

# **RAMP**

## **The Research Assistant for Maniplexes and Polytopes**

### **0.3**

28 July 2020

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## Acknowledgements

We appreciate very much all past and future comments, suggestions and contributions to this package and its documentation provided by **GAP** users and developers.

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# Chapter 1

## Graphs for Maniplexes

### 1.1 Graphs for maniplexes functions

#### 1.1.1 DirectedGraphFromListOfEdges (for IsList,IsList)

▷ DirectedGraphFromListOfEdges(*list*, *list*) (operation)

**Returns:** IsGraph. Note this returns a directed graph.

Given a list of vertices and a list of directed-edges (represented as ordered pairs), this outputs the directed graph with the appropriate vertex and directed-edge set.

Here we have a directed cycle on 3 vertices.

Example

```
gap> g:= DirectedGraphFromListOfEdges([1,2,3],[[1,2],[2,3],[3,1]]);
rec( adjacencies := [ [ 2 ], [ 3 ], [ 1 ] ], group := Group(),
    isGraph := true, names := [ 1, 2, 3 ], order := 3,
    representatives := [ 1, 2, 3 ], schreierVector := [ -1, -2, -3 ] )
```

#### 1.1.2 GraphFromListOfEdges (for IsList,IsList)

▷ GraphFromListOfEdges(*list*, *list*) (operation)

**Returns:** IsGraph. Note this returns an undirected graph.

Given a list of vertices and a list of (directed) edges (represented as ordered pairs), this outputs the simple underlying graph with the appropriate vertex and directed-edge set.

Here we have a simple complete graph on 4 vertices.

Example

```
gap> g:= GraphFromListOfEdges([1,2,3,4],[[1,2],[2,3],[3,1], [1,4], [2,4], [3,4]]);
rec(
    adjacencies := [ [ 2, 3, 4 ], [ 1, 3, 4 ], [ 1, 2, 4 ], [ 1, 2, 3 ] ],
    group := Group(), isGraph := true, isSimple := true,
    names := [ 1, 2, 3, 4 ], order := 4, representatives := [ 1, 2, 3, 4 ]
    , schreierVector := [ -1, -2, -3, -4 ] )
```

#### 1.1.3 UnlabeledFlagGraph (for IsGroup)

▷ UnlabeledFlagGraph(*group*) (operation)

**Returns:** IsGraph. Note this returns an undirected graph.

Given a group (assumed to be the connection group of a maniplex), this outputs the simple underlying flag graph.

Here we build the flag graph for the cube from its connection group.

Example

```
gap> g:= UnlabeledFlagGraph(ConnectionGroup(Cube(3)));
rec(
  adjacencies := [ [ 3, 11, 20 ], [ 7, 13, 18 ], [ 1, 4, 10 ],
    [ 3, 25, 34 ], [ 26, 28, 35 ], [ 7, 13, 41 ], [ 2, 6, 8 ],
    [ 7, 27, 32 ], [ 28, 33, 35 ], [ 3, 20, 45 ], [ 1, 14, 23 ],
    [ 15, 17, 24 ], [ 2, 6, 31 ], [ 11, 25, 44 ], [ 12, 45, 47 ],
    [ 18, 28, 40 ], [ 12, 19, 27 ], [ 2, 16, 21 ], [ 17, 22, 24 ],
    [ 1, 10, 38 ], [ 18, 32, 40 ], [ 19, 41, 48 ], [ 11, 35, 44 ],
    [ 12, 19, 34 ], [ 4, 14, 37 ], [ 5, 38, 42 ], [ 8, 17, 30 ],
    [ 5, 9, 16 ], [ 39, 41, 48 ], [ 27, 32, 47 ], [ 13, 33, 39 ],
    [ 8, 21, 30 ], [ 9, 31, 46 ], [ 4, 24, 37 ], [ 5, 9, 23 ],
    [ 43, 45, 47 ], [ 25, 34, 48 ], [ 20, 26, 43 ], [ 29, 31, 46 ],
    [ 16, 21, 42 ], [ 6, 22, 29 ], [ 26, 40, 43 ], [ 36, 38, 42 ],
    [ 14, 23, 46 ], [ 10, 15, 36 ], [ 33, 39, 44 ], [ 15, 30, 36 ],
    [ 22, 29, 37 ] ], group := Group(), isGraph := true,
  isSimple := true, names := [ 1 .. 48 ], order := 48,
  representatives := [ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,
    15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30,
    31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46,
    47, 48 ],
  schreierVector := [ -1, -2, -3, -4, -5, -6, -7, -8, -9, -10, -11,
    -12, -13, -14, -15, -16, -17, -18, -19, -20, -21, -22, -23, -24,
    -25, -26, -27, -28, -29, -30, -31, -32, -33, -34, -35, -36, -37,
    -38, -39, -40, -41, -42, -43, -44, -45, -46, -47, -48 ] )
```

This also works with a maniplex input. Here we build the flag graph for the cube.

Example

```
gap> g:= UnlabeledFlagGraph(Cube(3));
```

### 1.1.4 FlagGraphWithLabels (for IsGroup)

▷ FlagGraphWithLabels(group)

(operation)

**Returns:** a triple [IsGraph, IsList, IsList].

Given a group (assumed to be the connection group of a maniplex), this outputs a triple [graph,list,list]. The graph is the unlabeled flag graph of the connection group. The first list gives the undirected edges in the flag graphs. The second list gives the labels for these edges.

Here we again build the flag graph for the cube from its connection group, but this time keep track of labels of the edges.

Example

```
gap> g:= FlagGraphWithLabels(ConnectionGroup(Cube(3)));
[ rec(
  adjacencies := [ [ 3, 11, 20 ], [ 7, 13, 18 ], [ 1, 4, 10 ],
    [ 3, 25, 34 ], [ 26, 28, 35 ], [ 7, 13, 41 ], [ 2, 6, 8 ],
    [ 7, 27, 32 ], [ 28, 33, 35 ], [ 3, 20, 45 ], [ 1, 14, 23 ],
    [ 15, 17, 24 ], [ 2, 6, 31 ], [ 11, 25, 44 ], [ 12, 45, 47 ],
    [ 18, 28, 40 ], [ 12, 19, 27 ], [ 2, 16, 21 ],
    [ 17, 22, 24 ], [ 1, 10, 38 ], [ 18, 32, 40 ],
```

```

[ 19, 41, 48 ], [ 11, 35, 44 ], [ 12, 19, 34 ],
[ 4, 14, 37 ], [ 5, 38, 42 ], [ 8, 17, 30 ], [ 5, 9, 16 ],
[ 39, 41, 48 ], [ 27, 32, 47 ], [ 13, 33, 39 ],
[ 8, 21, 30 ], [ 9, 31, 46 ], [ 4, 24, 37 ], [ 5, 9, 23 ],
[ 43, 45, 47 ], [ 25, 34, 48 ], [ 20, 26, 43 ],
[ 29, 31, 46 ], [ 16, 21, 42 ], [ 6, 22, 29 ],
[ 26, 40, 43 ], [ 36, 38, 42 ], [ 14, 23, 46 ],
[ 10, 15, 36 ], [ 33, 39, 44 ], [ 15, 30, 36 ],
[ 22, 29, 37 ] ], group := Group()), isGraph := true,
isSimple := true, names := [ 1 .. 48 ], order := 48,
representatives := [ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13,
14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28,
29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43,
44, 45, 46, 47, 48 ],
schreierVector := [ -1, -2, -3, -4, -5, -6, -7, -8, -9, -10, -11,
-12, -13, -14, -15, -16, -17, -18, -19, -20, -21, -22, -23,
-24, -25, -26, -27, -28, -29, -30, -31, -32, -33, -34, -35,
-36, -37, -38, -39, -40, -41, -42, -43, -44, -45, -46, -47,
-48 ] ),
[ [ 1, 3 ], [ 1, 11 ], [ 1, 20 ], [ 2, 7 ], [ 2, 13 ], [ 2, 18 ],
[ 3, 4 ], [ 3, 10 ], [ 4, 25 ], [ 4, 34 ], [ 5, 26 ], [ 5, 28 ],
[ 5, 35 ], [ 6, 7 ], [ 6, 13 ], [ 6, 41 ], [ 7, 8 ], [ 8, 27 ],
[ 8, 32 ], [ 9, 28 ], [ 9, 33 ], [ 9, 35 ], [ 10, 20 ],
[ 10, 45 ], [ 11, 14 ], [ 11, 23 ], [ 12, 15 ], [ 12, 17 ],
[ 12, 24 ], [ 13, 31 ], [ 14, 25 ], [ 14, 44 ], [ 15, 45 ],
[ 15, 47 ], [ 16, 18 ], [ 16, 28 ], [ 16, 40 ], [ 17, 19 ],
[ 17, 27 ], [ 18, 21 ], [ 19, 22 ], [ 19, 24 ], [ 20, 38 ],
[ 21, 32 ], [ 21, 40 ], [ 22, 41 ], [ 22, 48 ], [ 23, 35 ],
[ 23, 44 ], [ 24, 34 ], [ 25, 37 ], [ 26, 38 ], [ 26, 42 ],
[ 27, 30 ], [ 29, 39 ], [ 29, 41 ], [ 29, 48 ], [ 30, 32 ],
[ 30, 47 ], [ 31, 33 ], [ 31, 39 ], [ 33, 46 ], [ 34, 37 ],
[ 36, 43 ], [ 36, 45 ], [ 36, 47 ], [ 37, 48 ], [ 38, 43 ],
[ 39, 46 ], [ 40, 42 ], [ 42, 43 ], [ 44, 46 ] ],
[ 3, 2, 1, 3, 1, 2, 2, 1, 3, 1, 2, 3, 1, 1, 3, 2, 2, 1, 3, 1, 2, 3,
3, 2, 3, 1, 2, 3, 1, 2, 2, 1, 1, 3, 1, 2, 3, 1, 2, 3, 2, 3, 2, 2,
1, 1, 3, 2, 3, 2, 1, 1, 3, 3, 2, 3, 1, 1, 2, 1, 3, 3, 3, 2, 3, 1,
2, 3, 1, 2, 1, 2 ] ]

