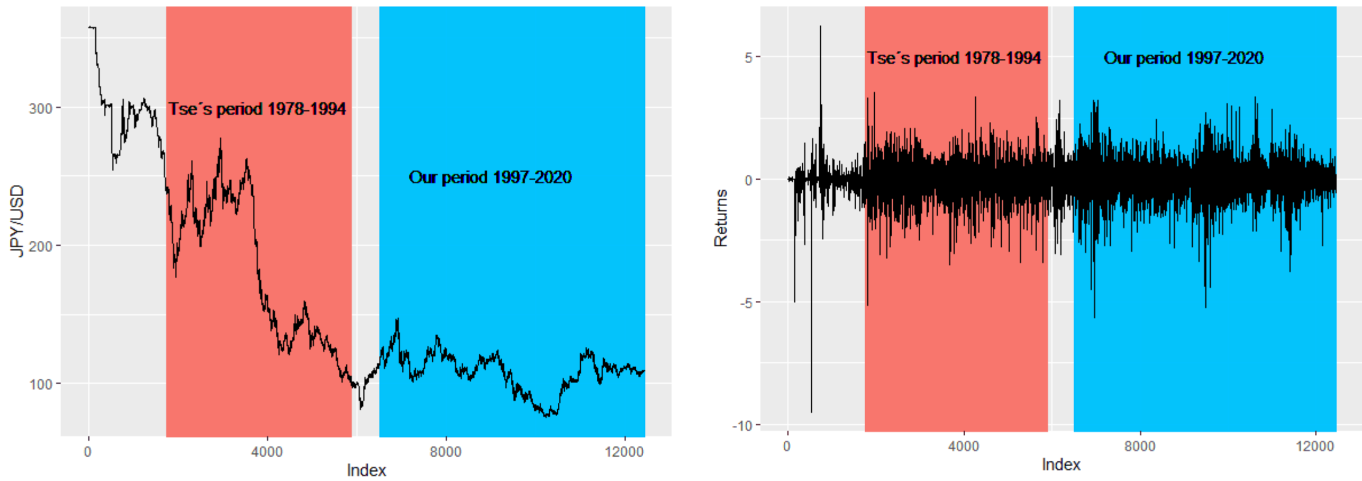
However, given that FIGARCH, FIAPARCH regressions are yet not supported by our R-package, we will instead use IGARCH to capture the effect of integration and determine whether it is more fitting to data than other (G)ARCH models.

2.1 Data

The data for this study comprises data from the daily trading days from 4th of January 1971 to the 3rd of January 2020 of the US Dollar -Yen (USD/YEN) exchange rate. However, in similar manner as the Tse and Tsui and Ho (TH) papers use various sampling periods, we will use the following sampling periods:

1. 1978 - 1994 (The period is used by Tse);
2. 1971 - 2020 (The entire sample available);
3. 1997 - 2020 (Our sample).

These intervals allow us to test Tse´s main findings in his time period (1), estimate the entire sample in order to capture as much information as possible from interval (2) and check the robustness when looking at a technologically advanced period (3). (3) is characterized by better computing power, more connectedness and ease of information access which may affect the data generating process. Period (1) and (3) can be seen in Figure 1 which shows the non-differenced, differenced and logged data.



Panel A Panel B

Figure 1. Non-differenced, differenced and logged USD/YEN exchange rate 1971-2020

We can observe periods of volatility clustering that would support the use of GARCH. In addition, there seems to be higher spikes of volatility in our period which may affect the result. Graphical analysis indicates that stationarity is obtained using differencing. So, we can continue with regression analysis. As can be seen from the level data in Panel B Figure 1, there seems to be a structural break in the data around observation 6000, corresponding to January 1995. In the level data, it is where the decline ends.

3. Our regression results vs Tse (1998)

The steps towards estimating the conditional heteroscedasticity start by estimating the mean equation. In order to estimate the returns of the exchange rate and to remove non-stationarity, we first difference and log the exchange rate which produces a stationary time series which additionally can be interpreted as the returns. From the ACF and PACF values as well as the p-values, we conclude that the mean equation is best described as an AR(1) process.

3.1 Regression Analysis (1978-1994)

We continue replication by estimating the GARCH and APARCH models as can be seen in Figure 2. All errors reported are robust standard errors.

