# The Research Assistant for Maniplexes and Polytopes

0.3

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### **Constructions**

#### 1.1 Extensions, amalgamations, and quotients

#### 1.1.1 UniversalPolytope (for IsInt)

▷ UniversalPolytope(n)

(operation)

Returns the universal polytope of rank n.

#### 1.1.2 FlatRegularPolyhedron (for IsInt, IsInt, IsInt, IsInt)

▷ FlatRegularPolyhedron(p, q, i, j)

(operation)

Returns the flat regular polyhedron with automorphism group [p, q] / (r2 r1 r0 r1 = (r0 r1) $^i$  (r1 r2) $^j$ ). This function does not currently validate the inputs to make sure that the output makes sense.

#### 1.1.3 QuotientPolytope (for IsManiplex, IsList)

▷ QuotientPolytope(M, rels)

(operation)

Returns the quotient of M by rels, which may be given as either a list of Tietze words, such as [[1,2,1,0,1,2,1,0]] or as a string like  $(r0 r1 r2 r1)^2$ ,  $(r0 r1 r2)^4$ .

#### 1.1.4 UniversalExtension (for IsManiplex)

▷ UniversalExtension(M)

(operation)

Returns the universal extension of M, i.e. the maniplex with facets isomorphic to M that covers all other maniplexes with facets isomorphic to M. Currently only defined for reflexible maniplexes.

#### 1.1.5 UniversalExtension (for IsManiplex, IsInt)

 $\triangleright$  UniversalExtension(M, k)

(operation)

Returns the universal extension of M with last entry of Schlafli symbol k. Currently only defined for reflexible maniplexes.

#### 1.1.6 TrivialExtension (for IsManiplex)

▷ TrivialExtension(M)

(operation)

Returns the trivial extension of M, also known as  $\{M/, 2\}$ .

#### 1.1.7 FlatExtension (for IsManiplex, IsInt)

 $\triangleright$  FlatExtension(M, k)

(operation)

Returns the flat extension of M with last entry of Schlafli symbol k. (As defined in "Flat Extensions of Abstract Polytopes".) Currently only defined for reflexible maniplexes.

#### 1.1.8 Amalgamate (for IsManiplex, IsManiplex)

▷ Amalgamate(M1, M2)

(operation)

Returns the amalgamation of M1 and M2. Implicitly assumes that M1 and M2 are compatible. Currently only defined for reflexible maniplexes.

#### 1.1.9 Medial (for IsManiplex)

<u>-</u>

(operation)

Given a 3-maniplex M, returns its medial.

#### 1.2 Duality

▷ Medial(M)

#### 1.2.1 Dual (for IsManiplex)

Dual(M)

(attribute)

**Returns:** The maniplex that is dual to *M*.

#### 1.2.2 IsSelfDual (for IsManiplex)

▷ IsSelfDual(P)

(property)

**Returns:** Whether this polytope is isomorphic to its dual.

#### 1.2.3 Petrial (for IsManiplex)

▷ Petrial(P)

(attribute)

**Returns:** The Petrial (Petrie dual) of P. Note that this is not necessarily a polytope.

#### 1.2.4 IsSelfPetrial (for IsManiplex)

▷ IsSelfPetrial(P)

(property)

**Returns:** Whether this polytope is isomorphic to its Petrial.

#### 1.3 Products

#### 1.3.1 PyramidOver (for IsManiplex)

▷ PyramidOver(M) (operation)

Returns the pyramid over M. Currently only works for finite maniplexes.

#### 1.3.2 PrismOver (for IsManiplex)

PrismOver(M) (operation)

Returns the prism over M. Currently only works for finite maniplexes.

### **Databases**

#### 2.1 Regular polyhedra

#### 2.1.1 DegeneratePolyhedra (for IsInt)

▷ DegeneratePolyhedra(maxsize)

(operation)

Returns all degenerate polyhedra (of type {2, q} and {p, 2}) with up to maxsize flags.

#### 2.1.2 FlatRegularPolyhedra (for IsInt)

⊳ FlatRegularPolyhedra(maxsize)

(operation)

Returns all nondegenerate flat regular polyhedra with up to maxsize flags. Currently supports a maxsize of 4000 or less.