```

This also works with a maniplex input. Here we build the flag graph for the cube.

Example

```
gap> g:= FlagGraphWithLabels(Cube(3));
```

### 1.1.5 LayerGraph (for IsGroup, IsInt, IsInt)

▷ LayerGraph([group, int, int])

(operation)

**Returns:** IsGraph. Note this returns an undirected graph.

Given a group (assumed to be the connection group of a maniplex), and two integers, this outputs the simple underlying graph given by incidences of faces of those ranks. Note: There are no warnings yet to make sure that  $i, j$  are bounded by the rank.

Here we build the graph given by the 6 faces and 12 edges of a cube from its connection group.

Example

```
gap> g:= LayerGraph(ConnectionGroup(Cube(3)),2,1);
rec(
  adjacencies := [ [ 7, 10, 12, 17 ], [ 8, 10, 15, 18 ],
    [ 7, 9, 13, 14 ], [ 8, 11, 13, 16 ], [ 9, 12, 16, 18 ],
    [ 11, 14, 15, 17 ], [ 1, 3 ], [ 2, 4 ], [ 3, 5 ], [ 1, 2 ],
    [ 4, 6 ], [ 1, 5 ], [ 3, 4 ], [ 3, 6 ], [ 2, 6 ], [ 4, 5 ],
    [ 1, 6 ], [ 2, 5 ] ], group := Group(), isGraph := true,
  isSimple := true, names := [ 1 .. 18 ], order := 18,
  representatives := [ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,
    15, 16, 17, 18 ],
  schreierVector := [ -1, -2, -3, -4, -5, -6, -7, -8, -9, -10, -11,
    -12, -13, -14, -15, -16, -17, -18 ] )
```

This also works with a maniplex input. Here we build the graph given by the 6 faces and 12 edges of a cube.

Example

```
gap> g:= LayerGraph(Cube(3),2,1);;
```

### 1.1.6 Skeleton (for IsManiplex)

▷ `Skeleton(maniplex)`

(operation)

**Returns:** `IsGraph`. Note this returns an undirected graph.

Given a maniplex, this outputs the 0-1 skeleton. The vertices are the 0-faces, and the edges are the 1-faces.

Here we build the skeleton of the dodecahedron.

Example

```
gap> g:= Skeleton(Dodecahedron());;
```

### 1.1.7 Hasse (for IsManiplex)

▷ `Hasse(group)`

(operation)

**Returns:** `IsGraph`. Note this returns a directed graph.

Given a group, assumed to be the connection group of a maniplex, this outputs the Hasse Diagram as a directed graph. Note: The unique minimal and maximal face are assumed.

Here we build the Hasse Diagram of a 3-simplex from its representation as a maniplex.

Example

```
gap> Hasse(Simplex(3));
rec(
  adjacencies := [ [ ], [ 1 ], [ 1 ], [ 1 ], [ 1 ], [ 2, 4 ],
    [ 2, 3 ], [ 3, 5 ], [ 2, 5 ], [ 4, 5 ], [ 3, 4 ], [ 6, 9, 10 ],
    [ 6, 7, 11 ], [ 8, 10, 11 ], [ 7, 8, 9 ], [ 12, 13, 14, 15 ] ],
  group := Group(), isGraph := true, names := [ 1 .. 16 ],
  order := 16,
  representatives := [ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,
    15, 16 ],
  schreierVector := [ -1, -2, -3, -4, -5, -6, -7, -8, -9, -10, -11,
    -12, -13, -14, -15, -16 ] )
```



### 1.1.8 QuotientByLabel (for IsObject,IsList, IsList, IsList)

▷ `QuotientByLabel(object, list, list, list)` (operation)

**Returns:** `IsGraph`. Note this returns an undirected graph.

Given a graph, its edges, and its edge labels, and a sublist of labels, this creates the underlying simple graph of the quotient identifying vertices connected by labels not in the sublist.

Here we start with the flag graph of the 3-cube (with edge labels 1,2,3), and identify any vertices not connected by edge by edges of label 1. We can then check that this new graph is bipartite.

Example

```
gap> P:=Cube(3);;
gap> f:=FlagGraphWithLabels(P);;
gap> g:=f[1];;
gap> ed:=f[2];;
gap> lab:=f[3]; #Note This triple is to be replace by a single object.
[ 3, 2, 1, 3, 1, 2, 1, 2, 3, 2, 1, 3, 2, 1, 1, 3, 2, 2, 3, 1, 3, 1, 2, 3, 2, 1, 1, 2, 2, 3, 1, 3,
  3, 1, 2, 1, 3, 2, 2, 1, 2, 2, 3, 1, 1, 3, 1, 3, 3, 2, 1, 2, 1, 3, 3, 1, 3, 2, 2, 2, 2, 3, 3, 1,
gap> Q:=QuotientByLabel(g,ed,lab,[1]);
rec( adjacencies := [ [ 5, 6, 8 ], [ 3, 4, 7 ], [ 2, 6, 8 ], [ 2, 5, 8 ], [ 1, 4, 7 ], [ 1, 3, 7 ],
  isSimple := true, names := [ 1 .. 8 ], order := 8, representatives := [ 1, 2, 3, 4, 5, 6, 7, 8 ]
gap> IsBipartite(Q);
true
```

## Chapter 2

# Constructions

### 2.1 Extensions, amalgamations, and quotients

#### 2.1.1 UniversalPolytope (for IsInt)

▷ `UniversalPolytope( $n$ )` (operation)

Returns the universal polytope of rank  $n$ .

