RAMP

The Research Assistant for Maniplexes and Polytopes

0.3

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Chapter 1

Constructions

1.1 Extensions, amalgamations, and quotients

1.1.1 UniversalPolytope (for IsInt)

▷ `UniversalPolytope(n)` (operation)

Returns the universal polytope of rank `n`.

1.1.2 FlatRegularPolyhedron (for IsInt, IsInt, IsInt, IsInt)

▷ `FlatRegularPolyhedron(p, q, i, j)` (operation)

Returns the flat regular polyhedron with automorphism group [p, q] / (r2 r1 r0 r1 = (r0 r1)^i (r1 r2)^j). This function does not currently validate the inputs to make sure that the output makes sense.

1.1.3 QuotientPolytope (for IsManiplex, IsList)

▷ `QuotientPolytope(M, rels)` (operation)

Returns the quotient of `M` by `rels`, which may be given as either a list of Tietze words, such as [[1,2,1,0,1,2,1,0]] or as a string like "(r0 r1 r2 r1)^2, (r0 r1 r2)^4".

1.1.4 UniversalExtension (for IsManiplex)

▷ `UniversalExtension(M)` (operation)

Returns the universal extension of `M`, i.e. the maniplex with facets isomorphic to `M` that covers all other maniplexes with facets isomorphic to `M`. Currently only defined for reflexible maniplexes.

1.1.5 UniversalExtension (for IsManiplex, IsInt)

▷ `UniversalExtension(M, k)` (operation)

Returns the universal extension of `M` with last entry of Schlafli symbol `k`. Currently only defined for reflexible maniplexes.

1.1.6 TrivialExtension (for IsManiplex)

▷ `TrivialExtension(M)` (operation)

Returns the trivial extension of `M`, also known as `{M/, 2}`.

1.1.7 FlatExtension (for IsManiplex, IsInt)

▷ `FlatExtension(M, k)` (operation)

Returns the flat extension of `M` with last entry of Schlafli symbol `k`. (As defined in "Flat Extensions of Abstract Polytopes".) Currently only defined for reflexible maniplexes.

1.1.8 Amalgamate (for IsManiplex, IsManiplex)

▷ `Amalgamate(M1, M2)` (operation)

Returns the amalgamation of `M1` and `M2`. Implicitly assumes that `M1` and `M2` are compatible. Currently only defined for reflexible maniplexes.

1.1.9 Medial (for IsManiplex)

▷ `Medial(M)` (operation)

Given a 3-maniplex `M`, returns its medial.

1.2 Duality

1.2.1 Dual (for IsManiplex)

▷ `Dual(M)` (attribute)

Returns: The maniplex that is dual to `M`.

1.2.2 IsSelfDual (for IsManiplex)

▷ `IsSelfDual(P)` (property)

Returns: Whether this polytope is isomorphic to its dual.

1.2.3 Petrial (for IsManiplex)

▷ `Petrial(P)` (attribute)

Returns: The Petrial (Petrie dual) of `P`. Note that this is not necessarily a polytope.

1.2.4 IsSelfPetrial (for IsManiplex)

▷ `IsSelfPetrial(P)` (property)

Returns: Whether this polytope is isomorphic to its Petrial.

1.3 Products

1.3.1 PyramidOver (for IsManiplex)

▷ `PyramidOver(`*`M`*`)` (operation)

Returns the pyramid over *M*. Currently only works for finite maniplexes.

1.3.2 PrismOver (for IsManiplex)

▷ `PrismOver(`*`M`*`)` (operation)

Returns the prism over *M*. Currently only works for finite maniplexes.

Chapter 2

Databases

2.1 Regular polyhedra

2.1.1 DegeneratePolyhedra (for IsInt)

▷ `DegeneratePolyhedra(`*`maxsize`*`)` (operation)

Returns all degenerate polyhedra (of type {2, q} and {p, 2}) with up to *maxsize* flags.

2.1.2 FlatRegularPolyhedra (for IsInt)

▷ `FlatRegularPolyhedra(`*`maxsize`*`)` (operation)

Returns all nondegenerate flat regular polyhedra with up to *maxsize* flags. Currently supports a maxsize of 4000 or less.

2.1.3 SmallRegularPolyhedra (for IsInt)

▷ `SmallRegularPolyhedra(`*`maxsize`*`)` (operation)

Returns all regular polyhedra with up to *maxsize* flags. Currently supports a maxsize of 4000 or less. You can also set options "nondegenerate" and "nonflat".