#### 2.1.3 SmallRegularPolyhedra (for IsInt)

▷ SmallRegularPolyhedra(maxsize)

(operation)

Returns all regular polyhedra with up to maxsize flags. Currently supports a maxsize of 4000 or less. You can also set options "nondegenerate" and "nonflat".

```
L1 := SmallRegularPolyhedra(500);;
L2 := SmallRegularPolyhedra(1000 : nondegenerate);;
L3 := SmallRegularPolyhedra(2000 : nondegenerate, nonflat);;
```

### **Combinatorics and Structure**

#### 3.1 Faces

#### 3.1.1 NumberOfIFaces (for IsManiplex, IsInt)

▷ NumberOfIFaces(M, i)

(operation)

Returns The number of i-faces of M.

#### 3.1.2 NumberOfVertices (for IsManiplex)

▷ NumberOfVertices(M)

(attribute)

Returns the number of vertices of M.

#### 3.1.3 NumberOfEdges (for IsManiplex)

▷ NumberOfEdges(M)

(attribute)

Returns the number of edges of M.

#### 3.1.4 NumberOfFacets (for IsManiplex)

▷ NumberOfFacets(M)

(attribute)

Returns the number of facets of M.

#### 3.1.5 NumberOfRidges (for IsManiplex)

▷ NumberOfRidges(M)

(attribute)

Returns the number of ridges ((n-2)-faces) of M.

#### 3.1.6 Fvector (for IsManiplex)

Returns the f-vector of M.

#### **3.1.7** Facets (for IsManiplex)

→ Facets (M) (attribute)

Returns the facet-types of M (i.e. the maniplexes corresponding to the facets). Currently only works for reflexible maniplexes.

#### 3.1.8 VertexFigures (for IsManiplex)

VertexFigures (M) (attribute)

Returns the types of vertex-figures of M (i.e. the maniplexes corresponding to the vertex-figures). Currently only works for reflexible maniplexes.

#### 3.2 Basics

#### 3.2.1 Size (for IsManiplex)

▷ Size(M) (attribute)

Returns the number of flags of the maniplex M.

#### 3.2.2 RankManiplex (for IsManiplex)

▷ RankManiplex(M) (attribute)

Returns the rank of the maniplex *M*.

#### 3.2.3 SchlafliSymbol (for IsManiplex)

▷ SchlafliSymbol(M) (attribute)

Returns the Schlafli symbol of the maniplex M. Currently only implemented for reflexible maniplexes.

#### 3.2.4 ComputeSchlafliSymbol (for IsManiplex)

▷ ComputeSchlafliSymbol(M)

Computes the Schlafli symbol of the maniplex M, ignoring any currently stored data about the Schlafli symbol. This is called by SchlafliSymbol(M) if no value is stored yet, but it can also be called

(operation)

directly by the user to double-check any optimizations. Currently only implemented for reflexible maniplexes.

#### 3.2.5 IsTight (for IsManiplex and IsPolytopal)

▷ IsTight(P) (property)

Returns: true or false

Returns whether the polytope P is tight, meaning that it has a Schlafli symbol  $\{k_1, ..., k_{n-1}\}$  and has  $2 k_1 ... k_{n-1}$  flags, which is the minimum possible. This property doesn't make any sense for non-polytopal maniplexes, which aren't constrained by this lower bound.

#### 3.2.6 PetrieLength (for IsReflexibleManiplex)

▷ PetrieLength(M) (attribute)

#### 3.2.7 HoleLength (for IsReflexibleManiplex)

→ HoleLength(M) (attribute)

#### 3.2.8 IsDegenerate (for IsManiplex)

▷ IsDegenerate(M) (property)

Returns: true or false

Returns whether the maniplex M has any sections that are digons. We may eventually want to include maniplexes with even smaller sections.

#### 3.2.9 SymmetryTypeGraph (for IsManiplex)

Returns the Symmetry Type Graph of the maniplex M, encoded as a permutation group on Rank(M) generators.

#### 3.2.10 NumberOfFlagOrbits (for IsManiplex)

NumberOfFlagOrbits(M) (attribute)

Returns the number of orbits of the automorphism group of M on its flags.

#### 3.2.11 IsReflexible (for IsManiplex)

**Returns:** Whether the maniplex M is reflexible (has one flag orbit).

#### 3.2.12 IsRotary (for IsManiplex)

▷ IsRotary(M) (property

**Returns:** Whether the maniplex M is rotary; i.e., whether it is orientable and has at most 2 flag orbits (making it either chiral or orientably regular).