#### 2.1.2 FlatRegularPolyhedron (for IsInt, IsInt, IsInt, IsInt)

▷ `FlatRegularPolyhedron( $p, q, i, j$ )` (operation)

Returns the flat regular polyhedron with automorphism group  $[p, q] / (r_2 r_1 r_0 r_1 = (r_0 r_1)^i (r_1 r_2)^j)$ . This function does not currently validate the inputs to make sure that the output makes sense.

#### 2.1.3 QuotientPolytope (for IsManiplex, IsList)

▷ `QuotientPolytope( $M, rels$ )` (operation)

Returns the quotient of  $M$  by  $rels$ , which may be given as either a list of Tietze words, such as  $[[1,2,1,0,1,2,1,0]]$  or as a string like  $"(r_0 r_1 r_2 r_1)^2, (r_0 r_1 r_2)^4"$ .

#### 2.1.4 UniversalExtension (for IsManiplex)

▷ `UniversalExtension( $M$ )` (operation)

Returns the universal extension of  $M$ , i.e. the maniplex with facets isomorphic to  $M$  that covers all other maniplexes with facets isomorphic to  $M$ . Currently only defined for reflexible maniplexes.

#### 2.1.5 UniversalExtension (for IsManiplex, IsInt)

▷ `UniversalExtension( $M, k$ )` (operation)

Returns the universal extension of  $M$  with last entry of Schläfli symbol  $k$ . Currently only defined for reflexible maniplexes.

### 2.1.6 TrivialExtension (for IsManiplex)

▷ `TrivialExtension( $M$ )` (operation)

Returns the trivial extension of  $M$ , also known as  $\{M/, 2\}$ .

### 2.1.7 FlatExtension (for IsManiplex, IsInt)

▷ `FlatExtension( $M, k$ )` (operation)

Returns the flat extension of  $M$  with last entry of Schlafli symbol  $k$ . (As defined in "Flat Extensions of Abstract Polytopes".) Currently only defined for reflexible maniplexes.

### 2.1.8 Amalgamate (for IsManiplex, IsManiplex)

▷ `Amalgamate( $M1, M2$ )` (operation)

Returns the amalgamation of  $M1$  and  $M2$ . Implicitly assumes that  $M1$  and  $M2$  are compatible. Currently only defined for reflexible maniplexes.

### 2.1.9 Medial (for IsManiplex)

▷ `Medial( $M$ )` (operation)

Given a 3-maniplex  $M$ , returns its medial.

## 2.2 Duality

### 2.2.1 Dual (for IsManiplex)

▷ `Dual( $M$ )` (attribute)

**Returns:** The maniplex that is dual to  $M$ .

### 2.2.2 IsSelfDual (for IsManiplex)

▷ `IsSelfDual( $P$ )` (property)

**Returns:** Whether this polytope is isomorphic to its dual.

### 2.2.3 Petrial (for IsManiplex)

▷ `Petrial( $P$ )` (attribute)

**Returns:** The Petrial (Petrie dual) of  $P$ . Note that this is not necessarily a polytope.

### 2.2.4 IsSelfPetrial (for IsManiplex)

▷ `IsSelfPetrial( $P$ )` (property)

**Returns:** Whether this polytope is isomorphic to its Petrial.

## 2.3 Products

### 2.3.1 PyramidOver (for IsManifold)

▷ `PyramidOver( $M$ )` (operation)

Returns the pyramid over  $M$ . Currently only works for finite manifolds.

### 2.3.2 PrismOver (for IsManifold)

▷ `PrismOver( $M$ )` (operation)

Returns the prism over  $M$ . Currently only works for finite manifolds.

## Chapter 3

# Databases

### 3.1 Regular polyhedra

#### 3.1.1 DegeneratePolyhedra (for IsInt)

▷ `DegeneratePolyhedra(maxsize)` (operation)

Returns all degenerate polyhedra (of type  $\{2, q\}$  and  $\{p, 2\}$ ) with up to *maxsize* flags.

#### 3.1.2 FlatRegularPolyhedra (for IsInt)

▷ `FlatRegularPolyhedra(maxsize)` (operation)

Returns all nondegenerate flat regular polyhedra with up to *maxsize* flags. Currently supports a maxsize of 4000 or less.

#### 3.1.3 SmallRegularPolyhedra (for IsInt)

▷ `SmallRegularPolyhedra(maxsize)` (operation)

Returns all regular polyhedra with up to *maxsize* flags. Currently supports a maxsize of 4000 or less. You can also set options "nondegenerate" and "nonflat".

Example

```
L1 := SmallRegularPolyhedra(500);;
L2 := SmallRegularPolyhedra(1000 : nondegenerate);;
L3 := SmallRegularPolyhedra(2000 : nondegenerate, nonflat);;
```

## Chapter 4

# Combinatorics and Structure

### 4.1 Faces

#### 4.1.1 NumberOfFaces (for IsManiplex, IsInt)

▷ `NumberOfFaces( $M$ ,  $i$ )` (operation)

Returns The number of  $i$ -faces of  $M$ .

#### 4.1.2 NumberOfVertices (for IsManiplex)

▷ `NumberOfVertices( $M$ )` (attribute)

Returns the number of vertices of  $M$ .

#### 4.1.3 NumberOfEdges (for IsManiplex)

▷ `NumberOfEdges( $M$ )` (attribute)

Returns the number of edges of  $M$ .

#### 4.1.4 NumberOfFacets (for IsManiplex)

▷ `NumberOfFacets( $M$ )` (attribute)

Returns the number of facets of  $M$ .

#### 4.1.5 NumberOfRidges (for IsManiplex)

▷ `NumberOfRidges( $M$ )` (attribute)

Returns the number of ridges (( $n-2$ )-faces) of  $M$ .

#### 4.1.6 Fvector (for IsManiplex)

▷ `Fvector( $M$ )` (attribute)

Returns the f-vector of  $M$ .

#### 4.1.7 Section (for IsManiplex, IsInt, IsInt)

▷ `Section( $M, j, i$ )` (operation)

Returns the section  $F_j / F_i$ , where  $F_j$  is the  $j$ -face of the base flag of  $M$  and  $F_i$  is the  $i$ -face of the base flag.

#### 4.1.8 Section (for IsManiplex, IsInt, IsInt, IsInt)

▷ `Section( $M, j, i, k$ )` (operation)

Returns the section  $F_j / F_i$ , where  $F_j$  is the  $j$ -face of flag number  $k$  of  $M$  and  $F_i$  is the  $i$ -face of the same flag.

#### 4.1.9 Sections (for IsManiplex, IsInt, IsInt)

▷ `Sections( $M, j, i$ )` (operation)

Returns all sections of type  $F_j / F_i$ , where  $F_j$  is a  $j$ -face and  $F_i$  is an incident  $i$ -face.

#### 4.1.10 Facets (for IsManiplex)

▷ `Facets( $M$ )` (attribute)

Returns the facet-types of  $M$  (i.e. the maniplexes corresponding to the facets).

#### 4.1.11 Facet (for IsManiplex, IsInt)

▷ `Facet( $M, k$ )` (operation)

Returns the facet of  $M$  that contains the flag number  $k$  (that is, the maniplex corresponding to the facet).

#### 4.1.12 Facet (for IsManiplex)

▷ `Facet( $M$ )` (attribute)

Returns the facet of  $M$  that contains flag number 1 (that is, the maniplex corresponding to the facet).

### 4.1.13 VertexFigures (for IsManiplex)

▷ `VertexFigures( $M$ )` (attribute)

Returns the types of vertex-figures of  $M$  (i.e. the maniplexes corresponding to the vertex-figures).

### 4.1.14 VertexFigure (for IsManiplex, IsInt)

▷ `VertexFigure( $M$ ,  $k$ )` (operation)

Returns the vertex-figures of  $M$  that contains flag number  $k$ .

