

## Decision Tree Induction: Algorithm

- Basic algorithm
  - Tree is constructed in a **top-down, recursive, divide-and-conquer manner**
  - At start, all the training examples are at the root
  - Examples are partitioned recursively based on selected attributes
  - On each node, attributes are selected based on the training examples on that node, and a heuristic or statistical measure (e.g., **information gain**)
- Conditions for stopping partitioning
  - All samples for a given node belong to the same class
  - There are no remaining attributes for further partitioning
  - There are no samples left
- Prediction
  - Majority voting** is employed for classifying the leaf

เลือก root node ที่ info gain สูงสุด  
เลือก Attributes ที่ info gain สูงสุด

เลือก majority class ใน node นั้น

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## How to Handle Continuous-Valued Attributes?

- Method 1: Discretize continuous values and treat them as categorical values
  - E.g., age: < 20, 20..30, 30..40, 40..50, > 50
- Method 2: Determine the **best split point** for continuous-valued attribute A
  - Sort the value A in increasing order; e.g. 15, 18, 21, 22, 24, 25, 29, 31, ...
  - Possible split point: the midpoint between each pair of adjacent values
    - $(a_i + a_{i+1})/2$  is the midpoint between the values of  $a_i$  and  $a_{i+1}$
    - e.g.,  $(15+18)/2 = 16.5$ , 19.5, 21.5, 23, 24.5, 27, 30, ...
  - The point with the **maximum information gain** for A is selected as the **split-point** for A
- Split: Based on split point P
  - The set of tuples in D satisfying  $A \leq P$  vs. those with  $A > P$

ตัวอย่าง = 15, 18, 21, 22, 24, 25, 29, 31, ...  
→ 9 จุดที่เป็นไปได้

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## Gain Ratio: A Refined Measure for Attribute Selection

- Information gain measure is biased towards attributes with a large number of values
- Gain ratio: Overcomes the problem (as a normalization to information gain)

$$SplitInfo_A(D) = - \sum_{j=1}^v \frac{|D_j|}{|D|} \times \log_2 \left( \frac{|D_j|}{|D|} \right)$$

- GainRatio(A) = Gain(A)/SplitInfo(A)
- The attribute with the maximum gain ratio is selected as the splitting attribute
- Gain ratio is used in a popular algorithm C4.5 (a successor of ID3) by R. Quinlan
- Example
  - $SplitInfo_{income}(D) = -\frac{4}{14} \log_2 \frac{4}{14} - \frac{6}{14} \log_2 \frac{6}{14} - \frac{4}{14} \log_2 \frac{4}{14} = 1.557$
  - GainRatio(income) =  $0.029/1.557 = 0.019$

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## Another Measure: Gini Index

- Gini index: Used in CART, and also in IBM IntelligentMiner
- If a data set D contains examples from n classes, gini index,  $gini(D)$  is defined as

$$gini(D) = 1 - \sum_{j=1}^n p_j^2$$

- $p_j$  is the relative frequency of class j in D

- If a data set D is split on A into two subsets  $D_1$  and  $D_2$ , the gini index  $gini(D)$  is defined as

$$gini_A(D) = \frac{|D_1|}{|D|} gini(D_1) + \frac{|D_2|}{|D|} gini(D_2)$$

- Reduction in Impurity:

$$\Delta gini(A) = gini(D) - gini_A(D)$$

- The attribute provides the smallest  $gini_{split}(D)$  (or the largest reduction in impurity) is chosen to split the node (**need to enumerate all the possible splitting points for each attribute**)

ตัวอย่าง  $1 - \left( \left( \frac{7}{14} \right)^2 + \left( \frac{7}{14} \right)^2 \right)$

สูตรลด  
 $\frac{8(3,5)}{14} = -\frac{3}{7} \log \frac{3}{7} - \frac{5}{7} \log \frac{5}{7}$