#### **3.2.13** Description (for IsManiplex)

▷ Description(M) (attribute)

Returns a short name for the maniplex M, if one is available. For example, Description(Simplex(3)) = "3-simplex".

#### 3.3 Posets

#### 3.3.1 PosetFromFaceListOfFlags (for IsList)

▷ PosetFromFaceListOfFlags(list)

(operation)

**Returns:** *IsPosetOfFlags*. Not that the function is INTENTIONALLY agnostic about whether it is being given full poset or not.

Given a list of lists of faces in increasing rank, where each face is described by the incident flags, gives you a IsPosetOfFlags object back.

#### 3.3.2 RankPoset (for IsPoset)

▶ RankPoset(arg) (attribute)

#### 3.3.3 IsFull (for IsPoset)

▷ IsFull(poset)

(attribute)

**Returns:** *true* or *false* 

Checks or creates the value of the attribute IsFull for an IsPoset.

#### 3.3.4 IsFlaggable (for IsPoset)

▷ IsFlaggable(arg)

(attribute)

#### 3.3.5 IsAtomic (for IsPoset)

▷ IsAtomic(poset)

(attribute)

**Returns:** *true* or *false* 

Checks if poset is atomic. Note, currently something that is computed, just declared.

#### 3.3.6 PartialOrder (for IsPoset)

▷ PartialOrder(poset)

(attribute)

**Returns:** partial order

HasPartialOrder Checks if poset has a declared partial order (binary relation). SetPartialOrder assigns a partial order to the poset. Note, currently something that is computed, just declared.

#### 3.3.7 ListIsFullPoset (for IsList)

▷ ListIsFullPoset(list)

(operation)

Returns: true or false

Given list, a poset as a list of faces ordered by rank, each face listing the flags on the face, this function will tell you if the poset is full or not.

#### 3.3.8 RankOfPoset (for IsPoset)

▷ RankOfPoset(poset)

(operation)

Returns: integer

Given a poset, returns the rank of the poset. Note: There may be hidden assumptions here to untangle later.

#### 3.3.9 IsNotFull (for IsPoset)

▷ IsNotFull(poset)

(operation)

**Returns:** *true* or *false* 

Lets me check to see if a poset is NOT full. For use in certain filtering operations.

#### 3.3.10 PosetOfConnectionGroup (for IsPermGroup)

▷ PosetOfConnectionGroup(g)

(operation)

**Returns:** *IsPosetOfFlags* with *IsFull*=false.

Given a group, returns a poset with an internal representation as a list of faces ordered by rank, where each face is represented as a list of the flags it contains. Note that this function does not include the minimal (empty) face nor the maximal face of the maniplex. Note that the i-faces correspond to the i+1 item in the list because of how GAP indexes lists.

#### 3.3.11 FullPosetOfConnectionGroup (for IsPermGroup)

⊳ FullPosetOfConnectionGroup(g)

(operation)

**Returns:** *IsPosetOfFlags* with *IsFull*=true.

Returns a full poset corresponding to the connection group g with an internal representation as a list of faces ordered by rank, where each face is represented as a list of the flags it contains. This function does include the minimal (empty) face nor the maximal face of the maniplex, so the list has n+2 ranks if the maniplex is of rank n. Note that the i-faces correspond to the i+1 item in the list because of how GAP indexes lists.

#### 3.3.12 PosetOfManiplex (for IsManiplex)

▷ PosetOfManiplex(mani)

(operation)

**Returns:** IsPosetOfFlags

Given a maniplex, returns a poset of the maniplex with an internal representation as a list of faces ordered by rank, where each face is represented as a list of the flags it contains. Note that this function does not include the minimal (empty) face nor the maximal face of the maniplex. Note that the i-faces correspond to the i+1 item in the list because of how GAP indexes lists.

#### 3.3.13 FullPosetOfManiplex (for IsManiplex)

⊳ FullPosetOfManiplex(mani)

(operation)

Returns: IsPosetOfFlags

Given a maniplex, returns a poset with the internal representation be a list of lists of faces ordered by rank, where each face is represented as a list of the flags it contains. Note that this function does include the minimal (empty) face and the maximal face of the maniplex. Note that the i-faces correspond to the i+1 item in the list because of how GAP indexes lists.