### 4.1.15 VertexFigure (for IsManiplex)

▷ `VertexFigure( $M$ )` (attribute)

Returns the vertex-figures of  $M$  that contains the base flag.

## 4.2 Flatness

### 4.2.1 IsFlat

▷ `IsFlat( $M$ )` (property)

▷ `IsFlat( $M$ ,  $i$ ,  $j$ )` (operation)

**Returns:** true or false

In the first form, returns true if every vertex of the maniplex  $M$  is incident to every facet. In the second form, returns true if every  $i$ -face of the maniplex  $M$  is incident to every  $j$ -face.

## 4.3 Schläfli symbol

### 4.3.1 SchläfliSymbol (for IsManiplex)

▷ `SchläfliSymbol( $M$ )` (attribute)

Returns the Schläfli symbol of the maniplex  $M$ . Each entry is either an integer or a set of integers, where entry number  $i$  shows the polygons that we obtain as sections of  $(i+1)$ -faces over  $(i-2)$ -faces.

### 4.3.2 IsEquivelar (for IsManiplex)

▷ `IsEquivelar( $M$ )` (property)

**Returns:** the the maniplex  $M$  is equivelar; i.e., whether its Schläfli Symbol consists of integers at each position (no lists).

## 4.4 Basics

### 4.4.1 Size (for IsManiplex)



▷ `Size( $M$ )` (attribute)

Returns the number of flags of the maniplex  $M$ .

#### 4.4.2 RankManiplex (for IsManiplex)

▷ `RankManiplex( $M$ )` (attribute)

Returns the rank of the maniplex  $M$ .

#### 4.4.3 IsTight (for IsManiplex and IsPolytopal)

▷ `IsTight( $P$ )` (property)

**Returns:** true or false

Returns whether the polytope  $P$  is tight, meaning that it has a Schläfli symbol  $\{k_1, \dots, k_{n-1}\}$  and has  $2 k_1 \dots k_{n-1}$  flags, which is the minimum possible. This property doesn't make any sense for non-polytopal maniplexes, which aren't constrained by this lower bound.

#### 4.4.4 IsDegenerate (for IsManiplex)

▷ `IsDegenerate( $M$ )` (property)

**Returns:** true or false

Returns whether the maniplex  $M$  has any sections that are digons. We may eventually want to include maniplexes with even smaller sections.

#### 4.4.5 SymmetryTypeGraph (for IsManiplex)

▷ `SymmetryTypeGraph( $M$ )` (attribute)

Returns the Symmetry Type Graph of the maniplex  $M$ , encoded as a permutation group on `Rank( $M$ )` generators.

#### 4.4.6 NumberOfFlagOrbits (for IsManiplex)

▷ `NumberOfFlagOrbits( $M$ )` (attribute)

Returns the number of orbits of the automorphism group of  $M$  on its flags.

#### 4.4.7 FlagOrbitRepresentatives (for IsManiplex)

▷ `FlagOrbitRepresentatives( $M$ )` (attribute)

Returns one flag from each orbit under the action of `AutomorphismGroup( $M$ )`.

#### 4.4.8 IsReflexible (for IsManiplex)

▷ `IsReflexible( $M$ )` (property)

**Returns:** Whether the maniplex  $M$  is reflexible (has one flag orbit).

#### 4.4.9 IsChiral (for IsManiplex)

- ▷ `IsChiral( $M$ )` (property)  
**Returns:** Whether the maniplex  $M$  is chiral.

#### 4.4.10 IsRotary (for IsManiplex)

- ▷ `IsRotary( $M$ )` (property)  
**Returns:** Whether the maniplex  $M$  is rotary; i.e., whether it is either reflexible or chiral.

#### 4.4.11 Description (for IsManiplex)

- ▷ `Description( $M$ )` (attribute)  
 Returns a short name for the maniplex  $M$ , if one is available. For example, `Description(Simplex(3))` = "3-simplex".

### 4.5 Zigzags and holes

#### 4.5.1 ZigzagLength (for IsManiplex, IsInt)

- ▷ `ZigzagLength( $M$ ,  $j$ )` (operation)  
**Returns:** The lengths of  $j$ -zigzags of the 3-maniplex  $M$ . This corresponds to the lengths of orbits under  $r_0$  ( $r_1 r_2$ ) <sup>$j$</sup> .

#### 4.5.2 ZigzagVector (for IsManiplex)

- ▷ `ZigzagVector( $M$ )` (attribute)  
**Returns:** The lengths of all zigzags of the 3-maniplex  $M$ . A rank 3 maniplex of type  $\{p, q\}$  has  $\text{Floor}(q/2)$  distinct zigzag lengths because the  $j$ -zigzags are the same as the  $(q-j)$ -zigzags.

#### 4.5.3 PetrieLength (for IsManiplex)

- ▷ `PetrieLength( $M$ )` (attribute)  
**Returns:** The length of the petrie polygons of the maniplex  $M$ .

#### 4.5.4 HoleLength (for IsReflexibleManiplex)

- ▷ `HoleLength( $M$ )` (attribute)

## Chapter 5

# Families of Polytopes

### 5.1 Classical Polytopes

#### 5.1.1 Vertex

▷ `Vertex()` (operation)

#### 5.1.2 Edge

▷ `Edge()` (operation)

#### 5.1.3 Pgon (for IsInt)

▷ `Pgon( $p$ )` (operation)

#### 5.1.4 Cube (for IsInt)

▷ `Cube( $n$ )` (operation)

#### 5.1.5 HemiCube (for IsInt)

▷ `HemiCube( $n$ )` (operation)

#### 5.1.6 CrossPolytope (for IsInt)

▷ `CrossPolytope( $n$ )` (operation)

### 5.1.7 HemiCrossPolytope (for IsInt)

▷ HemiCrossPolytope( $n$ ) (operation)

### 5.1.8 Simplex (for IsInt)

▷ Simplex( $n$ ) (operation)

### 5.1.9 CubicTiling (for IsInt)

▷ CubicTiling( $n$ ) (operation)

### 5.1.10 Dodecahedron

▷ Dodecahedron() (operation)

### 5.1.11 HemiDodecahedron

▷ HemiDodecahedron() (operation)

### 5.1.12 Icosahedron

▷ Icosahedron() (operation)

### 5.1.13 HemIcosahedron

▷ HemIcosahedron() (operation)

### 5.1.14 24Cell

▷ 24Cell() (operation)

### 5.1.15 Hemi24Cell

▷ Hemi24Cell() (operation)

### 5.1.16 120Cell

▷ 120Cell() (operation)

**5.1.17 Hemi120Cell**

▷ `Hemi120Cell()` (operation)

**5.1.18 600Cell**

▷ `600Cell()` (operation)

**5.1.19 Hemi600Cell**

▷ `Hemi600Cell()` (operation)

# Chapter 6

## Groups

### 6.1 Groups

#### 6.1.1 AutomorphismGroup (for IsManifold)

▷ `AutomorphismGroup( $M$ )` (attribute)

Returns the automorphism group of  $M$ . This group is not guaranteed to be in any particular form.

#### 6.1.2 AutomorphismGroupFpGroup (for IsManifold)

▷ `AutomorphismGroupFpGroup( $M$ )` (attribute)

Returns the automorphism group of  $M$  as a finitely presented group.

#### 6.1.3 AutomorphismGroupPermGroup (for IsManifold)

▷ `AutomorphismGroupPermGroup( $M$ )` (attribute)

Returns the automorphism group of  $M$  as a permutation group.

#### 6.1.4 ConnectionGroup (for IsManifold)

▷ `ConnectionGroup( $M$ )` (attribute)

Returns the connection group of  $M$  as a permutation group. We may eventually allow other types of connection groups.

#### 6.1.5 EvenConnectionGroup (for IsManifold)

▷ `EvenConnectionGroup( $M$ )` (attribute)

Returns the even-word subgroup of the connection group of  $M$  as a permutation group.

