Introduction to Algorithms, 2ed 's Notes

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Abstract

This is notes of my self-study course on Algorithms. It's taken when I was read the book "Introduction to Algorithms, 2ed"

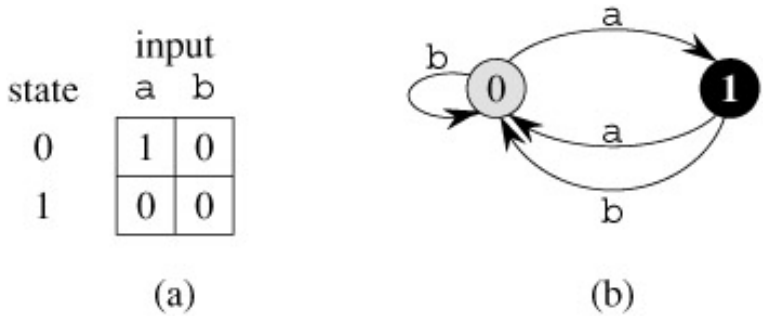
Contents

1 String Matching

1.1 String matching with finite automata

Finite automata
A finite automaton M is a 5-tuple $(Q, q_0, A, \Sigma, \delta)$, where

- Q is a finie set of ***states***
- $q_0 \in Q$ is the ***start state***
- $A \subseteq Q$ is a distinguished set of ***accepting states***
- Σ is a finite ***input alphabet***
- δ is a function from $Q \times \sum \rightarrow Q$, called the ***transition function*** of M.



Final-state function

A finite automaton M induces a function ϕ, called the ***final-state function***, from Σ^* to Q such that $\phi(w)$ is the state M ends up in after scanning the string w. Thus, M accepts a string w if and only if $\phi(w) \in A$. The function ϕ is defined by the recursive relation

$$\begin{aligned} \phi(\epsilon) &= q_0 \\ \phi(wa) &= \delta(\phi(w), a) \text{ for } w \in \Sigma^*, a \in \Sigma \end{aligned}$$

Suffix Function

We define an auxiliary function σ, called the ***suffix function*** corresponding to P. The function σ is a mapping from $\sum^* \rightarrow \{0, 1, \ldots, m\}$ such that $\sigma(x)$ is the length of the longest prefix of P that is suffix of x:
$\sigma(x) = max\{k : P_k \sqsupset x\}$
For example: for the pattern $P = ab$ we have $\sigma(\epsilon) = 0, \sigma(ccaca) = 1,$ and, $\sigma(ccab) = 2$

We define the string-matching automaton that corresponds to a given pattern $P[1 \ldots m]$ as follows.

- The state set Q is $\{0, 1, \ldots, m\}$. The start state q_0 is state 0, and state m is the only accepting state
- The transition function δ is defined by the following equation, for any state q and character a:
  $\delta(q, a) = \sigma(P_q a)$

After scanning the first i characters of the text string T, the machine is in state $\phi(T_i) = q$, where $q = \sigma(T_i)$ is the length of the longest suffix of T_i that

is also a prefix of the pattern P. If the next character scanned is $T[i+1] = a$, then the machine should make a transition to state $\sigma(T_{i+1}) = \sigma(T_i a)$. Theorem shows that $\sigma(T_i a) = \sigma(P_q a)$. That is, to compute the length of the longest suffix of $T_i a$ that is a prefix of P, we can compute the longest suffix of $P_q a$ that is a prefix of P. At each state, the machine only needs to know the length of the longest prefix of P that is a suffix of what has been read so far. Therefore, setting $\delta(q, a) = \sigma(P_q a)$ maintains the desired invariant

The Knuth-Morris-Pratt algorithm

This algorithm avoids the computation of the transition function δ altogether, and its matching time is $\ominus(n)$ using just an auxiliary function $\pi[1 \ldots m]$ precomputed from the pattern in time $\ominus(m)$.