Example

```
L1 := SmallRegularPolyhedra(500);;
L2 := SmallRegularPolyhedra(1000 : nondegenerate);;
L3 := SmallRegularPolyhedra(2000 : nondegenerate, nonflat);;
```

Chapter 3

Combinatorics and Structure

3.1 Faces

3.1.1 NumberOfIFaces (for IsManiplex, IsInt)

▷ `NumberOfIFaces(M, i)` (operation)

Returns The number of i-faces of M.

3.1.2 NumberOfVertices (for IsManiplex)

▷ `NumberOfVertices(M)` (attribute)

Returns the number of vertices of M.

3.1.3 NumberOfEdges (for IsManiplex)

▷ `NumberOfEdges(M)` (attribute)

Returns the number of edges of M.

3.1.4 NumberOfFacets (for IsManiplex)

▷ `NumberOfFacets(M)` (attribute)

Returns the number of facets of M.

3.1.5 NumberOfRidges (for IsManiplex)

▷ `NumberOfRidges(M)` (attribute)

Returns the number of ridges ((n-2)-faces) of M.

3.1.6 Fvector (for IsManiplex)

▷ `Fvector(M)` (attribute)

Returns the f-vector of `M`.

3.1.7 Facets (for IsManiplex)

▷ `Facets(M)` (attribute)

Returns the facet-types of `M` (i.e. the maniplexes corresponding to the facets). Currently only works for reflexible maniplexes.

3.1.8 VertexFigures (for IsManiplex)

▷ `VertexFigures(M)` (attribute)

Returns the types of vertex-figures of `M` (i.e. the maniplexes corresponding to the vertex-figures). Currently only works for reflexible maniplexes.

3.2 Posets

3.2.1 PosetFromFaceListOfFlags (for IsList)

▷ `PosetFromFaceListOfFlags(list)` (operation)

Returns: *IsPosetOfFlags*. Not that the function is INTENTIONALLY agnostic about whether it is being given full poset or not.

Given a `list` of lists of faces in increasing rank, where each face is described by the incident flags, gives you a IsPosetOfFlags object back.

3.2.2 IsFull (for IsPoset)

▷ `IsFull(poset)` (attribute)

Returns: *true* or *false*

Checks or creates the value of the attribute IsFull for an IsPoset.

3.2.3 IsFlaggable (for IsPoset)

▷ `IsFlaggable(arg)` (attribute)

3.2.4 IsFlaggablePoset (for IsPosetOfFlags)

▷ `IsFlaggablePoset(poset)` (operation)

Returns: *true* or *false*

Given a `poset` (whose elements are lists of flags) corresponding to a maniplex, this function will tell you if it is flaggable, i.e., if the flags can be recovered from the poset or not.

3.2.5 ListIsFullPoset (for IsList)

▷ `ListIsFullPoset(list)` (operation)

Returns: *true* or *false*

Given `list`, a poset as a list of faces ordered by rank, each face listing the flags on the face, this function will tell you if the poset is full or not.

3.2.6 RankOfPoset (for IsPoset)

▷ `RankOfPoset(poset)` (operation)

Returns: *integer*

Given a `poset`, returns the rank of the poset. Note: There may be hidden assumptions here to untangle later.

3.2.7 IsNotFull (for IsPoset)

▷ `IsNotFull(poset)` (operation)

Returns: *true* or *false*

Lets me check to see if a poset is NOT full. For use in certain filtering operations.

3.2.8 PosetOfConnectionGroup (for IsPermGroup)

▷ `PosetOfConnectionGroup(g)` (operation)

Returns: *IsPosetOfFlags* with *IsFull*=false.

Given a group, returns a poset with an internal representation as a list of faces ordered by rank, where each face is represented as a list of the flags it contains. Note that this function does not include the minimal (empty) face nor the maximal face of the maniplex. Note that the i-faces correspond to the $i+1$ item in the list because of how GAP indexes lists.

3.2.9 FullPosetOfConnectionGroup (for IsPermGroup)

▷ `FullPosetOfConnectionGroup(g)` (operation)

Returns: *IsPosetOfFlags* with *IsFull*=true.

Returns a full poset corresponding to the connection group g with an internal representation as a list of faces ordered by rank, where each face is represented as a list of the flags it contains. This function does include the minimal (empty) face nor the maximal face of the maniplex, so the list has $n+2$ ranks if the maniplex is of rank n. Note that the i-faces correspond to the $i+1$ item in the list because of how GAP indexes lists.

3.2.10 PosetOfManiplex (for IsManiplex)

▷ `PosetOfManiplex(mani)` (operation)

Returns: *IsPosetOfFlags*

Given a maniplex, returns a poset of the maniplex with an internal representation as a list of faces ordered by rank, where each face is represented as a list of the flags it contains. Note that this function does not include the minimal (empty) face nor the maximal face of the maniplex. Note that the i-faces correspond to the $i+1$ item in the list because of how GAP indexes lists.