### 6.1.6 RotationGroup (for IsManiplex)

▷ `RotationGroup( $M$ )` (attribute)

Returns the rotation group of  $M$ . This group is not guaranteed to be in any particular form.

### 6.1.7 ChiralityGroup (for IsRotaryManiplex)

▷ `ChiralityGroup( $M$ )` (attribute)

Returns the chirality group of the rotary maniplex  $M$ . This is the kernel of the group epimorphism from the rotation group of  $M$  to the rotation group of its maximal reflexible quotient. In particular, the chirality group is trivial if and only if  $M$  is reflexible.

### 6.1.8 ExtraRelators (for IsReflexibleManiplex)

▷ `ExtraRelators( $M$ )` (attribute)

For a reflexible maniplex  $M$ , returns the relators needed to define its automorphism group as a quotient of the string Coxeter group given by its Schläfli symbol. Not particularly robust at the moment.

### 6.1.9 ExtraRotRelators (for IsRotaryManiplex)

▷ `ExtraRotRelators( $M$ )` (attribute)

For a reflexible maniplex  $M$ , returns the relators needed to define its rotation group as a quotient of the rotation group of a string Coxeter group given by its Schläfli symbol. Not particularly robust at the moment.

### 6.1.10 IsStringC (for IsGroup)

▷ `IsStringC( $G$ )` (operation)

For an sggi  $G$ , returns whether the group is a string C group. It does not check whether  $G$  is an sggi.

### 6.1.11 IsStringCPlus (for IsGroup)

▷ `IsStringCPlus( $G$ )` (operation)

For a "string rotation group"  $G$ , returns whether the group is a string C+ group. It does not check whether  $G$  is a string rotation group.

# Chapter 7

## Properties

### 7.1 Orientability

#### 7.1.1 IsOrientable (for IsManiplex)

▷ `IsOrientable(p)` (property)

**Returns:** true or false

A polytope is orientable if its flag graph is bipartite. Currently only implemented for regular polytopes.

#### 7.1.2 IsIOrientable (for IsManiplex, IsList)

▷ `IsIOrientable(p, I)` (operation)

For a subset  $I$  of  $\{0, \dots, n-1\}$ , a polytope is  $I$ -orientable if every closed path in its flag graph contains an even number of edges with colors in  $I$ . Currently only implemented for regular polytopes.

#### 7.1.3 IsVertexBipartite (for IsManiplex)

▷ `IsVertexBipartite(p)` (property)

**Returns:** true or false

A polytope is vertex-bipartite if its 1-skeleton is bipartite. This is equivalent to being  $I$ -orientable for  $I = \{0\}$ .

#### 7.1.4 IsFacetBipartite (for IsManiplex)

▷ `IsFacetBipartite(p)` (property)

**Returns:** true or false

A polytope is facet-bipartite if the 1-skeleton of its dual is bipartite. This is equivalent to being  $I$ -orientable for  $I = \{n-1\}$ .



# Chapter 8

## Basics

### 8.1 Constructors

#### 8.1.1 UniversalSggi

- ▷ `UniversalSggi(n)` (operation)
- ▷ `UniversalSggi(sym)` (operation)

In the first form, returns the universal Coxeter Group of rank *n*. In the second form, returns the Coxeter Group with Schläfli symbol *sym*.

#### 8.1.2 ReflexibleManiplex (for IsGroup)

- ▷ `ReflexibleManiplex(g)` (operation)

Given a group *g* (which should be a string C-group), returns the abstract regular polytope with that automorphism group, where the privileged generators are those returned by `GeneratorsOfGroup(g)`.

#### 8.1.3 ReflexibleManiplex (for IsList, IsList)

- ▷ `ReflexibleManiplex(symbol, relations)` (operation)

Returns an abstract regular polytope with the given Schläfli symbol and with the given relations. The formatting of the relations is quite flexible. All of the following work:

Example

```
q := ReflexibleManiplex([4,3,4], "(r0 r1 r2)^3, (r1 r2 r3)^3");
q := ReflexibleManiplex([4,3,4], "(r0 r1 r2)^3 = (r1 r2 r3)^3 = 1");
p := ReflexibleManiplex([infinity], "r0 r1 r0 = r1 r0 r1");
```

If the option `set_schlafl` is set, then we set the Schläfli symbol to the one given. This may not be the correct Schläfli symbol, since the relations may cause a collapse, so this should only be used if you know that the Schläfli symbol is correct.

#### 8.1.4 ReflexibleManiplex (for IsString)

▷ `ReflexibleManiplex(name)` (operation)

Returns the regular polytope with the given symbolic name. Examples: `ReflexibleManiplex("{3,3,3}")`; `ReflexibleManiplex("{4,3}_3")`; If the option `set_schlafl` is set, then we set the Schlafli symbol to the one given. This may not be the correct Schlafli symbol, since the relations may cause a collapse, so this should only be used if you know that the Schlafli symbol is correct.

### 8.1.5 Maniplex (for IsGroup)

▷ `Maniplex(G)` (operation)

Returns a maniplex with connection group  $G$ , where  $G$  is assumed to be a permutation group on the flags.

### 8.1.6 IsPolytopal (for IsManiplex)

▷ `IsPolytopal(M)` (property)

**Returns:** `true` or `false`

Returns whether the maniplex  $M$  is a polytope.

# Chapter 9

## Actions

### 9.1 Faithfulness

#### 9.1.1 IsVertexFaithful (for IsReflexibleManifold)

▷ `IsVertexFaithful( $M$ )` (property)  
**Returns:** true or false  
Returns whether the reflexible manifold  $M$  is vertex-faithful; i.e., whether the action of the automorphism group on the vertices is faithful.

#### 9.1.2 IsFacetFaithful (for IsReflexibleManifold)

▷ `IsFacetFaithful( $M$ )` (property)  
**Returns:** true or false  
Returns whether the reflexible manifold  $M$  is facet-faithful; i.e., whether the action of the automorphism group on the facets is faithful.

#### 9.1.3 MaxVertexFaithfulQuotient (for IsReflexibleManifold)

▷ `MaxVertexFaithfulQuotient( $M$ )` (operation)  
Returns the maximal vertex-faithful reflexible manifold covered by  $M$ .

# Chapter 10

## Posets

I'm in the process of reconciling all of this, but there are going to be a number of ways to define a poset:

- As an `IsPosetOfFlags`, where the underlying description is an ordered list of length  $n + 2$ . Each of the  $n + 2$  list elements is a list of faces, and the assumption is that these are the faces of rank  $i - 2$ , where  $i$  is the index in the master list (e.g., `1 [1] [1]` would usually correspond to the unique  $-1$  face of a polytope – and there won't be an `1 [1] [2]`). Each face is then a list of the flags incident with that face.
- As an `IsPosetOfIndices`, where the underlying description is a binary relation on a set of indices, which correspond to labels for the elements of the poset.
- If the poset is known to be atomic, then by a description of the faces in terms of the atoms... usually we'll just need the list of the elements of maximal rank, from which all other elements may be obtained.
- As an `IsPosetOfElements`, where the elements could be anything, and we have a known function determining the partial order on the elements.

Usually, we assume that the poset will have a natural rank function on it.

### 10.1 Poset constructors

#### 10.1.1 PosetFromFaceListOfFlags (for IsList)

▷ `PosetFromFaceListOfFlags(list)` (operation)

**Returns:** `IsPosetOfFlags`. Note that the function is INTENTIONALLY agnostic about whether it is being given full poset or not.

Given a *list* of lists of faces in increasing rank, where each face is described by the incident flags, gives you a `IsPosetOfFlags` object back. Note that if you don't include faces or ranks, this function doesn't know about about them!

Here we have a poset using the `IsPosetOfFlags` description for the triangle.