#### 3.3.14 PosetFromPartialOrder (for IsBinaryRelation)

▷ PosetFromPartialOrder(partialOrder)

(operation)

Returns: IsPosetOfIndices

Given a partial order on a finite set of size n, this function will create a partial order on [1..n].

#### 3.3.15 PosetFromElements (for IsList,IsPartialOrderBinaryRelation)

▷ PosetFromElements(list\_of\_faces, partial\_order)

(operation)

**Returns:** *IsPosetOfElements* 

This is for gathering elements with a known ordering function into a poset. Note... you should expect to get complete garbage if you send it a list of faces of different types.

#### 3.3.16 AreIncidentFaces (for IsObject,IsObject)

▷ AreIncidentFaces(object1, object2)

(operation)

**Returns:** *true* or *false* 

Given two faces, will tell you if they are incident. Currently only supports faces as list of their incident flags.

#### 3.3.17 FlagsAsListOfFacesFromPoset (for IsPoset)

▷ FlagsAsListOfFacesFromPoset(poset)

(operation)

**Returns:** *IsList* 

Given a poset, this will give you a version of the list of flags in terms of the faces described in the poset. Note that the flag list does not include the empty face or the maximal face.

#### 3.3.18 AdjacentFlag (for IsPosetOfFlags,IsList,IsInt)

▷ AdjacentFlag(poset, flag, i)

(operation)

**Returns:** flag(s)

Given a flag (represented as chains of faces comprised of lists of flags) and a poset and a rank, this function will give you the *i*-adjacent flag. Note that adjacencies are listed from ranks 0 to one less than the dimension. You can replace *flag* with the integer corresponding to that flag. Appending *true* to the arguments will give the position of the flag instead of its description from *FlagsAsListOfFaces-FromPoset*.

#### 3.3.19 ConnectionGeneratorOfPoset (for IsPoset,IsInt)

▷ ConnectionGeneratorOfPoset(poset, i)

(operation)

**Returns:** A permutation on the flags.

Given a *poset* and an integer *i*, this function will give you the associated permutation for the rank *i*-connection.

#### 3.3.20 ConnectionGroupOfPoset (for IsPoset)

▷ ConnectionGroupOfPoset(poset)

(operation)

**Returns:** *IsPermGroup* 

Given a poset corresponding to a maniplex, this function will give you the connection group.

#### 3.3.21 FacesOfPosetAsBinaryRelationOnFaces (for IsPoset)

▷ FacesOfPosetAsBinaryRelationOnFaces(poset)

(operation)

**Returns:** A binary relation on the integers 1 through n, where n is the number of faces of the full poset.

FacesOfPosetAsBinaryRelationOnFaces

#### 3.3.22 FaceListOfPoset (for IsPoset)

▷ FaceListOfPoset(poset)

(operation)

Returns: list

Gives a list of faces collected into lists ordered by increasing rank.

#### 3.3.23 RankPosetElement (for IsPosetElement)

▷ RankPosetElement(posetelement, {face})

(attribute)

**Returns:** *true* or *false* 

The rank of a poset element. Alternately *RankFace*(IsPosetElement).

#### 3.3.24 FlagList (for IsPosetElement)

▷ FlagList(posetelement, {face})

(attribute)

Returns: list

Description of posetelement n as a list of incident flags (when present).

#### 3.3.25 FromPoset (for IsPosetElement)

▷ FromPoset(posetelement, {face})

(attribute)

Returns: poset

Gives the poset to which the face belongs (when present).

#### 3.3.26 AtomList (for IsPosetElement)

▷ AtomList(posetelement, {face})

(attribute)

Returns: list

Description of posetelement n as a list of atoms (when present).

#### 3.3.27 Index (for IsPosetElement)

▷ Index(arg) (attribute)

#### 3.3.28 PosetElementFromListOfFlags (for IsList,IsInt)

▷ PosetElementFromListOfFlags(list, n)

(operation)

Returns: IsPosetElement

This is used to create a face of rank n from a list of flags of poset. If an IsPoset object is appended to the input will tell the element what poset it belongs to.