1

	Estimate	Std. Error	t value	Pr(> | t|)
mu	-0.011	0.012	-0.967	0.333
omega	0.029	0.009	3.054	0.002
alpha1	0.085	0.018	4.617	0.00000
beta1	0.852	0.034	24.940	0

2

	Estimate	Std. Error	t value	Pr(> | t|)
mu	-0.012	0.012	-1.004	0.315
ar1	0.031	0.018	1.723	0.085
omega	0.029	0.009	3.039	0.002
alpha1	0.085	0.019	4.552	0.00001
beta1	0.852	0.034	24.736	0

3

	Estimate	Std. Error	t value	Pr(> | t|)
mu	-0.010	0.012	-0.879	0.379
omega	0.031	0.011	2.937	0.003
alpha1	0.089	0.020	4.579	0.00000
beta1	0.859	0.034	25.018	0
gamma1	-0.013	0.064	-0.198	0.843
delta	1.729	0.354	4.883	0.00000

4

	Estimate	Std. Error	t value	Pr(> | t|)
mu	-0.011	0.012	-0.891	0.373
ar1	0.031	0.018	1.765	0.078
omega	0.031	0.010	2.941	0.003
alpha1	0.089	0.020	4.519	0.00001
beta1	0.859	0.034	24.994	0
gamma1	-0.019	0.064	-0.288	0.773
delta	1.737	0.359	4.844	0.00000

Table 1. Replication of Tse (1998) Table 1[1,2]

From Regression (1), we observe that the sum of alpha and beta is 0.94 and statistically significant, implying that shocks to CV are persistent. Similar to Tse, we additionally run the GARCH-ARFIMA (1,0,0) model as seen in (2). This is based on plotting the AC and PAC functions as a diagnostic check. The estimates do not change sign, significance and the magnitude is very similar to the estimates in (1). Regressions (1) - (2) are nearly identical to those of Tse (1998) and Tsui and Ho (2004), with an exception of the coefficient value of AR in (2).

As APARCH models by nature attempt to capture the larger conditional volatility, the estimates for alpha1 and beta1 see a slight increase. Gamma1 in Regression (3) is insignificant demonstrating that the volatility in the model is largely symmetrical. Having run the Ljung box test values, we generate p value > 0.05 ensuring that the APARCH models do not have errors with serial correlation.

In regressions (3) - (4), we can observe that γ is also statistically insignificant as in Tse (2004), indicating absence of asymmetric CV and in line with Tsui and Ho (2004). In (4), additionally, we can see that again AR coefficient is different and positive as opposed to Tse.

[1] Here we do not report Regression (5) as our analysis package does not allow us to make such adjustments in the APARCH model.

[2] In order as provided - Regression (1) is a GARCH(1,1) model with ARFIMA (0,0,0) specification. Regression (2) is a GARCH (1,1) model with ARFIMA(1,0,0) specification. Regression (3) is an APARCH (1,1) model with ARFIMA (0,0,0) specification. Regression (4) is an APARCH (1,1) model with ARFIMA (1,0,0) specification.

Continuing our analysis and accounting for possible integration by using IGARCH, we find the following results (see Table 2). When forcing alpha and beta to sum to 1, the coefficients become larger as is expected. They stay within the same range of value, alpha being close to 0.1 and beta close to 0.9 and showing significance. Our software does not produce values for the standard errors of beta which we suspect arises because of the previously mentioned restriction.

1

	Estimate	Std. Error	t value	Pr(> | t|)
mu	-0.012	0.010	-1.228	0.219
omega	0.008	0.003	2.213	0.027
alpha1	0.094	0.026	3.640	0.0003
beta1	0.906			

2

	Estimate	Std. Error	t value	Pr(> | t|)
mu	-0.013	0.010	-1.247	0.212
ar1	0.033	0.016	2.075	0.038
omega	0.008	0.003	2.183	0.029
alpha1	0.094	0.026	3.590	0.0003
beta1	0.906			

3

	Estimate	Std. Error	t value	Pr(> | t|)
mu	-0.021	0.010	-2.088	0.037
omega	0.00000	0.00000	0.886	0.376
beta1	1			

Table 2. Replication of Tse (1998) Table 2

Given that we use IGARCH instead of FIGARCH or FIAPARCH, we can not directly compare our regression coefficient values to those of Tse. Generally, from Table 2 we can observe that in mean equation mu is not statistically significant in most cases, nevertheless, CV equation parameters are mostly significant and with adequately normal t-values.