Example

```
gap> poset:=PosetFromFaceListOfFlags([[], [[1,2],[3,6],[4,5]], [[1,4],[2,3],[5,6]], [[1,2,3,4,5,6]]), [[1,2,3,4,5,6]]
A poset using the IsPosetOfFlags representation with 8 faces.
gap> FaceListOfPoset(poset);
[[[ ]], [[1, 2], [3, 6], [4, 5]], [[1, 4], [2, 3], [5, 6]], [[1, 2, 3, 4, 5, 6]]]
```

### 10.1.2 PosetFromConnectionGroup (for IsPermGroup)

▷ PosetFromConnectionGroup(*g*) (operation)

**Returns:** IsPosetOfFlags with IsP1=true.

Given a group, returns a poset with an internal representation as a list of faces ordered by rank, where each face is represented as a list of the flags it contains. Note that this function includes the minimal (empty) face and the maximal face of the maniplex. Note that the  $i$ -faces correspond to the  $i + 1$  item in the list because of how GAP indexes lists.

Example

```
gap> g:=Group([(1,4)(2,3)(5,6),(1,2)(3,6)(4,5)]);
Group([ (1,4)(2,3)(5,6), (1,2)(3,6)(4,5) ])
gap> PosetFromConnectionGroup(g);
A poset using the IsPosetOfFlags representation with 8 faces.
```

### 10.1.3 PosetFromManiplex (for IsManiplex)

▷ PosetFromManiplex(*mani*) (operation)

**Returns:** IsPosetOfFlags

Given a maniplex, returns a poset of the maniplex with an internal representation as a list of faces ordered by rank, where each face is represented as a list of the flags it contains. Note that this function does include the minimal (empty) face and the maximal face of the maniplex. Note that the  $i$ -faces correspond to the  $i + 1$  item in the list because of how GAP indexes lists.

Example

```
gap> p:=HemiCube(3);
Regular 3-polytope of type [ 4, 3 ] with 24 flags
gap> PosetFromManiplex(p);
A poset using the IsPosetOfFlags representation with 15 faces.
```

### 10.1.4 PosetFromPartialOrder (for IsBinaryRelation)

▷ PosetFromPartialOrder(*partialOrder*) (operation)

**Returns:** IsPosetOfIndices

Given a partial order on a finite set of size  $n$ , this function will create a partial order on  $[1..n]$ .

Example

```
gap> l:=List([[1,1],[1,2],[1,3],[1,4],[2,4],[2,2],[3,3],[4,4]],x->Tuple(x));
gap> r:=BinaryRelationByElements(Domain([1..4]), l);
<general mapping: Domain([ 1 .. 4 ]) -> Domain([ 1 .. 4 ]) >
gap> poset:=PosetFromPartialOrder(r);
A poset using the IsPosetOfIndices representation
gap> h:=HasseDiagramBinaryRelation(PartialOrder(poset));
<general mapping: Domain([ 1 .. 4 ]) -> Domain([ 1 .. 4 ]) >
gap> UnderlyingRelation(h);
Domain([ DirectProductElement( [ 1, 2 ] ), DirectProductElement( [ 1, 3 ] ), DirectProductElement( [ 1, 4 ] ), DirectProductElement( [ 2, 4 ] ) ])
```

Note that what we've accomplished here is the poset containing the elements 1, 2, 3, 4 with partial order determined by whether the first element divides the second. The essential information about the poset can be obtained from the Hasse diagram.

### 10.1.5 PosetFromElements (for IsList)

▷ `PosetFromElements(list_of_faces, {func})` (operation)

**Returns:** `IsPosetOfElements`

This is for gathering elements with a known ordering *func* on two variables into a poset. Note... you should expect to get complete garbage if you send it a list of faces of different types. If your list of faces `HasFlagList` or `HasAtomList`, you may omit the function. Also note, the expectation is that *func* behaves similarly to `IsSubset`, i.e., *func* (x,y)=true means y is less than x in the order. Also worth noting, is that the internal representation of this kind of poset can and does keep both the partial order on the indices, and the list of faces corresponding to those indices, and the binary relation *func* (if the *list\_of\_faces* elements all have `HasFlagList` or `HasAtomList`, this will be the operation `PairCompareFlagsList` or `PairCompareAtomsList`).

Example

```
gap> g:=SymmetricGroup(3);
Sym( [ 1 .. 3 ] )
gap> asg:=AllSubgroups(g);
[ Group(()), Group([ (2,3) ]), Group([ (1,2) ]), Group([ (1,3) ]), Group([ (1,2,3) ]), Group([ (1,3,2) ]), Group([ (2,3,1) ]), Group([ (1,2,3) ]) ]
gap> poset:=PosetFromElements(asg,IsSubgroup);
A poset using the IsPosetOfIndices representation
gap> HasseDiagramBinaryRelation(PartialOrder(poset));
<general mapping: Domain([ 1 .. 6 ]) -> Domain([ 1 .. 6 ]) >
gap> UnderlyingRelation(last);
Domain([ DirectProductElement( [ 1, 2 ] ), DirectProductElement( [ 1, 3 ] ), DirectProductElement( [ 1, 2, 3 ] ), DirectProductElement( [ 2, 3 ] ), DirectProductElement( [ 1, 2, 3 ] ), DirectProductElement( [ 1, 2, 3 ] ) ]
gap> ElementsList(poset){[2,6]};
[ Group([ (2,3) ]), Group([ (1,2,3), (2,3) ]) ]
```

Here we have an example of how we can store a partially ordered set, and recover information about which objects are incident in the partial order. Another interesting example:

Example

```
gap> poset:=PosetFromManiplex(HemiCube(3));
A poset using the IsPosetOfFlags representation with 15 faces.
gap> rfl:=RankedFaceListOfPoset(poset);
[ [ [ ], -1 ], [ [ 1, 6, 2, 9, 3, 13 ], 0 ], [ [ 4, 14, 16, 23, 11, 21 ], 0 ], [ [ 5, 22, 8, 17, 10, 12 ], 0 ], [ [ 7, 10, 12, 24, 15, 18 ], 0 ], [ [ 1, 5, 2, 8 ], 1 ], [ [ 3, 11, 13, 21 ], 1 ], [ [ 4, 12, 17, 10, 12, 24 ], 1 ], [ [ 6, 15, 9, 18 ], 1 ], [ [ 10, 19, 24, 20 ], 1 ], [ [ 14, 22, 23, 17 ], 1 ], [ [ 1, 5, 6, 22, 15, 14, 12, 4 ], 2 ], [ [ 2, 8, 3, 19, 11, 10, 16, 7 ], 2 ], [ [ 9, 18, 13, 24, 21, 20, 23, 17 ], 2 ], [ [ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24 ], 3 ]
gap> Apply(rfl,PosetElementFromListOfFlags);
gap> pos2:=PosetFromElements(rfl);
A poset using the IsPosetOfIndices representation
gap> List(FacesList(pos2),x->Rank(x));
[ -1, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 2, 3 ]
```

### 10.1.6 Helper functions

The functions `PairCompareFlagsList` and `PairCompareAtomsList` are used in poset construction.

### 10.1.7 PairCompareFlagsList (for IsList,IsList)

▷ `PairCompareFlagsList(list1, list2)` (operation)

**Returns:** true or false

Function assumes *list1* and *list2* are of the form `[listOfFlags,i]` where `listOfFlags` is a list of flags in the face and *i* is the rank of the face. Allows comparison of `HasFlagList` elements.

### 10.1.8 PairCompareAtomsList (for IsList,IsList)

▷ `PairCompareAtomsList(list1, list2)` (operation)

**Returns:** true or false

Function assumes *list1* and *list2* are of the form `[listOfAtoms,int]` where `listOfAtoms` is a list of flags in the face and *int* is the rank of the face. Allows comparison of `HasAtomList` elements.

## 10.2 Poset attributes

### 10.2.1 ElementsList (for IsPoset)

▷ `ElementsList(poset)` (attribute)

Will recover the list of faces of the poset, format may depend on *type* of representation of poset.