3.2.11 FullPosetOfManiplex (for IsManiplex)

▷ `FullPosetOfManiplex(mani)` (operation)

Returns: *IsPosetOfFlags*

Given a maniplex, returns a poset with the internal representation be a list of lists of faces ordered by rank, where each face is represented as a list of the flags it contains. Note that this function does include the minimal (empty) face and the maximal face of the maniplex. Note that the i-faces correspond to the $i+1$ item in the list because of how GAP indexes lists.

3.2.12 AreIncidentFaces (for IsObject,IsObject)

▷ `AreIncidentFaces(object1, object2)` (operation)

Returns: *true* or *false*

Given two faces, will tell you if they are incident. Currently only supports faces as list of their incident flags.

3.2.13 FlagsAsListOfFacesFromPoset (for IsPoset)

▷ `FlagsAsListOfFacesFromPoset(poset)` (operation)

Returns: *IsList*

Given a `poset`, this will give you a version of the list of flags in terms of the faces described in the `poset`. Note that the flag list does not include the empty face or the maximal face.

3.2.14 AdjacentFlag (for IsPosetOfFlags,IsList,IsInt)

▷ `AdjacentFlag(poset, flag, i)` (operation)

Returns: *flag(s)*

Given a flag (represented as chains of faces comprised of lists of flags) and a poset and a rank, this function will give you the `i`-adjacent flag. Note that adjacencies are listed from ranks 0 to one less than the dimension. You can replace `flag` with the integer corresponding to that flag. AdjacentFlag(*poset*,*i*,*j*) returns the *j*-adjacent flag to the *i*th flag in *poset*. AdjacentFlag(*poset*,*i*,*j*,*true*) will give the position of the flag.

3.2.15 ConnectionGeneratorOfPoset (for IsPoset,IsInt)

▷ `ConnectionGeneratorOfPoset(poset, i)` (operation)

Returns: A permutation on the flags.

Given a `poset` and an integer *i*, this function will give you the associated permutation for the rank *i*-connection.

3.2.16 ConnectionGroupOfPoset (for IsPoset)

▷ `ConnectionGroupOfPoset(poset)` (operation)

Returns: *IsPermGroup*

Given a `poset` corresponding to a maniplex, this function will give you the connection group.

Chapter 4

Families of Polytopes

4.1 Classical Polytopes

4.1.1 Vertex

▷ `Vertex()` (operation)

4.1.2 Edge

▷ `Edge()` (operation)

4.1.3 Pgon (for IsInt)

▷ `Pgon(`*p*`)` (operation)

4.1.4 Cube (for IsInt)

▷ `Cube(`*n*`)` (operation)

4.1.5 HemiCube (for IsInt)

▷ `HemiCube(`*n*`)` (operation)

4.1.6 CrossPolytope (for IsInt)

▷ `CrossPolytope(`*n*`)` (operation)

4.1.7 HemiCrossPolytope (for IsInt)

▷ `HemiCrossPolytope(`*n*`)` (operation)

4.1.8 Simplex (for IsInt)

▷ `Simplex(`*n*`)` (operation)

4.1.9 CubicTiling (for IsInt)

▷ `CubicTiling(`*n*`)` (operation)

4.1.10 Dodecahedron

▷ `Dodecahedron()` (operation)

4.1.11 HemiDodecahedron

▷ `HemiDodecahedron()` (operation)

4.1.12 Icosahedron

▷ `Icosahedron()` (operation)

4.1.13 HemiIcosahedron

▷ `HemiIcosahedron()` (operation)

4.1.14 24Cell

▷ `24Cell()` (operation)

4.1.15 Hemi24Cell

▷ `Hemi24Cell()` (operation)

4.1.16 120Cell

▷ `120Cell()` (operation)

4.1.17 Hemi120Cell

▷ `Hemi120Cell()` (operation)

4.1.18 600Cell

▷ `600Cell()` (operation)

4.1.19 Hemi600Cell

▷ `Hemi600Cell()` (operation)

Chapter 5

Groups

5.1 Groups

5.1.1 AutomorphismGroup (for IsManiplex)

▷ `AutomorphismGroup(M)` (attribute)

Returns the automorphism group of M. This group is not guaranteed to be in any particular form.

5.1.2 AutomorphismGroupFpGroup (for IsManiplex)

▷ `AutomorphismGroupFpGroup(M)` (attribute)

Returns the automorphism group of M as a finitely presented group.

5.1.3 AutomorphismGroupPermGroup (for IsManiplex)

▷ `AutomorphismGroupPermGroup(M)` (attribute)

Returns the automorphism group of M as a permutation group.