3.2 Using full sample (1971-2020)

To verify the external validity and previous result persistence in time, the second part of our analysis contains the same models as before, yet using the full 1971-2020 sample. Taking into account almost 50 years of exchange rates reveals some strangeness in the data. Comparing regression (1) and (2) of Table 3, gives rise to two very different results. In (1) there is no significance in any of the variables whilst in (2) beta and alpha display massive significance of the level of more than 3000 in t-value. Whilst beta displays high t-values in all of the regressions in Table 1, it is dwarfed by the t-value found in Table 3 Regression (2). A very high t-value of the beta coefficient may be a sign of a unit root in the variance equation. This does not seem to be a good explanation either, due to the low t-values of the estimates´ t-values in the variance equation in Table 4 as we will observe later on.

Compared with Tse´s findings, there is a noticeable difference in the mean equation estimates, Tse´s estimates consistently are higher, except for our regression (4). When including an AR component in the mean equation, our results are more in line with Tse´s findings for the GARCH models (1) and (2) in Table 3 both for the value of the estimates and when they show significance. The standard errors for Tse are, however, much lower. For the alpha and beta parameters in Table 4, the results line up with Tse´s findings in his smaller sample. We both find low significance of the estimates, Tse´s having higher p-values. Tse´s parameter *d* captures the fractional integration which we are unable to do, due to software restrictions. The *d* parameter which represents some unit root in the data is instead captured by our beta parameter which displays a very high t-value.

1

	Estimate	Std. Error	t value	Pr(> | t|)
mu	0.009	0.112	0.077	0.939
omega	0.002	0.011	0.151	0.880
alpha1	0.100	1.300	0.077	0.939
beta1	0.897	5.813	0.154	0.877

2

	Estimate	Std. Error	t value	Pr(> | t|)
mu	0.004	0.006	0.672	0.502
ar1	0.026	0.013	1.954	0.051
omega	0.001	0.00000	7,697.448	0
alpha1	0.084	0.003	29.965	0
beta1	0.915	0.0003	3,461.790	0

3

	Estimate	Std. Error	t value	Pr(> | t|)
mu	-0.001	0.006	-0.099	0.921
omega	0.022	0.009	2.466	0.014
alpha1	0.137	0.038	3.645	0.0003
beta1	0.854	0.036	23.920	0
gamma1	0.113	0.054	2.079	0.038
delta	1.407	0.232	6.071	0

4

	Estimate	Std. Error	t value	Pr(> | t|)
mu	0.018	0.087	0.206	0.837
ar1	0.006	0.027	0.206	0.837
omega	0.0004	0.001	0.620	0.535
alpha1	0.065	0.315	0.205	0.837
beta1	0.893	1.445	0.618	0.537
gamma1	0.054	0.264	0.206	0.837
delta	2.702	4.428	0.610	0.542

Table 3. Replication of Tse´s (1998) Table 1 using full sample (1971-2020)

1

	Estimate	Std. Error	t value	Pr(> | t|)
mu	0.003	0.006	0.588	0.556
omega	0.002	0.008	0.204	0.838
alpha1	0.092	0.761	0.121	0.904
beta1	0.908			

2

	Estimate	Std. Error	t value	Pr(> | t|)
mu	0.002	0.00000	10,021.020	0
ar1	0.039	0.00000	8,034.878	0
omega	0.002	0.010	0.152	0.879
alpha1	0.092	0.603	0.152	0.879
beta1	0.908			

3

	Estimate	Std. Error	t value	Pr(> | t|)
mu	-0.010	0.005	-1.833	0.067
omega	0.00000	0.00000	1.678	0.093
beta1	1			

Table 4. Replication of Tse (1998) Table 2 using full sample (1971-2020)

The APARCH models in (3) and (4) of Table 3 also become very different depending on the inclusion of an AR-component in the mean equation. (3) show significance in all of the estimates for the variance equation and they display more reasonable t-values in terms of magnitude. Since they show vastly different levels of significance, it does not provide much evidence of robustness in the models since changing some parameters in the AR part should not have this big of an effect. A break in the data generating process might have happened given that the Tse finds significance in his time period and we don´t in the full time period. We therefore consider another time period.

3.3 Our sample (1997-2020)

With the former caveats of our sample, i.e., structural break around 1995, we continue our analysis by looking at the time period of 1997-2020. From Figure 1 Panel B, we can observe that within this period, volatility is lower and contains less outliers (extreme highs/lows). In the same manner, we continue by estimating GARCH and APARCH models as we can see in Table 5.