#### 3.3.29 PosetElementFromAtomList (for IsList,IsInt)

▷ PosetElementFromAtomList(list, n)

(operation)

Returns: IsFace

Creates a *face* with *list* of atoms at rank n. If an IsPoset object is appended to the input will tell the element what poset it belongs to.

#### 3.3.30 PosetElementFromIndex (for IsObject,IsInt)

▷ PosetElementFromIndex(obj, n)

(operation)

**Returns:** *IsFace* 

Creates a *face* with index obj at rank n. If an IsPoset object is appended to will tell the element what poset it belongs to.

#### 3.3.31 RankedFaceListOfPoset (for IsPoset)

▷ RankedFaceListOfPoset(poset)

(operation)

Returns: list

Gives a list of [face,rank] pairs for all the faces of poset.

### 3.3.32 IsSubface (for IsFace,IsFace)

▷ IsSubface([face1, face1])

(operation)

**Returns:** *true* or *false* 

face1 and face2 are IsFace or IsPosetElement. Subface will check to make sure face2 is a subface of face1.

# **Families of Polytopes**

4.1	Classical Polytopes		
4.1.1	Vertex		
<pre>     Vertex()     (operation) </pre>			
4.1.2	Edge		
⊳ Edge	e()	(operation)	
4.1.3	Pgon (for IsInt)		
⊳ Pgor	$\mathbf{n}(p)$	(operation)	
4.1.4	Cube (for IsInt)		
⊳ Cube	e(n)	(operation)	
4.1.5	HemiCube (for IsInt)		
⊳ Hemi	iCube(n)	(operation)	
4.1.6	CrossPolytope (for IsInt)		
⊳ Cros	$\operatorname{ssPolytope}(n)$	(operation)	

4.1.7	HemiCrossPolytope (for IsInt)	
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⊳ Simp	plex(n)	(operation)
4.1.9	CubicTiling (for IsInt)	
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4.1.10	Dodecahedron	
⊳ Dode	ecahedron()	(operation)
4.1.11	HemiDodecahedron	
⊳ Hemi	iDodecahedron()	(operation)
4.1.12	Icosahedron	
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4.1.14	24Cell	
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4.1.15	Hemi24Cell	
⊳ Hemi	i24Cell()	(operation)
4.1.16	120Cell	
⊳ 1200	Cell()	(operation)

#### 4.1.17 Hemi120Cell

→ Hemi120Cell() (operation)

#### 4.1.18 600Cell

#### 4.1.19 Hemi600Cell

### Groups

#### 5.1 Groups

#### **5.1.1** AutomorphismGroup (for IsManiplex)

▷ AutomorphismGroup(M)

(attribute)

Returns the automorphism group of M. This group is not guaranteed to be in any particular form.

#### 5.1.2 AutomorphismGroupFpGroup (for IsManiplex)

(attribute)

Returns the automorphism group of M as a finitely presented group.

#### **5.1.3** AutomorphismGroupPermGroup (for IsManiplex)

(attribute)

Returns the automorphism group of M as a permutation group.

#### **5.1.4** ConnectionGroup (for IsManiplex)

▷ ConnectionGroup(M)

(attribute)

Returns the connection group of M as a permutation group. We may eventually allow other types of connection groups.

#### **5.1.5** EvenConnectionGroup (for IsManiplex)

▷ EvenConnectionGroup(M)

(attribute)

Returns the even-word subgroup of the connection group of M as a permutation group.

#### **5.1.6** RotationGroup (for IsManiplex)

Returns the rotation group of M. This group is not guaranteed to be in any particular form.

#### **5.1.7** ExtraRelators (for IsReflexibleManiplex)

For a reflexible maniplex M, returns the relators needed to define its automorphism group as a quotient of the string Coxeter group given by its Schlafli symbol. Not particularly robust at the moment.

#### 5.1.8 IsStringC (for IsGroup)

▷ IsStringC(G) (operation)

For an sggi G, returns whether the group is a string C group. It does not check whether G is an sggi.

### **Properties**

#### 6.1 Orientability

#### **6.1.1** IsOrientable (for IsManiplex)

▷ IsOrientable(p)

(property)

Returns: true or false

A polytope is orientable if its flag graph is bipartite. Currently only implemented for regular polytopes.

