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Physics Assignment - 4

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1) Find the missing order for a double slit fraunhofer pattern if the width of each slit is 0.15 mm and they are separated by a distance 0.60 mm.

Solⁿ $\sin(\theta) = m.\frac{\lambda}{d}$,

where :-

- θ is the angle at which the constructive interference occurs.
- m is the order of the interference.
- λ is the wavelength of light.
- d is the distance between the slit.

Width of each slit (w) = 0.15 mm = 0.00015 m
Distance of between the slit (d) = 0.60 mm = 0.00060 m

$$\sin(\theta) = m.\frac{\lambda}{d}$$

$$= m.\frac{6.5 \times 10^{-7}\ m}{0.00060\ m}$$

$$= m. 1.0833 \times 10^{-3}$$

$$m = \frac{\sin\theta}{1.0833 \times 10^{-3}}$$

Now, let's calculate m for the first order maximum (m=1)

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$$m = \frac{a\sin\theta}{1.0833\times10^{-3}} = \frac{a\sin\theta}{1.0833\times10^{-3}} = 1$$

2. light of 5000 Å is incident normally on a slit having a width of 0.2 mm. Find the width of the centoral maxima of a diffraction pattern on a screen 9.0 m away.

Ans $\theta = \frac{1.22\ \lambda}{W}$

where,

- θ is the angular width of the centoral maximom.
- λ is the wavelength of light.
- w is the width of the slit.

wavelength (λ) = 5000 Å = 5000×10^{-10} meters
width of the slit (w) = 0.2 mm = 0.2×10^{-3} meters.

$$\theta = \frac{1.22\,(5000\times10^{-10}\,m)}{0.2\times10^{-3}\,m}$$

$$\theta = \frac{1.22\times5\times10^{-7}}{2\times10^{-4}}$$

$$\theta = \frac{6.1\times10^{-7}}{2\times10^{-4}}$$

$\theta = 3.05\times10^{-3}$ radians

width of centoral maximom = $2L\theta$
$L = 9.0\,m$

width of centoral maximum = $2.9\,m\times3.05\times10^{-3}$ radians.

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Width of Centeral maximum $= 18.0 \times 3.05 \times 10^{-3}$ m

Width of Centeral maximum $= 0.0549$ m

3. What is the maximum number of lines in a grating which will just resolve the 3rd order spectrum (formed by grating) of ~~the~~ two lines having wavelengths 5890 Å and 5896 Å ?

Sol^n ⇒ $\theta = \frac{m.\lambda}{d}$

For the 3rd order spectrum, $m = 3$

$$\Delta\theta = \frac{3.(\lambda_2 - \lambda_1)}{d}$$

where $\Delta\theta$ is the angular separation between the two lines with wavelength λ_1 and λ_2

$$\Delta\theta \geq \frac{\lambda}{d},$$

where, λ is the average wavelength of two lines, given by.

$$\lambda = \frac{\lambda_1 + \lambda_2}{2}$$

Now, let's plug in the value:

$$\Delta\theta \geq \frac{(\lambda_1 + \lambda_2)}{2d}$$

$$\frac{3.(\lambda_2 - \lambda_1)}{d} \geq \frac{(\lambda_1 + \lambda_2)}{2d}$$

Canceling out the d terms.

$$3 \cdot (\lambda_2 - \lambda_1) \geq \frac{(\lambda_1 + \lambda_2)}{2}$$

$$3 \cdot (5896 - 5890)\,\text{Å} \geq \frac{(5898 + 5890)}{2}\,\text{Å}$$

$$3.6\,\text{Å} \geq \frac{11786}{2}\,\text{Å}$$

$$18\,\text{Å} \geq 5893\,\text{Å}$$

Clearly, this inequality is not true. Therefore you cannot resolve the 3rd order spectrum of these two lines with given grating.

4. Write down the intensity expression for single slit double slit, and grating diffraction pattern and ~~show~~ draw corresponding pattern.

Ans 1 Single slit Diffraction pattern :-

Intensity Expression :-

$$I(\theta) \propto \left(\frac{\sin(\pi a \sin(\theta)/\lambda)}{\pi a \sin(\theta)/\lambda}\right)^2$$

where,

$I(\theta)$ is the intensity at an angle θ
a is the slit width.
λ is the wavelength of light.

2 Double slit interference pattern :-

Intensity Expression :-

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$$I(\theta) \propto \cos^2\left(\frac{\pi d \sin(\theta)}{\lambda}\right) \cdot \left(\frac{\sin(\pi a \sin(\theta)/\lambda)}{\pi a \sin(\theta)/\lambda}\right)^2$$

3. Grating Diffraction Pattern :-

Intensity Expression :-

$$I(\theta) \propto \left(\frac{\sin(N\pi d \sin(\theta)/\lambda)}{\sin(\pi d \sin(\theta)/\lambda)}\right)^2$$

5. The intensity expression for single slit diffraction is

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2}$$

Show that condition for maxima is $\tan\alpha = \alpha$ and condition for minima is $\alpha = n\pi$

Sol^n $I = I_0 \frac{\sin^2\alpha}{\alpha^2}$ with respect to α, and set the derivative equal to zero.

1. For maxima:

$$\frac{dI}{d\alpha} = I_0 \cdot \frac{d}{d\alpha}\left(\frac{\sin^2\alpha}{\alpha^2}\right)$$

$$\frac{dI}{d\alpha} = I_0 \cdot \left(\frac{2\sin\alpha \cdot \cos\alpha \cdot \alpha^2 - 2\sin^3\alpha}{\alpha^3}\right)$$

$$\frac{dI}{d\alpha} = 0$$

$$2\sin\alpha \cdot \cos\alpha \cdot \alpha^2 - 2\sin^3\alpha = 0$$

Dividing both side 2 sinα

$$\cos\alpha \cdot \alpha^2 - \sin^2\alpha = 0$$

$$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$$

$$\alpha^2 - \sin^2\alpha = 0$$
$$\alpha^2 = \sin^2\alpha$$
$$\alpha = \sin\alpha$$

2. For minima:

$$\alpha = n\lambda$$

where, n is an integer representing the order of the minimum.

So,
- The condition for maxima is $\alpha = \sin\alpha$
- The condition for minima is $\alpha = n\lambda$.

6. Show that in young's double slit experiment friges are of equal width

Ans:- 1. Conditions for Constructive Interference:

Constructive interference occurs when the path difference between the two slits to a point on the screen is an integer multiple of the wavelength (λ) of the light.

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2 Condition for Destructive Interference:-

Destructive interference occurs when the path difference between the two slits to a point on the screen is a half integer multiple of wavelength $(\lambda/2)$

$$d \cdot \sin(\theta) = \left(m + \frac{1}{2}\right) \cdot \lambda$$

For constructive interference : $d \cdot \sin(\theta) = m \cdot \lambda$ — ①
For destructive interference : $d \cdot \sin(\theta) = \left(m + \frac{1}{2}\right) \cdot \lambda$ — ②

Subtract the ② - ①

$$d \cdot \sin(\theta) - d \cdot \sin(\theta) = m \cdot \lambda - \left(m + \frac{1}{2}\right) \cdot \lambda$$

$$0 = m \cdot \lambda - \left(m + \frac{1}{2}\right) \cdot \lambda$$

$$0 = \frac{1}{2} \cdot \lambda$$

Since, λ is a constant, the equation $0 = \frac{1}{2} \cdot \lambda$ is always false.

Therefore, there are no conditions for which destructive interferce occurs. In other words. all the fringes. in the a young's down slit interference pattern are bright and have equal width. This is why you see a series of equally space bright fringes on the screen.