1

	Estimate	Std. Error	t value	Pr(> | t|)
mu	0.006	0.007	0.808	0.419
omega	0.002	0.001	2.903	0.004
alpha1	0.041	0.003	14.793	0
beta1	0.955	0.001	965.724	0

2

	Estimate	Std. Error	t value	Pr(> | t|)
mu	0.006	0.007	0.805	0.421
ar1	-0.012	0.014	-0.896	0.370
omega	0.002	0.001	2.897	0.004
alpha1	0.041	0.003	14.807	0
beta1	0.956	0.001	992.358	0

3

	Estimate	Std. Error	t value	Pr(> | t|)
mu	0	0.00001	0.00005	1.000
omega	0.008	0.003	2.631	0.009
alpha1	0.060	0.026	2.302	0.021
beta1	0.945	0.024	39.989	0
gamma1	0.248	0.126	1.966	0.049
delta	0.888	0.422	2.102	0.036

4

	Estimate	Std. Error	t value	Pr(> | t|)
mu	0.001	0.0002	2.709	0.007
ar1	-0.010	0.010	-1.054	0.292
omega	0.007	0.003	2.647	0.008
alpha1	0.060	0.022	2.763	0.006
beta1	0.945	0.020	47.539	0
gamma1	0.239	0.116	2.060	0.039
delta	0.900	0.353	2.552	0.011

Table 5. Replication of Tse (1998) Table 1 using our sample (1997-2020)

All coefficients in Table 5 in the CV equations are statistically significant, and again we observe relatively high t-values, however, this time, as we can see in next Table 6, our t-values are at least adequate, i.e., not above 1000 as we saw before and allows us to make inference on IGARCH results; in addition, alpha and beta coefficients are close to unity, thus supporting use of IGARCH.

For our new sample in regression (1) and (2), the addition of the AR component does not affect the significance or the estimated values of alpha1 and beta1. For regression (3) and (4), we obtain a significant gamma value implying that the volatility of our sample is asymmetrical, i.e., negative shocks have a greater effect than positive shocks on the volatility. This is largely in line with the results obtained by Tsui and Ho (2004) and contradicts the findings by Tse (1998).

1

	Estimate	Std. Error	t value	Pr(> | t|)
mu	0.006	0.008	0.783	0.433
omega	0.001	0.001	1.997	0.046
alpha1	0.043	0.009	4.587	0.00000
beta1	0.957			

2

	Estimate	Std. Error	t value	Pr(> | t|)
mu	0.006	0.008	0.787	0.431
ar1	-0.012	0.014	-0.886	0.376
omega	0.001	0.001	1.992	0.046
alpha1	0.043	0.009	4.576	0.00000
beta1	0.957			

3

	Estimate	Std. Error	t value	Pr(> | t|)
mu	-0.001	0.008	-0.133	0.894
omega	0.00000	0.00000	1.981	0.048
beta1	1			

Table 6. Replication of Tse (1998) Table 2 using our sample (1997-2020)

If we take a look at Table 6, we find similar patterns - mu from mean equation is still statistically insignificant and all the other parameters are statistically significant, with adequate t-values. All of the coefficients for beta and alpha in Table 6 are very similar to each other and also similar to their counterparts in Table 5 when applicable. Given the estimate and the high level of significance, this could potentially mean that the IGARCH overfits the estimates of the model as highlighted in the theory earlier.

Conclusion

We find that our results mostly match those of Tse´s, only deviating a bit from an AR coefficient. We also agree with Tse that there is no asymmetric power when modelled as an APARCH and during his selected period (1978-1994). When considering the entire time period from 1971 to 2020 we find oddities in the estimate in the sense that adding an AR component in the mean equation causes the t-value of the volatility estimates to become very large (above 3000). As such there may be integration or fractional integration in the process.

Applying the same models to another period (1997-2020) showed that there may be some asymmetric power, as also found by Tsui and Ho (2004) in their sample (1986-2003). The asymmetry of the conditional volatility may therefore be as a result of something happening to the data generating process after Tse´s period, although more research would be needed to confirm this. In general, the results found by Tse (1998) are consistent for the period that he investigated but looking at periods after that we agree on the findings of Tsui and Ho (2004).

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