5.1.4 ConnectionGroup (for IsManiplex)

▷ `ConnectionGroup(M)` (attribute)

Returns the connection group of M as a permutation group. We may eventually allow other types of connection groups.

5.1.5 EvenConnectionGroup (for IsManiplex)

▷ `EvenConnectionGroup(M)` (attribute)

Returns the even-word subgroup of the connection group of M as a permutation group.

5.1.6 RotationGroup (for IsManiplex)

▷ `RotationGroup(M)` (attribute)

Returns the rotation group of `M`. This group is not guaranteed to be in any particular form.

5.1.7 ExtraRelators (for IsReflexibleManiplex)

▷ `ExtraRelators(M)` (attribute)

For a reflexible maniplex `M`, returns the relators needed to define its automorphism group as a quotient of the string Coxeter group given by its Schlafli symbol. Not particularly robust at the moment.

5.1.8 IsStringC (for IsGroup)

▷ `IsStringC(G)` (operation)

For an sggi `G`, returns whether the group is a string C group. It does not check whether `G` is an sggi.

Chapter 6

Properties

6.1 Orientability

6.1.1 IsOrientable (for IsManiplex)

▷ `IsOrientable(p)` (property)

Returns: `true` or `false`

A polytope is orientable if its flag graph is bipartite. Currently only implemented for regular polytopes.

6.1.2 IsIOrientable (for IsManiplex, IsList)

▷ `IsIOrientable(p, I)` (operation)

For a subset I of {0, ..., n-1}, a polytope if I-orientable if every closed path in its flag graph contains an even number of edges with colors in I. Currently only implemented for regular polytopes.

6.1.3 IsVertexBipartite (for IsManiplex)

▷ `IsVertexBipartite(p)` (property)

Returns: `true` or `false`

A polytope is vertex-bipartite if its 1-skeleton is bipartite. This is equivalent to being I-orientable for I = {0}.

6.1.4 IsFacetBipartite (for IsManiplex)

▷ `IsFacetBipartite(p)` (property)

Returns: `true` or `false`

A polytope is facet-bipartite if the 1-skeleton of its dual is bipartite. This is equivalent to being I-orientable for I = {n-1}.

Chapter 7

Basics

7.1 Constructors

7.1.1 UniversalSggi

▷ `UniversalSggi(`*`n`*`)` (operation)
▷ `UniversalSggi(`*`sym`*`)` (operation)

In the first form, returns the universal Coxeter Group of rank n. In the second form, returns the Coxeter Group with Schlafli symbol sym.

7.1.2 ReflexibleManiplex (for IsGroup)

▷ `ReflexibleManiplex(`*`g`*`)` (operation)

Given a group g (which should be a string C-group), returns the abstract regular polytope with that automorphism group, where the privileged generators are those returned by GeneratorsOfGroup(g).

7.1.3 ReflexibleManiplex (for IsList, IsList)

▷ `ReflexibleManiplex(`*`symbol, relations`*`)` (operation)

Returns an abstract regular polytope with the given Schlafli symbol and with the given relations. The formatting of the relations is quite flexible. All of the following work:

Example

```
q := ReflexibleManiplex([4,3,4], "(r0 r1 r2)^3, (r1 r2 r3)^3");
q := ReflexibleManiplex([4,3,4], "(r0 r1 r2)^3 = (r1 r2 r3)^3 = 1");
p := ReflexibleManiplex([infinity], "r0 r1 r0 = r1 r0 r1");
```

7.1.4 ReflexibleManiplex (for IsString)

▷ `ReflexibleManiplex(`*`name`*`)` (operation)

Returns the regular polytope with the given symbolic name. Examples: ReflexibleManiplex("{3,3,3}"); ReflexibleManiplex("{4,3}_3");

Chapter 8

Comparing maniplexes

8.1 Quotients and covers

8.1.1 IsQuotientOf (for IsManiplex, IsManiplex)

▷ `IsQuotientOf(`*M1*`, `*M2*`)` (operation)

Returns whether *M1* is a quotient of *M2*.

8.1.2 IsCoverOf (for IsManiplex, IsManiplex)

▷ `IsCoverOf(`*M1*`, `*M2*`)` (operation)

Returns whether *M1* is a cover of *M2*.

8.1.3 IsIsomorphicTo (for IsManiplex, IsManiplex)

▷ `IsIsomorphicTo(`*M1*`, `*M2*`)` (operation)

Returns whether *M1* is isomorphic to *M2*.

8.1.4 SmallestRegularCover (for IsManiplex)

▷ `SmallestRegularCover(`*M*`)` (attribute)

Returns the smallest regular cover of *M*, which is the maniplex whose automorphism group is the connection group of *M*.

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