### 10.2.2 OrderingFunction (for IsPoset)

▷ `OrderingFunction(poset)` (attribute)

`OrderingFunction` is an attribute of a poset which stores a function for ordering elements.

### 10.2.3 RankPoset (for IsPoset)

▷ `RankPoset(poset)` (attribute)

If the poset `IsP1`, ranks are assumed to run from  $-1$  to  $n$ , and function will return  $n$ . If `IsP1(poset)=false`, ranks are assumed to run from 1 to  $n$ .

### 10.2.4 IsFlaggable (for IsPoset)

▷ `IsFlaggable(poset)` (property)

**Returns:** list of flags as lists of poset elements.

Gives the list of maximal chains in a poset in terms of the elements of the poset. Synonym function is `FlagsList`.

Example

```
gap> poset:=PosetFromManiplex(HemiCube(3));
A poset using the IsPosetOfFlags representation with 15 faces.
gap> rfl:=RankedFaceListOfPoset(poset);;
gap> Apply(rfl,PosetElementFromListOfFlags);
gap> pos2:=PosetFromElements(rfl);
A poset using the IsPosetOfIndices representation
gap> MaximalChains(pos2)[1];
[ An element of a poset., An element of a poset., An element of a poset., An element of a poset.,
```

```
An element of a poset. ]
gap> List(last, Rank);
[ -1, 0, 1, 2, 3 ]
```

Checks or creates the value of the attribute `IsFlaggable` for an `IsPoset`. Point here is to see if the structure of the poset is sufficient to determine the flag graph. For `IsPosetOfFlags` this is another way of saying that the intersection of the faces (thought of as collections of flags) containing a flag is that selfsame flag. (Might be equivalent to prepolytopal... but Gabe was tired and Gordon hasn't bothered to think about it yet.) Now also works with generic poset element types (not just `IsPosetOfFlags`).

### 10.2.5 IsAtomic (for IsPoset)

- ▷ `IsAtomic(poset)` (property)  
**Returns:** true or false  
 Checks if `poset` is atomic. *Not a computed value.*

### 10.2.6 PartialOrder (for IsPoset)

- ▷ `PartialOrder(poset)` (attribute)  
**Returns:** partial order  
 HasPartialOrder Checks if `poset` has a declared partial order (binary relation). `SetPartialOrder` assigns a partial order to the `poset`. Note, currently something that is not computed, just declared.

### 10.2.7 ListIsP1Poset (for IsList)

- ▷ `ListIsP1Poset(list)` (operation)  
**Returns:** true or false  
 Given `list`, comprised of sublists of faces ordered by rank, each face listing the flags on the face, this function will tell you if the list corresponds to a P1 poset or not.

### 10.2.8 IsP1 (for IsPoset)

- ▷ `IsP1(poset)` (property)  
**Returns:** true or false  
 Determines whether a poset has property P1 from ARP.

Example

```
gap> p:=PosetFromElements(AllSubgroups(AlternatingGroup(4)), IsSubgroup);
A poset using the IsPosetOfIndices representation
gap> IsP1(p);
true
gap> p2:=PosetFromFaceListOfFlags([[[1],[2]],[[1,2]]]);
A poset using the IsPosetOfFlags representation with 3 faces.
gap> IsP1(p2);
false
```



### 10.2.9 IsP2 (for IsPoset)

▷ `IsP2(poset)` (property)

**Returns:** true or false

Determines whether a poset has property P2 from ARP.

Example

```
gap> poset:=PosetFromManiplex(HemiCube(3));
gap> IsP2(poset);
true
```

Another nice example

Example

```
gap> g:=AlternatingGroup(4);;a:=AllSubgroups(g);;poset:=PosetFromElements(a,IsSubgroup);
A poset using the IsPosetOfIndices representation
gap> IsP2(poset);
false
```

### 10.2.10 IsP3 (for IsPoset)

▷ `IsP3(poset)` (property)

**Returns:** true or false

Determines whether a poset is strongly flag connected (property P3' from ARP). May also be called with command `IsStronglyFlagConnected`. If you are not working with a pre-polytope, expect this to take a LONG time. In fact, this is currently pretty slow in general. Being strongly flag connected (or in the case of posets that are P1 and P2, strongly connected) is a combinatorial explosion scenario.

Helper for `IsP3`

### 10.2.11 IsFlagConnected (for IsPoset)

▷ `IsFlagConnected(arg)` (operation)

### 10.2.12 IsP4 (for IsPoset)

▷ `IsP4(poset)` (property)

**Returns:** true or false

Determines whether a poset satisfies the diamond condition. May also be invoked using `IsDiamondCondition`.

### 10.2.13 IsPolytope (for IsPoset)

▷ `IsPolytope(poset)` (property)

**Returns:** true or false

Determines whether a poset is an abstract polytope.

Example

```
gap> poset:=PosetFromManiplex(Cube(3));
A poset using the IsPosetOfFlags representation with 28 faces.
```

```

gap> IsPolytope(poset);
true
gap> KnownPropertiesOfObject(poset);
[ "IsP1", "IsP2", "IsP3", "IsP4", "IsPolytope" ]
gap> poset2:=PosetFromElements(AllSubgroups(AlternatingGroup(4)),IsSubgroup);
A poset using the IsPosetOfIndices representation
gap> IsPolytope(poset2);
false
gap> KnownPropertiesOfObject(poset2);
[ "IsP1", "IsP2", "IsPolytope" ]

```

### 10.2.14 IsPrePolytope (for IsPoset)

- ▷ `IsPrePolytope(poset)` (property)  
**Returns:** true or false  
 Determines whether a poset is an abstract pre-polytope.

## 10.3 Working with posets

### 10.3.1 IsIsomorphicPoset (for IsPoset,IsPoset)

- ▷ `IsIsomorphicPoset(poset1, poset2)` (operation)  
**Returns:** true or false  
 Determines whether *poset1* and *poset2* are isomorphic by checking to see if their Hasse diagrams are isomorphic.

Example

```

gap> IsIsomorphicPoset( PosetFromManiplex( PyramidOver( Cube(3) ) ), PosetFromManiplex( PrismOverCube(3) ) );
false
gap> IsIsomorphicPoset( PosetFromManiplex( PyramidOver( Cube(3) ) ), PosetFromManiplex( PyramidOverCube(3) ) );
true

```

### 10.3.2 PosetIsomorphism (for IsPoset,IsPoset)

- ▷ `PosetIsomorphism(poset1, poset2)` (operation)  
**Returns:** map on face indices  
 When *poset1* and *poset2* are isomorphic, will give you a map from the faces of *poset1* to the faces of *poset2*.

### 10.3.3 FlagsAsListOfFacesFromPoset (for IsPoset)

- ▷ `FlagsAsListOfFacesFromPoset(poset)` (operation)  
**Returns:** `IsList`  
 Given a *poset*, this will give you a version of the list of flags in terms of the faces described in the *poset*. Note that the flag list does not include the empty face or the maximal face. In other words, this gives a list of flags where each face is described in terms of its (enumerated) list of incident flags.

### 10.3.4 AdjacentFlag (for IsPosetOfFlags,IsList,IsInt)

▷ AdjacentFlag(*poset*, *flag*, *i*) (operation)

**Returns:** flag(s)

Given a poset, a flag, and a rank, this function will give you the *i*-adjacent flag. Note that adjacencies are listed from ranks 0 to one less than the dimension. You can replace *flag* with the integer corresponding to that flag. Appending true to the arguments will give the position of the flag instead of its description from FlagsAsListOffacesFromPoset.

### 10.3.5 AdjacentFlags (for IsPoset,IsList,IsInt)

▷ AdjacentFlags(*poset*, *flagaslistoffaces*, *adjacencyrank*) (operation)

If your poset isn't P4, there may be multiple adjacent maximal chains at a given rank. This function handles that case. May substitute IsInt for flagaslistoffaces corresponding to position of flag in list of maximal chains.