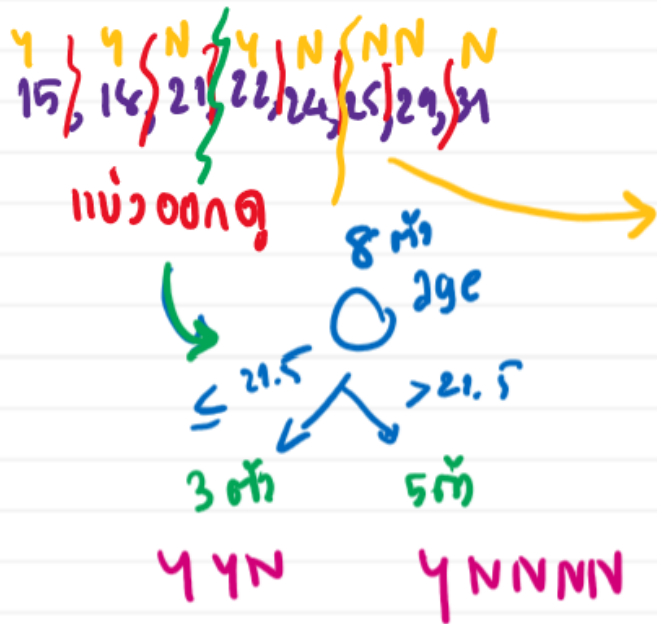
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## Method 1: Categorize

17, 18, 21, 22,

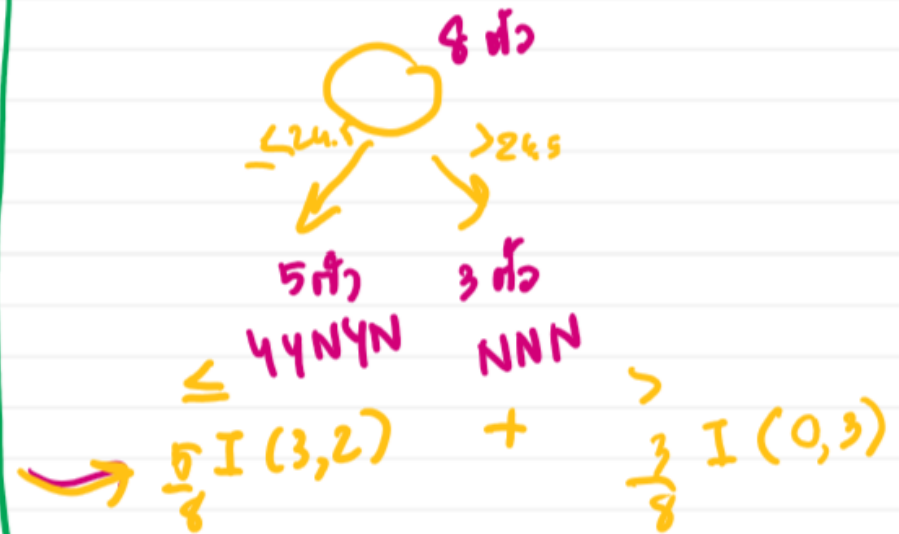
15-16, >18, >22

## Method 2: best split point



$$Age_{21.5} = \frac{3}{8} I(2,1) + \frac{5}{8} I(1,4)$$

## Method 3: Random



## Computation of Gini Index

- Example: D has 9 tuples in buys\_computer = "yes" and 5 in "no"  

$$gini(D) = 1 - \left(\frac{9}{14}\right)^2 - \left(\frac{5}{14}\right)^2 = 0.459$$
- Suppose the attribute income partitions D into 10 in  $D_1$ : {low, medium} and 4 in  $D_2$ 
  - $$gini_{income \in \{low, medium\}}(D) = \frac{10}{14} gini(D_1) + \frac{4}{14} gini(D_2)$$

$$= \frac{10}{14} \left(1 - \left(\frac{7}{10}\right)^2 - \left(\frac{3}{10}\right)^2\right) + \frac{4}{14} \left(1 - \left(\frac{2}{4}\right)^2 - \left(\frac{2}{4}\right)^2\right) = 0.443$$

$$= Gini_{income \in \{high\}}(D)$$
  - $Gini_{\{low, high\}}$  is 0.458;  $Gini_{\{medium, high\}}$  is 0.450
  - Thus, split on the {low, medium} (and {high}) since it has the lowest Gini index
- All attributes are assumed continuous-valued
- May need other tools, e.g., clustering, to get the possible split values
- Can be modified for categorical attributes

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## Comparing Three Attribute Selection Measures