#### 6.1.2 IsIOrientable (for IsManiplex, IsList)

 $\triangleright$  IsIOrientable(p, I)

(operation)

For a subset I of {0, ..., n-1}, a polytope if I-orientable if every closed path in its flag graph contains an even number of edges with colors in I. Currently only implemented for regular polytopes.

#### **6.1.3** IsVertexBipartite (for IsManiplex)

▷ IsVertexBipartite(p)

(property)

Returns: true or false

A polytope is vertex-bipartite if its 1-skeleton is bipartite. This is equivalent to being I-orientable for  $I = \{0\}$ .

#### **6.1.4** IsFacetBipartite (for IsManiplex)

▷ IsFacetBipartite(p)

(property)

Returns: true or false

A polytope is facet-bipartite if the 1-skeleton of its dual is bipartite. This is equivalent to being I-orientable for  $I = \{n-1\}$ .

### **Basics**

#### 7.1 Constructors

#### 7.1.1 UniversalSggi

```
▷ UniversalSggi(n) (operation)
▷ UniversalSggi(sym) (operation)
```

In the first form, returns the universal Coxeter Group of rank n. In the second form, returns the Coxeter Group with Schlafli symbol sym.

#### 7.1.2 ReflexibleManiplex (for IsGroup)

```
\triangleright ReflexibleManiplex(g) (operation)
```

Given a group g (which should be a string C-group), returns the abstract regular polytope with that automorphism group, where the privileged generators are those returned by GeneratorsOfGroup(g).

#### 7.1.3 ReflexibleManiplex (for IsList, IsList)

```
▶ ReflexibleManiplex(symbol, relations) (operation)
```

Returns an abstract regular polytope with the given Schlafli symbol and with the given relations. The formatting of the relations is quite flexible. All of the following work:

```
Example

q := ReflexibleManiplex([4,3,4], "(r0 r1 r2)^3, (r1 r2 r3)^3");

q := ReflexibleManiplex([4,3,4], "(r0 r1 r2)^3 = (r1 r2 r3)^3 = 1");

p := ReflexibleManiplex([infinity], "r0 r1 r0 = r1 r0 r1");
```

#### 7.1.4 ReflexibleManiplex (for IsString)

```
▶ ReflexibleManiplex(name) (operation)
```

Returns the regular polytope with the given symbolic name. Examples: ReflexibleManiplex("{3,3,3}"); ReflexibleManiplex("{4,3}\_3");

#### 7.1.5 Maniplex (for IsGroup)

▷ Maniplex(G)

Returns a maniplex with connection group G, where G is assumed to be a permutation group on the flags.

#### 7.1.6 IsPolytopal (for IsManiplex)

Returns: true or false

Returns whether the maniplex M is a polytope.

### **Actions**

#### 8.1 Faithfulness

#### 8.1.1 IsVertexFaithful (for IsReflexibleManiplex)

▷ IsVertexFaithful(M)

(property)

Returns: true or false

Returns whether the reflexible maniplex M is vertex-faithful; i.e., whether the action of the automorphism group on the vertices is faithful.

#### 8.1.2 IsFacetFaithful (for IsReflexibleManiplex)

▷ IsFacetFaithful(M)

(property)

Returns: true or false

Returns whether the reflexible maniplex *M* is facet-faithful; i.e., whether the action of the automorphism group on the facets is faithful.

#### 8.1.3 MaxVertexFaithfulQuotient (for IsReflexibleManiplex)

▷ MaxVertexFaithfulQuotient(M)

(operation)

Returns the maximal vertex-faithful reflexible maniplex covered by M.

## **Comparing maniplexes**

#### 9.1 Quotients and covers

#### 9.1.1 IsQuotientOf (for IsManiplex, IsManiplex)

▷ IsQuotientOf(M1, M2)

(operation)

Returns whether M1 is a quotient of M2.

#### 9.1.2 IsCoverOf (for IsManiplex, IsManiplex)

▷ IsCoverOf(M1, M2)

(operation)

Returns whether M1 is a cover of M2.

#### 9.1.3 IsIsomorphicTo (for IsManiplex, IsManiplex)

▷ IsIsomorphicTo(M1, M2)

(operation)

Returns whether M1 is isomorphic to M2.

#### 9.1.4 SmallestRegularCover (for IsManiplex)

▷ SmallestRegularCover(M)

(attribute)

Returns the smallest regular cover of M, which is the maniplex whose automorphism group is the connection group of M.

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