### 10.3.6 EqualChains (for IsList,IsList)

▷ EqualChains(*flag1*, *flag2*) (operation)

Determines whether two chains are equal.

### 10.3.7 ConnectionGeneratorOfPoset (for IsPoset,IsInt)

▷ ConnectionGeneratorOfPoset(*poset*, *i*) (operation)

**Returns:** A permutation on the flags.

Given a *poset* and an integer *i*, this function will give you the associated permutation for the rank *i*-connection.

### 10.3.8 ConnectionGroup (for IsPoset)

▷ ConnectionGroup(*poset*) (attribute)

**Returns:** IsPermGroup

Given a *poset* that is IsPrePolytope, this function will give you the connection group.

### 10.3.9 AutomorphismGroup (for IsPoset)

▷ AutomorphismGroup(*poset*) (attribute)

Given a *poset*, gives the automorphism group of the poset as an action on the maximal chains.

### 10.3.10 AutomorphismGroupOnElements (for IsPoset)

▷ AutomorphismGroupOnElements(*poset*) (attribute)

Given a *poset*, gives the automorphism group of the poset as an action on the elements.

### 10.3.11 FaceListOfPoset (for IsPoset)

- ▷ `FaceListOfPoset(poset)` (operation)  
**Returns:** list  
 Gives a list of faces collected into lists ordered by increasing rank.

### 10.3.12 FacesByRankOfPoset (for IsPoset)

- ▷ `FacesByRankOfPoset(poset)` (operation)  
**Returns:** list  
 Gives lists of faces ordered by rank. Also sets the rank for each of the faces.

### 10.3.13 HasseDiagramOfPoset (for IsPoset)

- ▷ `HasseDiagramOfPoset(poset)` (operation)  
**Returns:** directed graph

## 10.4 Elements of posets, also known as faces.

### 10.4.1 RankPosetElement (for IsPosetElement)

- ▷ `RankPosetElement(posetelement, {face})` (attribute)  
**Returns:** true or false  
 The rank of a poset element. Alternately `RankFace(IsPosetElement)`.

### 10.4.2 FlagList (for IsPosetElement)

- ▷ `FlagList(posetelement, {face})` (attribute)  
**Returns:** list  
 Description of `posetelement` *n* as a list of incident flags (when present).

### 10.4.3 FromPoset (for IsPosetElement)

- ▷ `FromPoset(posetelement, {face})` (attribute)  
**Returns:** poset  
 Gives the poset to which the face belongs (when present).

### 10.4.4 AtomList (for IsPosetElement)

- ▷ `AtomList(posetelement, {face})` (attribute)  
**Returns:** list  
 Description of `posetelement` *n* as a list of atoms (when present).

## 10.5 Element Constructors

### 10.5.1 PosetElementWithOrder (for IsObject, IsFunction)

- ▷ `PosetElementWithOrder(obj, func)` (operation)  
**Returns:** `IsFace`  
 Creates a face with *obj* and ordering function *func*.

### 10.5.2 `PosetElementFromListOfFlags` (for `IsList,IsInt`)

- ▷ `PosetElementFromListOfFlags(list, n)` (operation)  
**Returns:** `IsPosetElement`  
 This is used to create a face of rank *n* from a *list* of flags of poset. If an `IsPoset` object is appended to the input will tell the element what poset it belongs to.

### 10.5.3 `PosetElementFromAtomList` (for `IsList,IsInt`)

- ▷ `PosetElementFromAtomList(list, n)` (operation)  
**Returns:** `IsFace`  
 Creates a face with *list* of atoms at rank *n*. If an `IsPoset` object is appended to the input will tell the element what poset it belongs to.

### 10.5.4 `PosetElementFromIndex` (for `IsObject,IsInt`)

- ▷ `PosetElementFromIndex(obj, n)` (operation)  
**Returns:** `IsFace`  
 Creates a face with index *obj* at rank *n*. If an `IsPoset` object is appended to will tell the element what poset it belongs to.

## 10.6 Element operations

### 10.6.1 `RankedFaceListOfPoset` (for `IsPoset`)

- ▷ `RankedFaceListOfPoset(poset)` (operation)  
**Returns:** `list`  
 Gives a list of [*face*,*rank*] pairs for all the faces of *poset*.

### 10.6.2 `IsSubface` (for `IsFace,IsFace`)

- ▷ `IsSubface([face1, face1])` (operation)  
**Returns:** `true` or `false`  
*face1* and *face2* are `IsFace` or `IsPosetElement`. Subface will check to make sure *face2* is a subface of *face1*.

### 10.6.3 `AreIncidentElements` (for `IsObject,IsObject`)

- ▷ `AreIncidentElements(object1, object2)` (operation)  
**Returns:** `true` or `false`  
 Given two poset elements, will tell you if they are incident.

# Chapter 11

## Comparing maniplexes

### 11.1 Quotients and covers

#### 11.1.1 IsQuotient (for IsManiplex, IsManiplex)

▷ `IsQuotient( $M1$ ,  $M2$ )` (operation)

Returns whether  $M1$  is a quotient of  $M2$ .

#### 11.1.2 IsCover (for IsManiplex, IsManiplex)

▷ `IsCover( $M1$ ,  $M2$ )` (operation)

Returns whether  $M1$  is a cover of  $M2$ .

#### 11.1.3 IsIsomorphicManiplex (for IsManiplex, IsManiplex)

▷ `IsIsomorphicManiplex( $M1$ ,  $M2$ )` (operation)

Returns whether  $M1$  is isomorphic to  $M2$ .

#### 11.1.4 SmallestRegularCover (for IsManiplex)

▷ `SmallestRegularCover( $M$ )` (attribute)

Returns the smallest regular cover of  $M$ , which is the maniplex whose automorphism group is the connection group of  $M$ .

## Chapter 12

# ramp automatic generated documentation

### 12.1 ramp automatic generated documentation of methods

#### 12.1.1 UniversalRotationGroup (for IsInt)

▷ `UniversalRotationGroup(n)` (operation)

Returns the rotation subgroup of the universal Coxeter Group of rank *n*.

#### 12.1.2 UniversalRotationGroup (for IsList)

▷ `UniversalRotationGroup(sym)` (operation)

Returns the rotation subgroup of the Coxeter Group with Schläfli symbol *sym*.

#### 12.1.3 RotaryManiplex (for IsGroup)

▷ `RotaryManiplex(arg)` (operation)

#### 12.1.4 RotaryManiplex (for IsList)

▷ `RotaryManiplex(arg)` (operation)

#### 12.1.5 RotaryManiplex (for IsList, IsList)

▷ `RotaryManiplex(arg1, arg2)` (operation)

#### 12.1.6 EnantiomorphicForm (for IsRotaryManiplex)

▷ `EnantiomorphicForm(arg)` (operation)

## 12.2 ramp automatic generated documentation of global functions

### 12.2.1 AbstractPolytope

▷ `AbstractPolytope(args)` (function)

Calls `Maniplex(args)` and marks the output as polytopal.

### 12.2.2 AbstractRegularPolytope

▷ `AbstractRegularPolytope(args)` (function)

Calls `ReflexibleManiplex(args)` and marks the output as polytopal. Also available as `ARP(args)`.

### 12.2.3 AbstractRotaryPolytope

▷ `AbstractRotaryPolytope(args)` (function)

Calls `RotaryManiplex(args)` and marks the output as polytopal.

### 12.2.4 AddOrAppend

▷ `AddOrAppend(L, x)` (function)

Given a list  $L$  and an object  $x$ , this calls `Append(L, x)` if  $x$  is a list; otherwise it calls `Add(L, x)`. Note that since strings are internally represented as lists, `AddOrAppend(L, "foo")` will append the characters 'f', 'o', 'o'.



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