- The three measures, in general, return good results but
  - Information gain:**
    - biased towards multivalued attributes
  - Gain ratio:**
    - tends to prefer unbalanced splits in which one partition is much smaller than the others
  - Gini index:**
    - biased to multivalued attributes
    - has difficulty when # of classes is large
    - tends to favor tests that result in equal-sized partitions and purity in both partitions

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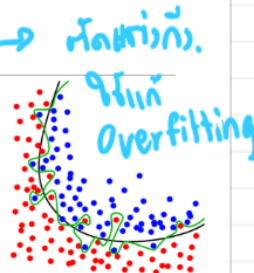
## Other Attribute Selection Measures

- Minimal Description Length (MDL) principle
  - Philosophy: The simplest solution is preferred
  - The best tree as the one that requires the fewest # of bits to both (1) encode the tree, and (2) encode the exceptions to the tree
- CHAID: a popular decision tree algorithm, measure based on  $\chi^2$  test for independence
- Multivariate splits (partition based on multiple variable combinations)
  - CART: finds multivariate splits based on a linear combination of attributes
- There are many other measures proposed in research and applications
  - E.g., G-statistics, C-SEP
- Which attribute selection measure is the best?
  - Most give good results, none is significantly superior than others

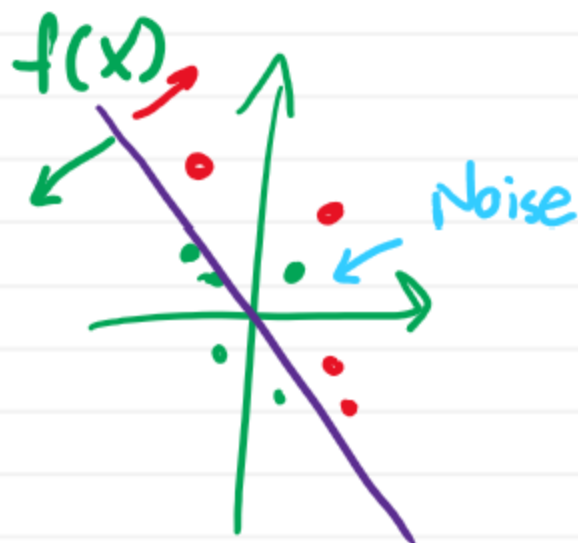
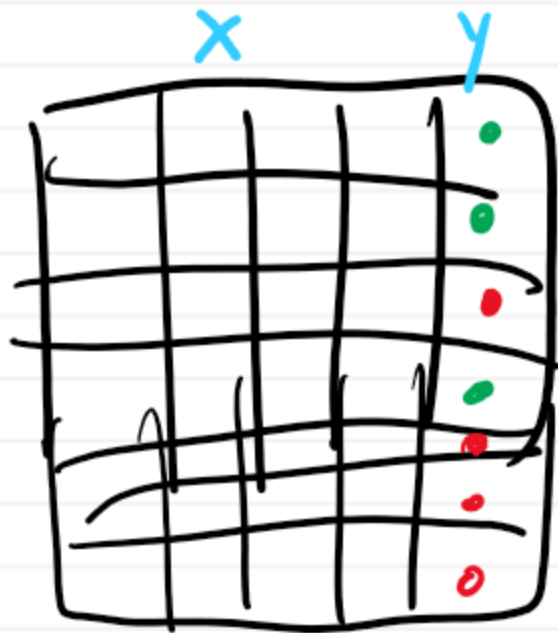
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## Overfitting and Tree Pruning

- Overfitting: An induced tree may overfit the training data
  - Too many branches, some may reflect anomalies due to noise or outliers
  - Poor accuracy for unseen samples
- Two approaches to avoid overfitting
  - Prepruning: Halt tree construction early—do not split a node if this would result in the goodness measure falling below a threshold
    - Difficult to choose an appropriate threshold
  - Postpruning: Remove branches from a "fully grown" tree—get a sequence of progressively pruned trees
    - Use a set of data different from the training data to decide which is the "best pruned tree"



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Occam's Razor

Criterion = เกณฑ์

splitter = วิธีการ node

max\_depth = จำนวนชั้นของ decision tree

min\_samples\_split = จำนวน data ที่น้อยที่สุด  
ที่ split data ใน node

min\_samples\_leaf = เกณฑ์ขั้นต่ำของ node

max\_leaf\_node = จำนวน max node หนึ่ง

max\_features = จำนวน features ที่